Dyakonov-Perel spin relaxation near the metal-insulator transition and in hopping transport

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In a heavily doped semiconductor with weak spin-orbital interaction the Dyakonov-Perel spin relaxation rate is known to be proportional to the Drude conductivity. We argue that in the case of weak spin-orbital interaction this proportionality goes beyond the Drude mechanism: At low temperatures it stays valid through the metal-insulator transition and in the range of exponentially small hopping conductivity.

Spin relaxation processes in semiconductors continue to attract attention in connection with various spintronics applications.[1–4] In crystals lacking a center of inversion, for example, in GaAs, spin of a free electron experiences precession with the Larmor frequency $\boldsymbol{\Omega}_k$, which is cubic in terms of components of the wave vector $\boldsymbol{k}$. Scattering of the electron randomly changes direction of its wave vector $\boldsymbol{k}$ and, therefore, the direction of $\boldsymbol{\Omega}_k$ leading to the angular diffusion of spin magnetization $\boldsymbol{S}$. This results in the Dyakonov-Perel mechanism of spin relaxation,[5] which was predicted 35 years ago and now is widely used to interpret spin relaxation data in doped semiconductors.[1–4] The spin relaxation time, τ_s, is determined[5] by

$$\tau_s^{-1} = \int_0^\infty \langle \boldsymbol{\Omega}_k(0)\boldsymbol{\Omega}_k(t)\rangle dt = \Omega^2 \tau. \qquad (1)$$

Here Ω is the effective Larmor frequency averaged over the electron energy distribution, $\langle \boldsymbol{\Omega}_k(0)\boldsymbol{\Omega}_k(t)\rangle$ is the correlator of Larmor frequencies for an electron at time difference t and τ is the relaxation time of the third order moment of the distribution function, which we assume to be close to the electron momentum relaxation time. Equation (1) is valid only for $\Omega\tau \ll 1$. The Drude conductivity $\sigma = ne^2\tau/m$, where n is the concentration of electrons, e is the charge of an electron and m is its effective mass. This gives

$$\tau_s^{-1} = A\sigma, \qquad (2)$$

where $A \simeq \Omega^2 m/ne^2$ is the dimensionless coefficient.

The goal of this paper is to show that for a small enough spin-orbit interaction Eq. (2) is valid beyond the limits of validity of the Drude mechanism of conduction. Let us imagine that at a low temperature T we vary the concentration of donors N in the semiconductor from $N \gg N_c$ to $N \ll N_c$, where N_c is the critical concentration of the metal-insulator transition. Then at $N \gg N_c$ we deal with the Drude conductivity and Eq. (2) is valid. In the critical range of the metal-insulator transition where $N > N_c$, but $N - N_c \ll N_c$ the conductivity decreases as $e^2/\hbar\xi(N)$, where the correlation length $\xi(N) = a[N_c/(N-N_c)]^\nu$ and a is the donor Bohr radius. This gives

$$\sigma \sim \frac{e^2}{\hbar a}[(N-N_c)/N_c]^\nu. \qquad (3)$$

We argue below that for such "critical metal" Eq. (2) is still valid. At low temperatures one can define a narrow range

$\Delta N \ll N_c$, around N_c, where at $|N - N_c| < \Delta N$ metallic conductivity crosses over to the variable range hopping conductivity (see calculation of ΔN below). Coulomb interaction of electrons leads to the variable range hopping following the Efros-Shklovskii (ES) law[6]

$$\sigma(T) = \sigma_0 \exp[-(T_0/T)^{1/2}], \qquad (4)$$

where $T_0 = Ce^2/\kappa(N)\xi(N)$, C is the numerical coefficient, $\xi(N) = a[N_c/(N_c-N)]^\nu$ is the localization length and $\kappa(N) = \kappa[N_c/(N_c-N)]^\zeta$ is the dielectric constant enhanced near the transition with respect to its clean crystal value κ. We argue below that Eq. (2) is valid for the ES conductivity both near the transition or in the lightly doped semiconductor, where $N < N_c/2$, $\xi = a$ and $\kappa(N) = \kappa$.

Let us start from the metallic side of the transition, where the conductivity of the critical metal is given by Eq. (3). The reason of the conductivity reduction near the metal-insulator transition is the interference leading to the non-Gaussian diffusion. (One can say that electron dwells on some close loop trajectories.) Still one can define electron trajectories, wave vectors and velocities $\boldsymbol{v} = \hbar\boldsymbol{k}/m$ at each trajectory (the dominating quadratic part of the electron spectrum is isotropic). Then the conductivity is proportional to the diffusion coefficient

$$D = \int_0^\infty \langle \boldsymbol{v}(0)\boldsymbol{v}(t)\rangle dt, \qquad (5)$$

where $\langle \boldsymbol{v}(0)\boldsymbol{v}(t)\rangle$ is the correlator of electron velocities. On the other hand, one can write a scaling estimate

$$\tau_s^{-1} = \int_0^\infty \langle \boldsymbol{\Omega}_k(0)\boldsymbol{\Omega}_k(t)\rangle dt \sim \Omega^2\tau \sim D\frac{\Omega^2}{\langle v^2\rangle}. \qquad (6)$$

This proves Eq. (2) for the critical metal case.

