Microscopic description of the critical behavior of three-dimensional Ising-like systems in an external field

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We study the behavior of a three-dimensional Ising-like system in an external field near the critical point by using the non-Gaussian spin-density fluctuations, namely, the quartic measure density with the even and odd powers of the variable (the asymmetric ρ^4 model). The basic idea of the analytic method for deriving complete expressions of the thermodynamic characteristics (including the scaling functions) is described. The proposed method allows us to perform the calculations on the microscopic level without any adjustable parameters. Explicit expressions for the total free energy, order parameter, susceptibility, entropy, and specific heat of the system are obtained as functions of the temperature and field. The regions of the so-called weak and strong fields are considered for temperatures above and below T_c (T_c is the phase-transition temperature in the absence of an external field). The average spin moment and susceptibility, depending on the field variation and the proximity to T_c, are investigated. It is confirmed that the temperature and field fluctuations for the order parameter play the leading roles in the weak and strong fields, respectively.

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I. INTRODUCTION

A central problem in statistical physics is the description of phase transitions and critical phenomena. The most spectacular achievement in the theory of critical phenomena, confirming the concept of universality and scaling, is the development of the renormalization-group (RG) ideas.[1] The application of the Feynman diagram technique by Wilson as well as the introduction of the expansion parameter $\epsilon=4-d$ and the constructive idea that the critical exponents vary smoothly with varying space dimension d opened paths for the advancement of field-theory RG methods in the theory of critical phenomena. The use of the RG approach in this theory provided a deep insight into the processes in the neighborhood of the critical temperature and made it possible to calculate the critical exponents, the equation of state, the correlation functions, and some other important characteristics of various model systems. At present, there are different statements of RG approaches to critical phenomena. The essences of some of them are presented in Refs. 2–5. Much success was achieved in the theory of critical phenomena using renormalized perturbation theory series[6] and the resummation procedure for asymptotic series.[7,8] The RG transformation within the framework of the collective variables (CV) method, generalized by Yukhnovskii[9–12] to the case of spin systems and used in the present paper, can also be related to the Wilson type. This theoretical investigation method for a system of charged particles was initially presented by Bohm and Pines (see, for example, Ref. 13) and also by Zubarev.[14] The main results of investigating the critical behavior of the three-dimensional (3D) Ising model using the CV method are indicated in Ref. 15.

The study of the effect of an external magnetic field on the critical behavior of spin systems, in particular, of 3D Ising-like systems, is an actual problem. Due to their simplicity, convenience for mathematical analysis, and the many physically relevant applications,[16] the 3D Ising model and members of the corresponding universality class belong to the most extensively studied systems. Despite the great successes in the investigation of 3D Ising-like systems made by means of various methods (see, for example, Refs. 16–18), the statistical description of the behavior of the mentioned systems near the critical point in terms of the temperature and field variables and the calculation of scaling functions are still of interest.[19] The direct theoretical computation of the total thermodynamic characteristics (including the scaling functions) is our aim in this research work. The calculations within the method of CV $\rho_{\mathbf{k}}$ (Refs. 11 and 20–22) are performed using the non-Gaussian spin-density fluctuations, namely, the quartic measure density. The latter is represented as an exponential function of the CV whose argument includes the even and odd powers up to the fourth power of the variable (the asymmetric ρ^4 model). The asymmetric ρ^4 model has not been used deeply enough so far. In previous studies (see, for example, above-mentioned Refs. 11 and 20–22), the non-Gaussian measure densities contained only the even powers of the variable in exponential functions. An infinitely weak external field was introduced in the course of calculation of the contribution from the long-wave spin-density oscillation modes to the 3D Ising system thermodynamic characteristics. In this paper, we introduce an external field in the Hamiltonian from the outset. Such an approach leads to the appearance of odd powers of the CV in the expression for the partition function and allows us to describe a lot of quantities (cumulants, coefficients of the partition function, etc.) as functions of an external field.

The term collective variables is applied to a special class of variables specific for each individual physical system. The set of CV contains variables associated with order parameters. For this reason, the phase space of CV is most natural for describing a phase transition. For magnetic systems, the CV $\rho_{\mathbf{k}}$ are the variables associated with modes of spin moment density oscillations, while the order parameter is associated with the variable ρ_0, in which the subscript 0 corresponds to the peak of the Fourier transform of the interaction potential.

II. BASIC RELATIONS

We consider a 3D Ising-like system on a simple cubic lattice with N sites and period c in a homogeneous external field h. The Hamiltonian of a system has the form

$$H = -\frac{1}{2}\sum_{\mathbf{j},\mathbf{l}} \Phi(r_{\mathbf{jl}})\sigma_{\mathbf{j}}\sigma_{\mathbf{l}} - h\sum_{\mathbf{j}} \sigma_{\mathbf{j}}. \tag{1}$$

Here $r_{\mathbf{jl}}$ is the distance between particles at sites $\mathbf{j}$ and $\mathbf{l}$, $\sigma_{\mathbf{j}}$ is the operator of the z component of spin at the $\mathbf{j}$th site with two eigenvalues $+1$ and -1. The interaction potential is an exponentially decreasing function

$$\Phi(r_{\mathbf{jl}}) = A\exp\left(-\frac{r_{\mathbf{jl}}}{b}\right), \tag{2}$$

where A is a constant, b is the radius of effective interaction. For the Fourier transform of the interaction potential, we use the following approximation:[20,21,23]

$$\Phi(k) = \begin{cases} \Phi(0)(1-2b^2k^2), & k \leqslant B_0, \\ \Phi(0)\bar{\Phi}, & B_0 < k \leqslant B. \end{cases} \tag{3}$$

Here $\bar{\Phi}$ is the small constant, $B=\pi/c$ is the boundary of the Brillouin half-zone, $\Phi(0)=8\pi A(b/c)^3$, and $B_0=B/s_0$. The parameter s_0 determines the region of the wave vector values $k\leqslant B_0$, where the parabolic approximation for $\Phi(k)$ is effective [see Eq. (3)]. The model potential (3) is based on the fact that large values of the wave vector are inessential for calculating critical characteristics.

The integration over the layers of the CV phase space[21,22] leads to the representation of the partition function of the system in the form of a product of the partial partition functions Q_n (Refs. 23–25) of individual layers and the integral $I_{n_p+1}=\int\mathcal{W}_4^{(n_p+1)}(\rho)(d\rho)^{N_{n_p+1}}$ of the "smoothed" effective measure density

$$Z = Z_0Q_0Q_1\ldots Q_{n_p}j_{n_p+1}[Q(P^{(n_p)})]^{N_{n_p+1}}I_{n_p+1}. \tag{4}$$

Here $Z_0=2^N(\cosh h')^N\exp[\beta\Phi(0)\bar{\Phi}N/2]$, $h'=\beta h$ is the dimensionless field. The quantity $\beta=1/(kT)$ is the inverse temperature, k is the Boltzmann constant. The expression for $Q(P^{(n)})$ is presented in Refs. 25 and 26, and $j_{n_p+1}=\sqrt{2}^{N_{n_p+1}-1}$. The quartic measure density of the (n_p+1)th block structure

$$\mathcal{W}_4^{(n_p+1)}(\rho) = \exp\Bigg[-\tilde{a}_1^{(n_p+1)}N_{n_p+1}^{1/2}\rho_0 - \frac{1}{2}\sum_{k\leqslant B_{n_p+1}} d_{n_p+1}(k)\rho_{\mathbf{k}}\rho_{-\mathbf{k}} - \frac{1}{3!}a_3^{(n_p+1)}N_{n_p+1}^{-1/2}\sum_{\substack{\mathbf{k}_1,\ldots,\mathbf{k}_3\\ k_i\leqslant B_{n_p+1}}} \rho_{\mathbf{k}_1}\cdots\rho_{\mathbf{k}_3}\delta_{\mathbf{k}_1+\cdots+\mathbf{k}_3} - \frac{1}{4!}a_4^{(n_p+1)}N_{n_p+1}^{-1}\sum_{\substack{\mathbf{k}_1,\ldots,\mathbf{k}_4\\ k_i\leqslant B_{n_p+1}}} \rho_{\mathbf{k}_1}\cdots\rho_{\mathbf{k}_4}\delta_{\mathbf{k}_1+\cdots+\mathbf{k}_4}\Bigg] \tag{5}$$

