Generalization of a circuit theory for current perpendicular to plane magnetoresistance and current-driven torque

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Extensions of an existing circuit theory for current perpendicular to plane magnetoresistance and currentdriven torque in noncollinear magnetic-multilayer pillar devices are presented. Our expressions for monodomain critical-current threshold J_c and giant magnetoresistance ΔR are firstly derived in terms of assumed spin-channel resistances for each of the two ferromagnets. Spinflips are thus neglected. We find a class of closed linear relationships connecting J_c^{-1} and ΔR . We then derive more general expressions for these quantities which take into account spin-flip relaxation. In this case, we assume analytically calculable linear 2×2 current-voltage matrices for the separate two-channel ferromagnets. These expressions again lead to a class of closed linear relationships connecting J_c^{-1} and ΔR . The latter generalization gives a simple theoretical framework to take into account bulk and interfacial spin flip and more complicated multilayer structures often used in experiments.

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I. INTRODUCTION

The discovery of current-in-plane (CIP) giant magnetoresistance (GMR) by Baibich *et al.*¹ in 1988 opened a new field of research known as spintronics. It led to new applications for magnetic devices such as commercial read heads² in data storage and exploratory integrated magnetic random access memory³ (MRAM). Recent attention is additionally focused on the alternative current perpendicular to plane (CPP) spin-dependent transport in a magnetic multilayer (MML), which is dominated by spin-dependent scattering.⁴ This strong spin dependence permits control of the current flowing through a spin valve⁵ by means of the relative orientation of the magnetization of the ferromagnetic electrodes.

An inverse effect of CPP-GMR was predicted $6,7$ in 1996. These authors showed that an electrical current spin polarized by a first ferromagnetic layer can induce excitations in a second ferromagnetic layer. Subsequently, many experiments have confirmed the resulting current-induced magnetization oscillations and switching (CIMS) using point contacts⁸ and device pillars.⁹ This phenomenon, known as spin transfer torque (STT), is now studied experimentally in spin-valve structures¹⁰ and magnetic tunnel junctions.¹¹ More advanced theories for STT through metallic¹² and tunneling¹³ spacers have appeared.

One STT theory for the case of a metallic spacer combines a statistical-density operator description of the spacer layer and a classical resistive spin-channel model description of the ferromagnetic electrodes.14 It applies particularly to certain magnetic metals and alloys, including Co, CoFe, NiFe, and Ni, which lie on the negative-slope portion of the Slater-Neél-Pauling curve of atomic moment versus atomic number. This theory extends the common two-channel resistor model of collinear CPP-GMR to the case of noncollinear magnetizations. Later, there appeared a general expression for critical current based on this theory.15 We propose here a more comprehensive study of its implications, and give more general expressions for currents and torques.

In Sec. II we present the main results of the original circuit model for the case of two simple ferromagnetic electrodes neglecting all spin-flip processes within the device pillar. Section III digresses briefly to an experimental study of a different case which includes spin-flip relaxation within the spacer. In Sec. IV, we generalize to the case where the electrodes are more complicated in order to introduce bulk and interfacial spin-memory loss. In Sec. V, we discuss the validity of this model.

II. SIMPLE CIRCUIT THEORY

Throughout this paper, we consider a unit cross-sectional area of a magnetic trilayer, composed of two ferromagnetic electrodes (left pinned layer and right free layer), separated by a nonmagnetic spacer layer. In a simple illustration of the method, Refs. 14 and 15 use the circuit similar to Fig. 1. Our object is to evaluate the left and right spin-channel electric currents J_L and J_R and the spin torque induced on the right electrode (say) by a total electrical current J flowing from left to right. Spin-flip relaxation, also called spin diffusion, is

FIG. 1. Effective electrical circuit for a simple trilayer neglecting interfacial spin flip.

