Effect of gap suppression by superfluid current on the nonlinear microwave response of *d*-wave superconductors

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Recently several works have focused on the intrinsic nonlinear current in passive microwave filters as a tool for identifying *d*-wave order parameter symmetry in the high-T_c cuprates. Evidence has been found for *d*-wave pairing in YBCO and further work has ensued. Most of the theoretical work has been limited to low temperatures because it has not included the effect of the superfluid current on the energy gap. We find that this effect leads to important corrections above $T \sim 0.2T_c$, while leaving the $1/T$ low-temperature behavior intact. A twofold increase in the nonlinear coefficient at temperatures of the order of $\sim 0.75T_c$ is found, and as $T \rightarrow T_c$ the nonlinearity comes entirely from the effects of the superfluid current on the gap. Impurity scattering has been included and, in addition, signatures for the case of $d+s$ wave are presented.

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I. INTRODUCTION

Interest in generating novel methods for probing the order parameter symmetry in superconductors has driven the development of new techniques and experimental configurations, such as angle-resolved electron tunneling[1] or angle-resolved magnetic field dependence of specific heat,[2] which can provide unambiguous signatures of order parameter nodes and their location. One such proposal is associated with the measurement of the intrinsic nonlinear current in passive microwave filters. Such devices give rise to third-order intermodulation effects which, while detrimental for practical applications in superconducting communication filter technology, make these devices useful for examining issues of order parameter symmetry. Excellent progress in the field of high-temperature superconductivity has been made in this area.

Initially, Yip and Sauls[3,4] proposed the examination of the nonlinear Meissner effect (NLME) for evidence of *d*-wave gap symmetry in the cuprates. Their predictions were not confirmed by experiments at the time;[5] however, Dahm and Scalapino[6,7] (DS) proposed to examine a related quantity: the intermodulation distortion (IMD) which arises from the nonlinear inductance resulting from a quadratic dependence of the penetration depth on the superfluid current. In the experiment, when two input signals at frequencies ω_1 and ω_2 lie within the width of the resonance for a microstrip cavity, the nonlinearity of the device generates a signal at what is called the intermodulation frequency $2\omega_1-\omega_2$. The power of this signal can be related to the intrinsic nonlinear current.[7] In the DS work, a signature of the *d*-wave gap is found in an upturn in the temperature dependence of the nonlinear coefficient at low temperatures, in contrast to exponential decay for an *s*-wave gap. Such evidence of an upturn has been found in YBCO films by several groups[8–10] with excellent agreement with the DS theory. This success has led to further calculations to examine the issue of nonlocal effects for this quantity[11] (thought to be the possible reason[12] for not seeing the NLME proposed by Yip and Sauls) and the *d*-wave signature remains robust. More recently, measurements of the nonlinear current have been made near T_c on a range of different films and the data has been analyzed in terms of the DS approach with impurity scattering being used to explain variations between the films.[13] However, in this work and others[14,15] which extrapolate the DS theory to high temperature, the effect of the superfluid current on the gap has been neglected as an approximation. We show here that inclusion of this effect has significant impact on the nonlinear coefficient for $T \gtrsim 0.2T_c$, requiring a reanalysis of previous results. Indeed, when the current dependence of the gap is considered, the nonlinear coefficient shows a twofold increase at $T \sim 0.75T_c$ over the value calculated using the approximation of a current-independent gap.

Finally, we note that other theoretical works have examined the intrinsic nonlinear current for two-band superconductors and MgB_2 (Refs. 15 and 16) and for one-band *s*-wave superconductors.[4,16] Effects of the superfluid on the gap have been included in Refs. 4 and 16.

Our paper is structured as follows: in the next section, we summarize our theoretical approach which allows for both the inclusion of the current dependence in the gap and impurity scattering (from unitary to Born limit). Strong electron-boson coupling effects are also available in this formalism. In Sec. III, we discuss the results illustrating the corrections to the simplified theory at finite temperature and we revisit the experimental situation, including the issue of impurities. We end by providing predictions for an admixture of *d*- and *s*-wave symmetry as has been recently suggested by angle-resolved electron tunneling experiments on YBCO.[1] We form our conclusions briefly in Sec. IV.