includes odd powers of the variable in addition to even powers. Here $B_{n_p+1}=B_0s^{-(n_p+1)}$, $N_{n_p+1}=N_0s^{-3(n_p+1)}$, $N_0=Ns_0^{-3}$, s is the RG parameter, and $\delta_{\mathbf{k}_1+\cdots+\mathbf{k}_l}$ is the Kronecker symbol. The coefficients $\tilde{a}_1^{(n)}=s^{-n}w_n$, $d_n(0)=s^{-2n}r_n$ [appearing in $d_n(k)=d_n(0)+2\beta\Phi(0)b^2k^2$], $a_3^{(n)}=s^{-3n}v_n$, and $a_4^{(n)}=s^{-4n}u_n$ are related to the coefficients of the $(n+1)$th layer in the recurrence relations[24,25] (RR) whose solutions[23,24]

$$w_n = -c_{h1}\mathcal{M}_1(h')E_1^n - c_{h2}\mathcal{M}_1(h')T_{13}^{(0)}\times\varphi_0^{-1/2}[\beta\Phi(0)]^{-1}E_3^n, \quad \mathcal{M}_1 = \tanh h',$$

$$r_n = r^* + c_{k1}^{(0)}\beta\Phi(0)\tau E_2^n + c_{k2}T_{24}^{(0)}\varphi_0^{-1/2}[\beta\Phi(0)]^{-1}E_4^n,$$

$$v_n = -c_{h2}\mathcal{M}_1(h')E_3^n,$$

$$u_n = u^* + c_{k1}^{(0)}[\beta\Phi(0)]^2T_{42}^{(0)}\varphi_0^{1/2}\tau E_2^n + c_{k2}E_4^n \tag{6}$$

are used to calculate the energy of the system. In the region of the critical regime, the variables w_n, r_n, v_n, and u_n are close to the coordinates of the fixed point $w^*=0$, $r^*=-f_0\beta\Phi(0)$, $v^*=0$, and $u^*=\varphi_0[\beta\Phi(0)]^2$. The reduced temperature is defined by $\tau=(T-T_c)/T_c$ (T_c is the phase-transition temperature in the absence of an external field). The eigenvalues E_l $(E_1,E_2,E_3>1,E_4<1)$ of the RG linear transformation matrix and other quantities appearing in Eqs. (6) are given in Refs. 23–26.

The analytic method for evaluating the thermodynamic functions of the system in the vicinity of the critical point is developed for the weak and strong fields. The weak and strong fields $\tilde{h}$ are defined on the basis of comparison with the value of the limiting field $\tilde{h}_c$. The relation for the limiting field can be written in the form[23]

$$\tilde{h}_c = |\tilde{\tau}|^{p_0}, \quad p_0 = \frac{5}{2}\nu, \tag{7}$$

where ν is the critical exponent of the correlation length. The designations are introduced as

$$\tilde{\tau} = \tau\frac{c_{k1}^{(0)}}{f_0}, \quad \tilde{h} = \frac{h'}{f_0}. \tag{8}$$

It should be noted that the critical behavior of the system in an external field and the calculation of the free energy depend on the trajectory of tending the system to the critical point in the field-temperature plane. The diagram of the regions, defined by different trajectories of the system tending to the critical point $(\tilde{\tau}=0,\tilde{h}=0)$, is shown in Fig. 1.

III. EXPLICIT EXPRESSIONS FOR THERMODYNAMIC CHARACTERISTICS OF A 3D ISING-LIKE SYSTEM IN REGIONS OF THE WEAK AND STRONG EXTERNAL FIELDS

For calculating the free energy of the system in an external field, we take into account two fluctuation processes, which exist for the order parameter near the critical point. These fluctuation processes are described by a non-Gaussian

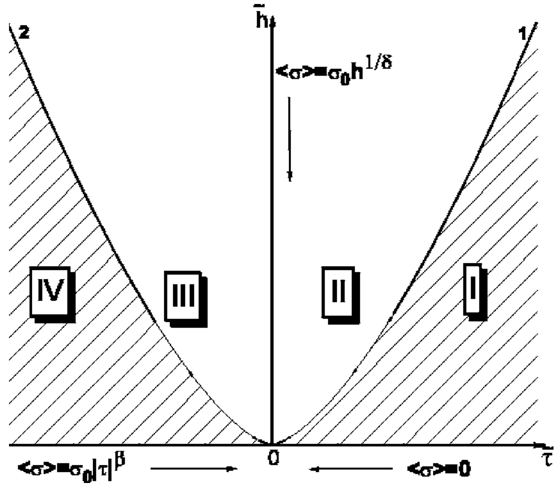

FIG. 1. Regions of the possible location of the trajectories of the system tending to the critical point ($\tilde{\tau}=0, \tilde{h}=0$). The curves 1 and 2 correspond to the limiting value of the field $\tilde{h}_c$ (7) when $T>T_c$ and $T<T_c$, respectively. The regions I and IV correspond to small values of the field ($\tilde{h}<\tilde{h}_c$) when $T>T_c$ and $T<T_c$, and the regions II and III are characterized by large values of the field ($\tilde{h}>\tilde{h}_c$). The expressions for the order parameter are indicated for the trajectories along axes.

distribution. The first of them is characterized by the quantity $m_\tau=-\ln\tilde{\tau}/\ln E_2-1$ at $T>T_c$ (or $\mu_\tau=-\ln|\tilde{\tau}|/\ln E_2-1$ at $T<T_c$) and is associated with the temperature variable. This process is observed for the effective spin blocks whose sizes do not prevail over period $c_m=cs_0 s^{m_\tau}$, which is commensurate with the system correlation length $\xi=\xi_0\tilde{\tau}^{-\nu}$ at the fixed value of $\tilde{\tau}$. The second fluctuation process is described by the quantity $n_h=-\ln\tilde{h}/\ln E_1-1$ and is related to the field variable. For small values of the field ($\tilde{h}<\tilde{h}_c$), the behavior of the system is determined by the first fluctuation process since $m_\tau<n_h$ (Ref. 23). For large values of the field ($\tilde{h}>\tilde{h}_c$), the inequality $n_h<m_\tau$ is performed and the main contribution to behavior of the system is ensured by the field fluctuations of the order parameter. At $\tilde{h}=\tilde{h}_c$, the temperature and field effects on the system near the critical point are equivalent ($m_\tau=n_h$). The quantity $\tilde{h}_c$ satisfies the condition of equality of the spontaneous moment and the moment induced by the field. The relation for $\tilde{h}_c$ (7) corresponds to the equation of the pseudocritical line.[16,27,28] The conditions $m_\tau<n_h$ (or $\mu_\tau<n_h$ for $T<T_c$) and $n_h<m_\tau$ define the different ways of calculating the free energy of the system.