neglected in this illustration everywhere except in the large nonmagnetic external contact regions. In these regions resistance is neglected and spin flip is taken into account, as a limiting case, by the two direct interchannel connections indicated in the figure. The spacer is a normal diamagnetic layer metal having negligible bulk resistivity. Here, R_L (respectively, R_R) are the left-spin-channel resistances (respectively, right-spin-channel resistances). They are composed additively of bulk and interfacial terms. W_L (respectively, W_R) are the corresponding electrochemical voltages within the spacer, appropriate to the spin quantization axes of the left (respectively, right) magnets. We use the general definition $W=V\pm\Delta\mu/e$, where *V* is ordinary electric voltage within the spacer and $2\Delta\mu$ is the difference in chemical potential, associated with spin accumulation, between any two spin channels and $-e$ is the electron charge. The use of W_L and W_R , just as one would normally use *V*, when applying Kirchoff's laws to this effective resistive network, takes account of spin accumulation effects. To solve the network, one needs four relations connecting the variables J_L and W_L to J_R and W_R . Two of these relations are those of ordinary electric continuity: $2V_1 = W_{L+} + W_{L-} = W_{R+} + W_{R-}$ and $J = J_{L+} + J_{L-}$ $=J_{R+}+J_{R-}$. For the two remaining relations we refer to the quantum-mechanically determined connection formulae (13) and (14) of Ref. 14. Setting $\Delta J_i = J_{i+} - J_{i-}$ and $\Delta W_i = W_{i+}$ $-W_{i-}$, $(i=R,L)$, these relations are:

$$
2\Delta J_R \cos \theta = \Delta J_L [1 + \cos^2 \theta] - G \Delta W_L \sin^2 \theta, \qquad (1)
$$

$$
2\Delta W_R \cos \theta = -\Delta J_L \sin^2 \theta / G + \Delta W_L [1 + \cos^2 \theta], \quad (2)
$$

where θ is the angle between the magnetizations of the right and left electrodes. The parameter *G* is given by the formula $G=e^2k_F^2/\sqrt{3}\pi h$ for an assumed spherical Fermi surface in the spacer. It is the only additional parameter introduced while advancing from the collinear $(\theta=0,\pi)$ to noncollinear (general θ) magnetic configuration. Note that $-\hbar \Delta J_i/2e$ is the spin-angular-momentum current across the interface between the spacer and the *i*th electrode and ΔW_i is proportional to the spin accumulation of the spacer with respect to the spinquantization axis in this electrode. Equations (1) and (2) are specialized to electrode/spacer interfaces with vanishing majority-spin reflection. This approximation is suited to interfaces joining Cu to such weakly spin-flip relaxing electrode compositions as Co, CoFe, NiFe, Ni, and others of magnetic elements or alloys lying on the negative-slope side of the Slater-Neél-Pauling curve.

Furthermore, a macroscopic relation $14,15$ gives the expression for the torque exerted by the current on the right electrode:

$$
L_R = \hbar \left[-\Delta J_R \cos \theta + \Delta J_L \right] / 2e \sin \theta. \tag{3}
$$

Thus the key variables to calculate the torque are ΔJ_R and ΔJ_L . To evaluate these quantities, Eqs. (1) and (2) can be combined with Kirchoff's equations

$$
W_{R+} = R_{R+} J_{R+},
$$
\n(4)

$$
W_{R-} = R_{R-}J_{R-},\tag{5}
$$

$$
W_{L+} = V_1 - R_{L+} J_{L+},\tag{6}
$$

$$
W_{L-} = V_1 - R_{L-}J_{L-},\tag{7}
$$

to give, after some algebra:

$$
\alpha \Delta J_R + \beta \Delta J_L = \gamma J,\tag{8}
$$

$$
\delta \Delta J_R + \zeta \Delta J_L = \eta J,\tag{9}
$$

where:

$$
\alpha = -4 \cos \theta, \tag{10}
$$

$$
\beta = 2[1 + \cos^2 \theta] + G \sin^2 \theta [R_{L-} + R_{L+}], \quad (11)
$$

$$
\gamma = G \sin^2 \theta [R_{L-} - R_{L+}], \qquad (12)
$$

$$
\delta = 2[R_{R+} + R_{R-}] \cos \theta, \qquad (13)
$$

$$
\zeta = 2\sin^2\theta/G + [R_{L-} + R_{L+}][1 + \cos^2\theta],\tag{14}
$$

$$
\eta = [1 + \cos^2 \theta][R_{L-} - R_{L+}] + 2[R_{R-} - R_{R+}] \cos \theta. \quad (15)
$$

Then, ΔJ_R and ΔJ_L can be expressed as a function of the electric current $J = J_{L,R+} + J_{L,R-}$:

$$
\Delta J_R = J[\gamma \zeta - \beta \eta]/D, \qquad (16)
$$

$$
\Delta J_L = J[\alpha \eta - \gamma \delta]/D, \qquad (17)
$$

where $D = \alpha \zeta - \beta \delta$ is the determinant of the coefficients in Eqs. (8) and (9) .