II. THEORY

In this work, we evaluate the full current from the standard expression given for the imaginary-axis Matsubara rep-

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resentation and modified for a d-wave order parameter $\Delta(\theta)=\Delta\cos(2\theta)$ in two dimensions:[4,16–19]

$$j_s(q_s,\alpha)=\frac{2en}{mv_F}\pi T\sum_{n=-\infty}^{+\infty}\int_0^{2\pi}\frac{d\theta}{2\pi}\times\frac{i[\tilde{\omega}_n-is\cos(\theta-\alpha)]\cos(\theta-\alpha)}{\sqrt{[\tilde{\omega}_n-is\cos(\theta-\alpha)]^2+\tilde{\Delta}_n^2\cos^2(2\theta)}}. \quad (1)$$

This expression contains both the condensate current and the quasiparticle current due to excitations.[20] The angle α measures the direction of the current with respect to the order parameter antinode, with $\alpha=0$ indicating the current in the antinodal direction and $\alpha=\pi/4$ for the nodal direction. The other notation is standard with e the electric charge, m the electron mass, T the temperature, n the electron density, and v_F the Fermi velocity. The superfluid momentum q_s enters through $s=v_Fq_s$, and, importantly for the results of this paper, it also enters the equations for the Matsubara gaps and renormalized frequencies and, as a result, the current will decay the gap. The equations for the Matsubara gaps $\tilde{\Delta}_n=Z_n\Delta_n$ and renormalized frequencies $\tilde{\omega}_n=\omega_nZ_n$ modified for d-wave symmetry are

$$\tilde{\Delta}_n=\pi T\sum_{m=-\infty}^{+\infty}\lambda(m-n)\int_0^{2\pi}\frac{d\theta}{2\pi}\times\frac{\tilde{\Delta}_m\cos^2(2\theta)}{\sqrt{[\tilde{\omega}_m-is\cos(\theta-\alpha)]^2+\tilde{\Delta}_m^2\cos^2(2\theta)}} \quad (2)$$

and

$$\tilde{\omega}_n=\omega_n+g\pi T\sum_{m=-\infty}^{+\infty}\lambda(m-n)\Omega_m+\pi\Gamma^+\frac{\Omega_n}{c^2+\Omega_n^2}, \quad (3)$$

with

$$\Omega_n=\int_0^{2\pi}\frac{d\theta}{2\pi}\frac{\tilde{\omega}_n-is\cos(\theta-\alpha)}{\sqrt{[\tilde{\omega}_n-is\cos(\theta-\alpha)]^2+\tilde{\Delta}_n^2\cos^2(2\theta)}}, \quad (4)$$

where the Matsubara frequencies are $\omega_n=\pi T(2n-1)$, for integer n, and we have included the possibility of impurity scattering from the unitary ($c=0$) to Born ($c\to\infty$) limit via the last term in Eq. (3) which requires self-consistency through Ω_n. Here, Γ^+ is proportional to the impurity scattering rate and c is related to the scattering phase shift.[13] Note that an impurity term does not appear in Eq. (2) in d-wave as it does in s-wave. This is because in d-wave it averages to zero. Finally, in general, the kernel of these equations would normally be based on a momentum-dependent electron-boson spectrum. To mimic this unknown spectrum in an approximate manner, a parameter g is introduced to represent that the interaction in the ω channel could be different from that in the Δ channel in the case of a momentum-dependent interaction that would give rise to a d-wave order parameter symmetry. Likewise, there is no $\cos(2\theta)$ factor in the numerator of the ω channel as there is in the Δ channel, reflecting the fact that the interaction in the renormalization channel is taken to be isotropic to first order.[21,22] Thus, the electron-boson spectral function, which we denote by $\alpha^2F(\Omega)$, enters $\lambda(n-m)$ as follows:

$$\lambda(m-n)\equiv 2\int_0^{\infty}\frac{\Omega\alpha^2F(\Omega)}{\Omega^2+(\omega_n-\omega_m)^2}d\Omega. \quad (5)$$

Here, to take the limit of these equations to give the standard BCS result for d-wave in our numerical evaluation, we take the electron-boson spectrum to be a δ function at high frequency. Specifically, we take $\alpha^2F(\Omega)=A\delta(\Omega-\omega_E)$, with $\omega_E=200$ meV and A about 40 to obtain this limit. Likewise, we exclude renormalization effects which are not based on impurities by taking g to be zero. This gives the BCS gap ratio in d-wave to be $2\Delta_0/kT_c=4.28$. When we wish to consider strong-coupling effects corresponding to $2\Delta_0/kT_c=5$, for example, then we take g to be a finite value and use a δ function at lower frequency for the boson spectrum.[22]