The free energy of the system in the regions of both weak and strong fields is calculated by separating the contributions from the short- and long-wave modes of spin-density oscillations. The cases of temperatures $T>T_c$ and $T<T_c$ are considered. Short- and long-wave modes are separated by the layer number n_p [see Eq. (4)] determining the point of exit of the system from the critical-regime region by the temperature or by the field variable [$n_p=m_\tau$ (the weak-field region and $T>T_c$), $n_p=\mu_\tau$ (the weak-field region and $T<T_c$), and $n_p=n_h$ (the strong-field region)] (Ref. 23). Our calculations are performed for some fixed value of the parameter $s=s^*$ $=3.3783$. For such a preferred value of s nullifying the quantities

$$h_2^{(n)}=\sqrt{6}(r_n+q)u_n^{-1/2}, \quad h_3^{(n)}=h_{30}v_n u_n^{-3/4} \tag{9}$$

at the fixed point, the mathematical description becomes less complicated. Here $q=\bar{q}\beta\Phi(0)$, $\bar{q}=\pi^2(b/c)^2 s_0^{-2}(1+s^{-2})$, $h_{30}=24^{3/4}/6$. When $s=s^*$, we have $\nu=\ln s/\ln E_2=0.609$ (Ref. 25). To simplify our calculations, we neglect the critical exponent η (characterizing the behavior of the pair-correlation function for $T=T_c$), although it can be taken into account if necessary.[11,20] The short-wave part of partition function (the contribution from fluctuations of $\rho_{\mathbf{k}}$ with $k\in[B_{n_p},B_0]$ or the partial partition functions Q_n in Eq. (4) for $n\leqslant n_p$)is obtained using the quartic measure density with the even and odd powers of the variable. The calculation of the expression describing the contribution to free energy from short-wave modes of spin-density oscillations involves the summation of partial free energies over the layers of the phase space of CV up to the point, at which the system leaves the critical-regime region. In this case, it is important to obtain an explicit dependence on the number of the layer. For this purpose, the solutions of RR (6) are used. We attract your attention to the behavior of the quantities $h_2^{(n)}$ and $h_3^{(n)}$ near the critical point. Each of them takes on small values when $n<n_p$. It is easy to make sure of this using the solutions of RR (6). We have

$$h_2^{(n)}=h_{22}\left\{c_{k1}^{(0)}\tau E_2^n-\frac{1}{2}\varphi_0^{-1/2}T_{42}^{(0)}(c_{k1}^{(0)}\tau E_2^n)^2\right\},$$

$$h_3^{(n)}=h_{32}\mathcal{M}_1(h')E_3^n(1-h_{34}c_{k1}^{(0)}\tau E_2^n), \tag{10}$$

where $h_{22}=(6/\varphi_0)^{1/2}$, $h_{32}=-h_{30}c_{h_2}(u^*)^{-3/4}$, $h_{34}=3T_{42}^{(0)}\varphi_0^{-1/2}/4$. For all values $n<n_p$, we find that $h_2^{(n)}\ll 1$. The analogous inequality takes place for $h_3^{(n)}$ since $\mathcal{M}_1(h')\propto h'\ll 1$. Therefore, for all $n<n_p$, the quantities Q_n can be presented in the form of series in powers of $h_2^{(n)}$ and $h_3^{(n)}$. Using Eqs. (10), we arrive at their explicit expressions. The calculation of the long-wave part of partition function [the contribution from fluctuations of $\rho_{\mathbf{k}}$ with $k\in(0,B_{n_p})$ or the quantity I_n in Eq. (4) for $n>n_p$] is based on using the Gaussian measure density as the basis one. It is related with coefficients $a_3^{(n)}$ and $a_4^{(n)}$, which begin to decrease fast [in comparison with $d_n(k)$] when the number n increases. Here, we have developed a direct method of calculations with the results obtained by taking into account the short-wave modes as initial parameters. The final integration with respect to the variable ρ_0 is performed in the approximation of the non-Gaussian measure density with the renormalized coefficients using the steepest-descent method.

An introduction of an external field in our analysis leads to the generalized description of the critical behavior of the system on the basis of ideas and procedures presented in Refs. 11 and 20–22. The calculation of the partition function of a 3D one-component spin system in an external field as well as the RR and their explicit solutions near the critical point are given in Refs. 24–26. The proposed method for calculating the thermodynamic characteristics of 3D Ising-like systems in an external field is described in detail in Refs.

29 and 30 for the high-temperature ($T>T_c$) and low-temperature ($T<T_c$) regions, respectively. References 24 and 26 have been written on the basis of the unpublished Ref. 25. The further subsections of this section include the main points of Refs. 29 and 30. The basic idea of the calculational method is presented above. Calculating separately and summing the contributions to free energy from short- and long-wave spin-density oscillation modes, we can obtain the complete expression for the free energy of the system. Other thermodynamic characteristics (average spin moment, susceptibility, entropy, and specific heat) are defined by direct differentiation of the total free energy with respect to field or temperature. We shall consider the cases of some values of the temperature and field (regions I–IV in Fig. 1).

A. The case of $T>T_c$ and $\tilde{h}\ll\tilde{h}_c$

According to the formula (4), it is convenient to present the free energy of the system in the form[23]

$$F^{(+)}=F_0+F_{CR}+F_{TR}+F_I. \tag{11}$$

The term

$$F_0=-kTN\left(\ln 2+\ln\cosh h'+\frac{1}{2}\beta\Phi(0)\bar{\Phi}\right) \tag{12}$$

corresponds to the contribution from noninteracting spins when $\bar{\Phi}=0$. The quantity $F_{CR}=-kT\ln Q_0-kT\sum_{n=1}^{n_p}\ln Q_n$ (where $n_p=m_\tau$) is the contribution to free energy from short-wave oscillation modes at $T>T_c$. It corresponds to the critical-regime region. Using Eqs. (10) and the approximate relations[29]

$$\ln Q_0=N_0(e''_{c0}+e''_{c1}\tilde{\tau}+e''_{c2T}\tilde{\tau}^2+e''_{c2}\tilde{h}^2),$$

$$\ln Q_n=N_n\Bigg[H_{20}+H_{21}h_2^{(n-1)}-\gamma h_2^{(n)}-H_{22}(h_3^{(n-1)})^2+\frac{3}{8}\gamma(h_3^{(n)})^2\Bigg], \tag{13}$$

taking into account the sum of geometric series and the equalities $s^{-3(m_\tau+1)}=\tilde{\tau}^{3\nu}$, $\tilde{\tau}E_2^{m_\tau+1}=1$, $E_3^{m_\tau+1}=\tilde{\tau}^{-\nu/2}$, we can write the following expression for F_{CR} accurate to within $\tilde{\tau}^2$ and $\tilde{h}^2$:

$$F_{CR}=-kTN_0(e_{0p}+e_{1p}\tilde{\tau}+e_{2p}\tilde{\tau}^2+e_{3p}\tilde{h}^2+e_{4p}\tilde{\tau}^{3\nu}). \tag{14}$$

The contribution to free energy

$$F_{TR}=-kT\left\{\ln Q_{m_\tau+1}+N_{m_\tau+2}\left[\frac{1}{2}\ln 2+\ln Q(P^{(m_\tau+1)})\right]\right\}$$

from the layer of the CV space immediately beyond the point of exit m_τ from the critical regime is related to the transition region.[20,22] This is the free energy of the regime, which corresponds to the transition from short-wave to long-wave oscillation modes for the order parameter. For F_{TR}, we obtain

$$F_{TR}=-kTN_0\tilde{\tau}^{3\nu}\{f_{mp}+s^{-3}[f_{p2}-(m_\tau+1)\ln s]\}. \tag{15}$$

The quantities f_{mp} and f_{p2} as well as the coefficients in Eqs. (13) and (14) are independent of the field.[29]

