Guided and quasiguided elastic waves in phononic crystal slabs

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Guided and quasiguided elastic waves in a glass plate coated on one side with a periodic monolayer of polymer spheres, immersed in water, are studied by means of accurate numerical calculations using the on-shell layer-multiple-scattering method. This system supports, in addition to the modes of the bare plate, almost dispersionless, slow modes which originate from the array of spheres. We calculate and analyze in detail the dispersion diagrams of the interacting modes of the composite slab, and provide a consistent interpretation of the underlying physics.

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I. INTRODUCTION

The propagation of surface elastic waves can be efficiently manipulated by periodic structuring on a length scale comparable to the corresponding wavelength. Surface elastic waves, and in particular Rayleigh waves, at a periodically corrugated surface of a semi-infinite isotropic elastic medium have been studied both theoretically[1] and experimentally.[2] More recently, surface modes of composite materials with elastic coefficients varying periodically in space on a macroscopic scale, so-called phononic crystals,[3] attracted also considerable interest.[4–12] However, these studies refer, almost exclusively, to semi-infinite phononic crystals having a two-dimensional (2D) periodicity, while elastic modes of periodically structured finite slabs received much less attention. Guided and quasiguided elastic waves in thin homogeneous slabs, so-called Lamb waves,[13] have been extensively studied for a long time for a variety of reasons, not least of which are technological applications in nondestructive testing and quantitative evaluation,[14,15] acoustic sensors,[16] etc. Introducing a periodic modulation in a thin slab, will cause a folding of the dispersion curves of the Lamb waves, accompanied by the opening of Bragg gaps at the Brillouin-zone boundaries. Moreover, additional modes may appear thus offering further possibilities for tailoring the propagation of slab elastic waves.

The aim of the present paper is to study guided and quasiguided elastic waves in thin slabs of phononic crystals. For this purpose we consider a composite slab consisting of a 2D periodic array of polymer spheres on a glass plate, immersed in water. This system supports, in addition to the modes of the bare plate, almost dispersionless, slow modes which originate from the array of spheres. We report a thorough analysis of all these guided and quasiguided modes, by means of first-principles calculations using the layer-multiple-scattering method.[17,18] This method is ideally suited for the system under consideration because it can treat efficiently, besides an infinite phononic crystal, also a slab of the crystal of finite thickness (the slab may consist of a number of layers, which can be either planes of spheres with the same 2D periodicity or homogeneous plates). The eigenfrequency and the lifetime of the above slab modes are deduced directly from the corresponding spectral density of states of the elastic field. The corresponding dispersion curves and their symmetry are analyzed in conjunction with relevant transmission spectra, for normal incidence as well as for incidence at an angle. Our analysis elucidates the complex spectra associated with these modes and provides a transparent picture of the underlying physics.

The paper is organized as follows. In Sec. II we summarize the essentials of our method of calculation, the emphasis being placed on aspects of it which are directly related to the calculations carried out in this work. Section III is devoted to the discussion and analysis of our results, and the last section concludes the paper.

II. METHOD OF CALCULATION

We consider, to begin with, a plane of nonoverlapping spheres at $z=0$: an array of spheres centered on the sites of a 2D lattice specified by $\mathbf{R}_n=n_1\mathbf{a}_1+n_2\mathbf{a}_2$, where $\mathbf{a}_1$ and $\mathbf{a}_2$ are primitive vectors in the xy plane and $n_1,n_2=0,\pm1,\pm2,\pm3,\ldots$. The corresponding 2D reciprocal lattice is obtained, in the usual manner, as follows: $\mathbf{g}=m_1\mathbf{b}_1+m_2\mathbf{b}_2$, with $m_1,m_2=0,\pm1,\pm2,\pm3,\ldots$ and $\mathbf{b}_1,\mathbf{b}_2$ defined by $\mathbf{b}_i\cdot\mathbf{a}_j=2\pi\delta_{ij}$, where $i,j=1,2$. We assume that an elastic plane wave (it can be longitudinal or transverse) of angular frequency ω is incident on the plane of spheres from the left ($z<0$). Due to the 2D periodicity of the structure under consideration, the component of the wave vector of the incident wave parallel to the plane of spheres, $\mathbf{q}_\parallel$, can always be written as $\mathbf{q}_\parallel=\mathbf{k}_\parallel+\mathbf{g}'$, where the reduced wave vector $\mathbf{k}_\parallel$ lies in the surface Brillouin zone (SBZ) and $\mathbf{g}'$ is an appropriate reciprocal vector of the given lattice. Therefore, the wave vector of the incident wave has the form $\mathbf{K}^+_{\mathbf{g}'\nu'}=\mathbf{k}_\parallel+\mathbf{g}'+[q^2_{\nu'}-(\mathbf{k}_\parallel+\mathbf{g}')^2]^{1/2}\hat{\mathbf{e}}_z$, where $\hat{\mathbf{e}}_z$ is the unit vector along the z axis, and ν' specifies the polarization of the wave: $q_{\nu'}=q_l=\omega/c_l$ for a longitudinal wave and $q_{\nu'}=q_t=\omega/c_t$ for a transverse wave. The displacement vector $\mathbf{u}_{\rm in}(\mathbf{r})$ corresponding to the incident plane wave, expressed with respect to an origin $\mathbf{A}_l$ on the left of the plane of spheres, has the form

$$\mathbf{u}_{\rm in}(\mathbf{r})=[u_{\rm in}]^+_{\mathbf{g}'i'}\exp[i\mathbf{K}^+_{\mathbf{g}'\nu'}\cdot(\mathbf{r}-\mathbf{A}_l)]\hat{\mathbf{e}}_{i'}. \tag{1}$$