Let us now consider the Dyakonov-Perel mechanism of spin relaxation for the hopping conductivity. Any hopping transport can be considered as a result of fast tunnel hops from one localized state to another alternating with exponentially long waiting periods in each localized state. While waiting an electron has $k = v = 0$ and, therefore, is not relaxing its spin via the Dyakonov-Perel mechanism. On the other hand, an electron tunneling between two localized states has the real trajectory and the real displacement, which it traverses in imaginary time and, therefore, it has the imagi-

nary k and v. Therefore, its spin experiences precession in the course of tunneling. Its Larmor frequency $\Omega \propto k^3$ is imaginary, too. But because time is imaginary the angle of rotation in the course of the hop is real. This real angle is proportional to the real displacement of the electron and the direction of rotation is related to the direction of the hop.

The fraction of time during which the electron hops or, in other word, tunnels is proportional to $\exp[-(T_0/T)^{1/2}]$. This is why the hopping conductivity has this small exponential factor. But τ_s^{-1} should have the same small factor because as we explained relaxation happens only during hops. It is clear, therefore, Eq. (2) should be valid for the ES law, at least in the exponential sense.

One can improve these arguments using the language of redefined correlators $\langle \boldsymbol{\Omega}_k^*(0)\boldsymbol{\Omega}_k^*(t)\rangle$ and $\langle \boldsymbol{v}^*(0)\boldsymbol{v}^*(\boldsymbol{t})\rangle$. In this correlators $\boldsymbol{\Omega}_k^*(t)$ and $\boldsymbol{v}^*(t)$ are the rotation angle during a hop and the hop displacement divided by the waiting time, respectfully. These correlators now decay on exponentially large times because all the waiting times are included in their definition. In the hopping conductivity regime the first correlator is responsible for the spin relaxation rate τ_s^{-1}, while the second one calculated for a long enough time history of an electron is related to the diffusion coefficient and the conductivity. These correlators are obviously proportional to each other, what again leads to Eq. (2). Note that our approach to spin relaxation in a lightly doped semiconductor is completely different from the one suggested by Kavokin[7] and based on the role of the anisotropic exchange between electrons localized on different donors. While we are talking about Dyakonov-Perel relaxation related to a single electron diffusion in space, Kavokin relies on rotation of spin of a localized electron in the collective field of other localized electrons.

Let us make a comment about the range of concentrations, where crossover between Eqs. (3) and (4) takes place, while staying away from any discussion of the mechanism of conductivity in this range. At low temperature the relative width of this range is small, $\Delta N/N_c \ll 1$. Indeed, one can estimate ΔN equating $T_0(N)$ to T and identifying ΔN with $N_c - N$. This gives $\Delta N/N_c = [T/(e^2/\kappa a)]^{1/(\nu+\zeta)}$. It is known from experiments[6] that $\nu + \zeta \simeq 2$. As we argued above Eq. (2) is valid on both sides of the crossover range ΔN. This means that Eq. (2) is valid in the crossover range as well.

Above we have concentrated on the three-dimensional case. In two dimensions validity of Eq. (2) for the hopping conductivity can be demonstrated even more transparently. Let us consider the two-dimensional electron gas (2DEG) without structural inversion asymmetry in the (001)-plane of GaAs crystal and assume that initially electron spins are polarized along z-axis perpendicular to 2DEG plane. Then at times smaller than τ_s the spin magnetization $\boldsymbol{S}$ evolves following to the equation

$$dS_x/dt = \Omega_y S_z, \quad dS_y/dt = -\Omega_x S_z, \quad (7)$$

where

$$\Omega_x = \gamma k_y(k_x^2 - k_z^2), \quad \Omega_y = -\gamma k_x(k_y^2 - k_z^2). \quad (8)$$

For a narrow quantum well the momentum components k_x^2, k_y^2 are much smaller than k_z^2 and, therefore, can be neglected in the right sides of Eqs. (8).[8] Then one can easily calculate the change of the spin magnetization δS during the time $\delta t \ll \tau_s$. This gives

$$\delta S/S_z = \gamma(m/\hbar)\boldsymbol{v}\,\delta t, \quad (9)$$

i.e., the angle of rotation of the spin magnetization is proportional to the electron displacement in the plane of quantum well. This leads directly to Eq. (2), both for the case of metallic conductivity and for the hopping transport. While in the latter both $\boldsymbol{v}$ and t are imaginary quantities, the angle of rotation of the spin magnetization and the electron displacement are real and as we see the initial rotational diffusion of S and the diffusion in real space are related as tightly as for the metallic conduction.

