

# Hierarchy of critical temperatures in four-layered ferromagnet/superconductor nanostructures and control devices

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The four-layered F/S/F'/S' nanostructure consisting of rather dirty superconducting (S) and ferromagnetic (F) metals is studied within the theory of the proximity effect taking detailed account of the boundary conditions. The F/S structures with four F and S layers are shown to have considerably richer physics than the F/S/F trilayer (due to the interplay between the 0 and  $\pi$  phase superconductivity and the 0 and  $\pi$  phase magnetism) and even the F/S superlattices. The extra  $\pi$  phase superconducting states obtained for the four-layered F/S/F'/S' system are found to be different from the known “superlattice” states. The dependence of the critical temperatures versus the F layer thicknesses is investigated. An optimal set of parameters is determined, for which the difference between the critical temperatures for different states becomes significant, and the corresponding phase diagrams are plotted. It is proven that this system can have different critical temperatures for different S and S' layers. A conceptual scheme of a control device with superconducting and magnetic recording channels that can be controlled separately using a weak external magnetic field is proposed on the basis of the F/S/F'/S' nanostructure. The devices with four, five, six, and seven different states are explored.

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## I. INTRODUCTION

Superconductivity and ferromagnetism are antagonistic ordering phenomena and their coexistence in homogeneous materials requires special conditions that are hard to realize. One possible explanation of the superconductivity suppression by ferromagnetic ordering in transition metals was given by Ginzburg,<sup>1</sup> who noted that the magnetic induction exceeds the critical field. This antagonism is also clear from the viewpoint of the microscopic theory: the attraction between electrons creates Cooper pairs in a singlet state, whereas the exchange interaction producing ferromagnetism tends to arrange electronic spins parallel to each other. Therefore, when the Zeeman energy of the electrons of a Cooper pair in the exchange field  $I$  exceeds the coupling energy, the measure of which is the superconducting gap  $\Delta$ , the superconducting state is destroyed. In contrast to critical field  $H_c$  acting on orbital states of the electrons of a pair, the exchange field acts on electronic spins (spin degrees of freedom), therefore the destruction of superconductivity due to this field is called the paramagnetic effect.<sup>2,3</sup>

However, the above-mentioned coexistence of superconducting and ferromagnetic order parameters is easily achievable in fabricated or natural F/S heterostructures consisting of alternating ferromagnetic metal (F) and superconducting (S) layers. In this case *superconducting* and *ferromagnetic* electronic systems are *spatially* separated. Due to the proximity effect, the superconducting order parameter can be induced in the F layer; on the other hand, the neighboring pair of the F layers can interact with each other via the S layer. Such systems exhibit rich physics, which can be controlled by varying the thicknesses of the F and S layers, or by placing the F/S structure in an external magnetic field.

The modern technologies of production of the layered structures, such as molecular-beam epitaxy, allow one to deposit layers of atomic thickness and to study the properties of such heterogeneous F/S systems as a function of the ferromagnetic ( $d_f$ ) or superconducting ( $d_s$ ) layer thickness. Numerous experiments on the F/S structures (contacts, trilayers, and superlattices) have revealed nontrivial dependences of the superconducting transition temperature  $T_c$  on the thickness of the ferromagnetic layer (see the reviews<sup>4-6</sup> and references therein).

The boundary value problem for the pair amplitude (the Cooper pair wave function) in a dirty superconductor for the F/S superlattice was formulated in pioneering works by Radović *et al.*<sup>7</sup> and Buzdin *et al.*<sup>8</sup> The critical temperature  $T_c$  that was also calculated as the  $d_f$  function in Refs. 7 and 8 exhibited both monotonic and nonmonotonic dependences. Oscillations of  $T_c(d_f)$  were related to periodical switching of the ground superconducting state between the 0 and  $\pi$  phases, so that the system chooses the state with higher transition temperature  $T_c$ . In the  $\pi$  phase state the superconducting order parameter  $\Delta$  in the neighboring S layers of the F/S superlattice has the opposite sign, contrary to the 0 phase state in which  $\Delta$  has the same sign for all S layers. The experimental evidence of the  $\pi$  superconducting state in the F/S systems has been discussed in the review.<sup>5</sup> The concept of a  $\pi$  junction was proposed by Bulaevskii *et al.*<sup>9</sup>

However, the boundary conditions used in Refs. 7 and 8 are correct only in the limit of high transparency of the F/S interface. In subsequent studies<sup>10-13</sup> the boundary conditions have been derived from the microscopic theory, and they are valid for arbitrary transparency of the F/S interface. The solution of the boundary value problem<sup>10-17</sup> has revealed an additional mechanism of nonmonotonic dependence of  $T_c$

due to modulation of the pair amplitude flux from the S layer to the F layer. This modulation is caused by the change of the FM layer thickness  $d_f$ . Moreover, it has also resulted in a prediction of different types of behavior  $T_c(d_f)$  such as reentrant<sup>10–12,18</sup> and periodically reentrant superconductivity.<sup>10–12</sup> Note that both the oscillations and the reentrant behavior of  $T_c(d_f)$  can appear not only in the F/S superlattice but also in simple F/S bilayer and F/S/F trilayer systems in which the  $\pi$  phase superconductivity is impossible in principle. The reentrant character of superconductivity that we have predicted has been recently observed experimentally in the Fe/V/Fe trilayer.<sup>19</sup>

Now it may be considered as proven<sup>4</sup> that superconductivity in the layered F/S systems is a combination of the BCS pairing with a zero total momentum of the pairs in the S layers and the pairing due to the Larkin-Ovchinnikov-Fulde-Ferrell (LOFF) mechanism<sup>20,21</sup> with a nonzero three-dimensional (3D) momentum of the pairs  $\mathbf{k}$  in the F layer. The LOFF pairs momentum  $k \approx 2I/v_f$  is determined by the Fermi surface splitting caused by the internal exchange field  $I$  (where  $v_f$  is the Fermi velocity in the F layers). Usually it is assumed<sup>7,8,10–17,22</sup> that the momentum of the LOFF pairs is directed across the F/S interface [the so-called one-dimensional (1D) case]. In our recent papers<sup>4,18,23</sup> we took into account the spatial variations of the pair amplitude not only across the F/S nanostructure but also along the F/S boundary (the 3D case). In the general case, this leads to the increase of the critical temperature  $T_c$  and to the smoothing of the  $T_c(d_f)$  oscillations, in comparison with the 1D version of the theory, due to the 3D-1D-3D phase transitions. The appearance of the 3D-1D-3D phase transition cascade is associated with the umklapp processes at which the LOFF pairs momentum  $\mathbf{k}$  is exactly conserved up to a minimal reciprocal lattice vector  $\mathbf{g}$  of the 2D surface LOFF states. Therefore  $\mathbf{k}$  is actually a quasimomentum and this fact is reflected in the revised F/S boundary conditions.<sup>4,23</sup> The use of the latter in turn can result in a  $T_c(d_f)$  dependence with one local minimum, which is a typical experimental non-monotonic behavior.<sup>4</sup>

Of special interest is the study of the multilayered F/S structures, in which various types of magnetic order can arise in the F layers due to their indirect interaction via the S layers. Recently the theory of the proximity effect has been developed for the F/S structures taking into account the inverse influence of superconductivity on magnetism of the F layers and on mutual orientation of their magnetizations. This aspect of the proximity effect has been studied for the F/S/F trilayer “spin-switch”<sup>24,25</sup> and exploring the possibility of the cryptoferromagnetic state in the F/S bilayer.<sup>26,27</sup> The long-range proximity effect due to triplet superconductivity that arises in the case of noncollinear alignment of magnetizations in the F layers has been studied for the F/S/F trilayer system in Refs. 28–31.

An interplay between the 0 and  $\pi$  phase types of superconductivity in the S layers should be included in the above-mentioned magnetic mutual accommodation in the F/S superlattices. This added competition leads to two layered antiferromagnetic superconducting (AFMS) states.<sup>4,32,33</sup> In the AFMS state the phases of the magnetic order parameter in the neighboring F layers are shifted by  $\pi$ , i.e., the ex-

change fields  $I$  have opposite signs in the neighboring F layers. This state with antiparallel alignment of the corresponding magnetizations may be considered as a manifestation of the  $\pi$  phase magnetism. Similar to the F/S/F trilayer,<sup>24,25</sup> in the case of the F/S superlattice the AFM ordering of the magnetizations of all F layers leads to the significant *reduction* of the pair-breaking effect of the exchange field  $I$  for the S layers, and to the *raising* of the critical temperature of the layered system. This theoretical prediction has been experimentally confirmed for the Gd/La superlattices.<sup>34</sup> Goff *et al.*<sup>34</sup> have observed that the superlattices with prepared antiferromagnetic ordering of the magnetizations in the adjacent Gd layers undergo the transition into a superconducting state at considerably higher temperatures in comparison with the superlattices with ferromagnetic ordering of the Gd layers. This mutual accommodation between the superconducting and magnetic order parameters reflects a quantum coupling between the boundaries. The competition between the 0 and  $\pi$  phase superconductivity and the 0 and  $\pi$  phase magnetism leads to a change in the classification of the F/S superlattice state<sup>4,32,33</sup>.