For $\nu'=l$, $i'=1$ denotes the only nonzero component of the displacement vector, $\hat{\mathbf{e}}_1$ being the radial unit vector along the direction of $\mathbf{K}^+_{\mathbf{g}'l}$. For $\nu'=t$, $i'=2$ or 3 denotes the only nonzero component of the displacement vector, $\hat{\mathbf{e}}_2$ and $\hat{\mathbf{e}}_3$ being the polar and azimuthal unit vectors, respectively, which are perpendicular to $\mathbf{K}^{+}_{\mathbf{g}'t}$.

Since ω and $\mathbf{k}_{\parallel}$ are conserved quantities in the scattering process, the scattered by the plane of spheres field will consist of a series of plane waves with wave vectors

$$\mathbf{K}^{\pm}_{\mathbf{g}\nu} = \mathbf{k}_{\parallel} + \mathbf{g} \pm [q_\nu^2 - (\mathbf{k}_{\parallel} + \mathbf{g})^2]^{1/2}\hat{\mathbf{e}}_z, \tag{2}$$

for all reciprocal-lattice vectors $\mathbf{g}$ and polarizations along $\hat{\mathbf{e}}_1$, $\hat{\mathbf{e}}_2$, and $\hat{\mathbf{e}}_3$ (radial, polar, and azimuthal unit vectors, respectively, associated with every $\mathbf{K}^{s}_{\mathbf{g}\nu}$, $s=\pm$). When $(\mathbf{k}_{\parallel}+\mathbf{g})^2 > q_\nu^2$ the corresponding wave decays to the right for $s=+$ and to the left for $s=-$, and the corresponding unit vectors $\hat{\mathbf{e}}_i$ become complex. The transmitted (incident+scattered) wave, expressed with respect to an origin $\mathbf{A}_r$ on the right of the plane of spheres, has the form

$$\mathbf{u}^{+}_{\mathrm{tr}}(\mathbf{r}) = \sum_{\mathbf{g}i} Q^{\mathrm{I}}_{\mathbf{g}i;\mathbf{g}'i'}[u_{\mathrm{in}}]^{+}_{\mathbf{g}'i'} \exp[i\mathbf{K}^{+}_{\mathbf{g}\nu}\cdot(\mathbf{r}-\mathbf{A}_r)]\hat{\mathbf{e}}_i, \quad z>0, \tag{3}$$

and the reflected wave, expressed with respect to $\mathbf{A}_l$, has the form

$$\mathbf{u}^{-}_{\mathrm{rf}}(\mathbf{r}) = \sum_{\mathbf{g}i} Q^{\mathrm{III}}_{\mathbf{g}i;\mathbf{g}'i'}[u_{\mathrm{in}}]^{+}_{\mathbf{g}'i'} \exp[i\mathbf{K}^{-}_{\mathbf{g}\nu}\cdot(\mathbf{r}-\mathbf{A}_l)]\hat{\mathbf{e}}_i, \quad z<0. \tag{4}$$

The above equations define the elements of the transmission ($\mathbf{Q}^{\mathrm{I}}$) and reflection ($\mathbf{Q}^{\mathrm{III}}$) matrices for a plane wave incident on the plane of spheres from the left. They depend on the scattering properties of the individual scatterer, on the geometry of the plane, and of course on the frequency, the angle of incidence, and the polarization of the incident wave. Similarly we can define the transmission matrix elements $Q^{\mathrm{IV}}_{\mathbf{g}i;\mathbf{g}'i'}$, and the reflection matrix elements $Q^{\mathrm{II}}_{\mathbf{g}i;\mathbf{g}'i'}$ for a plane wave incident on the plane of spheres from the right. One can use the same notation (the calculation is of course much easier) to describe the scattering properties of a homogeneous plate. In this case the Q matrices are diagonal in $\mathbf{g}$ because of the translation invariance parallel to the xy plane. Explicit expressions for the Q matrices in the different cases can be found elsewhere.[18]

We obtain the transmission and reflection matrices for a pair of consecutive layers, say 1 on the left and 2 on the right, to be denoted by $\mathbf{Q}$, by combining the matrices $\mathbf{Q}(1)$ and $\mathbf{Q}(2)$ of the two layers. Taking $\mathbf{A}_r(1)$, the origin on the right of the left layer, at the same point as $\mathbf{A}_l(2)$, the origin on the left of the right layer, in the host region, one can easily show that

$$\mathbf{Q}^{\mathrm{I}} = \mathbf{Q}^{\mathrm{I}}(2)[\mathbf{I} - \mathbf{Q}^{\mathrm{II}}(1)\mathbf{Q}^{\mathrm{III}}(2)]^{-1}\mathbf{Q}^{\mathrm{I}}(1),$$