With the help of the term

$$F_I=-kT\ln I_{m_\tau+2}, \tag{16}$$

where

$$I_{m_\tau+2}=\int(d\rho)^{N_{m_\tau+2}}\times\exp\Bigg[a_{1m}N^{1/2}\tilde{h}\rho_0-\frac{1}{2}\sum_{k\le B_{m_\tau+2}}d_{m_\tau+2}(k)\rho_{\mathbf{k}}\rho_{-\mathbf{k}}-\frac{1}{3!}a_3^{(m_\tau+2)}N_{m_\tau+2}^{-1/2}\sum_{\substack{\mathbf{k}_1,\ldots,\mathbf{k}_3\\ k_i\le B_{m_\tau+2}}}\rho_{\mathbf{k}_1}\cdots\rho_{\mathbf{k}_3}\delta_{\mathbf{k}_1+\cdots+\mathbf{k}_3}-\frac{1}{4!}a_4^{(m_\tau+2)}N_{m_\tau+2}^{-1}\sum_{\substack{\mathbf{k}_1,\ldots,\mathbf{k}_4\\ k_i\le B_{m_\tau+2}}}\rho_{\mathbf{k}_1}\cdots\rho_{\mathbf{k}_4}\delta_{\mathbf{k}_1+\cdots+\mathbf{k}_4}\Bigg], \tag{17}$$

we take into account long-wave fluctuations of the order parameter. Now, after the integration over 0th,1st,…,($m_\tau+1$)th layers of the phase space of CV, the coefficient in the quadratic term of an exponential function in Eq. (17) changes sign and takes on the positive value.[29] This term dominates over other terms for all $k\neq 0$. Thus, we perform the integration with respect to CV with these values of indices **k** using the Gaussian distribution. In this case

$$I_{m_\tau+2}=\prod_{k\neq 0}^{B_{m_\tau+2}}\left(\frac{\pi}{d_{m_\tau+2}(k)}\right)^{1/2}I_{m_\tau+2}^{(0)}, \tag{18}$$

where

$$I_{m_\tau+2}^{(0)}=\int(d\rho_0)\exp\Bigg[a_{1m}N^{1/2}\tilde{h}\rho_0-\frac{1}{2}d_{m_\tau+2}(0)\rho_0^2-\frac{1}{3!}a_3^{(m_\tau+2)}N_{m_\tau+2}^{-1/2}\rho_0^3-\frac{1}{4!}a_4^{(m_\tau+2)}N_{m_\tau+2}^{-1}\rho_0^4\Bigg]. \tag{19}$$

Here $a_{1m}=f_0\mathcal{M}_{20}/\mathcal{M}_2$, $\mathcal{M}_2=1-\tanh^2 h'$, and $\mathcal{M}_{20}$ is given in Refs. 26 and 29. The coefficient in the cubic term $[a_3^{(m_\tau+2)}N_{m_\tau+2}^{-1/2}\propto\tilde{\tau}^{\nu}\tilde{h},\ \tilde{h}\ll\tilde{h}_c=\tilde{\tau}^{5\nu/2},\ \tilde{\tau}\ll 1]$ is vanishingly small in comparison with other coefficients $[d_{m_\tau+2}(0)\propto\tilde{\tau}^{2\nu}$, $a_4^{(m_\tau+2)}N_{m_\tau+2}^{-1}\propto\tilde{\tau}^{\nu}]$, and in following calculations, we shall neglect this term. For evaluating the integral (19) with respect to the variable ρ_0 associated with the order parameter, we use the steepest-descent method. It is convenient to carry out the substitution of the variable

$$\rho_0=\sqrt{N}\bar{\rho}_0.$$

The quantity $\bar{\rho}_0$ can be determined from the extremum condition for an exponential function in Eq. (19). As a result, we arrive at the equation

$$-a_{1m}\tilde{h}+d_{m_\tau+2}(0)\bar{\rho}_0+\frac{1}{6}a_4^{(m_\tau+2)}s_0^3 s^{3(m_\tau+2)}\bar{\rho}_0^3=0, \quad (20)$$

in which the substitution of the variable

$$\bar{\rho}_0=\sigma\tilde{\tau}^{\nu/2} \quad (21)$$

leads to the equation

$$-a_{1m}\tilde{h}+\left[r_{m_\tau+2}s^{-2}\sigma+\frac{1}{6}s_0^3 s^{-1}u_{m_\tau+2}\sigma^3\right]\tilde{\tau}^{5\nu/2}=0. \quad (22)$$

It should be recalled that the factor $\tilde{\tau}^{5\nu/2}$ in Eq. (22) satisfies the relation $\tilde{\tau}^{5\nu/2}=\tilde{h}_c$ [see Eq. (7)]. This cubic equation has only one real solution, which assumes the approximate form[29]

$$\sigma\approx a_{1m}s^2 r_{m_\tau+2}^{-1}\tilde{h}/\tilde{h}_c \quad (23)$$

for $\tilde{h}\ll\tilde{h}_c$. For quantity $I_{m_\tau+2}^{(0)}$, we have

$$I_{m_\tau+2}^{(0)}=\sqrt{\frac{2\pi}{E''(\bar{\rho}_0)}}\exp[E_0(\bar{\rho}_0)],$$

where

$$E_0(\bar{\rho}_0)=N\left[a_{1m}\tilde{h}\bar{\rho}_0-\frac{1}{2}r_{m_\tau+2}s^{-2}\tilde{\tau}^{2\nu}\bar{\rho}_0^{2}-\frac{1}{4!}s_0^3 s^{-1}u_{m_\tau+2}\tilde{\tau}^{\nu}\bar{\rho}_0^{4}\right]. \quad (24)$$

Expression (24) corresponds to a microscopic analog of the Landau free energy. The relation (21) with solution of the equation (22) forms the equation of state of the system. With allowance for Eqs. (21), (23), and (7), the quantity $E_0(\bar{\rho}_0)$ can be presented as

$$E_0(\bar{\rho}_0)=NE_0^{(2)}\tilde{\tau}^{3\nu}\left(\frac{\tilde{h}}{\tilde{h}_c}\right)^2\left[1-E_0^{(4)}\left(\frac{\tilde{h}}{\tilde{h}_c}\right)^2\right]. \quad (25)$$

Here

$$E_0^{(2)}=a_{1m}^2\frac{s^2}{2r_{m_\tau+2}},\quad E_0^{(4)}=\frac{1}{12}a_{1m}^2 s^5 s_0^3\frac{u_{m_\tau+2}}{r_{m_\tau+2}^3}. \quad (26)$$

Thus, possessing the solution of Eq. (22) and taking into account the contribution from CV $\rho_{\mathbf{k}}$ with nonzero values of the wave vector ($0<k\leqslant B_{m_\tau+2}$), we can find the free energy of the regime of long-wave fluctuations

$$F_I=-\frac{1}{2}kTN_{m_\tau+2}\ln\pi+\frac{1}{2}kT\sum_{k\neq 0}^{B_{m_\tau+2}}\ln d_{m_\tau+2}(k)-kTE_0(\bar{\rho}_0). \quad (27)$$

Calculating the second term in Eq. (27) with the help of a transition to the spherical Brillouin zone and integration with respect to k, we arrive at the formula

$$F_I=-kTN_0 s^{-3}\tilde{\tau}^{3\nu}\left[\frac{1}{2}\ln\pi+(m_\tau+2)\ln s-\frac{1}{2}I_0''\right]-kTN\tilde{\tau}^{3\nu}E_0^{(2)}\left(\frac{\tilde{h}}{\tilde{h}_c}\right)^2\left[1-E_0^{(4)}\left(\frac{\tilde{h}}{\tilde{h}_c}\right)^2\right], \quad (28)$$

where

$$I_0''=\ln(D_0'+D_1')-\frac{2}{3}+2\frac{D_0'}{D_1'}-2\left(\frac{D_0'}{D_1'}\right)^{3/2}\arctan\left(\frac{D_1'}{D_0'}\right)^{1/2},$$

$$D_0'=\beta\Phi(0)[f_0(E_2-1)],\quad D_1'=2\beta\Phi(0)\left(\frac{\pi b}{s_0 c}\right)^2.$$

The contributions to free energy of the system from all regimes of fluctuations, obtained above at $T>T_c$ and $\tilde{h}\ll\tilde{h}_c$, allow us to write the total free energy, Eq. (11), in the form

$$F^{(+)}=-kTN\left\{\ln\cosh h'+l_0+\tilde{\tau}^{3\nu}\left[l_{1T}+l_{12T}\left(\frac{\tilde{h}}{\tilde{h}_c}\right)^2+l_{14T}\left(\frac{\tilde{h}}{\tilde{h}_c}\right)^4\right]+l_2\tilde{h}^2+l_3\tilde{\tau}+l_4\tilde{\tau}^2\right\}. \quad (29)$$

Here

$$l_0=\ln 2+\frac{1}{2}\beta_c\Phi(0)\bar{\Phi}+s_0^{-3}e_{0p},$$

$$l_{1T}=s_0^{-3}\left[e_{4p}+f_{mp}+s^{-3}\left(f_{p2}+\frac{1}{2}\ln\pi+\ln s-\frac{1}{2}I_0''\right)\right],$$

$$l_{12T}=E_0^{(2)},\quad l_{14T}=-E_0^{(2)}E_0^{(4)},$$

$$l_2=s_0^{-3}e_{3p},\quad l_3=s_0^{-3}e_{1p}-\frac{1}{2}\beta_c\Phi(0)\bar{\Phi}\frac{f_0}{c_{k1}^{(0)}},$$

$$l_4=s_0^{-3}e_{2p}+\frac{1}{2}\beta_c\Phi(0)\bar{\Phi}\left(\frac{f_0}{c_{k1}^{(0)}}\right)^2, \quad (30)$$

and β_c is the inverse critical temperature.