The F/S nanostructures possess two data-recording channels: the superconducting one determined by conducting properties of the S layers, and the magnetic one determined by ordering of the F layer magnetizations. The F/S/F trilayer devices, proposed in Refs. 24, 25, and 35, operate through transitions between the superconducting (S) and normal (N) states that are induced by changes of the mutual ordering of the magnetizations of the adjacent F layers. These changes are controlled by an external magnetic field  $H$ . The data stored in the superconducting and magnetic channels of this *switch* device change *simultaneously*, and the magnetic order completely determines the “superconducting information.” The scheme of a complex device on the basis of the F/S superlattices, in which the superconducting and magnetic data-recording channels can be controlled separately, has been proposed in Refs. 32 and 33.

In Sec. II we briefly discuss the earlier proposed control devices (“spin switches”) based on the F/S nanostructures. In Sec. III we explore the four-layered F/S/F'/S' system assuming the competition between the 0 and  $\pi$  phase magnetism and the 0 and  $\pi$  phase superconductivity takes place. We solve the Usadel equations for this structure taking into account the boundary conditions. In Sec. IV we construct the phase diagrams with an optimal set of parameters. In Sec. V we propose a scheme of a control device based on the studied F/S/F'/S' system and discuss its few possible operating regimes.

## II. SPIN SWITCHES FOR CURRENT ON THE BASIS OF F/S HETEROSTRUCTURES

A conceptual scheme of a spin switch device for current based on a F/S/F trilayer was proposed by Buzdin *et al.*<sup>24</sup> and Tagirov<sup>25</sup> for the case of the “Cooper limit” when the thicknesses  $d_s$  and  $d_f$  of the S and F layers are much less than the corresponding coherence lengths  $\xi_s$  and  $\xi_f$ , respectively. It has been theoretically shown that the “antiferromagnetic” (AFM) configuration of such a three-layered system with an

antiparallel arrangement of the magnetizations of the F layers has a higher transition temperature  $T_c$  in comparison with the one for the “ferromagnetic” (FM) configuration. In other words the AFM configuration is energetically more favorable and the AFMS state is the ground state of this system at  $T < T_c$  in the absence of an external magnetic field. The nature of this behavior of  $T_c$  is related to a reduction of the pair-breaking action of the exchange field of the F layers in the AFM configuration on superconducting pairs, i.e., to a partial compensation of the paramagnetic effect.

Applying small magnetic field  $H$  higher than the coercivity  $H_{\text{coer}}$  of the F layer one can change the AFM orientation of the magnetizations to the FM one. Under certain conditions the trilayer system can undergo a transition from a superconducting (AFMS) state to a normal (FMN) one, i.e., from a state with zero resistance to a resistive one. As the magnetic field is turned off, the AFM orientation of the magnetizations (the  $\pi$  phase magnetic state) and the superconducting properties of the system are restored. Note that in a certain sense the F/S/F switch operates in the same manner as a usual isolated superconductor which turns into the normal state if the applied field  $H$  exceeds the critical field  $H_c$ . Moreover, since the value of the critical field  $H_c$  is determined by the difference between the critical temperature and the temperature of a sample ( $T_c - T$ ), the  $H_c$  can be made arbitrarily small by choosing the temperature  $T$  close enough to the critical temperature.

Considering the switches for current it is necessary to note a few earlier papers<sup>35,36</sup> on similar devices with one channel of data recording that operate on the basis of transition between superconducting and normal states. A model of a superconducting switch device has been proposed in Ref. 36 on the basis of the F/I/S structure (I is an insulator) in which the magnetic fringe field of a ferromagnetic film arising due to special switch geometry is used to control the critical current in an underlying superconducting film.

The switch device on the basis of the *three-layered* F'/F''/S structure in which the direction of the magnetization in the relatively thin internal F'' layer is changed by a weak magnetic field was theoretically studied in Ref. 35. As the mutual ordering of the magnetizations  $\mathbf{M}'$  and  $\mathbf{M}''$  changes from an antiparallel arrangement to a parallel one, the device undergoes a transition from the S state to the N one.

We would like also to note that an experimental attempt was made to observe the “spin switch” effect in the *three-layered* CuNi/Nb/CuNi system<sup>37</sup>. Authors succeeded in showing that the critical transition temperature is higher for the AFMS state than it is for the FMS state. However, due to a nonoptimal choice of parameters of the system the measured difference between  $T_c(\text{AFMS})$  and  $T_c(\text{FMS})$  did not exceed 0.005 K. Despite this small difference the experimental setup allows one to clearly identify the two states.

The *multilayered* F/S systems in which there is an additional competition between the 0 and  $\pi$  phase types of superconductivity have much greater potential in fundamental studies and future device applications. In fact, in the F/S superlattices the pair amplitude  $F$  should satisfy to periodical conditions  $F(z+L, I) = e^{i\varphi} F(z, I e^{i\chi})$ , where  $L = d_s + d_f$  is the superlattice period, and  $\varphi$  and  $\chi$  are the phases of supercon-

ducting and magnetic order parameters, respectively. As it follows from the detailed analysis carried out in works<sup>4,32,33,38</sup> on the basis of the theory of the proximity effect<sup>10–12,18</sup> for the case of a *contact* between the dirty S and F metals, the superconducting states of the F/S *superlattice* can be described using four different sets  $\varphi\chi: 00, \pi 0, 0\pi,$  and  $\pi\pi$ . This leads to a considerably greater number of combinations of the magnetic (FM or AFM) and conducting (S or N) properties of the F/S superlattices (up to five different ones) in comparison with the *trilayer* case, in which only two states (AFMS and FMN) have been considered<sup>24,25</sup>. Hence there is a much larger variety of regimes at which the control devices on the basis of the F/S *superlattice* can operate. The conditions of the *separate* control of the magnetic and superconducting channels of data recording were determined. Note that the critical thickness of the S layers  $d_s^c$ , at which  $T_c$  vanishes, is always less for the AFMS state than for the FMS state. More recently, Fominov *et al.*<sup>31</sup> have shown for the F/S/F trilayer that superconductivity has merely the AFMS nature at any thickness  $d_s > d_s^c$ .

Thus, as we can see, the F/S superlattices possess a number of *theoretical advantages* in comparison with the three-layered structures that make them a better choice for the development of a conceptual scheme of the proposed control devices to be used for data storage and processing. However, from a *practical* point of view in the case of the superlattices it is quite difficult to control orientation of the magnetization separately of each F layer by an external magnetic field.<sup>39</sup> The simplest F/S system that assumes a competition between the 0 and  $\pi$  phase states both in magnetism and superconductivity is a four-layered F/S/F'/S' system with two superconducting layers and two ferromagnetic layers that differ in their boundary conditions for the inner and outer layers in contrast to the F/S *superlattice* approach,<sup>4,38</sup> where the four-layered F/S/F/S system was considered only as an *elementary cell* with *periodic* boundary condition.

### III. FOUR-LAYERED F/S/F'/S' STRUCTURE

Consider the four-layered F/S/F'/S' system with alternating layers along the  $z$  axis (see Fig. 1). Assume that both outer F and S' layers have thicknesses which are half the thicknesses of the corresponding inner F' and S layers,  $d_f/2$  and  $d_s/2$ , respectively. This would allow us to simplify the solutions, and, thereafter, to compare the obtained results for the four-layered structure with the ones for the trilayer and superlattice cases.