$$\mathbf{Q}^{\mathrm{II}} = \mathbf{Q}^{\mathrm{II}}(2) + \mathbf{Q}^{\mathrm{I}}(2)\mathbf{Q}^{\mathrm{II}}(1)[\mathbf{I} - \mathbf{Q}^{\mathrm{III}}(2)\mathbf{Q}^{\mathrm{II}}(1)]^{-1}\mathbf{Q}^{\mathrm{IV}}(2),$$

$$\mathbf{Q}^{\mathrm{III}} = \mathbf{Q}^{\mathrm{III}}(1) + \mathbf{Q}^{\mathrm{IV}}(1)\mathbf{Q}^{\mathrm{III}}(2)[\mathbf{I} - \mathbf{Q}^{\mathrm{II}}(1)\mathbf{Q}^{\mathrm{III}}(2)]^{-1}\mathbf{Q}^{\mathrm{I}}(1),$$

$$\mathbf{Q}^{\mathrm{IV}} = \mathbf{Q}^{\mathrm{IV}}(1)[\mathbf{I} - \mathbf{Q}^{\mathrm{III}}(2)\mathbf{Q}^{\mathrm{II}}(1)]^{-1}\mathbf{Q}^{\mathrm{IV}}(2). \tag{5}$$

All Q matrices refer of course to the same ω and $\mathbf{k}_{\parallel}$, and $\mathbf{I}$ is the unit matrix. It is obvious that by the same procedure we can obtain the Q matrices for a slab consisting of any finite number N_{L} of layers. The transmittance $\mathcal{T}(\omega,\mathbf{k}_{\parallel}+\mathbf{g}',i')$ and the reflectance $\mathcal{R}(\omega,\mathbf{k}_{\parallel}+\mathbf{g}',i')$ of the slab are defined as the ratio of the transmitted, respectively the reflected, energy flux to the energy flux associated with the incident wave. For incidence from the left we obtain

$$\mathcal{T} = \frac{\sum_{\mathbf{g}i} c_\nu^2 |Q^{\mathrm{I}}_{\mathbf{g}i;\mathbf{g}'i'}|^2 K^{+}_{\mathbf{g}\nu z}}{c_{\nu'}^2 K^{+}_{\mathbf{g}'\nu' z}} \tag{6}$$

and

$$\mathcal{R} = \frac{\sum_{\mathbf{g}i} c_\nu^2 |Q^{\mathrm{III}}_{\mathbf{g}i;\mathbf{g}'i'}|^2 K^{+}_{\mathbf{g}\nu z}}{c_{\nu'}^2 K^{+}_{\mathbf{g}'\nu' z}}. \tag{7}$$

We remember that only propagating beams (those with $K^{+}_{\mathbf{g}\nu z}$ real) enter the numerators of the above equations. Finally we note that if absorption is present it can be calculated from the requirement of energy conservation: $\mathcal{A}=1-\mathcal{T}-\mathcal{R}$.

The difference in the number of states up to a given frequency ω, between the slab embedded in a homogeneous medium and that of the homogeneous medium extending over all space is given by[19]

$$\Delta N(\omega) = \frac{N}{A}\int\int_{\mathrm{SBZ}} d^2k_{\parallel}\Delta N(\omega,\mathbf{k}_{\parallel}), \tag{8}$$

where N is the number of surface unit cells of the plane of spheres, A is the area of the SBZ, and

$$\Delta N(\omega,\mathbf{k}_{\parallel}) = \frac{1}{2\pi}\mathrm{Im}\ln\det\mathbf{S}, \tag{9}$$

with the elements of the S matrix in the representation $\{s\mathbf{g}i\}$ given by

$$S^{++}_{\mathbf{g}i;\mathbf{g}'i'} = \exp\{i[\mathbf{K}^{+}_{\mathbf{g}'}\cdot\mathbf{A}_l(1) - \mathbf{K}^{+}_{\mathbf{g}}\cdot\mathbf{A}_r(N_{\mathrm{L}})]\}Q^{\mathrm{I}}_{\mathbf{g}i;\mathbf{g}'i'},$$

$$S^{+-}_{\mathbf{g}i;\mathbf{g}'i'} = \exp\{i[\mathbf{K}^{-}_{\mathbf{g}'}\cdot\mathbf{A}_r(N_{\mathrm{L}}) - \mathbf{K}^{+}_{\mathbf{g}}\cdot\mathbf{A}_r(N_{\mathrm{L}})]\}Q^{\mathrm{II}}_{\mathbf{g}i;\mathbf{g}'i'},$$

$$S^{-+}_{\mathbf{g}i;\mathbf{g}'i'} = \exp\{i[\mathbf{K}^{+}_{\mathbf{g}'}\cdot\mathbf{A}_l(1) - \mathbf{K}^{-}_{\mathbf{g}}\cdot\mathbf{A}_l(1)]\}Q^{\mathrm{III}}_{\mathbf{g}i;\mathbf{g}'i'},$$

$$S^{--}_{\mathbf{g}i;\mathbf{g}'i'} = \exp\{i[\mathbf{K}^{-}_{\mathbf{g}'}\cdot\mathbf{A}_r(N_{\mathrm{L}}) - \mathbf{K}^{-}_{\mathbf{g}}\cdot\mathbf{A}_l(1)]\}Q^{\mathrm{IV}}_{\mathbf{g}i;\mathbf{g}'i'} \tag{10}$$