Note that solution (16) , (17) satisfies the collinear relations

$$
\left[\frac{d\Delta J_L}{d\theta}\right]_{\theta=0,\pi} = \left[\frac{d\Delta J_R}{d\theta}\right]_{\theta=0,\pi} = 0.
$$

Also, these collinear conditions satisfy the equality ΔJ_L $=\Delta J_R$. Therefore, differentiation of Eq. (3), with proper treatment of limits having the form $0 \div 0$, gives the equation

$$
\left[\frac{dL_R}{d\theta}\right]_{\theta=0,\pi} = \frac{\hbar}{4e} \left[\Delta J_R \pm \frac{d^2 \Delta J_L}{d\theta^2} - \frac{d^2 \Delta J_R}{d\theta^2}\right]_{\theta=0,\pi} \quad (18)
$$

evaluated at the limits $\theta = 0$ or $\theta = \pi$ depending on the initial P or AP state of the magnetizations. The efficiency of initial conversion of electric current to torque is defined by the formula (Ref. 15 and 16).

$$
\epsilon = \frac{2e}{\hbar J} \left[\frac{dL_R}{d\theta} \right]_{\theta = 0, \pi} . \tag{19}
$$

Carrying this out, one finds the efficiencies

$$
\epsilon_{\uparrow\uparrow(\uparrow\downarrow)} = \frac{1}{2} \frac{R_{L\mp} - R_{L\pm} + R_{R-} - R_{R+} + G[R_{L\mp}R_{R+} - R_{L\pm}R_{R-}]}{R_{L+} + R_{L-} + R_{R+} + R_{R-}}.
$$
\n(20)

The complete expression for critical current threshold, following Ref. 16 in the single-domain approximation to a small free element, is:

$$
J_{\uparrow\uparrow(\uparrow\downarrow)} = \frac{4\alpha_G M_s d_R e H_{eff}}{\hbar \epsilon_{\uparrow\uparrow(\uparrow\downarrow)}},\tag{21}
$$

where $H_{eff} = H_{coupling} + H_{ext} + H_k$, H_{ext} is the external field, *Hcoupling* is the combined effect of exchange, dipole, and Neél coupling fields between the ferromagnetic electrodes, H_k is the uniaxial anisotropy field including demagnetizating effects of shape. It is written $H_k = \frac{1}{2} H_{shape} + 2 \pi M_s$ for the usual experimental case of in-plane equilibrium magnetization, where H_{shape} is the in-plane shape-induced anisotropy field and *Ms* the saturation magnetization.

The expression of the absolute magnetoresistance of the pillar circuit is:

$$
\Delta R = \frac{[R_{L-} - R_{L+}][R_{R-} - R_{R+}]}{R_{L+} + R_{L-} + R_{R+} + R_{R-}}.
$$
\n(22)

We can also express the current polarization for collinear moments. As we neglected the resistivity of the spacer, as well as spin accumulation inside it, we get from (16) and $(17):$

$$
\frac{\Delta J_R(\theta=0)}{J} \equiv \frac{\Delta J_L(\theta=0)}{J} \equiv \frac{R_{R-} - R_{R+} + R_{L-} - R_{L+}}{R_{R-} + R_{R+} + R_{L-} + R_{L+}}.
$$
\n(23)

Using Eq. (23), one can express any parameter of the MML as a function of $\Delta J/J$. Substituting this latter expression in Eqs. (21) and (22), one can find a relationship between the collinear polarization and the absolute CPP-GMR, where the coefficients will depend on all the material parameters except the one which was substituted in Eq. (23).