The choice of this particular system allows one to take into account a possible phase change of the superconducting and magnetic order parameters while traversing through the F or S layers, and to investigate mutual accommodation of the competing BCS and LOFF types of electron pairing on the one hand, and of superconductivity and magnetism on the other. For simplicity we will use the 1D model when both order parameters and the pair amplitude depend only on  $z$ . An extension of the results to the 3D case is straightforward (see, for example, Refs. 4 and 23). We note here that the distinction between solutions obtained within the framework of the 1D and 3D models is marginal for the choice of parameters studied below.<sup>32,33</sup>

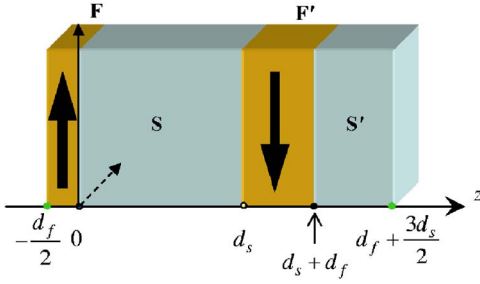


FIG. 1. (Color online) The geometry of the studied four-layered F/S/F'/S' system. Vertical arrows show the directions of the magnetizations that play the role of the magnetic order parameter (they are in a plane perpendicular to the  $z$ -axis).

To find the critical temperature we assume the usual relation between the energy parameters of the system  $\varepsilon_F \gg 2I \gg T_{cs}$ , where  $\varepsilon_F$  is the Fermi energy and  $T_{cs}$  is the critical temperature of the S material. We also suppose the dirty limit conditions

$$l_s \ll \xi_s \ll \xi_{s0}, \quad l_f \ll a_f \ll \xi_f.$$

Here  $l_{s(f)} = v_{s(f)} \tau_{s(f)}$  is the mean free path length for the S(F) layer;  $v_{s(f)}$  is the Fermi velocity;  $\xi_{s(f)} = (D_{s(f)}/2\pi T_{cs})^{1/2}$  is the superconducting coherence length;  $a_f = v_f/2I$  is the spin stiffness length;  $\xi_{s0}$  is the BCS coherence length; and  $D_{s(f)} = v_{s(f)} l_{s(f)}/3$  is the diffusion coefficient.

In this case, the common boundary value problem<sup>12</sup> for each layer is reduced to the Gor'kov self-consistency equations for the "pair amplitudes"<sup>40</sup>  $F(z, \omega)$

$$\begin{aligned} \Delta_s(z) &= 2\lambda_s \pi T \operatorname{Re} \sum_{\omega>0} 'F_s(z, \omega), \\ \Delta_f(z) &= 2\lambda_f \pi T \operatorname{Re} \sum_{\omega>0} 'F_f(z, \omega) \end{aligned} \quad (1)$$

and to the Usadel equations, that appear for the S and F layers as follows:

$$\begin{aligned} \left[ \omega - \frac{D_s}{2} \frac{\partial^2}{\partial z^2} \right] F_s(z, \omega) &= \Delta_s(z), \\ \left[ \omega + iI - \frac{D_f}{2} \frac{\partial^2}{\partial z^2} \right] F_f(z, \omega) &= \Delta_f(z). \end{aligned} \quad (2)$$

In Eqs. (1) and (2)  $\omega = \pi T(2n+1)$  is the Matsubara frequency;  $\Delta_{s(f)}$  and  $\lambda_{s(f)}$  are the superconducting order parameter and the electron-electron coupling constant in the S(F) layers, respectively. The prime on the summation sign indicates cutoff at the Debye frequency  $\omega_D$ . The diffusion coefficient  $D_f$  in the F layer is assumed to be real rather than complex<sup>4</sup> since the difference between its two values is insignificant under the conditions  $2I\tau_f \ll 1$  used below (see discussion in Ref. 23).

The coupling between the superconducting and ferromagnetic layers is provided by corresponding boundary conditions, which connect the pair amplitude fluxes with the pair amplitude jumps on the interfaces of the layers, and are written in the following form:<sup>10–12</sup>

$$\begin{aligned} \frac{4}{\sigma_s v_s} D_s \left. \frac{\partial F_s(z, \omega)}{\partial z} \right|_{z=z_i} &= \frac{4}{\sigma_f v_f} D_f \left. \frac{\partial F_f(z, \omega)}{\partial z} \right|_{z=z_i} \\ &= \pm [F_s(z_i \pm 0, \omega) - F_f(z_i \mp 0, \omega)]. \end{aligned} \quad (3a)$$

Here index  $i$  numbers the interfaces, and  $z_i$  takes the following values:  $z_1=0$ ,  $z_2=d_s$ ,  $z_3=d_s+d_f$ . The upper signs are chosen at  $i=1,3$ , the lower ones are chosen at  $i=2$ . The pair amplitude flux through the outside boundaries ( $z_0=-d_f/2$ ,  $z_4=3d_s/2+d_f$ ) is absent

$$\left. \frac{\partial F_f(z, \omega)}{\partial z} \right|_{z=z_0} = \left. \frac{\partial F_s(z, \omega)}{\partial z} \right|_{z=z_4} = 0. \quad (3b)$$

The last conditions (3b) distinguish the F/S/F'/S' case from the F/S superlattice case<sup>4,38</sup> in which the periodical boundary conditions are imposed. In Eq. (3a)  $\sigma_{s(f)}$  is the boundary transparency at the S(F) side ( $0 \leq \sigma_{s,f} < \infty$ ).<sup>4,12</sup> They satisfy the detailed balance condition:  $\sigma_s v_s N_s = \sigma_f v_f N_f$ , where  $N_{s(f)}$  is the density of states at the Fermi level.

In order to calculate the critical temperatures of this F/S system taking into consideration the boundary transparencies, thicknesses of layers, etc. we should solve the system of equations (2) and (3) together with the self-consistency equations (1).

The powerful pair-breaking action of the exchange field  $I$  ( $I \gg \pi T_{cs}$ ) is the basic mechanism for the destruction of superconductivity in the F/S systems. For simplicity<sup>12</sup> assume that  $\lambda_f=0$  ( $\Delta_f=0$ ) in the F layers. We will search the solutions of Eqs. (1)–(3) for the inner layers as a linear combination of symmetric and antisymmetric functions relative to the centers of the S and F' layers. The pair amplitudes look the same as in the superlattice case.<sup>33</sup> The zero flux of the pair amplitude through the outside boundaries (3b) determines only the even cosinelike functions for the outer layers. At these boundaries the antinodes should be fixed. Thus the simplest solutions of the boundary value problem for the F/S/F'/S' system have the following form:

$$F_f = B \frac{\cos k_f(z + d_f/2)}{\cos(k_f d_f/2)}, \quad -d_f/2 < z < 0, \quad (4a)$$

$$F_s = A \frac{\cos k_s(z - d_s/2)}{\cos(k_s d_s/2)} + C \frac{\sin k_s(z - d_s/2)}{\sin(k_s d_s/2)}, \quad 0 < z < d_s, \quad (4b)$$

$$\begin{aligned} F_f' &= B' \frac{\cos k_f'(z - d_s - d_f/2)}{\cos(k_f' d_f/2)} + D' \frac{\sin k_f'(z - d_s - d_f/2)}{\sin(k_f' d_f/2)}, \\ & \quad d_s < z < (d_f + d_s), \end{aligned} \quad (4c)$$

$$F'_s = A' \frac{\cos k'_s(z - d_f - 3d_s/2)}{\cos(k'_s d_s/2)}, \quad (d_f + d_s) < z < (d_f + 3d_s/2). \quad (4d)$$

Here  $k_{s(f)}$  and  $k'_{s(f)}$  are the components of the wave vector that describe spatial changes of the pair amplitudes  $F_{s(f)}$  and  $F'_{s(f)}$  across the layers (along the  $z$  axis) independently of the frequency  $\omega$ . The chosen form of the  $F_s$  and  $F'_f$  pair amplitudes is related to the symmetry of the F/S/F'/S' system. In Eqs. (4b) and (4c) the first terms are responsible for the symmetric superconducting 0 phase solutions, while the second terms are responsible for the appearance of superconducting anti-symmetric  $\pi$  phase solutions (see discussion below in Sec. IV).

Since we are mainly interested in performing qualitative studies of the properties of the F/S/F'/S' nanostructure, the single-mode approximation (4) is used to obtain the analytical solution of the complicated boundary value problem. However, when quantitative estimates are needed (to fit theoretical results to experimental data) the latter approximation works well only for a certain range of the values of the parameters in the problem.<sup>15,16</sup> According to our estimates<sup>12</sup> the optimal set of parameters used below is close to this range ( $d_{s(f)} > \xi_{s(f)}$ ). Note that in any approximation (single-mode, multimode, etc.<sup>16</sup>) the symmetry of the problem solutions will be different for the different S and S' (F and F') layers. This fact reflects the general property of the studied system: the nonequivalence of layers of the same type, which results in different superconducting properties of the internal S and outer S' layers.