To extract the nonlinear coefficient that is relevant to the passive microwave filters and can be used as a sensitive probe of order parameter symmetry, we can assume that j_s can be expanded to third order for small q_s as

$$j_s=j_0\left[\frac{n_s(T)}{n}\left(\frac{q_sv_F}{\Delta_0}\right)-\beta(T)\left(\frac{q_sv_F}{\Delta_0}\right)^3\right], \quad (6)$$

where $j_0=ne\Delta_0/(mv_F)$. Note that for strong coupling, we replace the q_sv_F in this formula by $q_sv_F/(1+\lambda)$, where λ is the mass renormalization parameter.[16] From this Dahm and Scalapino define[6]

$$b(T)\equiv\frac{\beta(T)}{[n_s(T)/n]^3}, \quad (7)$$

the square of which is related to the third-order intermodulation power in microwave filters. Hence, measuring the intermodulation power provides a measure of the nonlinear coefficient. As we calculate the full q_s dependence of j_s, using Eqs. (1)–(5) with no approximation of taking the gap to be independent of q_s, as was done in previous works, it is easiest to extract these quantities directly from our numerical data. To do this we form the quantity of j_s/q_s versus q_s^2 which is a straight line at low q_s, and from this we obtain the superfluid density from the intercept and the nonlinear coefficient from the slope. By this method, we have confirmed previous results at low temperature and can proceed to examine the issue of higher temperatures where the current reduces the gap even at low q_s. We have also demonstrated this method for one-band and two-band s-wave superconductors.[16]

III. RESULTS

In Fig. 1, we show the current as a function of q_s in the two major directions, along the node and antinode, at both low and high temperatures. This was done using the equations above and illustrates that we can reproduce correctly the $T=0$ results in the literature[23,24] and that we can also evaluate the current at high temperature in this formalism. The Matsubara formalism is also ideally suited for including impurity scattering. There are few points to note in this fig-

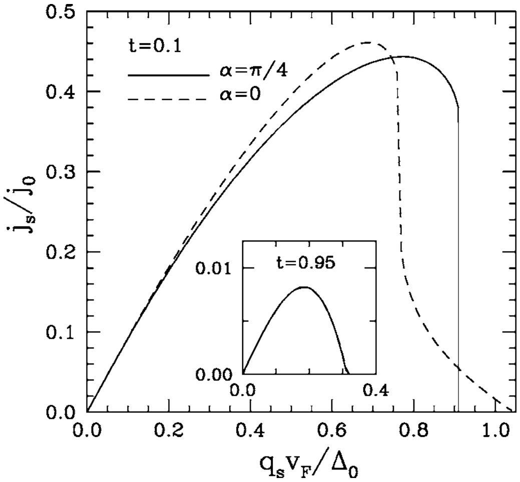

FIG. 1. The normalized current j_s/j_0 as a function of $q_s v_F/\Delta_0$, where $j_0 = ne\Delta_0/(mv_F)$ and Δ_0 is the energy gap at $T=0$. Shown are the low-temperature BCS curves for two directions: the current in the antinodal direction with $\alpha=0$ (dashed line) and the nodal direction with $\alpha=\pi/4$ (solid line), given for a reduced temperature $t = T/T_c = 0.1$. The inset shows that the curves overlap for T near T_c (in this case, $t=0.95$).

ure. The q_s in the order parameter is essential to obtain these curves [see Refs. 23 and 24 for curves of $\Delta(q_s)$ versus q_s] and it is the decay of the order parameter by the superfluid current that causes the current j_s to drop dramatically beyond the peak [otherwise, if $\Delta(q_s=0)$ is used, the curves decay slowly to zero as $q_s \rightarrow \infty$, for example, as $1/q_s$ for $T=0$ BCS s-wave]. Furthermore, the effect of q_s in the gap becomes even more important at low q_s when the temperature is approaching T_c, as it is the current in the gap that causes the downturn which now starts at much lower q_s. Finally, for high temperatures near T_c, the current is fairly independent of the angle α.