for the given ω and $\mathbf{k}_{\parallel}$. The phase factors in Eq. (10) arise from the need to refer all waves to a common origin. We note that the size of the S matrix in Eq. (9) is restricted to those reciprocal-lattice vectors which correspond to propagating beams, and that the resulting $\Delta N(\omega,\mathbf{k}_{\parallel})$ does not include possible bound states of the system. The eigenfrequencies of the bound states can be obtained, separately, from the condition for the existence of a wave field localized within the slab. Dividing the slab into a left and a right part, this condition leads to the following secular equation:

$$\det[\mathbf{I} - \mathbf{Q}^{\mathrm{II}}_{\mathrm{left}}\mathbf{Q}^{\mathrm{III}}_{\mathrm{right}}] = 0. \tag{11}$$

The S matrix is defined, in general terms, as the matrix which transforms the incoming wave field into the outgoing wave field, and it is a unitary matrix because of flux conservation. The causality condition implies that the eigenvalues of the S matrix are analytic functions in the upper complex frequency half-plane but they may have poles in the lower half-plane at $\omega_i - i\gamma_i$, $\gamma_i \geqslant 0$, which correspond to zeros at $\omega_i + i\gamma_i$; ω_i is the eigenfrequency while γ_i denotes the inverse of the lifetime of the respective mode. Among all possible solutions, those with $\gamma_i/\omega_i \ll 1$ are of particular physical interest. Considering such a simple pole at $\omega_1 - i\gamma_1$, separated from the other poles, in the vicinity of this point, on the real axis, the corresponding eigenvalue of the S matrix has the form: $\exp(2i\alpha)(\omega-\omega_1-i\gamma_1)/(\omega-\omega_1+i\gamma_1)$. Assuming that the phase angle α as well as the other eigenvalues do not vary considerably with frequency in the vicinity of the pole, we obtain from Eq. (9),

$$\Delta n(\omega,\mathbf{k}_\parallel) = \frac{\partial \Delta N(\omega,\mathbf{k}_\parallel)}{\partial \omega} \simeq \frac{1}{\pi}\frac{\gamma_1}{(\omega-\omega_1)^2+\gamma_1^2}, \tag{12}$$

i.e., the change in the density of states, $\Delta n(\omega,\mathbf{k}_\parallel)$, has the form of a Lorentzian resonance centered at ω_1. Its width is determined by γ_1 and its integral from $-\infty$ to ∞ equals unity. Since $\gamma_1/\omega_1 \ll 1$, this resonant mode resembles a bound state: it has a long (though not infinite) lifetime and the field intensity associated with it is mostly concentrated within the slab (though it leaks, to some minor degree, in the host region). Such states are referred to as *virtual bound states*.

The virtual bound states of a phononic-crystal slab manifest themselves as resonance structures also in the corresponding transmission spectrum. Let us restrict ourselves, for simplicity, to the case of a fluid host medium and to frequencies below the first Bragg diffraction threshold, so that the scattered wave field consists of only one longitudinal propagating beam. Assuming further that the slab has a parallel plane of mirror symmetry, the S matrix has the form

$$\mathbf{S} = \begin{pmatrix} t & r \\ r & t \end{pmatrix}, \tag{13}$$

where t and r, the corresponding transmission and reflection matrices with the proper phase factors [see Eq. (10)], are reduced to scalar quantities. It is straightforward to show that the eigenvalues of the S matrix are $t+r$ and $t-r$ and, since $\mathbf{S}$ is unitary, these can be written as

$$t + r = \exp(2i\delta_+), \quad t - r = \exp(2i\delta_-), \tag{14}$$

where δ_+ and δ_-, the so-called scattering phase shifts, are real functions of frequency. Using Eqs. (14), the transmittance of the slab can be expressed in terms of the scattering phase shifts as follows:

$$\mathcal{T} \equiv |t|^2 = \cos^2(\delta_+ - \delta_-). \tag{15}$$

If in a relatively short range of frequency there is a number, $i=1,2,\ldots$, of virtual bound states, introducing δ_i, the corresponding resonant parts of the phase shifts, through

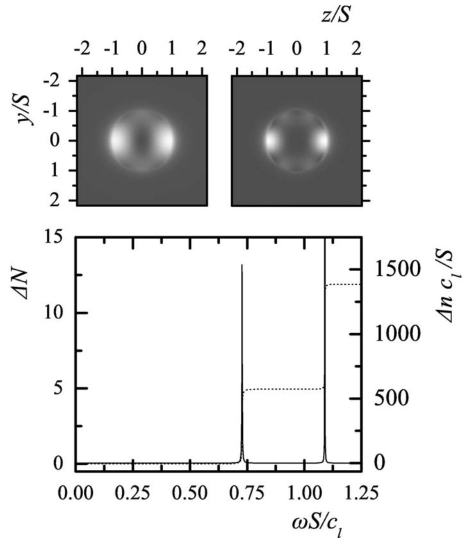

FIG. 1. Change in the number of states ΔN (dotted line) and in the density of states Δn (solid line) induced by a polyethylene sphere in water. The field intensity distribution associated with the two resonant states is shown at the top.

$$\sin \delta_i = \frac{\gamma_i}{[(\omega-\omega_i)^2+\gamma_i^2]^{1/2}},$$

$$\cos \delta_i = -\frac{\omega-\omega_i}{[(\omega-\omega_i)^2+\gamma_i^2]^{1/2}}, \quad i=1,2,\ldots, \tag{16}$$

and a roughly constant phase ϕ which contains the contribution of the nonresonant parts of the phase shifts, the transmittance of the slab takes the form

$$\mathcal{T} \simeq \cos^2(\pm\delta_1 \pm \delta_2 \pm \cdots - \phi). \tag{17}$$

The + or − sign of δ_i in Eq. (17) corresponds to a pole of $t+r$ or of $t-r$, respectively.