Substituting the solutions (4) into the self-consistency equations (1) and performing the standard summation over  $\omega$  we derive the usual Abrikosov-Gor'kov type equation for the reduced superconducting transition temperatures  $t_c, t'_c$  of the S and S' layers, respectively,

$$\ln t_c = \Psi\left(\frac{1}{2}\right) - \text{Re} \Psi\left(\frac{1}{2} + \frac{D_s k_s^2}{4\pi T_{cs} t_c}\right), \quad (5)$$

where  $t_c = T_c/T_{cs}$ ;  $\Psi(x)$  is the digamma function, and the pair-breaking parameter  $D_s k_s^2$  is the solution of the other transcendental equation [see Eqs. (10), (11), and (7) below], which may differ not only for each of the possible phases, but also for each superconducting layer (S and S') as well. To get an equation for  $t'_c$  it is necessary to exchange  $t_c$  for  $t'_c = T'_c/T_{c,s}$  and  $k_s$  for  $k'_s$  in Eq. (5) [see also Eq. (7) and its discussion below].

Substituting Eq. (4) in Eqs. (3) we obtain a set of six equations for factors  $A, B, C, A', B', D'$

$$\begin{aligned} B + \alpha A + \beta C &= 0, & \gamma B - A + C &= 0, \\ \alpha A - \beta C + B' - D' &= 0, & -A - C + \gamma' B' - \delta' D' &= 0, \\ B' + D' + \alpha' A' &= 0, & \gamma' B' + \delta' D' - A' &= 0, \end{aligned} \quad (6)$$

where the following notations are introduced

$$\alpha = \frac{4D_s k_s}{\sigma_s v_s} \tan \frac{k_s d_s}{2} - 1, \quad \beta = \frac{4D_s k_s}{\sigma_s v_s} \cot \frac{k_s d_s}{2} + 1,$$

$$\gamma = -\frac{4D_f k_f}{\sigma_f v_f} \tan \frac{k_f d_f}{2} + 1, \quad \delta = \frac{4D_f k_f}{\sigma_f v_f} \cot \frac{k_f d_f}{2} + 1. \quad (7)$$

Quantities related to the outer superconducting S' layer ( $\alpha'$ ) or the inner F' layer ( $\gamma', \delta'$ ) are marked with a *prime*. The prime also appears at corresponding wave vectors ( $k'_s$  or  $k'_f$ ) on the right side of the expressions (7). In the framework of the made approximations the complex value of the wave vectors is defined as follows:

$$k_f^2 = (k'_f)^2 = -\frac{2iI}{D_f}, \quad (8a)$$

$$k_f^2 = -\frac{2iI}{D_f}, \quad (k'_f)^2 = \frac{2iI}{D_f} = (k_f^2)^*. \quad (8b)$$

Equation (8a) is valid for the case of the mutual ferromagnetic ordering of the magnetizations in the F and F' layers, and Eqs. (8b) are valid for the case of the antiferromagnetic ordering when  $I' = -I$ , i.e., the phase  $\chi$  of the magnetic order parameter equals  $\pi$ .

Note that within the 3D model<sup>4,18,23</sup>  $k_f^2$  and  $(k'_f)^2$  in Eqs. (8) should be replaced by  $(k_f)^2 + q_f^2$  and  $(k'_f)^2 + q_f^2$ , respectively. Here  $q_f$  is a wave vector in the F/S boundary plane, which is responsible for the 2D interface LOFF states with the spatial oscillations of the pair amplitude in the  $x$ - $y$  plane. The concrete value of  $q_f$  is found from the condition for maximum of  $T_c$ . However, in our case the difference between the 1D ( $q_f \equiv 0$ ) and 3D ( $q_f \neq 0$ ) approaches is unessential for the selected set of the system parameters (see Sec. IV below).

Thus the expressions obtained above include a competition between the 0 phase and the  $\pi$  phase types of superconductivity. They also take into account interaction of the localized moments of layers F and F' through the superconducting layer S. The quantum coupling both between the adjacent S and S' layers through the F' layer and between the adjacent F and F' layers through the S layer is provided by superconducting correlations of conduction electrons. It is known<sup>41</sup> that the role of the *true* superconducting order parameter for the heterogeneous systems discussed in the paper is played by a pair amplitude  $F(z) = \Delta(z)/\lambda(z)$ . In contrast to a parameter  $\Delta(z)$ , the pair amplitude does not vanish in the ferromagnetic layer, but provides a quantum coupling between the layers via superconducting correlations which have an inhomogeneous oscillatory behavior due to the LOFF-like type of pairing.

The pair-breaking parameters  $D_s k_s^2$  for the S layer and  $D_s (k'_s)^2$  for the S' layer should be determined from the condition of nontrivial compatibility of the set of equations (6). It is possible to factorize the corresponding determinant and to obtain the following equation:

$$\begin{aligned} (\alpha' \delta' + 1)[(\alpha\gamma + 1)(\beta\gamma' - 1) + (\beta\gamma - 1)(\alpha\gamma' + 1)] \\ + (\alpha' \gamma' + 1)[(\alpha\gamma + 1)(\beta\delta' - 1) + (\beta\gamma - 1)(\alpha\delta' + 1)] = 0. \end{aligned} \quad (9)$$

Equation (9) can be simplified by taking into account the independence of the solutions for the S and S' layers and

knowing the solutions for the superlattice case. It is possible to obtain the following sets of equations for  $k_s$  and  $k'_s$ , which are different for the FM and AFM configurations. Note that only equations leading to the finite nonzero critical temperature are kept in these sets [see Eqs. (10) and (11) below].

For the FM ordering of the magnetizations we obtain two cases FM( $a$ ) and FM( $b$ )

$$\text{FM}\left(\begin{array}{c} a \\ a' \end{array}\right) \Rightarrow \begin{cases} \alpha\gamma + 1 = 0 \leftarrow \text{layer S} \\ \alpha'\gamma + 1 = 0 \leftarrow \text{layer S}' \end{cases},$$

$$\text{FM}\left(\begin{array}{c} b \\ b' \end{array}\right) \Rightarrow \begin{cases} 2\alpha\gamma\beta\delta + (\beta - \alpha)(\gamma + \delta) = 2 \leftarrow \text{layer S} \\ \alpha'\delta + 1 = 0 \leftarrow \text{layer S}' \end{cases}. \quad (10)$$

Here  $\gamma'$  and  $\delta'$  for the F' layer are substituted by  $\gamma$  and  $\delta$ , respectively, due to Eq. (8a).

According to Eq. (8b), when  $k'_f = k_f^*$  (leading to  $\gamma' = \gamma^*$  and  $\delta' = \delta^*$ ), for the AFM ordering we have two other cases AFM( $c$ ) and AFM( $d$ )

$$\text{AFM}\left(\begin{array}{c} c \\ c' \end{array}\right) \Rightarrow \begin{cases} \alpha\beta|\gamma|^2 + (\beta - \alpha)\text{Re } \gamma = 1 \leftarrow \text{layer S} \\ \alpha'\gamma^* + 1 = 0 \leftarrow \text{layer S}' \end{cases},$$

$$\text{AFM}\left(\begin{array}{c} d \\ d' \end{array}\right) \Rightarrow \begin{cases} 2\alpha\gamma\beta\delta^* + (\beta - \alpha)(\gamma + \delta^*) = 2 \leftarrow \text{layer S} \\ \alpha'\delta^* + 1 = 0 \leftarrow \text{layer S}' \end{cases}. \quad (11)$$

In the general case, there are four different solution sets FM( $a, a'$ ), FM( $b, b'$ ), AFM( $c, c'$ ), and AFM( $d, d'$ ) for the S and S' layers, each of which completely defines the state of *both* layers and, hence, the corresponding reduced transition temperatures  $t_c$  and  $t'_c$  [Eq. (5)]. However, since the solution  $t_c$  of Eq. (5) does not change when  $k_s$  is replaced by its complex conjugate the solutions  $t'_c$  for the S' layer do not depend on relative orientation of the magnetizations: the solution of Eq. (5) for the FM( $a'$ ) case coincides with the solution for the AFM( $c'$ ) case. The same is true for the solutions for the FM( $b'$ ) and AFM( $d'$ ) cases.

Moreover, two different solutions for the outer S' layer [FM( $a'$ )=AFM( $c'$ ) and FM( $b'$ )=AFM( $d'$ )] always coincide with the solutions for the F/S superlattice,<sup>4,32,33</sup> in which an orientation of the magnetizations is common to all F layers (the FM case). We try to classify the solutions (10) and (11) for the F/S/F'/S' system following the classification scheme proposed for a *superlattice* in Refs. 4, 32, and 33 (see also Sec. II). As it follows from that classification there are four possible states for a *superlattice*, which are described using two possible values (0 and  $\pi$ ) for the phases of the superconducting and magnetic order parameters. Two pairs of equations for the  $k'_s$  in the S' layer that coincide FM( $a'$ )=AFM( $c'$ ) and FM( $b'$ )=AFM( $d'$ ) lead to the 00 and  $\pi 0$  solutions, respectively: the first symbol corresponds to the superconducting order parameter phase ( $\varphi$ ), the latter one corresponds to the magnetic order parameter phase ( $\chi$ ). Thus we have only two distinguishable solutions  $a'$  and  $b'$  for the S' layer.