This latter feature is seen more clearly in Fig. 2 where we show the nonlinear coefficient $\beta(T)$ for d-wave in the clean limit as a function of temperature for the two directions just discussed (solid and short-dashed curves). Once again, it is seen that while the two curves are different at low T, as $T \rightarrow T_c$, the anisotropy is reduced and disappears at T_c. Indeed, we can obtain an analytical value for $\beta(T)$ at $T=T_c$ which is 0.651 and this is confirmed by the numerics shown in Fig. 2. It is also important to note that the same procedure which gives rise to these curves also provides the normalized superfluid density which is shown as the solid curve in the inset of Fig. 3. This superfluid density curve is independent of the direction of the current as expected as it arises from the $q_s \rightarrow 0$ limit, and this curve is exactly the same as that which is calculated by standard formulas for the penetration depth. Here it is extracted from our j_s versus q_s curve as explained in the theory section, verifying the accuracy of our method.

Also shown in Fig. 2 by dotted line type are the curves for $\beta(T)$ for $\alpha=0$ and $\pi/4$ when the approximation of $\Delta(q_s=0)$ is taken [i.e., the s is set equal to zero in Eqs. (2)–(4) so that the gap is not modified by the superfluid current]. This is a central point of our paper that while this approximation works well at low temperatures, one sees from a comparison

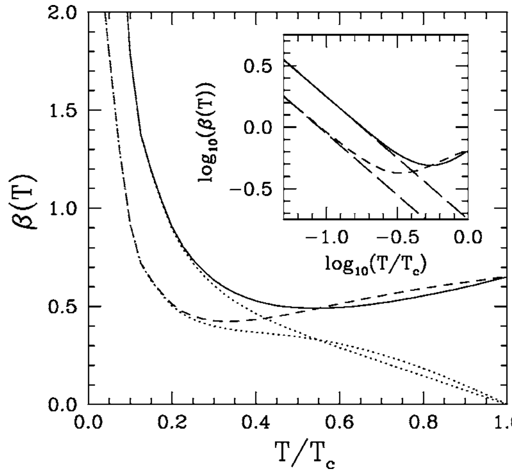

FIG. 2. The nonlinear coefficient $\beta(T)$ as a function of T/T_c, shown for two directions: $\alpha=\pi/4$ (solid line) and $\alpha=0$ (short-dashed line). The dotted curves are for the same two directions but with the approximation of neglecting the q_s dependence in the gap—i.e., $\Delta(q_s=0)$. The inset illustrates via a log-log plot that the low-temperature behavior varies as $\Delta_0/24T$ for $\alpha=0$ and $\Delta_0/12T$ for $\alpha=\pi/4$ (these expressions are shown as the long-dashed lines in both cases).

of the dotted curves with their respective short-dashed and solid ones that this approximation breaks down above $T \sim 0.2T_c$ and produces significant deviations as $T \rightarrow T_c$. Indeed, on physical grounds one does not expect the nonlinear current to go to zero as $T \rightarrow T_c$, but rather to increase. This is well known from the Ginzburg-Landau analysis of the critical current at T_c where the nonlinear term is essential.[18,19] From the point of view of device applications which would typically operate at about $0.5T_c$ or higher, this effect of decaying the gap by the superfluid can introduce a factor of 1.5–2 increase in the nonlinear response of the device. Finally, we confirm in Fig. 2 that the original use of this ap-

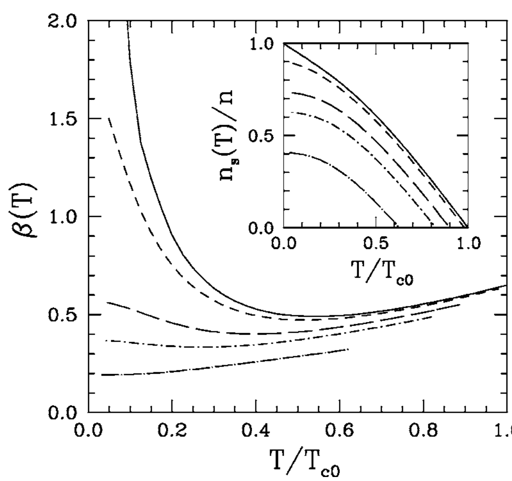

FIG. 3. The nonlinear coefficient $\beta(T)$ versus T/T_{c0} for varying impurity scattering in the unitary limit ($c=0$) and with $\alpha=\pi/4$. The inset shows the corresponding curves for the superfluid density $n_s(T)/n$. Curves are given for the pure limit (solid line), $\Gamma^+/T_{c0} = 0.0082$ (short-dashed line), 0.0404 (long-dashed line), 0.0697 (dot-short-dashed line), and 0.1432 (dot-long-dashed line).