III. RESULTS AND DISCUSSION

We consider, to begin with, a monolayer of a phononic crystal: a 2D periodic array of spheres. This may support guided and quasiguided modes which originate from resonant states of the individual spheres. Polymer spheres in a liquid host exhibit, in general, such resonant modes and are good candidates for our purposes. The displacement field associated with these modes has a mixed longitudinal-transverse ($L-N$) character in the sphere and is purely longitudinal in the host medium. Let us assume a square array, with lattice constant a_0, of polyethylene spheres ($\rho_p = 900\ \mathrm{kg/m^3}$, $c_{lp}=1950$ m/s, and $c_{tp}=540$ m/s), with radius $S=0.23a_0$, immersed in water ($\rho=1000\ \mathrm{kg/m^3}$ and $c_l = 1490$ m/s).

In Fig. 1 we show the change $\Delta N(\omega)$ in the number of states and the change $\Delta n(\omega)$ in the density of states of the elastic field induced by a single polyethylene sphere in water. These quantities have been calculated from the general equation[19] $\Delta N(\omega)=(1/\pi)\mathrm{Im\,Tr\,ln}[\mathbf{I}+\mathbf{T}]$, using the appropriate T matrix.[18] The density of states in the frequency region under consideration is characterized by two sharp peaks, at $\omega S/c_l=0.727$ and 1.090 or, for the given $S=0.23a_0$, at $\omega a_0/c_l=3.163$ and 4.737. These peaks are associated with a $\ell=2$ and a $\ell=3$ resonant state, respectively. We see that $\Delta N(\omega)$ increases by nearly five states over the $\ell=2$ resonance, and by nearly seven states over the $\ell=3$ resonance, as expected from the $(2\ell+1)$-fold degeneracy of these modes. The sharpness of the peaks implies a long lifetime for these states and a high amplitude of the associated displacement field in the sphere, as shown at the top of Fig. 1.

Assembling polyethylene spheres on a (square) lattice, we expect that their resonant states of given ℓ, interacting weakly between them, will form $2\ell+1$ relatively narrow bands, $\omega_\nu(\mathbf{k}_\parallel)$, $\nu=1,2,\ldots,2\ell+1$, about the corresponding eigenfrequency of the single sphere. At the symmetry points and along the symmetry lines of the SBZ, these bands can be classified in terms of the irreducible representations of the point group of the corresponding wave vector. For example, for $\mathbf{k}_\parallel=(0,0)$ ($\bar{\Gamma}$ point), the modes have the symmetry of the irreducible representations of the C_{4v} group: Δ_1, Δ_2, $\Delta_{1'}$, $\Delta_{2'}$, Δ_5.[20] The Δ_1, Δ_2, $\Delta_{1'}$, $\Delta_{2'}$ modes are nondegenerate and Δ_5 are doubly degenerate. According to a group-theory analysis,[21] a mixed longitudinal-transverse $(L-N)$ quadrupole state of the spheres gives at the $\bar{\Gamma}$ point a Δ_1, a Δ_2, a $\Delta_{2'}$, and a Δ_5 mode. We note that a longitudinal plane acoustic wave propagating in the water host, normal to the plane of spheres ($\mathbf{q}_\parallel=\mathbf{0}$), has the Δ_1 symmetry and, therefore, only modes of the plane of spheres with the same symmetry can be excited by an externally incident wave. The modes of different symmetry are inactive; they are guided modes and decay exponentially to zero away from the plane of spheres on either side of it. The inactive modes show up as delta functions and the active modes, of Δ_1 symmetry, show up as Lorentzian resonances in the corresponding density of states. The integral of each such Lorentzian equals one, while its center and width determine the eigenfrequency and inverse lifetime, respectively, of the respective quasiguided mode. As can be seen from Fig. 2(a), in agreement with the above discussion, the quadrupole resonant state of the single sphere gives, for $\mathbf{k}_\parallel=\mathbf{0}$, guided modes at $\omega a_0/c_l=3.147(\Delta_2)$, $3.177(\Delta_5)$, $3.183(\Delta_{2'})$, and a Δ_1 quasiguided mode at $\omega_1 a_0/c_l=3.152$ with an inverse lifetime $\gamma_1 a_0/c_l=0.013\,94$. In Fig. 2(b) we show the transmittance of the plane of spheres, at normal incidence, in the frequency region about the quadrupole resonant state of the single sphere. It can be seen that the transmission spectrum is characterized by a sharp dip which originates from the excitation of the corresponding quasiguided mode and is very well described by the function [see Eqs. (16) and (17)] $\mathcal{T}=\cos^2(\delta_1-\phi)$ with a single adjustable parameter $\phi=1.23°$.