The latter can be easily understood from the physical

point of view. Only *one* ferromagnetic layer (F') acts on the outer S' layer. As a result the state of the layer depends only on the magnitude of the exchange field in the F' layer and does not depend either on its sign or on mutual ordering of the magnetizations. In other words, the S' layer is *always* in the *local ferromagnet* (FM) environment, therefore the  $\pi$  magnetic solutions do not exist for this layer.

For the S layer we have also two known superlattices solutions, namely, the FM( $a$ ) solution, that leads to the 00 solution, and the AFM( $c$ ) solution, that leads to the 0 $\pi$  solution (the 0 phase superconductivity and the  $\pi$  phase magnetism). Finally, there are two *extra* solutions FM( $b$ ) and AFM( $d$ ). Their presence is related to the external boundary conditions (3b) since the pair amplitudes (4a) and (4d) contain only even cosine solutions. These states are the  $\pi$  superconducting states, and in order to distinguish them from the earlier mentioned *superlattice* solutions we will denote these ones with a tilde  $\widetilde{\pi\chi}$  ( $\chi=0, \pi$  is the phase of the magnetic order parameter). Thus the FM( $b$ ) solution determines the  $\widetilde{\pi 0}$  state of the four-layered system and the AFM( $d$ ) solution corresponds to the  $\widetilde{\pi\pi}$  one. In the next section we will examine the obtained solutions and clarify the winners in the interplay of the four states (10) and (11).

#### IV. DISCUSSION OF THE PHASE DIAGRAMS

Taking into account the notations of Eq. (7) the sets of Eqs. (5) and (8)–(11) can be used to study the dependence of critical temperatures ( $t_c$  and  $t'_c$ ) of the four-layered F/S/F'/S' system on the reduced thicknesses of the superconducting and magnetic layers,  $d_s/\xi_{s0}$  and  $d_f/a_f = \widetilde{d}$ .

There are four more theoretical parameters of the system ( $\sigma_s, 2I\tau_f, n_{sf}, l_s$ ) that are necessary to consider in a general case. Keeping in mind a possibility of an application of the system as a “control device” we have searched for such a set of parameters for which the difference between the various states of the F/S/F'/S' system is sufficiently large to be observed. After performing numerous computer experiments we have found a range for the values of the parameters that satisfies these conditions. The *optimal range of parameters* should be as follows: the boundary should be sufficiently transparent ( $\sigma_s \gtrsim 5 \gg 1$ ), the ferromagnetic metal should be sufficiently dirty or (and) weak enough in regard to its magnetic properties ( $2I\tau_f = l_f/a_f \lesssim 0.15 \ll 1$ ), and the parameter  $n_{sf} = N_s v_s / N_f v_f > 1$ . From an experimental viewpoint this constraint on the values of the parameters does not look unreasonable. Note, the stronger an implementation of each inequality is, the larger a difference can be between critical temperatures for the 0 and  $\pi$  magnetic states. The choice of any one of these parameters lying out-of-range leads to significant decreasing of that difference. Parameters  $d_s$  and  $l_s$  should satisfy to the “dirty limit” conditions. Their influence on the noted above difference is minimal, though all values are important for a shape of the  $T_c(d_f)$  dependence (this has been detailed in the recent review<sup>4</sup>).

A set of phase curves  $t_c(d_f)$  and  $t'_c(d_f)$  for the *optimal* values of the parameters is shown in Fig. 2. The notation used for the curves corresponds to the notation used in Eqs.

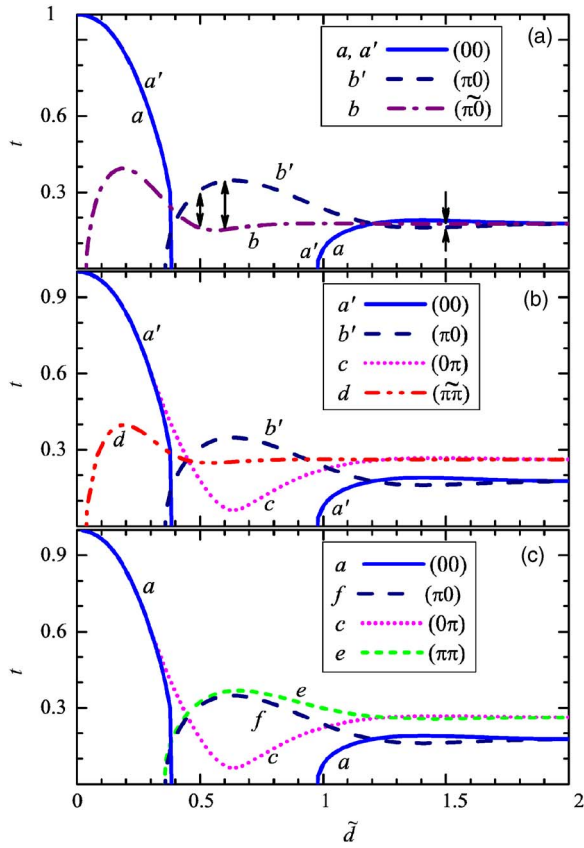


FIG. 2. (Color online) The phase diagrams ( $t-\tilde{d}$ ) of the F/S nanostructures for the following values of parameters:  $\sigma_s=15$ ,  $2l\tau_f=0.1$ ,  $n_{sf}=1.4$ ,  $l_s=0.25\xi_{s0}$ , and  $d_s=0.72\xi_{s0}$ . In the figure  $t=T/T_{cs}$  is the reduced temperature, and  $\tilde{d}=d_f/a_f$  is the reduced F layer thickness. The dependences of the reduced critical temperatures  $t_c$  and  $t'_c$  for the F/S/F'/S' system versus  $\tilde{d}=d_f/a_f$  are presented in panels (a) and (b). The  $t'_c$  curves for the outer S' layer are denoted using letters with a prime. The letters without a prime indicate the  $t_c$  curves for the inner S layer. (a) The phase diagram of the four-layered system for the ferromagnetic (FM) configuration of the magnetizations of both F layers. The arrows show the  $(t_c-t'_c)$  difference between the states which are discussed in the paper. (b) The phase diagram of the four-layered system for the AFM configurations. (c) The phase diagram of the F/S superlattice where the thicknesses of all F layers equal  $d_f$ , and the thicknesses of all S layers equal  $d_s$ . In this case all S layers have the same critical temperature. That is why only the  $t_c(\tilde{d})$  curves are shown here. The  $a$  and  $c$  curves form the phase diagram for the F/S/F trilayer in which the thicknesses of both F layers are equal to  $d_f/2$ .

(10) and (11). The curves  $c'$  and  $d'$  are not shown in Fig. 2 since  $c' \equiv a'$  and  $d' \equiv b'$ .

As one might expect, the  $a'$  and  $b'$  curves for the S' layer in Figs. 2(a) and 2(b) are identical for both FM and AFM configurations, and the  $a'$  curve for the S' layer coincides completely with the  $a$  curve for the S layer since both of them describe the same 00 state according to the superlattice classification scheme.<sup>4,32,33</sup> The rest of the states for the inner S layer (the  $b$  curve for the FM configuration, and the  $c$  and  $d$  curves for the AFM one) have different dependencies as compared with the ones for the S' layer and each other.

The phase diagram for the F/S superlattice [Fig. 2(c)] is obtained following procedures developed in Refs. 4, 32, and 33. Contrary to the four-layered F/S/F'/S' system in the superlattice case the different S layers have the same critical temperatures due to the periodicity condition imposed on the pair amplitudes. The  $a$ ,  $c$ , and  $f$  curves for the F/S superlattice completely coincide with the  $a$ ,  $c$ , and  $b'$  ones for the four-layered system, respectively. The  $\pi\pi$  superlattice state (the  $e$  curve), which is not found for our four-layered system, can be obtained from Eq. (5) where corresponding parameter  $D_s k_s^2$  is determined by the following equation:

$$\alpha\beta|\delta|^2 + (\beta - \alpha)\text{Re } \delta = 1.$$

To obtain the phase diagram for the F/S/F trilayer we have to exclude the states with the  $\pi$  phase superconductivity (the  $f$  and  $e$  curves) from this figure. Hence as it follows from Fig. 2 and discussions presented here the four-layered system has more physically different states than the F/S/F trilayer and even the F/S superlattice.