The average spin moment $\sigma^{(+)}=-(\partial F^{(+)}/\partial h)_T/N$ and the susceptibility $\chi^{(+)}=-(\partial^2F^{(+)}/\partial h^2)_T/N=(\partial\sigma^{(+)}/\partial h)_T$ can be found proceeding from the free energy $F^{(+)}$, Eq. (29). Their coefficients are expressed in terms of those of the free energy.

B. The case of $T>T_c$ and $\tilde{h}\gg\tilde{h}_c$

The expression (29) is valid for values of the field $\tilde{h}\ll\tilde{h}_c$ only. For describing the behavior of the system in the region II (see Fig. 1), we apply the similar technique using a few modifications. As in the previous subsection, the free energy consists of the several terms

$$F_h^{(+)}=F_0+F_{CR,h}+F_{TR,h}+F_{I,h}. \quad (31)$$

The main difference between expressions (31) and (11) is determined by the exit point n_p. In the case of the strong fields, the equality $n_p = n_h$ is valid. The field variable becomes dominant for the determination of the system critical behavior. Thus, for $\tilde{h} \gg \tilde{h}_c$, the term $F_{CR,h} = -kT \ln Q_0 - kT\sum_{n=1}^{n_p} \ln Q_n$ (where $n_p = n_h$) has the form

$$F_{CR,h} = -kTN_0[e_{0p} + e_{1p}\tilde{\tau} + e_{2p}\tilde{\tau}^2 + e_{3p}\tilde{h}^2 - \tilde{h}^{6/5}(F_{10} + F_{11}\varphi_h + F_{12}\varphi_h^2)], \tag{32}$$

where

$$\varphi_h = \tilde{\tau}\tilde{h}^{-1/p_0} = (\tilde{h}_c/\tilde{h})^{1/p_0}. \tag{33}$$

It should be noted that the coefficient $e_{4p} = -(F_{10} + F_{11} + F_{12})$ in Eq. (14) is formed of coefficients[29] in the last three terms from Eq. (32). The quantity φ_h is equal to unity for $\tilde{h} = \tilde{h}_c$ and reduces to zero with increasing the field. The free energy of the transition region $F_{TR,h} = -kTN_{n_h+1} \ln[\sqrt{2}Q(P^{(n_h)})]$ is defined as

$$F_{TR,h} = -kTN_0\tilde{h}^{6/5}[f_{p1c} - n_h \ln s + f_{p11c}\varphi_h - f_{p12c}\varphi_h^2]. \tag{34}$$

The coefficients

$$f_{p1c} = f_{p0} + \frac{1}{4}\ln u^*,$$

$$f_{p11c} = f_0\varphi_0^{-1/2}E_2^{-1}\left(\frac{1}{4}T_{42}^{(0)} + \sqrt{6}H_{211}\right),$$

$$f_{p12c} = \frac{1}{2}(f_0\varphi_0^{-1/2}E_2^{-1})^2 T_{42}^{(0)}\left(\frac{1}{4}T_{42}^{(0)} + \sqrt{6}H_{211}\right)$$

do not depend on the field variable. The constants f_{p0} and H_{211} are given in Ref. 29. As one can see, the contributions to free energy $F_{CR,h}$ and $F_{TR,h}$ [see Eqs. (32) and (34)] become essential in the region of the strong fields.

The quantity I_{n_h+1} in the term

$$F_{I,h} = -kT \ln I_{n_h+1}, \tag{35}$$

representing the contribution from the long-wave fluctuations, has the form similar to the integral I_{n_p+1} appearing in Eq. (4). In contrast to the calculation of the long-wave contribution in the case of the weak fields, we do not perform the additional step of integration after the exit from the critical-regime region since the coefficient in the quadratic term in the exponent of the effective measure density would be still negative. This quantity I_{n_h+1} is calculated using the substitution of variables

$$\rho_{\mathbf{k}} = \eta_{\mathbf{k}} + \sigma_h\sqrt{N}\delta_{\mathbf{k}}. \tag{36}$$

As a result, we obtain

$$I_{n_h+1} = \exp[E_0(\sigma_h)]\int (d\eta)^{N_{n_h+1}} \times\exp\Bigg[A_0\sqrt{N}\eta_0 - \frac{1}{2}\sum_{k\le B_{n_h+1}} d_h(k)\eta_{\mathbf{k}}\eta_{-\mathbf{k}} - \frac{1}{3!}b_h N_{n_h+1}^{-1/2}\sum_{\substack{\mathbf{k}_1,\ldots,\mathbf{k}_3\\ k_i\le B_{n_h+1}}} \eta_{\mathbf{k}_1}\cdots\eta_{\mathbf{k}_3}\delta_{\mathbf{k}_1+\cdots+\mathbf{k}_3} - \frac{1}{4!}a_h N_{n_h+1}^{-1}\sum_{\substack{\mathbf{k}_1,\ldots,\mathbf{k}_4\\ k_i\le B_{n_h+1}}} \eta_{\mathbf{k}_1}\cdots\eta_{\mathbf{k}_4}\delta_{\mathbf{k}_1+\cdots+\mathbf{k}_4}\Bigg]. \tag{37}$$

Here

$$A_0 = a_{1m}\tilde{h} - d_{n_h+1}(0)\sigma_h - \frac{1}{6}a_4^{(n_h+1)}\sigma_h^3\frac{N}{N_{n_h+1}},$$

$$d_h(k) = d_h(0) + 2\beta\Phi(0)b^2k^2,$$

$$d_h(0) = d_{n_h+1}(0) + \frac{1}{2}\sigma_h^2 a_4^{(n_h+1)}\frac{N}{N_{n_h+1}},$$

$$b_h = \sigma_h a_4^{(n_h+1)}\left(\frac{N}{N_{n_h+1}}\right)^{1/2}, \quad a_h = a_4^{(n_h+1)}, \tag{38}$$

and

$$E_0(\sigma_h) = N\left[a_{1m}\tilde{h}\sigma_h - \frac{1}{2}d_{n_h+1}(0)\sigma_h^2 - \frac{a_4^{(n_h+1)}}{4!}\sigma_h^4\frac{N}{N_{n_h+1}}\right]. \tag{39}$$

The quantity σ_h can be found using the condition $\partial E_0(\sigma_h)/\partial\sigma_h = 0$. Taking into account the relations $d_{n_h+1}(0) = s^{-2(n_h+1)}r_{n_h+1}$, $a_4^{(n_h+1)} = s^{-4(n_h+1)}u_{n_h+1}$, $N_{n_h+1} = Ns_0^{-3}s^{-3(n_h+1)}$, $s^{-a(n_h+1)} = \tilde{h}^{2a/5}$ and carrying out the substitution of the variable

$$\sigma_h = \sigma_0\tilde{h}^{1/5}, \tag{40}$$

we arrive at the cubic equation

$$\tilde{h}\left[a_{1m} - r_{n_h+1}\sigma_0 - \frac{1}{6}u_{n_h+1}\sigma_0^3 s_0^3\right] = 0. \tag{41}$$