Thus, and this is one important result of this paper, we find a class of linear relationships between the collinear polarization and the absolute CPP-GMR. For example, expressing R_{L+} as a function of the collinear polarization and replacing it in $\Delta R \equiv V_1 / J(\pi) - V_1 / J(0)$, one finds:

$$
\Delta R = \frac{[R_{R-} - R_{R+}][2R_{L-} + R_{R+} + R_{R-}]}{2[R_{R-} + R_{L-}]} \frac{\Delta J_R}{J}(\theta = 0)
$$

$$
- \frac{[R_{R-} - R_{R+}]}{2[R_{R-} + R_{L-}]}.
$$
(24)

By the same method, the inverse critical current $J_{\uparrow\uparrow(\uparrow\downarrow)}$ can be expressed as a function of the collinear polarization:

$$
\frac{1}{J_{\uparrow\uparrow}} = \left[\frac{4\alpha_G M_s d_R e H_{eff}}{\hbar}\right]^{-1} \left[\frac{\left[2 + G(R_{R+} + R_{R-})\right] \Delta J_R}{4} (\theta = 0) - \frac{G[R_{R-} - R_{R+}]}{4}\right]
$$
\n(25)

and

$$
\frac{1}{J_{\uparrow\downarrow}} = \left[\frac{4\alpha_G M_s d_R e H_{eff}}{\hbar}\right]^{-1} \times \left[\frac{\left[2 + G(R_{R+} + R_{R-})\right] \Delta J}{4} (\theta = ??) - \frac{G[R_{R-} - R_{R+}]}{4}\right].
$$
\n(26)

Combining Eqs. (24) and (25) , we find then a linear relationship between the inverse critical current and ΔR :

$$
\frac{1}{J_{\uparrow\uparrow}} = \left[\frac{4\alpha_G M_s d_R e H_{eff}}{\hbar} \right]^{-1} \times \left[\frac{\Delta R \left[R_{R-} + R_{L-} \right] \left[2 + G(R_{R-} + R_{R+}) \right]}{2 \left[R_{R-} - R_{R+} \right] \left[2R_{L-} + R_{R-} + R_{R+} \right]} - \frac{\left[R_{R-} - R_{R+} \right] \left[1 - GR_{R-} \right]}{2 \left[2R_{L-} + R_{R-} + R_{R+} \right]} \right]. \tag{27}
$$

In summary, our derived Eq. (27) has the linear form

$$
1/J_{\uparrow\uparrow(\uparrow\downarrow)} = \mathbf{P}\Delta R + \mathbf{Q} \tag{28}
$$

for the inverse critical current threshold on the absolute magnetoresistance whenever any one of the four-channel resistances $R_{L\pm}$, $R_{R\pm}$, is varied without affecting α_G , M_s , or d_R .

III. ALTERNATIVE CASE OF SPIN-FLIP IN SPACER

A linear dependence, superficially resembling Eq. (28), has been proposed recently^{17,18}. Urazhdin *et al.*, in Ref. 17 varied the spacer layer spin-diffusion length and observed a linear variation of the inverse critical current as a function of the absolute magnetoresistance. Whereas we assume no spin diffusion inside the spacer layer in our model, it remains interesting to compare it with this experiment. As a matter of fact in Ref. 17, increasing the spacer spin diffusion length inside the spacer layer decreases the collinear polarization due to the pinned layer (here called $\Delta J_L / J$) impinging on the free layer. This variation of collinear polarization induces a variation of both CPP-GMR and spin torque amplitude illustrated by the linear dependence. This experiment shows the important role of the collinear polarization in the linear relationship between CPP-GMR and spin torque amplitude in a context different from that of this paper.

This linear dependence can be understood phenomenologically. The general behaviour of the inverse critical current as a function of the CPP-GMR depends on a material parameter that does not appear in Eqs. (24) and (25). It means that the linearity can be observed when varying experimentally one parameter while keeping all the others unchanged. Although the authors of Ref. 17 varied the spacer layer spin-diffusion length, we show a similar dependence varying other material or structure parameters like layer thickness or spin asymmetry.

Another explanation of this experiment¹⁷ has been proposed by one of the authors,¹⁹ using a model based on the Valet and Fert theory of CPP collinear spin-dependent transport. It is shown that, in a spin-valve, it is possible to link the spin torque amplitude to the absolute magnetoresistance through the collinear polarization due to the pinned layer alone. The simulations presented in this model show another interesting feature that can be seen in Eq. (28) .

As a matter of fact, we note that **Q** is generally nonzero. This means that one might adjust the varying parameter in order to obtain $\Delta R = -Q/P$ in order to cancel the spin transfer effect. Alternatively, a value of this parameter may cause cancellation of the magnetoresistance preserving a torque of amplitude **Q**.