Thus the 00,  $\pi 0$ , and  $0\pi$  states of the four-layered system [Figs. 2(a) and 2(b)] correspond to the same states of the superlattice [Fig. 2(c)]. The  $\pi 0$  state [the  $b$  curve in Fig. 2(a)] and the  $\pi\pi$  one [the  $d$  curve in Fig. 2(b)] are the states associated with the  $\pi$  phase superconductivity. These are two extra solutions which are not found in the superlattice case [Fig. 2(c)]. The main difference between these  $\pi\chi$  and the known  $\pi\chi$  states is the peak position. For the inner S layer it is shifted to the lower values of the  $d_f$  thickness compared with the superlattice case due to the implementation of the external boundary conditions (3b).

The above-mentioned peculiarities of the four-layered system lead to different critical temperatures of different S layers. To show this consider the FM configuration [Fig. 2(a)] in detail. If there is no difference between  $t_c$  and  $t'_c$  for the 00 state then the case of the  $\pi$  phase superconductivity is more interesting since there is a difference between  $t'_c(\pi 0)$  and  $t_c(\pi 0)$ . Actually for each superconducting layer the upper envelope curve is realized due to the free energy minimum condition. In the case of the FM configuration that will be an  $a'-b'-a'$  curve and an  $a-b-a$  one for the S' and S layers, respectively. This leads to switching the ground state between the states with the 0 and  $\pi$  superconducting phases as the thickness  $\tilde{d}$  changes (at  $\tilde{d} \sim 0.4$  and  $\tilde{d} \sim 1.2$ , respectively).

In the  $\pi$  phase superconductivity case, the order parameter  $\Delta$  has opposite signs for the S and S' layers. Accordingly, the pair amplitude in the inner F' layer [Eq. (4c)] has a sinelike behavior ( $B'=0$ ) and is antisymmetric with respect to the layer center at which the sign change of the pair amplitude takes place while traversing the F' layer. The above-mentioned different  $T_c$  behavior in the S and S' layers [the  $b$  and  $b'$  curves in Fig. 2(a), respectively] leads to a difference between critical temperatures  $t$  and  $t'$ . For instance, at  $\tilde{d}=1.5$  the reduced critical temperature of the S layer  $t_c$  is equal to 0.177, and  $t'_c=0.163$ , at  $\tilde{d}=0.5$  the difference is larger:  $t'_c=0.308$  and  $t_c=0.16$ . If the reduced thickness  $\tilde{d}$  were

equal to 0.6, the difference would be almost maximal:  $t'_c = 0.346$ , and  $t_c = 0.154$ .

The reduced critical temperatures  $t_c$  and  $t'_c$  that correspond to these three values of the reduced thickness  $\tilde{d}$  are shown in Fig. 2(a) by arrows. The difference between two critical temperatures  $t_c$  and  $t'_c$  should be observed in experiments with the special field-cooled samples prepared with the FM ordering of the magnetizations (see Ref. 34 for experimental details).

The appearance of the critical temperature difference in the four-layered F/S system is a manifestation of the *critical temperatures hierarchy* in its clearest form. The origin of the  $T_c$  difference is obvious because, firstly, the S and S' layers are in a different magnetic environment and, secondly, they have different boundary conditions. In particular it is expressed in the above-mentioned shift of the peak of the  $\pi 0$  dependence due to the outside boundary conditions.

For the AFM configuration of the F/S/F'/S' system we have a similar picture [Fig. 2(b)], but in this case there are four different curves. Note that all above-mentioned peculiarities take place as well. As it has been discussed above, the phase curves for the S' layer are the same for both the FM and the AFM orientations. Two different solutions are obtained for the inner S layer. One of them is the known "superlattice" solution  $0\pi$  (curve *c*) while the second one is the  $\pi\pi$  solution (curve *d*). There is also a competition between the  $0$  and  $\pi$  phase superconductivity that leads to a appearance of the corresponding envelope curves of the second-order phase transition for the S and S' layers ( $c-d$  -  $c$  and  $a'-b'-a'$ , respectively).

The  $0\pi$  solution corresponds to the  $D'=0$  and  $B' \neq 0$  case, and the *A* and *C* factors are not equal to 0, i.e., the pair amplitude in the S layer does not possess any parity. The admixture of the sine solutions to the cosine ones in expression (4b) reflects the partial compensation of the paramagnetic effect of exchange field *I* for the S layer in the AFM state with antiparallel alignment of the F layer magnetizations. The previous statement applies to the  $\pi\pi$  state in the S' layer too. As in the stated above FM case, the difference between  $t_c$  and  $t'_c$  can be observed in experiments with the special field-cooled AFM samples.

Let us take up the common case, when there is the interplay of all four states (10) and (11), to clarify the winners in this competition. For convenience all the phase curves are shown in Figs. 2(a) and 2(b) in one combined diagram (Fig. 3).

Note that at  $\tilde{d}=0.5$  the  $\pi\pi$  state [the *d* curve in Figs. 2(b) and 3] has the highest  $T_c$  among all possible states for the S layer  $t_c \approx 0.25$ , but that it is lower than the appropriate temperature for the  $\pi 0$  state of the S' layer  $t'_c \approx 0.31$ . According to the theory of second-order phase transitions, the state possessing the lower free energy (higher  $T_c$ ) is realized. Thus for the samples with the reduced thickness  $\tilde{d}=0.5$  the S and S' layers are both in the normal (N) state if the temperature  $t > t'_c \approx 0.31$ . Below  $t'_c$  the S' layer becomes superconducting (S) but the S layer remains in the N state while  $t > t_c \approx 0.25$ . Finally, at  $t < t_c$  the AFMS state [AFM( $d, d' \equiv b'$ )] wins and for the whole system we have the case with the  $\pi$  phase superconductivity and the  $\pi$  phase magnetism.

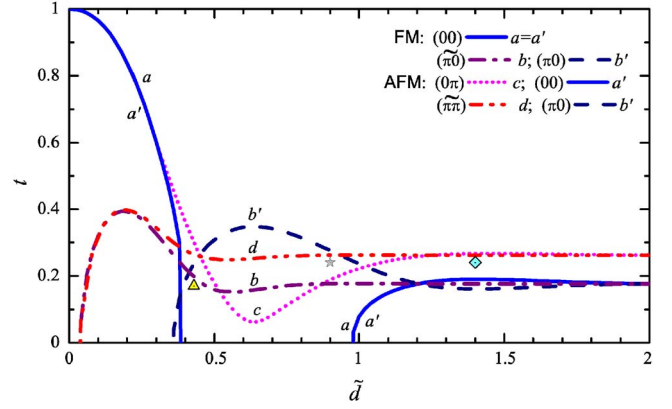


FIG. 3. (Color online) The combined phase diagram of the four-layered F/S/F'/S'. All parameters and notations correspond to the ones used in Fig. 2. The symbols discussed in Sec. V correspond to the "working" points of the proposed control device with a various number of possible states.

At  $\tilde{d}=1.5$  we have the following chain of the second-order phase transitions:

$$\uparrow N \downarrow N (\text{or } \uparrow N \downarrow N) \xrightarrow{t_c=0.27} \uparrow S \downarrow N \xrightarrow{t'_c=0.19} \uparrow S \downarrow S$$

(see caption to Fig. 4 for an explanation of the notation). Thus at low temperatures the AFMS state [AFM( $c, c' \equiv a'$ )] wins too, but this state is associated with the  $0$  phase superconductivity. Note in the framework of our theory only transition temperatures can be found and it is not possible to determine what state inside the "normal" state region is preferable.