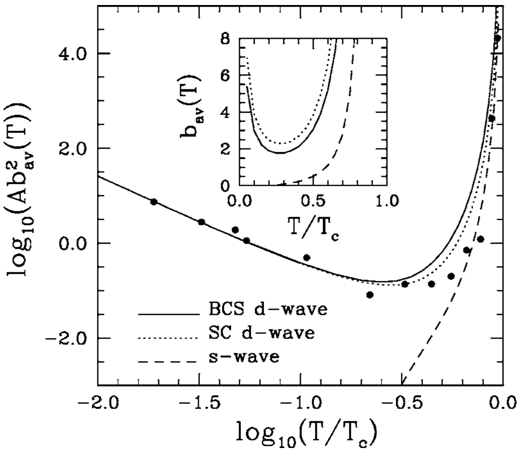

FIG. 4. Plot of $\log_{10}[Ab_{av}^2(T)]$ versus $\log_{10}(T/T_c)$ for s-wave (dashed curve), BCS d-wave with $2\Delta_0/kT_c=4.28$ (solid line), and strong-coupling (SC) d-wave with $2\Delta_0/kT_c=5$ (dotted line). The curves are compared with the experimental data (solid dots) taken from Ref. 9. The inset shows the same curves for $b(T)$ versus T/T_c.

proximation for low temperatures is robust and the $1/T$ signature of the d-wave gap discussed by Dahm and Scalapino[6,7] remains intact. We find by our procedure the same result as determined by Dahm and Scalapino analytically, that $\beta(T,\alpha=0)\simeq\Delta_0/24T$ and $\beta(T,\alpha=\pi/4)\simeq\Delta_0/12T$ for $T\to 0$, which is illustrated by the log-log plot in the inset of Fig. 2.

Turning to the case of impurities which were discussed previously by Dahm and Scalapino[6] and Andersen *et al.*,[13] we comment on modifications that occur due to the current in the gap. Again, the results of Dahm and Scalapino which are given for $T<0.2T_c$ remain robust; however, the results of Andersen *et al.* which focus on high temperature are necessarily modified when the approximation of a q_s-independent order parameter is removed. This is shown in Fig. 3, where we show both $\beta(T)$ and $n_s(T)/n$ for varying impurity content (here we use the unitary limit for illustration, as was done in Ref. 13). The $\beta(T)$ curves are shown only for $\alpha=\pi/4$. Once again, for $T\gtrsim 0.2T_c$, deviations occur in the manner discussed before. The significant feature is that the impurity scattering reduces the T_c relative to the pure case which has T_{c0}, and hence for fixed temperature of $T\sim 0.75T_{c0}$, for example, with varying impurity content, the decay of the gap by the superfluid current has an even greater effect for increasing impurity content and so it must not be neglected.

We now turn to a reexamination of the comparison of theory with some of the data that exists in the literature, maintaining the same analysis that was done previously. First, we consider the clean-limit YBCO data presented by Oates and co-workers.[9] In Fig. 4, we reproduce these data for the normalized IMD along with our calculations for BCS d-wave, where $2\Delta_0=4.28kT_c$. The IMD power has been shown to be related to $b^2(T)$—i.e.,