As in the region of the weak fields, the equation (41) has only one real solution. This solution, nullifying the quantity A_0 from Eqs. (38), can be approximated by the following power series in $\varphi_h \ll 1$ ($\tilde{h} \gg \tilde{h}_c$) with known coefficients:[29]

$$\sigma_0 = \sigma_0^{(0)} + \sigma_0^{(1)}\varphi_h - \sigma_0^{(2)}\varphi_h^2. \tag{42}$$

After the substitution (36), the quadratic term in the expression of the exponent in Eq. (37) becomes positive and dominates for all $k \ne 0$. Thus, we can perform the integration in Eq. (37) with respect to the variables $\eta_{\mathbf{k}}$ except the variable η_0. Next step in our calculations lies in the return to the variable ρ_0 with the help of the relation $\eta_0=\rho_0-\sigma_h\sqrt{N}$. As a result, the quantity I_{n_h+1} assumes the following form:

$$I_{n_h+1}=\prod_{k\neq 0}^{B_{n_h+1}}\left(\frac{\pi}{d_h(k)}\right)^{1/2}\int d\rho_0 \exp\Bigg[\sqrt{N}a_{1m}\tilde{h}\rho_0-\frac{1}{2}d_{n_h+1}(0)\rho_0^2 - \frac{1}{4!}\frac{a_4^{(n_h+1)}}{N_{n_h+1}}\rho_0^4\Bigg]. \tag{43}$$

As in the case of $\tilde{h}\ll\tilde{h}_c$, performing the substitution $\rho_0=\sqrt{N}\rho_h$, which leads to the appearance of a sharp maximum of the integrand in

$$I_{n_h+1}=\prod_{k\neq 0}^{B_{n_h+1}}\left(\frac{\pi}{d_h(k)}\right)^{1/2}\sqrt{N}\int_{-\infty}^{+\infty}e^{-NE(\rho_h)}d\rho_h \tag{44}$$

due to the factor N in the exponent, and using the steepest-descent method, we find

$$I_{n_h+1}=\sqrt{\frac{2\pi}{E''(\bar{\rho}_h)}}e^{-NE(\bar{\rho}_h)}\prod_{k\neq 0}^{B_{n_h+1}}\left(\frac{\pi}{d_h(k)}\right)^{1/2}. \tag{45}$$

Here $\bar{\rho}_h$ is the extremum point of the expression

$$E(\rho_h)=-a_{1m}\tilde{h}\rho_h+\frac{1}{2}d_{n_h+1}(0)\rho_h^2+\frac{1}{4!}a_4^{(n_h+1)}\frac{N}{N_{n_h+1}}\rho_h^4, \tag{46}$$

which defines the fraction of free energy associated with the order parameter. This point is determined from the condition of extremum $\partial E(\rho_h)/\partial\rho_h=0$ or

$$a_{1m}\tilde{h}-d_{n_h+1}(0)\bar{\rho}_h-\frac{1}{6}a_4^{(n_h+1)}\frac{N}{N_{n_h+1}}\bar{\rho}_h^3=0. \tag{47}$$

Presenting the solution of Eq. (47) in the form

$$\bar{\rho}_h=\bar{\rho}_{h0}\tilde{h}^{1/5}, \tag{48}$$

we obtain the same equation (41), where the role of σ_0 plays

$$\bar{\rho}_{h0}=\bar{\rho}_{h0}^{(0)}+\bar{\rho}_{h0}^{(1)}\varphi_h-\bar{\rho}_{h0}^{(2)}\varphi_h^2. \tag{49}$$

Taking the logarithm, transiting to the spherical Brillouin zone, and integrating with respect to k in the expression (45), we arrive at the formula for the free energy of long-wave fluctuations

$$F_{I,h}=-kTN\left\{\left[(n_h+1)\ln s-\frac{1}{2}I_0'+\frac{1}{2}\ln\pi\right]s_0^{-3}+E_{00}\right\}\tilde{h}^{6/5}. \tag{50}$$

The quantity I_0' has the same form as I_0'' appearing in Eq. (28), but the coefficient D_0' is defined as

$$D_0'=r_{n_h+1}+\frac{1}{2}s_0^3u_{n_h+1}\sigma_0^2.$$

Expressions (49) and

$$r_{n_h+1}=r^*(1-\varphi_h),$$

$$u_{n_h+1}=u^*(1+f_0T_{42}^{(0)}\varphi_0^{-1/2}\varphi_h) \tag{51}$$

allow us to obtain the approximate relations for

$$E_{00}=a_{1m}\bar{\rho}_{h0}-\frac{1}{2}r_{n_h+1}\bar{\rho}_{h0}^2-\frac{1}{4!}s_0^3u_{n_h+1}\bar{\rho}_{h0}^4 \tag{52}$$

and for $F_{I,h}$, Eq. (50).

Collecting the contributions from all regimes of fluctuations according to Eq. (31), we can now write the complete expression for the free energy of the system in the case of $T>T_c$ and $\tilde{h}\gg\tilde{h}_c$

$$F_h^{(+)}=-kTN[\ln\cosh h'+l_0+\tilde{h}^{6/5}(l_1+l_{11}\varphi_h+l_{12}\varphi_h^2) + l_2\tilde{h}^2+l_3\tilde{\tau}+l_4\tilde{\tau}^2]. \tag{53}$$

Here

$$l_1=E_{00}^{(0)}+s_0^{-3}\left(-F_{10}+f_{p1c}+\frac{1}{2}\ln\pi+\ln s-\frac{1}{2}I_{00}'\right),$$

$$l_{11}=E_{00}^{(1)}+s_0^{-3}\left(-F_{11}+f_{p11c}-\frac{1}{2}I_{01}'\right),$$

$$l_{12}=-E_{00}^{(2)}+s_0^{-3}\left(-F_{12}-f_{p12c}-\frac{1}{2}I_{02}'\right). \tag{54}$$

The coefficients l_0, l_2, l_3, and l_4 are defined in Eqs. (30). The quantities $E_{00}^{(l)}$ and I_{0l}' are the components[29] of the approximate representations

$$E_{00}=E_{00}^{(0)}+E_{00}^{(1)}\varphi_h-E_{00}^{(2)}\varphi_h^2 \tag{55}$$

and

$$I_0'=I_{00}'+I_{01}'\varphi_h+I_{02}'\varphi_h^2, \tag{56}$$

respectively.

Differentiating the expression (53) for $F_h^{(+)}$ with respect to field, we can find the average spin moment and the susceptibility of the system. Using Eq. (53), the entropy $S_h^{(+)}=-(\partial F_h^{(+)}/\partial T)_h$ and the specific heat $C_h^{(+)}=-T(\partial^2F_h^{(+)}/\partial T^2)_h=T(\partial S_h^{(+)}/\partial T)_h$ of the system can be obtain also.

C. The cases of $T<T_c$, $\tilde{h}\ll\tilde{h}_c$, and $T<T_c$, $\tilde{h}\gg\tilde{h}_c$

A calculation technique for the thermodynamic characteristics of the system in the regions IV and III (see Fig. 1) is similar to that elaborated in the case of $T>T_c$ and $\tilde{h}\gg\tilde{h}_c$ (the previous section). In order to obtain explicit dependences for the thermodynamic functions, we assume that the point of exit n_p of the system from the critical-regime region is only a function of one of variables τ and h'. Such an assumption is valid for $\tilde{h}\ll\tilde{h}_c$ and $\tilde{h}\gg\tilde{h}_c$. We choose the variable, which has the stronger influence on the critical behavior of the system. Hence, there are two cases: the weak-field region $\tilde{h}\ll\tilde{h}_c$ is characterized by the equality

$$n_p = \mu_\tau = -\frac{\ln \tilde{\tau}_1}{\ln E_2} - 1, \quad (57)$$

where the quantity μ_τ defines the exit point by the temperature value, and the strong-field region $\tilde{h} \gg \tilde{h}_c$ is determined by

$$n_p = n_h = -\frac{\ln \tilde{h}}{\ln E_1} - 1, \quad (58)$$

where n_h is the exit point controlled by the field. Here $\tilde{\tau}_1 = -\tilde{\tau}$. For $T < T_c$, we have $\tilde{h}_c = \tilde{\tau}_1^{p_0}$.