For $\mathbf{k}_\parallel\neq(0,0)$ the symmetry is lower. For example, along the $\overline{\Gamma X}$ direction: $\mathbf{k}_\parallel=(k_x,0)$, $0<k_x<\pi/a_0$, the point group of the wave vector is the C_{1h} group. This group has two one-dimensional irreducible representations, with basis functions which are even (Q_1) and odd (Q_2) upon reflection with respect to the xz plane. Group theory tells us that, along $\overline{\Gamma X}$, each of the Δ_1 and Δ_2 modes develops into a Q_1 band, the

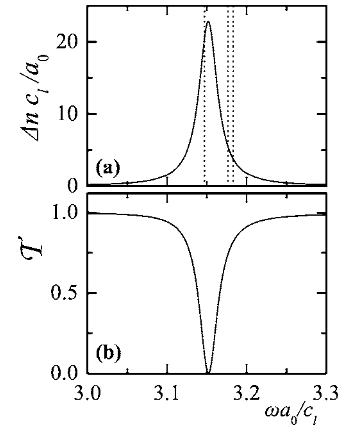

FIG. 2. A square array, with lattice constant a_0, of polyethylene spheres, with radius $S=0.23a_0$, in water. (a) Change of the density of states of the system with respect to water, for $\mathbf{k}_\parallel=\mathbf{0}$. The dotted vertical lines indicate the position of the bound states. (b) Transmittance at normal incidence.

$\Delta_{2'}$ mode develops into a Q_2 band, and the doubly degenerate Δ_5 mode splits into one Q_1 and one Q_2 band.[20] Since a longitudinal elastic plane wave incident on the plane of spheres with $\mathbf{q}_\parallel=(q_x,0)$ has the Q_1 symmetry, it can excite only Q_1 modes. Therefore, for a given $\mathbf{k}_\parallel$, along the $\overline{\Gamma X}$ direction, we expect two guided (Q_2) and three quasiguided (Q_1) modes about the quadrupole resonance of the single sphere. This is indeed shown in Fig. 3(a) for $\mathbf{k}_\parallel=(0.24\pi/a_0,0)$: we obtain two guided modes at $\omega a_0/c_l=3.177$ and 3.182; and three quasiguided modes at $\omega_1 a_0/c_l=3.149$, $\omega_2 a_0/c_l=3.153$, and $\omega_3 a_0/c_l=3.174$ with inverse lifetimes $\gamma_1 a_0/c_l=0.000\,12$, $\gamma_2 a_0/c_l=0.011\,89$, and $\gamma_3 a_0/c_l=0.002\,34$, respectively. The corresponding transmission spectrum exhibits various types of resonance structures at $\omega_1,\omega_2,\omega_3$, as can be seen in Fig. 3(b), and is very accurately reproduced by the function [see Eqs. (16) and (17)]

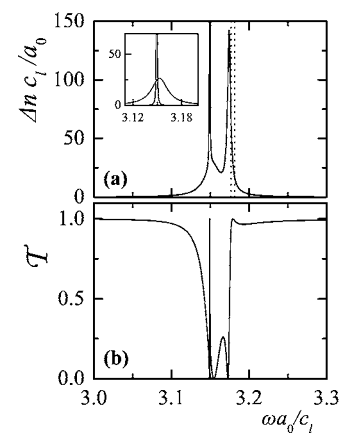

FIG. 3. A square array, with lattice constant a_0, of polyethylene spheres, with radius $S=0.23a_0$, in water. (a) Change of the density of states of the system with respect to water, for $\mathbf{k}_\parallel=(0.24\pi/a_0,0)$. The dotted vertical lines indicate the position of the bound states. In the frequency region about $\omega a_0/c_l=3.15$, Δn is analyzed in the two Lorentzian curves of unit area shown in the inset. (b) Transmittance at off-normal incidence for $\mathbf{q}_\parallel=(0.24\pi/a_0,0)$.

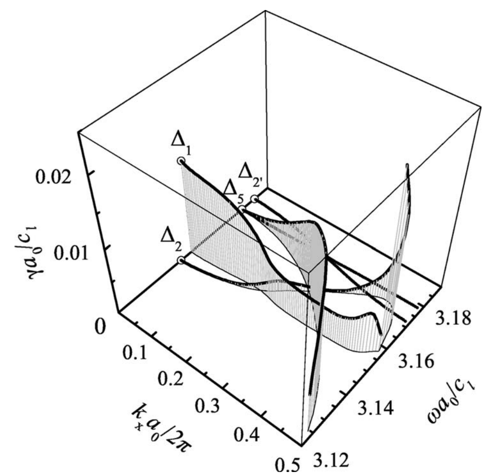

FIG. 4. Dispersion curves of the guided and quasiguided modes of a square array, with lattice constant a_0, of polyethylene spheres, with radius $S=0.23a_0$, in water, along the $\overline{\Gamma X}$ direction. There are two bands of guided modes with real eigenfrequencies which have the Q_2 symmetry, and three bands of quasiguided modes with complex eigenfrequencies which have the Q_1 symmetry.