FIG. 2. Effective electrical circuit for a general multiplayer with interfacial spin flip.

Let us focus now on the critical current threshold expressed in Eq. (21). Substituting one parameter by its expression as a function of $J_{\uparrow\downarrow}$ for example, one can find another linear relationship in the form:

$$
1/J_{\uparrow\uparrow} = S/J_{\uparrow\downarrow} + T. \tag{29}
$$

We find the same explanation as exposed above for this linearity and the same kind of conclusion: is seems to be theoretically possible to cancel one inverse critical current, keeping the other one nonzero.

IV. GENERALIZATION

A weakness of the Sec. II model is that it does not consider any spin-flip transitions within the device pillar. (Only within the large external contact region.). Jiang et al.^{18,19} for example, inserted a Ru capping layer on the top of the active part of a spin-valve and observed a reduction of the critical current as well as an enhancement of the absolute magnetoresistance. This experiment emphasizes the importance of spin-flip processes 20 in CPP-GMR devices. Let us now consider a more general case, where both electrodes are more complex electrical subcircuits (which allows us to introduce spin flip or synthetic antiferromagnets for example) as shown in Fig. 2. In order to include this case, we use Eqs. (1) and

(2) together with general Kirchoff's relations. (We still disallow spin flips within the spacer and its interfaces.):

$$
W_{R+} = aJ_{R+} + bJ_{R-},
$$

\n
$$
W_{R-} = cJ_{R+} + dJ_{R-},
$$

\n
$$
W_{L+} = V_1 + kJ_{L+} + lJ_{L-},
$$

\n
$$
W_{L-} = V_1 + mJ_{L+} + nJ_{L-}.
$$

Solution of the Valet-Fert diffusion equations in seperated *L* and *R* regions can provide the eight new parameters a, b, \ldots, n in these equations. The coefficients of Eqs. (8) and (9) are now given by:

$$
\alpha = -4 \cos \theta,
$$

\n
$$
\beta = 2[1 + \cos^2 \theta] + G \sin^2 \theta [-k + m + l - n],
$$

\n
$$
\gamma = G \sin^2 \theta [k - m + l - n]
$$

\n
$$
\delta = 2[a - c + d - b] \cos \theta,
$$

\n
$$
\zeta = 2 \sin^2 \theta / G + [-k + m + l - n][1 + \cos^2 \theta],
$$

\n
$$
\eta = [1 + \cos^2 \theta][k - m + l - n] - 2[a - c - d + b] \cos \theta.
$$

In this framework, the general expression of the absolute magnetoresistance is:

$$
\Delta R = \frac{\left[k-n\right]\left[a-d\right] + \left[l-m\right]\left[c-b\right]}{k-l-m+n-a+b+c-d}.\tag{30}
$$

We note that k , l , m , n are generally negative for positive spin asymmetry in the right electrode. The expression of the critical current when the magnetizations are initially parallel is now:

$$
J_{\uparrow\uparrow} = \frac{4\alpha_G M_s d_R e H_{eff}}{\hbar} \frac{k - l - m + n - a + b + c - d}{-k - l + m + n + a + b - c - d + G[(b - d)(k - m) + (a - c)(n - l)]}
$$
(31)

and when the magnetizations are antiparallel:

$$
J_{\uparrow\downarrow} = \frac{4\alpha_G M_s d_R e H_{eff}}{\hbar} \frac{k - l - m + n - a + b + c - d}{k + l - m - n + a + b - c - d + G[(n - l)(b - d) + (a - c)(k - m)]}.
$$
(32)

Once again it is possible to find a linear relationship, as proposed in Sec. II, between the inverse critical current and the absolute CPP-GMR. Using Eq. (30), we express *l*, for example, as a function of ΔR and substitute it in Eq. (31). One finds a linear relationship between STT amplitude and CPP-GMR:

$$
1/J_{\uparrow\uparrow} = \mathbf{P}\Delta R + \mathbf{Q},\tag{33}
$$

where:

$$
\mathbf{P} = \frac{4\alpha_G M_s d_R e H_{eff}}{\hbar} \frac{[-2 + G(-d + c + b - a)][k - m + c - a]}{[b - c][2m - k - n + a - c + d - b] + [a - d][k - n]},
$$

$$
Q = \frac{4 \alpha_G M_s d_R e H_{eff}}{\hbar} \frac{[d+c-b-a][n-k+b-c+G[(a-c)(n-m)+(b-a)(k-m)]]}{[b-c][2m-k-n+a-c+d-b]+[a-d][k-n]}.
$$

A relation of the form (33) is obeyed whenever any one of the eight circuit parameters *a*,*b*,...,*n* varies without altering α_G , M_s , or d_R .