Let us consider the case of $T < T_c$ and $\tilde{h} \ll \tilde{h}_c$. The contributions to free energy

$$F^{(-)} = F_0 + \tilde{F}_{CR} + \tilde{F}_{TR} + \tilde{F}_I \quad (59)$$

have the following forms:[30]

$$\tilde{F}_{CR} = -kTN_0(e_{0p} - e_{1p}\tilde{\tau}_1 + e_{2p}\tilde{\tau}_1^2 + e_{3p}\tilde{h}^2 + \tilde{e}_{4p}\tilde{\tau}_1^{3\nu}),$$

$$\tilde{F}_{TR} = -kTN_0\tilde{\tau}_1^{3\nu}(\tilde{f}_{p1} - \mu_\tau \ln s),$$

$$\tilde{F}_I = -kTN(\tilde{E}_{022}\tilde{\tau}_1^{3\nu} + a_{1m}\tilde{h}\bar{\eta}) - kTN_0\tilde{\tau}_1^{3\nu}\mu_\tau \ln s. \quad (60)$$

The term F_0 is defined in Eq. (12), and

$$\tilde{e}_{4p} = -(F_{10} - F_{11} + F_{12}),$$

$$\tilde{f}_{p1} = f_{p0} + H_{211}h_2^{(\mu_\tau)} + \tfrac{1}{4}\ln u_{\mu_\tau}. \quad (61)$$

The coefficient

$$\tilde{E}_{022} = -\frac{1}{2}r_{\mu_\tau+1}\bar{\eta}_0^2 - \frac{1}{4!}u_{\mu_\tau+1}s_0^3\bar{\eta}_0^4 + \frac{1}{2}s_0^{-3}\ln\pi + \left(\ln s - \frac{1}{2}\tilde{I}_0'\right)s_0^{-3}$$

as well as the quantities $\bar{\eta}_0$ and $\tilde{I}_0'$ can by given in the form of the approximate representations[30]

$$\tilde{E}_{022} = \tilde{E}_{022}^{(0)} - \tilde{E}_{022}^{(1)}\frac{\tilde{h}}{\tilde{h}_c} - \tilde{E}_{022}^{(2)}\left(\frac{\tilde{h}}{\tilde{h}_c}\right)^2,$$

$$\bar{\eta}_0 = \bar{\eta}_0^{(0)} - \bar{\eta}_0^{(1)}\frac{\tilde{h}}{\tilde{h}_c} - \bar{\eta}_0^{(2)}\left(\frac{\tilde{h}}{\tilde{h}_c}\right)^2,$$

$$\tilde{I}_0' = \tilde{I}_{00}' - \tilde{I}_{01}'\frac{\tilde{h}}{\tilde{h}_c} + \tilde{I}_{02}'\left(\frac{\tilde{h}}{\tilde{h}_c}\right)^2. \quad (62)$$

The quantity $\bar{\eta}_0$ characterizes the solution $\bar{\eta} = \bar{\eta}_0\tilde{\tau}_1^{\nu/2}$ of the corresponding cubic equation

$$a_{1m}\tilde{h} - d_{\mu_\tau+1}(0)\bar{\eta} - \frac{1}{6}a_4^{(\mu_\tau+1)}\frac{N}{N_{\mu_\tau+1}}\bar{\eta}^3 = 0.$$

On the basis of Eq. (59), we obtain the total free energy

$$F^{(-)} = -kTN\left\{\ln\cosh h' + l_0 + \tilde{\tau}_1^{3\nu}\left[l_{1\mu} + l_{11\mu}\frac{\tilde{h}}{\tilde{h}_c} + l_{12\mu}\left(\frac{\tilde{h}}{\tilde{h}_c}\right)^2\right] + l_2\tilde{h}^2 - l_3\tilde{\tau}_1 + l_4\tilde{\tau}_1^2\right\}. \quad (63)$$

Here

$$l_{1\mu} = \tilde{E}_{022}^{(0)} + s_0^{-3}(\tilde{e}_{4p} + \tilde{f}_{p1}),$$

$$l_{11\mu} = a_{1m}\bar{\eta}_0^{(0)} - \tilde{E}_{022}^{(1)}, \quad l_{12\mu} = -a_{1m}\bar{\eta}_0^{(1)} - \tilde{E}_{022}^{(2)}. \quad (64)$$

Let us describe the results of the calculation of the total free energy $F_h^{(-)}$ in the case of $T < T_c$ and $\tilde{h} \gg \tilde{h}_c$. The contributions to free energy of the system as well as the total free energy are found by analogy with the above-presented case of $T > T_c$ and $\tilde{h} \gg \tilde{h}_c$. Using the equalities[30]

$$\tilde{h}E_1^{n_h+1} = 1, \quad E_2^{n_h+1} = \tilde{h}^{-1/p_0},$$

$$E_3^{n_h+1} = \tilde{h}^{-1/5}, \quad N_{n_h+1} = N_0\tilde{h}^{6/5}$$

and taking into account the designations $\tilde{\tau}_1 = -\tilde{\tau}$ and $\varphi_h = \tilde{\tau}_1\tilde{h}^{-1/p_0} = (\tilde{h}_c/\tilde{h})^{1/p_0}$, we arrive at the analogous expressions, but signs before the terms proportional to φ_h in Eqs. (32), (34), (42), (49), (51), (53), (55), and (56) must be replaced by opposite signs.

Let us now write the final formulas for the total free energy $F^{(\pm)}$, order parameter (average spin moment) $\sigma^{(\pm)}$, susceptibility $\chi^{(\pm)}$, entropy $S_h^{(\pm)}$, and specific heat $C_h^{(\pm)}$ of a 3D uniaxial magnet. Explicit expressions assume the following forms:

(a) in the weak-field regions ($\tilde{h} \ll \tilde{h}_c$)

$$F^{(+)} = -kTN\left[\ln\cosh h' + l_0 + l_{1T}\tilde{\tau}^{3\nu} + l_{12T}\tilde{\tau}^{3\nu}\left(\frac{\tilde{h}}{\tilde{h}_c}\right)^2 + l_{14T}\tilde{\tau}^{3\nu}\left(\frac{\tilde{h}}{\tilde{h}_c}\right)^4 + l_2\tilde{h}^2 + l_3\tilde{\tau} + l_4\tilde{\tau}^2\right],$$

$$F^{(-)} = -kTN\left[\ln\cosh h' + l_0 + l_{1\mu}|\tilde{\tau}|^{3\nu} + l_{11\mu}|\tilde{\tau}|^{3\nu}\frac{\tilde{h}}{\tilde{h}_c} + l_{12\mu}|\tilde{\tau}|^{3\nu}\left(\frac{\tilde{h}}{\tilde{h}_c}\right)^2 + l_2\tilde{h}^2 - l_3|\tilde{\tau}| + l_4|\tilde{\tau}|^2\right],$$

$$\sigma^{(+)} = \tanh h' + \sigma_0 h' + \sigma_2^{(+)}\tilde{\tau}^{\nu/2}\frac{h'}{\tilde{h}_c} + \sigma_3^{(+)}\tilde{\tau}^{\nu/2}\left(\frac{h'}{\tilde{h}_c}\right)^3,$$

$$\sigma^{(-)} = \tanh h' + \sigma_0 h' + \sigma_1^{(-)}|\tilde{\tau}|^{\nu/2} + \sigma_2^{(-)}|\tilde{\tau}|^{\nu/2}\frac{h'}{\tilde{h}_c},$$

$$\chi^{(\pm)} = \beta[1 - \tanh^2 h' + \chi_0 + \chi_1^{(\pm)}|\tilde{\tau}|^{-\gamma}], \quad \gamma = 2\nu; \quad (65)$$