$\mathcal{T}=\cos^2(\delta_1+\delta_2-\delta_3-\phi)$ with a single fitting parameter $\phi=1.35°$. It is interesting to note that a resonance structure in the transmission spectrum, which originates from a corresponding quasiguided mode, can be a simple peak or dip, but also an abrupt variation between a minimum and a maximum (asymmetric Fano resonance[22]) such as the sharp structure in Fig. 3(b) at ω_1, depending on the values of the phase angles entering in Eq. (17).

By varying $\mathbf{k}_{\parallel}$ along $\overline{\Gamma X}$, we deduce the corresponding dispersion curves of the guided and quasiguided modes of the plane of spheres. As shown in Fig. 4, we obtain five narrow bands about the eigenfrequency of the quadrupole resonant state of the single sphere. It can be seen that states of the same symmetry interact and repel each other. It is worth remembering that the quasiguided modes are virtual bound states and not true bound states of the system. In other words, they have a long but finite lifetime, which corresponds to an imaginary part of the eigenfrequency typically a few orders of magnitude smaller than the real part, as shown in Fig. 4.

We now place the square array of polyethylene spheres on top of a homogeneous plate of glass ($\rho_g=2500$ kg/m^3, $c_{lg}=5700$ m/s, and $c_{tg}=3400$ m/s), of thickness $d=a_0$, and the whole system is immersed in water. Figure 5 shows the corresponding change in the density of states with respect to the water host, for $\mathbf{k}_{\parallel}=(0.24\pi/a_0,0)$, as well as the transmission coefficient of a (longitudinal) elastic plane wave incident with $\mathbf{q}_{\parallel}=(0.24\pi/a_0,0)$. It can be seen that, compared with the corresponding case of the unsupported plane of spheres shown in Fig. 3, we now have more resonance structures, and this implies the existence of additional modes. More precisely, we find the modes of the plane of spheres somewhat shifted to lower frequencies: we obtain two guided modes at $\omega a_0/c_l=2.994$ and 3.142; and three quasiguided modes at $\omega_1 a_0/c_l=2.879$, $\omega_2 a_0/c_l=2.990$, and $\omega_3 a_0/c_l=3.095$ with inverse lifetimes $\gamma_1 a_0/c_l=0.017\,20$, $\gamma_2 a_0/c_l=0.000\,69$, and $\gamma_3 a_0/c_l=0.000\,06$, respectively. In addition, we obtain one quasiguided mode at $\omega_4 a_0/c_l=2.756$ with $\gamma_4 a_0/c_l=0.005\,08$ as well as three modes, two guided and one quasiguided, at low frequencies (not shown in Fig. 5).

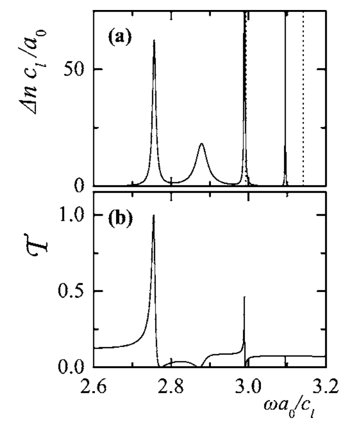

FIG. 5. A square array, with lattice constant a_0, of polyethylene spheres, with radius $S=0.23a_0$, on top of a glass plate, with thickness $d=a_0$, in water. (a) Change of the density of states of the system with respect to water, for $\mathbf{k}_{\parallel}=(0.24\pi/a_0,0)$. The dotted vertical lines indicate the position of the bound states. (b) Transmittance at off-normal incidence for $\mathbf{q}_{\parallel}=(0.24\pi/a_0,0)$.

In order to understand the physical origin of the additional modes, let us consider the glass plate alone. A bare glass plate in water supports, besides the nonresonant scattering states, also guided and quasiguided modes. These modes can be of transverse s type (the displacement field oscillates parallel to the plate) or of mixed longitudinal-transverse p type (the displacement field oscillates in a plane normal to the plate). The s modes cannot be matched continuously with an acoustic wave in the water region and represent guided waves confined within the plate (Love modes).[13] There is one band of Love modes starting from $\omega=0$ as $q_{\parallel}\rightarrow 0$. On the other hand, the mixed longitudinal-transverse p modes can be excited by an externally incident (longitudinal) wave, provided they are inside the propagation cone in water defined by $\omega=c_l q_{\parallel}$, and represent quasiguided waves. These modes can be classified as antisymmetric (A) and symmetric (S) upon reflection with respect to a parallel plane of mirror symmetry at the middle of the plate. There are four bands of such modes starting from $\omega=0$ as $q_{\parallel}\rightarrow 0$. These correspond to the water-borne A and S Scholte-Stoneley waves, and to the lowest A and S Lamb-type waves.[13] The Scholte-Stoneley modes lie outside the propagation cones in both glass and water (the dispersion curve of the S Scholte-Stoneley mode coincides with the line $\omega=c_l q_{\parallel}$); they represent guided waves which propagate along the water-glass interfaces and decrease exponentially away from each interface, on either side of it. Obviously, these modes cannot be matched continuously with a propagating wave in the water region and, therefore, they cannot be excited by an externally incident wave: energy and momentum cannot be conserved simultaneously. On the contrary, the Lamb-type modes lie inside the propagation cone in water; they represent quasiguided waves which leak outside the plate and can be excited by an externally incident wave.