V. VALIDITY OF THIS MODEL

This more general model has been validated by Xiao *et al.*, ²⁰ using a different notation. They showed that the circuit theory, taking into account bulk scattering, fitted very well with their model based on the Boltzmann transport equation, provided that the thickness of the left ferromagnetic layer (which polarizes the current) is lower than the spin diffusion length. For Co for example, experimental values give a spin diffusion length of 38 nm, and usual MML used in experiments have often a 5 nm-thick pinned layer. In such a case, the authors give the following equivalence for the material parameters:

$$
[R_{R,L+} + R_{R,L-}]/2 = \overline{\rho}_N l_{sf}^N [1 - \exp(-t_N^{R,L} / l_{sf}^N)] + \overline{\rho}_F t_F^{R,L} + \overline{R}_I + \overline{R}_C,
$$
\n(34)

$$
[R_{R,L+} - R_{R,L-}]/2
$$

=
$$
\frac{\Delta \rho_F t_F^{R,L} + \Delta R_I}{\overline{\rho}_N l_{sf}^N [1 - \exp(-t_N^{R,L}/l_{sf}^N)] + \overline{\rho}_F t_F^{R,L} + \overline{R}_I + \overline{R}_C},
$$
(35)

where $\bar{\rho}_{N,F}$, l_{sf}^N and $t_{N,F}$ are, respectively, the bulk resistivity, the spin-diffusion length, and the thickness of the nonmagnetic lead (respectively, ferromagnetic electrode), $\bar{\rho} = [\rho_+$ $+ \rho$ |/2 is the average resistivity, $\Delta \rho = [\rho_+ - \rho_-]/2$ is the resistivity difference. R_I and R_C are the interface and contact resistances. Replacing the material parameters by their equivalent formulae (34) and (35), the authors find an excellent agreement with the Boltzmann-based description of spin torque in MML. Furthermore, when the ferromagnetic electrodes are thicker than their spin diffusion length, it remains possible to evaluate the equivalent values of the circuit parameter using first principles calculations. Nevertheless, evaluation of the circuit parameters using Valet and Fert theory gives accurate results for thin pinned layers.

VI. CONCLUSION

We have extended a previous theory of CPP magnetoresistance and current-driven torque.^{14,15} This theory is based on a circuit model utilizing voltage-current connection formulae (1) and (2) whose assumption of perfect majority-spin transparency generally applies to electron structures of ferromagnetic electrodes whose compositions lie on the negativeslope side of the Slater-Neél-Pauling curve. It leads to closed expressions for current and torques in a noncollinear metallic magnetic multilayer expressed in terms of the same transport parameters which determine the magnetoresistance when the moments are aligned. We demonstrate that this theory can readily take into account complications due to spin flip in the bulk and at interfaces other than those of the spacer.

One special result is a class of linear relationships between STT amplitude and CPP-GMR. Each such relationship is found when varying only one CPP parameter, keeping all the others unchanged. The same kind of relationship can be demonstrated for the inverse critical current for monodomain excitation. In particular, we find that the torque is not necessarily zero when CPP-GMR is zero, and inversely.

Finally, we emphasize that this model accords well with a more complicated theory based on the Boltzmann transport equation, and can be used in designing magnetoelectronic devices when one wants to enhance or reduce STT effects.

Note added. A recent paper by L. Berger²¹ also relating excitation threshold to magnetoresistance, has come to our attention. Our relations are some equivalent to his. One reason for any differences may arise from a difference in models of ferromagnetic electron structures. Berger assumes the s-d model which distinguishes sp conduction electrons from localized d electrons comprising the spontaneous magnetization.

Our present treatment applies to the model of majorityspin transparency at interfaces 14 which is founded on results of first-principles electron structure calculations. The electron structure of fcc Co and Ni, to which our results apply, have strong hybridization between sp and d electrons in minority-spin band.

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