$$P_{\mathrm{IMD}} \propto b^2(T), \tag{8}$$

and hence we plot $b^2(T)$ in the figure and use an adjustable factor A to match the data with the theory as was done by Oates *et al.* Likewise, we have averaged the two directions for $\alpha=0$ and $\alpha=\pi/4$ after the manner of Oates *et al.* as their measurement averages over all directions. The low-temperature part of this log-log plot does not change from previous d-wave calculations but the high-temperature part is increased and the agreement with the data is not as good in this regime (we have chosen to fit the low-temperature part of the curve to the data). However, compared to the s-wave calculation, the evidence in support of d-wave remains striking. Oates *et al.* obtained a better fit with the d-wave theory because Dahm and Scalapino used a larger gap ratio in their BCS theory which is supported by other experiments. To illustrate this within our formalism, we can do a strong-coupling calculation where we take $g=1$ in Eq. (3) and move the electron-boson spectral function to lower frequency. In this manner, we can obtain $2\Delta_0/kT_c=5$ in d-wave corresponding to a mass renormalization parameter $\lambda=5$. In this case, the result (shown as the dotted curve and plotted with a new A to fit the low T region) does indeed move toward a better fit with the data over the full temperature range. As a technical note, to obtain a gap ratio of $2\Delta_0=6kT_c$ in this Eliashberg formalism, we would have to take λ to extremely large values and we would then be in the asymptotic regime which limits the value of the gap ratio to go no higher than about 6.5.[22] A much better fit is obtained with a gap ratio of about 6 but in our model the λ is unphysical. To overcome this, in order to produce higher gap ratios with more physical values of λ, would require a spin-fluctuation calculation of the type given in Ref. 25, which would include feedback on the electron-boson spectral density itself, which suppresses it at low ω and introduces coupling to a new resonant mode that grows in amplitude as T is reduced. This is beyond the scope of this work. Other work which has discussed this data has been that of Agassi and Oates[11] where they considered nonlocal effects; however, the calculation is a BCS one with no inclusion of the effect of the superfluid current on the gap and so it is difficult to say how both this and strong coupling might modify their results.

Finally in Fig. 4, in the inset, we give for reference the unscaled $b(T)$ versus T curves. The $1/T$ divergence is clear in comparison with the exponential decay of the s-wave case and the strong-coupling calculation is above the BCS one as is expected from previous work on s-wave superconductors.[16]

We revisit the issue of impurities in Fig. 5. This figure is based on a similar one presented by Andersen *et al.*[13] Several films were examined at 75 K, and the nonlinear coefficient was found to vary from sample to sample, as did also the penetration depth. The analysis in the paper used the DS theory with impurity scattering to argue that the data could be understood by assuming varying impurity content from one film to another. We reexamine this issue because at this high temperature the pairbreaking effect of the superfluid current on the gap should cause significant changes (our Fig. 3 should be contrasted with Fig. 3 of Ref. 13) and this was not included in the original analysis. In Fig. 5, we show our calculation at $T=0.75T_{c0}$ for the variation of the nonlinear coefficient with respect to the variation of the penetration depth when unitary scattering is included (like Andersen *et al.*, we have checked Born scattering and find there is essen-

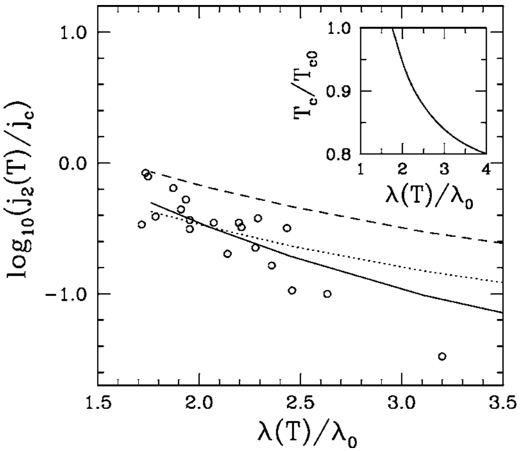

FIG. 5. Plot of $\log_{10}(j_2(T)/j_c)$ versus $\lambda(T)/\lambda_0$ for $T=0.75T_{c0}$. Andersen *et al.* (Ref. 13) have defined a quantity $(j_2/j_c)^2$ which is equal to our $1/b(T)$. The data (open circles) is taken from Ref. 13. The solid curve is for BCS d-wave with $\Delta(q_s)$ and the dashed line is for the case where the approximation $\Delta(q_s=0)$ has been used. The dotted curve shows the $\Delta(q_s=0)$ curve shifted down to overlap with the data. The inset shows the reduction in T_c from the pure value of T_{c0} that is implied by the values of $\lambda(T)$ at $T=0.75T_{c0}$ normalized to the pure value for $T=0$, λ_0.