The analogous analysis can be carried out for the entire range of the reduced F layer thicknesses ( $0 < \tilde{d} < 2$ ). Assume the system can choose its own state according to the theory

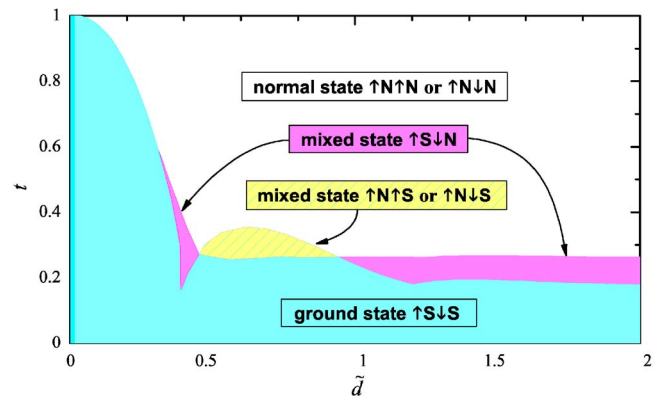


FIG. 4. (Color online) The generalized phase diagram of the four-layered F/S/F'/S' system with the same parameters as in Figs. 2 and 3. Vertical arrows show the direction of the magnetization in the corresponding ferromagnetic layer. The letters S and N stand for the superconducting and normal states of the superconducting layers, respectively. For simplicity the magnetization is assumed to be fixed and directed "upwards" in the outer F layer (see also the next section).



of the second-order phase transitions. The state with higher critical temperature wins, and one of four states defined by Eqs. (10) and (11) [see also Figs. 2(a), 2(b), and 3] is realized for the system. A complete phase diagram constructed for the system is presented in Fig. 4. Four different regions can be defined for this diagram: at high temperatures both S and S' layers are in normal state and the mutual ordering of the magnetizations in the F and F' layers is unimportant. As it follows from the phase diagram, there are two regions marked in a dark gray color (magenta in color online version) with a mixed antiferromagnetic state for which the inner S layer is superconducting (S), and the outer S' one is normal (N). The striped light gray (yellow) marked region corresponds to the mixed state with the superconducting outer S' layer and the normal S layer. Finally, at low temperatures and/or at small  $d_f$  thicknesses the system is in the ground AFMS state [gray (blue) marked region].

Thus if the inner S layer is in the *superconductive* state then ordering of the magnetizations should be *antiferromagnetic*. This is the result of the *inverse* action of *superconductivity on magnetism*.

Note, the details of the phase diagram significantly depend on the choice of the system parameters and the above analysis was carried out assuming the absence of an external magnetic field ( $H=0$ ).

## V. CONTROL DEVICE SCHEME

In this section we propose a conceptual scheme of a “control device” based on the four-layered F/S/F'/S' structure. Following the previous studies performed on spin valves<sup>25,42</sup> and for technical convenience, we add to the left external layer of the system one extra layer of a magnetic insulator (MI), whose role is to pin the direction of the magnetization  $\mathbf{M}$  in the outer F layer. One of the possible consequences of that is the return of our system to the initial state (in a magnetic sense) after switching off the magnetic field. Otherwise it is necessary to use additional sequences of the switching-off fields<sup>39</sup> to achieve that. Note that it is more convenient to use the four-layered system than the F/S superlattice<sup>4,32,33</sup> since it is easier to change the mutual ordering of the magnetizations of the F layers for this system. Thus, formally, our system becomes the MI/F/S/F'/S' one. However, this practically does not affect the preceding computations performed for the four-layered system, and we will use the notations earlier introduced for the four-layered system.

We can control the state of the F/S structure by applying a small external magnetic field  $\mathbf{H}$ , which slightly changes the phase diagram of the sample at the fixed temperature.<sup>24,25,32,33</sup> In this case there are specific values of the magnetic field: the coercivity  $H_{\text{coer}}$  at which the orientation of the magnetization  $\mathbf{M}'$  in the F' layer can be reversed, the critical field  $H_c$  which destroys superconductivity, and the pinning field  $H_p$  at which the direction of the pinned magnetization  $\mathbf{M}$  in the outer F layer can be reoriented. Assume that the ordering of localized spins in the F and F' layers is of a uniaxial type and the magnitudes of these fields are related as follows:<sup>43</sup>  $H_{\text{coer}} < H_c < H_p$ .

The study of the combined phase diagram (Fig. 3) helps in optimizing the choice of parameters of the four-layered

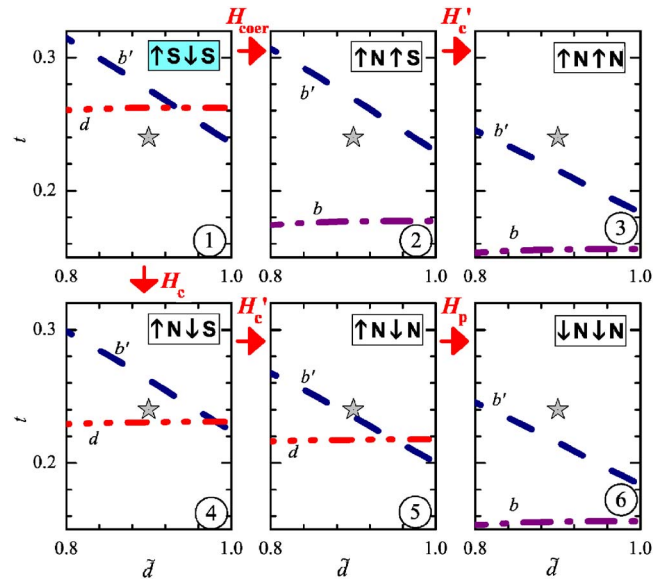


FIG. 5. (Color online) The qualitative scheme of the chain of phase transitions in the system in the initial “star” state under the influence of the external magnetic field  $\mathbf{H}$ . Only the envelope phase curves corresponding to the winning states are shown in each panel for the S and S' layers. Panel 1 corresponds to the “star” point vicinity in Fig. 3 with  $H=0$ . Panels 2 and 3 (4–6) correspond to the “upward” (“downward”) orientation of  $\mathbf{H}$ . All parameters and notations correspond to the ones used in Figs. 3 and 4.

F/S/F'/S' system, making it possible to control its superconducting and magnetic states. Assume the system is in one of the “working” points shown in the diagram. Each of these “working” points characterizes a sample that is described by a concrete set of parameters ( $d_f$ ,  $\sigma_s$ ,  $2I\tau_f$ , etc.), including the current temperature of a sample  $T$ . At zero magnetic field the system is in the initial AFM state. By applying an external magnetic field, we can change the state of the system. The changes in the magnetic field lead to transitions of the system between these states. Note that the system in the shown points ( $\star$ ,  $\diamond$ ,  $\triangle$ ) can have a number of logically different states (up to seven ones in the  $\triangle$  case).

Choose one of them, the “star,” which is located below the curves  $b'$  and  $d$  in Fig. 3 (the reduced “working” temperature of the sample  $t_\star = T_\star/T_{\text{cs}} \approx 0.24$ ,  $\tilde{d}_\star \approx 0.9$ ). The system in this point can have up to six different states. Changing first the external magnetic field  $H$  applied in the direction of the pinning field, one can induce transitions of the system between the ground AFMS state, the mixed FM state, and the normal FM state. Applying the external magnetic field in the opposite direction one can induce three other transitions between the ground AFMS state, the mixed AFM state, the antiferromagnetic normal (AFMN) state, and, finally, the ferromagnetic normal (FMN) one.

To show this in more detail, we assume that the orientation of the magnetization of the outer F layer pinned, for example, upwards ( $\uparrow$ ) as shown in Fig. 1. At  $H=0$  the system is in the initial AFMS state (see panel 1 of Fig. 5). If we apply the small external magnetic field  $\mathbf{H}$  that is larger than coercivity ( $H_{\text{coer}} < H < H_c$ ) in the direction of the magnetization  $\mathbf{M}$  of the F layer and the pinning field ( $\mathbf{H} \uparrow \uparrow \mathbf{H}_p$ ), then

the direction of the magnetization  $\mathbf{M}'$  in the  $F'$  layer is turned up. The system is transferred into the state with the ferromagnetic ordering of the magnetizations, and the AFM state curves ( $c$  and  $d$ ) disappear from the diagram. The remaining curves ( $a=a'$ ,  $b'$ , and  $b$ ) are only slightly changed (panel 2 of Fig. 5). As a result, the data stored using the superconducting property of the  $S'$  layer are kept unchanged while the information stored on the basis of the orientation of the magnetizations in the  $F$  and  $F'$  layers and the supercurrent in the middle  $S$  layer is changed.