This means that in the range of the ES variable range hopping both in three and two dimensions the Dyakonov-Perel spin relaxation rate is very small and exponentially decreases with temperature.

$$\tau_s^{-1} \propto \exp[-(T_0/T)^{1/2}]. \quad (10)$$

As a function of donor concentration N the rate has to exponentially decrease with the growth of T_0, while N is still in the critical range of transition $N_c - N \ll N_c$. At $N < N_c/2$ the temperature T_0 saturates at $T_0 = Ce^2/a\kappa$ and τ_s^{-1} saturates at a very small level exponentially dependent on T.

Of course, other mechanisms of spin relaxation can take over at weak doping and at low temperatures,[1–3] but because the Dyakonov-Perel relaxation typically is the dominating mechanism this crossover may happen only at very small relaxation rates.

Let us make a comment about the measurement of τ_s in the hopping regime. In a typical experiment polarized electrons are created in the conduction band and may experience few scattering events before being captured by donors with characteristic capture time τ_c. Thus, they may loose a fraction of their polarization by with DP spin relaxation time of free electrons τ_f. If $\tau_c < \tau_f$ they get captured before loosing spin in the conduction band. Then τ_s calculated above describes relaxation of practically all the polarization. In the opposite case, when $\tau_c > \tau_f$ only a small fraction of polarization of the order of τ_f/τ_c relaxes via hopping, while majority of the polarization relaxes faster.

In a pump-probe experiments[1] this means that hopping relaxation dominates only at times larger than $\tau_f \ln \tau_c/\tau_f$. Thus, in this case, hopping τ_s describes the tail of the spin relaxation. On the other hand, in continuous wave excitation experiments spin relaxation also happens first in the conduction band and then via hopping on donors. For $\tau_c < \tau_f$ a standard way[2] to measure τ_s directly leads to the hopping spin relaxation time. On the other hand, measuring hopping spin relaxation time by this method at $\tau_c > \tau_f$ is difficult and one needs more delicate methods like a direct optical readout of the donor spins.

In n-type GaAs the dependence of low temperature spin relaxation on the doping level was recently studied.[2] It was interpreted[2] with the help of the mechanism of anisotropic

exchange (immediately below the metal insulator transition) and by the hyperfine interaction with nuclei (at very small doping). These data look as if there is no substantial range of doping, where hopping Dyakonov-Perel relaxation dominates and τ_s^{-1} decreases with the concentration of donors proportionally to the hopping conductivity. This could be a result of the masking effect of spin loss during energy relaxation in the conduction band discussed above. If this is true, a pump-probe experiment should reveal the DP hopping relaxation in the long time tail of relaxation.

[1] J. M. Kikkawa and D. D. Awschalom, Phys. Rev. Lett. **80**, 4313 (1998); D. D. Awschalom, Physica E (Amsterdam) **10**, 1 (2001); A. W. Holleitner, V. Sih, R. C. Myers, A. G. Gossard, and D. D. Awschalom, cond-mat/0602155 (unpublished).

[2] R. I. Dzhioev, V. L. Korenev, I. A. Merkulov, B. P. Zakharchenya, D. Gammon, Al. L. Efros, and D. S. Katzer, Phys. Rev. Lett. **88**, 256801 (2002); R. I. Dzhioev, K. V. Kavokin, V. L. Korenev, M. V. Lazarev, B. Ya. Meltser, M. N. Stepanova, B. P. Zakharchenya, D. Gammon, and D. S. Katzer, Phys. Rev. B **66**, 245204 (2002).

[3] I. Zutic, J. Fabian, and S. Das Sarma, Rev. Mod. Phys. **76**, 323 (2004).

[4] C. Adelmann, X. Lou, J. Strand, C. J. Palmstrom, and P. A. Crowell, Phys. Rev. B **71**, 121301(R) (2005).

[5] M. I. Dyakonov and V. I. Perel, Sov. Phys. JETP **33**, 1053 (1971); Sov. Phys. Solid State **13**, 3023 (1972).

[6] B. I. Shklovskii and A. L. Efros, *Electronic Properties of Doped Semiconductors* (Springer-Verlag, Berlin, 1984).

[7] K. V. Kavokin, Phys. Rev. B **64**, 075305 (2001).

[8] M. I. Dyakonov and V. Yu. Kachorovskii, Sov. Phys. Solid State **20**, 110 (1986).