tially no difference in the result presented here). We have also taken $\alpha=\pi/4$ as at this temperature the anisotropy is almost completely gone, which we have checked. It should be noted that, once again, there is an adjustable parameter j_c which can be used to scale the theory to overlap with the data. Andersen *et al.* kept the theory fixed and used a value of j_c to adjust the data. Here we choose to keep the scaled data as they were presented in the original paper, and so we adjust our theory by a scale factor to overlap with the data (if we had adjusted the data instead, this would have increased the j_c value used for scaling the data by a factor of about 1.35). Doing so, we find with the solid curve that there is reasonable agreement between theory and data. However, we note, referring the reader to the inset of the figure, that the range of variation in penetration depth, if explained via impurity scattering, would imply a reduction in T_c of about 15%–20% from the pure case[26] and yet the experimental data indicate a T_c variation of only about 2%. This discrepancy was not noted in Ref. 13 and it does pose a problem for this interpretation. However, continuing with this explanation, we have compared our result with the case of taking $\Delta(q_s=0)$, shown as the dashed curve and using the same j_c scale factor, and we find that while some deviation may be corrected by choosing a different j_c, which would shift the curve (shown as the dotted curve), the slope is also different. While the interpretation of the data via impurity scattering may be open to question due to the large variation in T_c required, more experimental data may help to reduce the scatter and provide further tests of the theory.

Recently, a probe of angle-resolved electron tunneling was used to determine the order parameter symmetry in YBCO and this work has suggested that the order parameter is not pure d wave but an admixture of $d+s$ with about 15% of s-wave.[1] This is not the first time that it has been suggested that YBCO may have a small s-wave component,[27] and it is important to assess the consequences for the intrinsic nonlinear current and whether there would be any signatures unique to a $(d+s)$-wave order parameter. As a result, we now briefly consider such a symmetry, where we assume that the s-wave component is small compared with the d-wave part, such that nodes still exist but they are shifted from $\pi/4$. Taking $\Delta(\theta)/\Delta=a\cos(2\theta)+c$, we evaluate analytically the main features expected in the limits of $T\to T_c$ and $T\to 0$. As $T\to T_c$, we find that we can define a quantity related to $b(T)$:

$$\bar{b}(T_c) \equiv \lim_{T\to T_c} b(T)\left(1-\frac{T}{T_c}\right)^3, \tag{9}$$

which gives $(T_c/\Delta_0)^2\bar{b}(T_c)=0.0599$ for d-wave and 0.0266 for two-dimensional s-wave. In both cases, this quantity is isotropic with respect to the direction of the current; however, for $(d+s)$-wave, we have

$$\bar{b}(T_c)\left(\frac{T_c}{\Delta_0}\right)^2 = \frac{7\zeta(3)}{64\pi^2}\frac{\gamma_2^2\gamma_3}{\gamma_1^3(\gamma_1-\gamma_3)^2}, \tag{10}$$

where

$$\gamma_1 = \int_0^{2\pi}\frac{d\theta}{2\pi}\Delta^2(\theta), \tag{11}$$

$$\gamma_2 = \int_0^{2\pi}\frac{d\theta}{2\pi}\Delta^4(\theta), \tag{12}$$

$$\gamma_3 = \int_0^{2\pi}\frac{d\theta}{2\pi}\cos^2(\theta-\alpha)\Delta^2(\theta), \tag{13}$$

and there arises an anisotropy as a function of α in the nonlinear coefficient as $T\to T_c$, which is not there in pure d-wave [α enters only through γ_3 and for pure s- or pure d-wave γ_3 is a constant as α drops out of Eq. (13)]. Indeed, it can be sizable as shown in Fig. 6 (upper frame) for different percentages of the s-wave component, defined as $[c/(a+c)]\times 100\%$. For 15% s-wave, the anisotropy is 20%. Note also that the anisotropy is twofold. At low temperature, a d-wave order parameter already shows a fourfold anisotropy in $\beta(T)$ as a function of α. If we plot $\beta(T)$ normalized to $\Delta_0/24T$ as is shown in the lower frame of Fig. 6, we see that the fourfold d-wave anisotropy shifts to twofold for $(d+s)$-wave and also exhibits a shift in the position of the maxima (indicating a shift in the position of the nodes). The magnitude of the anisotropy, which was a factor of 2 for d-wave, is now even greater in $(d+s)$-wave. The analytic formula for this anisotropy, assuming $c<a$, is

$$\beta(T) = \frac{\Delta_0}{24T}\left(\left[1-\frac{c}{a}\cos(2\alpha)\right]^2+\left[1-\left(\frac{c}{a}\right)^2\right]\sin^2(2\alpha)\right)\frac{1}{\sqrt{1-(c/a)^2}}. \tag{14}$$