If the applied field  $H$  is greater than the critical field for the outer  $S'$  layer ( $H > H'_c$ ), the superconductivity is destroyed, and the system undergoes a transition into the FMN state (panel 3 of Fig. 5). In other words, the information written using the  $S'$  supercurrent changes as well. Note that one can call the transition from superconducting to normal state that is controlled by the external magnetic field to be the transition with practically infinite magnetoresistance.<sup>46</sup>

Thus these transitions can be written as follows (see also panels 1–3 of Fig. 5 for details):

$$\uparrow S \downarrow S \xrightarrow{H_{\text{coer}}} \uparrow N \uparrow S \xrightarrow{H'_c} \uparrow N \uparrow N.$$

Applying the external magnetic field in the opposite direction ( $\mathbf{H} \downarrow \uparrow \mathbf{H}_p$ ) it is possible to induce the other three additional transitions of the system. In principle it is necessary to distinguish the critical fields for the  $S$  and  $S'$  layers. Moreover, these fields can be different for the same four-layered samples in the AFM and FM configurations. For the “star” working point and the AFM ordering, we have to put that  $H'_c > H_c$ , since the difference  $[t'_c(\pi_0) - t_*]$  is larger than  $[t_c(\overline{\pi\pi}) - t_*]$  (in Fig. 3 and in panel 1 of Fig. 5 the  $b'$  curve is above the  $d$  one at  $\tilde{d} = \tilde{d}_*$ ). Note that generally the required magnitudes of the corresponding critical fields  $H_c$  and  $H'_c$  are determined by an appropriate choice of the working point position relative to the curves of the superconducting transition  $T_c(d_f)$  and  $T'_c(d_f)$ , respectively (see Fig. 3). In addition, by changing  $T$  and  $d_f$  one can always make  $H_c$  smaller than the field  $H_p$ , which is necessary to remove pinning.

If  $H$  is a bit larger than  $H_c$  but is less than  $H'_c$  the system undergoes a transition from the ground AFMS state into the mixed AFM state:

$$\uparrow S \downarrow S \xrightarrow{H_c} \uparrow N \downarrow S$$

(see also panels 1 and 4 of Fig. 5). Only the information that is stored using the superconducting property of the  $S$  layer changes, while the stored data associated with ordering of the magnetizations of both  $F$  and  $F'$  layers and the  $S'$  supercurrent remain unchanged. The final transitions in this series take place at a further increase of the magnetic field (from being  $H'_c < H < H_p$  to  $H > H_p$ ) in the same “downward” direction:

$$\uparrow N \downarrow S \xrightarrow{H'_c} \uparrow N \downarrow N \xrightarrow{H_p} \downarrow N \downarrow N$$

(see also panels 4–6 of Fig. 5).

Thus the  $F/S/F'/S'$  system prepared in such a way has six logically different states:  $\uparrow S \downarrow S$ ,  $\uparrow N \uparrow S$ ,  $\uparrow N \uparrow N$ ,  $\uparrow N \downarrow S$ ,

$\uparrow N \downarrow N$ ,  $\downarrow N \downarrow N$  (see also Fig. 5). Recall that in the case of the symmetric  $F/S$  superlattice the superconducting properties vary *synchronously* in all  $S$  layers, and a similar device on its base has up to *five* logically different states<sup>4,32,33,38</sup>. Note also that only *two* states were proposed for the trilayer spin switch.<sup>24,25</sup>

Consider another choice of parameters ( $t_\diamond \approx 0.24$ ,  $\tilde{d}_\diamond \approx 1.4$ ) in Fig. 3 that corresponds to the “diamond” working point with four operating states. At zero magnetic field the system is in the region that corresponds to the mixed AFM state (see Fig. 4), for which the inner  $S$  layer is superconducting, and the outer  $S'$  one is normal. In this case a “short” chain of transitions can be obtained by changing the directed “upwards” magnetic field (i.e.,  $\mathbf{H} \uparrow \uparrow \mathbf{H}_p$ ):

$$\uparrow S \downarrow N \xrightarrow{H_{\text{coer}}} \uparrow N \uparrow N.$$

The “longer” chain of transitions is obtained when the direction of magnetic field is changed to the opposite (i.e.,  $\mathbf{H} \downarrow \uparrow \mathbf{H}_p$ ):

$$\uparrow S \downarrow N \xrightarrow{H_c} \uparrow N \downarrow N \xrightarrow{H_p} \downarrow N \downarrow N.$$

Thus there are altogether only *four* different states.

Finally, if we choose the “triangle” working point ( $t_\Delta \approx 0.17$ ,  $\tilde{d}_\Delta \approx 0.43$ ) in Fig. 3, it is possible to obtain up to *seven* logically different states. The successive change of the magnitude of the directed “upwards” magnetic field leads to the following chain of transitions of the system:

$$\uparrow S \downarrow S \xrightarrow{H_{\text{coer}}} \uparrow S \uparrow S \xrightarrow{H_c} \uparrow N \uparrow S \xrightarrow{H'_c} \uparrow N \uparrow N.$$

Changing the direction of the field to the opposite one can obtain the following chain:

$$\uparrow S \downarrow S \xrightarrow{H'_c} \uparrow S \downarrow N \xrightarrow{H_c} \uparrow N \downarrow N \xrightarrow{H_p} \downarrow N \downarrow N.$$

As it follows from the phase diagram in Fig. 3, we have used the assumption that  $H_c < H'_c$  for the FM configuration and  $H_c > H'_c$  for the AFM one while considering the last working point. Moreover, if the “triangle” position were moved a little to the left ( $\tilde{d}_{\Delta, \text{new}} \approx 0.42$ ), then  $H_c \approx H'_c$  for the FM configuration and we would get the chain of transitions consisting of six different states.

It follows from our studies that the “spin switch” device proposed on the basis of a  $F/S/F$  trilayer<sup>24,25</sup> (even with the optimum set of parameters that we have found) has a much less number of logically different ways of data recording than the above studied four-layered  $F/S/F'/S'$  system.

## VI. CONCLUSIONS

The four-layered  $F/S/F'/S'$  system has been consistently studied within the modern theory of the proximity effect with a detailed account of the given boundary conditions. Theoretical studies of the critical temperature dependence on the thicknesses of the  $F$  layers have been performed for a wide range of parameters, and a physically interesting range of

their values has been determined. The latter should be of help in choosing materials and technology for preparation of the F/S systems with predetermined properties.

It has been shown that when the  $\pi$  phase superconductivity coexists with the nonequivalence of all layers the physics of the four-layered systems is considerably richer in comparison with one for the earlier studied three-layered F/S/F system<sup>24,25</sup> and even the F/S superlattices.<sup>4,32,33</sup> The extra  $\pi$  phase superconducting states obtained for the four-layered F/S/F'/S' system have been found to be different from the analogous  $\pi$  phase superconducting superlattice states. The hierarchy of critical temperatures has been shown to manifest itself mainly through the occurrence of the different critical temperatures in the different S and S' layers (space-separated or decoupled superconductivity). This prediction can be experimentally verified both for the common case and for the specially prepared field-cooled FM and AFM samples.

Theoretical studies performed in this paper have shown that the four-layered F/S/F'/S' system has the best prospects for its use in superconducting spin electronics (*superconducting spintronics*). This system can be used for a creation of the nanoelectronic devices combining within the same layered sample the advantages of the superconducting and magnetic channels of data recording that are associated with the conducting properties of both S and S' layers and the magnetic ordering of the magnetizations of the ferromagnetic layers. It has been emphasized that both these channels can be *separately* controlled by an external magnetic field. These magnetic fields can be made sufficiently weak<sup>43</sup> due to a choice of materials and parameters of the system. A few versions of the principal scheme of such a four-layered F/S/F'/S' device have been proposed and explored. It has been shown that the proposed control device can have up to *seven* different states, and transitions between these states can be controlled by a magnetic field. It should be noted that advantages of such spin devices are also associated with their small enough sizes (thickness  $d_f$  is about 0.5–5 nm, thick-

ness  $d_s \sim 25\text{--}80$  nm), relatively high switching speed (according to estimates<sup>36</sup> its range is from  $10^{-10}$  s<sup>-1</sup> up to  $10^{-11}$  s<sup>-1</sup> depending on used materials), and relatively large critical currents (they approximately coincide with currents for isolated superconductors<sup>25</sup>). Certainly, low temperatures at which usual “cold” superconductivity is possible would be a condition for the use of this type of control device. However, similar superconducting devices on the basis of the F/S structures with S layers made out of high-temperature superconducting materials<sup>47</sup> should work at much higher temperatures.

Finally, it should be noted that certain simplifications that were introduced in the performed studies do not impose any serious restrictions on the applicability of the main results obtained in the present paper (hierarchy of critical temperatures, phase diagram in Fig. 4, proposed schemes of a control device, etc.) that provide qualitative understanding and principal solutions of the problem. The model approach used in our studies adequately describes physics of the F/S/F'/S' system. Though taking into account the multimode approach<sup>15,16</sup> and/or the 3D treatment<sup>4,18,23</sup> could change the shape of the curves in the above discussed diagrams for the *same values* of parameters, it is always possible to *change* this set of values in such a way that *similar graphs* can be constructed. The effect of the influence of each parameter change on the shape of the phase curves was discussed in the review.<sup>4</sup>

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