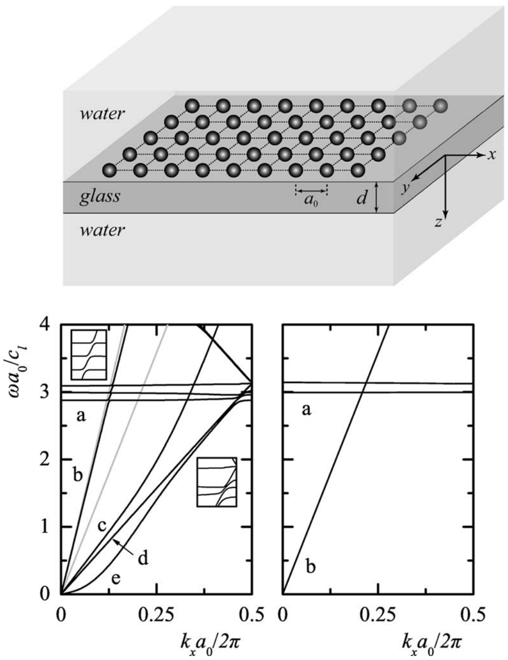

FIG. 6. Dispersion curves of the guided and quasiguided modes of a square array, with lattice constant a_0, of polyethylene spheres, with radius $S=0.23a_0$, on top of a glass plate, with thickness $d=a_0$, in water (see top diagram), along the $\overline{\Gamma X}$ direction. The left-hand panel shows the modes of Q_1 symmetry: modes of the plane of spheres (a), symmetric and antisymmetric Lamb-type modes (b), (c), and symmetric and antisymmetric Scholte-Stoneley modes (d), (e). In grey we show the dispersion lines $\omega=c_{tg}q_\parallel$ and $\omega=c_{lg}q_\parallel$. The insets show a detail view of level repulsion about the crossing points. The right-hand panel shows the modes of Q_2 symmetry: modes of the plane of spheres (a) and Love modes (b).

When we put the 2D array of polyethylene spheres on the plate, the dispersion curves associated with the plate modes are folded within the SBZ of the given lattice and (small) Bragg gaps open up at the Brillouin-zone boundaries. This is shown in Fig. 6 for $\mathbf{k}_\parallel$ along the $\overline{\Gamma X}$ direction, in which case the transverse s and the longitudinal-transverse p plate modes belong to the irreducible representations Q_2 and Q_1 of the C_{1h} group, respectively. It can be seen that the folded bands of the Scholte-Stoneley modes, above $\omega a_0/c_l=3.14$, lie inside the propagation cone in water and, therefore, represent quasiguided waves. These modes can be excited by an externally incident wave through an Umklapp process and give rise to resonance structures in the corresponding transmission spectrum. Moreover, the plate modes interact with those of the plane of spheres, of the same symmetry, and the corresponding bands repel each other if they are in close proximity in the complex frequency plane (see Fig. 6). We note that the A Lamb-type modes, which have a very short lifetime (the corresponding eigenfrequencies have a large imaginary part), essentially do not interact with the other Q_1 modes. The interaction between bands of Q_2 symmetry is also not observed, because there is no spatial overlap between the Love modes and the corresponding modes of the plane of spheres.

IV. CONCLUSION

In summary, we presented a theoretical study of guided and quasiguided elastic waves in a glass plate coated on one side with a periodic monolayer of polymer spheres, immersed in water, using the layer-multiple-scattering method. We found that this system supports, in addition to the modes of the bare plate, almost dispersionless, slow modes which originate from the array of spheres. Moreover, the periodic monolayer causes a folding of the bands within the SBZ and, in this manner, guided plate modes become quasiguided. Finally, the interaction between modes of the same symmetry induces a repulsion of the corresponding bands if they are in close proximity to each other in the complex frequency plane. The eigenfrequency and the lifetime of the guided and quasiguided modes of the composite slab are calculated directly from the corresponding spectral density of states of the elastic field. The associated dispersion curves and their symmetry are analyzed in conjunction with relevant transmission spectra, and we developed a simple analytic model which explains the complex line shapes of the observed transmission resonances. Our results demonstrate the efficiency of the layer-multiple-scattering method, which constitutes a powerful tool also for the analysis of the elastic modes of composite phononic-crystal slabs, even if there is large elastic mismatch and/or dissipative losses in the constituent materials. We note that the method can be extended to phononic crystals of nonspherical scatterers: the properties of the individual scatterer enter only through the corresponding T matrix, and this matrix for nonspherical scatterers can be evaluated efficiently by, e.g., a surface-integral method. Such an extension of the layer-multiple-scattering method in the corresponding problem of photonic crystals has been reported recently.[23] However, in systems similar to that studied in the present work, substitution of the spheres by cylinders, spheroids, or scatterers of other shape would not lead to qualitatively different results although, in some cases, degeneracies that arise from the high symmetry of the sphere may be removed. We expect that the present work will open new possibilities in the design of Lamb-wave delay lines, filters and resonators, based on phononic-crystal slabs, which might be useful for acoustoelastic devices,[24] telecommunication applications,[25,26] etc.

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