Thus, there should be observable signatures of an s-wave

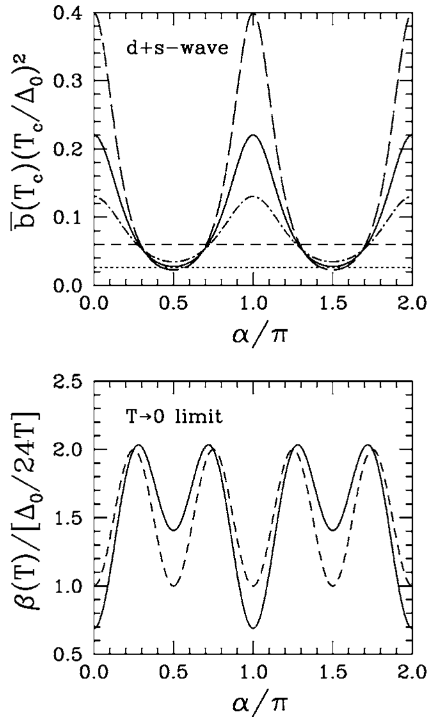

FIG. 6. The effect of a $(d+s)$-wave gap on the anisotropy near T_c as measured via $\bar{b}(T_c)$ (upper frame) and at low temperature as measured by $\beta(T)$ normalized to $\Delta_0/24$ T (lower frame). The anisotropy is plotted as a function of the current direction relative to the antinode, α. Curves are shown for s-wave (dotted line), d-wave (short-dashed line), and varying percentage of s-wave: 10% (dot-dashed line), 15% (solid line), and 20% (long-dashed line).

component should it be possible to do angle-resolved measurements of intrinsic nonlinear current.

IV. CONCLUSIONS

In summary, we have calculated the intrinsic nonlinear current of a d-wave superconductors, including the effect of the superfluid current on the order parameter. We find that the low-temperature $1/T$ behavior of the nonlinear coefficient, proposed as a test for d-wave symmetry by Dahm and Scalapino, remains unchanged. However, at temperatures above about $0.2T_c$, the approximation of taking the gap to be q_s independent fails and large corrections are found. Indeed, at temperatures of order $0.75T_c$, there is a twofold increase in the nonlinear coefficient over that obtained with a q_s-independent gap. This could have implications for the technological application of the cuprate materials as passive microwave devices for the communication industry. We also find that these corrections remain large in the presence of impurity scattering.

A reexamination of the comparison of theory and data from the intermodulation power with this approximation removed finds that strong coupling effects would still be required to obtain a fit over the entire temperature range. However, this requires calculations of such sophistication that they are beyond the scope of this paper. The signature in support of d-wave symmetry remains clear, regardless. More importantly, a recent analysis of data from several films assumes that the explanation of the variation in the nonlinear coefficient and penetration depth results from impurity scattering. As the data are taken at high temperature, where the decay of the gap by the current is significant, we find upon reevaluation of the theory that the relationship between these two quantities can vary markedly and a different slope is obtained along with a different overall scale factor. Unfortunately, we must point out that, while a possible fit to the data still remains, the basic assumption that impurity scattering is the source of the changes seen in the data implies a large variation in the T_c of the different samples, which is not seen experimentally. Further experiments along these lines might help to resolve this issue more completely.

Finally, due to a recent experimental observation of possible $(d+s)$-wave gap symmetry in YBCO measured by angled-resolved electron tunneling, we have examined the nonlinear coefficient at both low T and $T\rightarrow T_c$ to determine signatures of the s-wave component. Indeed, at $T\rightarrow T_c$, the isotropic behavior found for pure s- and pure d-wave is lost and a large anisotropy develops for $(d+s)$-wave as a function of the direction of the current in the plane. At low temperatures, the $1/T$ dependence remains but the fourfold anisotropy of the coefficient is altered to twofold and the magnitude is modified. Therefore, should it become possible to do experiments where the direction of the current can be varied with respect to the antinodes or nodes; then, we predict that signatures of a $(d+s)$-wave order parameter would be observable.

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