(b) in the strong-field regions ($\tilde{h} \gg \tilde{h}_c$)

$$F_h^{(+)} = -kTN\left[\ln\cosh h' + l_0 + l_1\tilde{h}^{6/5} + l_{11}\tilde{h}^{6/5}\left(\frac{\tilde{h}_c}{\tilde{h}}\right)^{1/p_0} + l_{12}\tilde{h}^{6/5}\left(\frac{\tilde{h}_c}{\tilde{h}}\right)^{2/p_0} + l_2\tilde{h}^2 + l_3\tilde{\tau} + l_4\tilde{\tau}^2\right],$$

$$F_h^{(-)} = -kTN\left[\ln\cosh h' + l_0 + l_1\tilde{h}^{6/5} - l_{11}\tilde{h}^{6/5}\left(\frac{\tilde{h}_c}{\tilde{h}}\right)^{1/p_0} + l_{12}\tilde{h}^{6/5}\left(\frac{\tilde{h}_c}{\tilde{h}}\right)^{2/p_0} + l_2\tilde{h}^2 - l_3|\tilde{\tau}| + l_4|\tilde{\tau}|^2\right],$$

$$\sigma_h^{(\pm)} = \tanh h' + \sigma_{h_0}h' + \sigma_{h_1}(h')^{1/5} + \sigma_{h_2}\tilde{\tau}(h')^{1/5-1/p_0} + \sigma_{h_3}\tilde{\tau}^2(h')^{1/5-2/p_0},$$

$$\chi_h^{(\pm)} = \beta[1 - \tanh^2 h' + \chi_{h_0} + \chi_{h_1}(h')^{-4/5} + \chi_{h_2}\tilde{\tau}(h')^{-4/5-1/p_0} + \chi_{h_3}\tilde{\tau}^2(h')^{-4/5-2/p_0}],$$

$$S_h^{(\pm)} = kN[s_{h_0} + s_{h_1}\tilde{h}^{\psi} + s_{h_2}\tilde{\tau}\tilde{h}^{-\varphi} + s_{h_3}\tilde{\tau}],$$

$$C_h^{(\pm)} = kN[c_{h_0} + c_{h_1}\tilde{h}^{-\varphi}],$$

$$\psi = \frac{6}{5} - \frac{1}{p_0}, \quad \varphi = -\left(\frac{6}{5} - \frac{2}{p_0}\right). \tag{66}$$

Here the subscript h corresponds to strong fields. The + and − signs refer to temperatures above and below T_c, respectively. For each of four regions of the temperature and field values ($T>T_c$ and $\tilde{h}\ll\tilde{h}_c$; $T>T_c$ and $\tilde{h}\gg\tilde{h}_c$; $T<T_c$ and $\tilde{h}\ll\tilde{h}_c$; $T<T_c$ and $\tilde{h}\gg\tilde{h}_c$), we obtain and solve some equation corresponding to the equation of state of the system.The coefficients in the expressions for the total free energy are defined by the relations (30), (54), and (64). For the remaining coefficients in Eqs. (65) and (66), we have

$$\sigma_0 = 2l_2 f_0^{-2}, \quad \sigma_2^{(+)} = 2l_{12T}f_0^{-2}, \quad \sigma_3^{(+)} = 4l_{14T}f_0^{-4},$$

$$\sigma_1^{(-)} = l_{11\mu}f_0^{-1}, \quad \sigma_2^{(-)} = 2l_{12\mu}f_0^{-2},$$

$$\chi_0 = \sigma_0, \quad \chi_1^{(\pm)} = \sigma_2^{(\pm)},$$

$$\sigma_{h_0} = \sigma_0, \quad \sigma_{h_1} = \tfrac{6}{5}l_1 f_0^{-6/5},$$

$$\sigma_{h_2} = \left(\frac{6}{5} - \frac{1}{p_0}\right)l_{11}f_0^{-(6/5-1/p_0)},$$

$$\sigma_{h_3} = \left(\frac{6}{5} - \frac{2}{p_0}\right)l_{12}f_0^{-(6/5-2/p_0)},$$

$$\chi_{h_0} = \sigma_{h_0}, \quad \chi_{h_1} = \tfrac{1}{5}\sigma_{h_1},$$

$$\chi_{h_2} = \left(\frac{1}{5} - \frac{1}{p_0}\right)\sigma_{h_2}, \quad \chi_{h_3} = \left(\frac{1}{5} - \frac{2}{p_0}\right)\sigma_{h_3},$$

$$s_{h_0} = l_0 + l_3\frac{c_{k1}^{(0)}}{f_0}, \quad s_{h_1} = l_{11}\frac{c_{k1}^{(0)}}{f_0},$$

$$s_{h_2} = 2l_{12}\frac{c_{k1}^{(0)}}{f_0}, \quad s_{h_3} = 2\left(l_3 + l_4\frac{c_{k1}^{(0)}}{f_0}\right),$$

$$c_{h_0} = s_{h_3}\frac{c_{k1}^{(0)}}{f_0}, \quad c_{h_1} = s_{h_2}\frac{c_{k1}^{(0)}}{f_0}. \tag{67}$$

As is seen from Eqs. (65), the equality $\sigma^{(+)}=0$ is valid for $h'=0$. In the case of $\tilde{h}\ll\tilde{h}_c$, the last term in the expression for $\sigma^{(+)}$ can be neglected. As a result, the average spin moment $\sigma^{(+)}$ is proportional to h'. The third term, corresponding to the moment induced by the field h', determines the main contribution to $\sigma^{(+)}$. The first and second terms in the expression for $\sigma^{(-)}$ are smaller than the third and fourth terms. The third term of $\sigma^{(-)}$ corresponds to the spontaneous moment of the system (for $h'=0$, we have $\sigma^{(-)}=\sigma_1^{(-)}|\tilde{\tau}|^{\nu/2}$), while the fourth term characterizes the moment induced by the field. In the case of $\tilde{h}\gg\tilde{h}_c$, the main contribution to $\sigma_h^{(\pm)}$ [see Eqs. (66)] is ensured by the term proportional to $(h')^{1/5}$. The term $s_{h_1}\tilde{h}^{\psi}$ of the entropy $S_h^{(\pm)}$ is more significant than the term $s_{h_2}\tilde{\tau}\tilde{h}^{-\varphi}$. The leading term of the specific heat $C_h^{(\pm)}$ is proportional to $\tilde{h}^{-\varphi}$.

Using the explicit expressions presented here, we can investigate the field dependences of thermodynamic characteristics of a 3D Ising-like system for various values of the temperature τ. Our calculations are illustrated by the case of $s_0=2$, $b/c=0.3$, $\bar{\Phi}=0.05$. The field dependences of the average spin moment and susceptibility for $T>T_c$ and $T<T_c$ are demonstrated in Fig. 2 (the region of weak fields $\tilde{h}\leqslant 0.1\tilde{h}_c$) and in Fig. 3 (the region of strong fields $\tilde{h}\geqslant 10\tilde{h}_c$). The plots in Fig. 2 show that the above-mentioned dependences in the weak fields $\tilde{h}\ll\tilde{h}_c$ are linear. Our value for the universal ratio of the susceptibility amplitudes $\chi_1^{(+)}/\chi_1^{(-)}=l_{12T}/l_{12\mu}\approx 5.85$, determining the ratio of the susceptibilities at temperatures above and below T_c in the weak-field regions [Figs. 2(c) and 2(d)], agrees with the other authors' data around 5 for $h=0$ (see, for example, Ref. 16). As we see from Fig. 3, the average spin moment and susceptibility of the system in the strong fields $\tilde{h}\gg\tilde{h}_c$ depend on the temperature weakly (the scatter of curves for various values of the temperature is small). Our estimates for the average spin moment for h' from the regions of the small and large values of the field[30] accord with the results[31,32] obtained with the help of Monte Carlo simulations for the Ising model on a simple cubic lattice with the interaction between nearest neighbors. In our calculations, this interaction is determined by the set of parameters $s_0=2$, $b/c=0.3$, $\bar{\Phi}=0.092$ (Ref. 25). Tables I (the high-temperature region, $\tau=0.0050$) and II (the low-temperature region, $\tau=-0.0047$) present our results and numerical data from Refs. 31 and 32 corresponding to peaks of

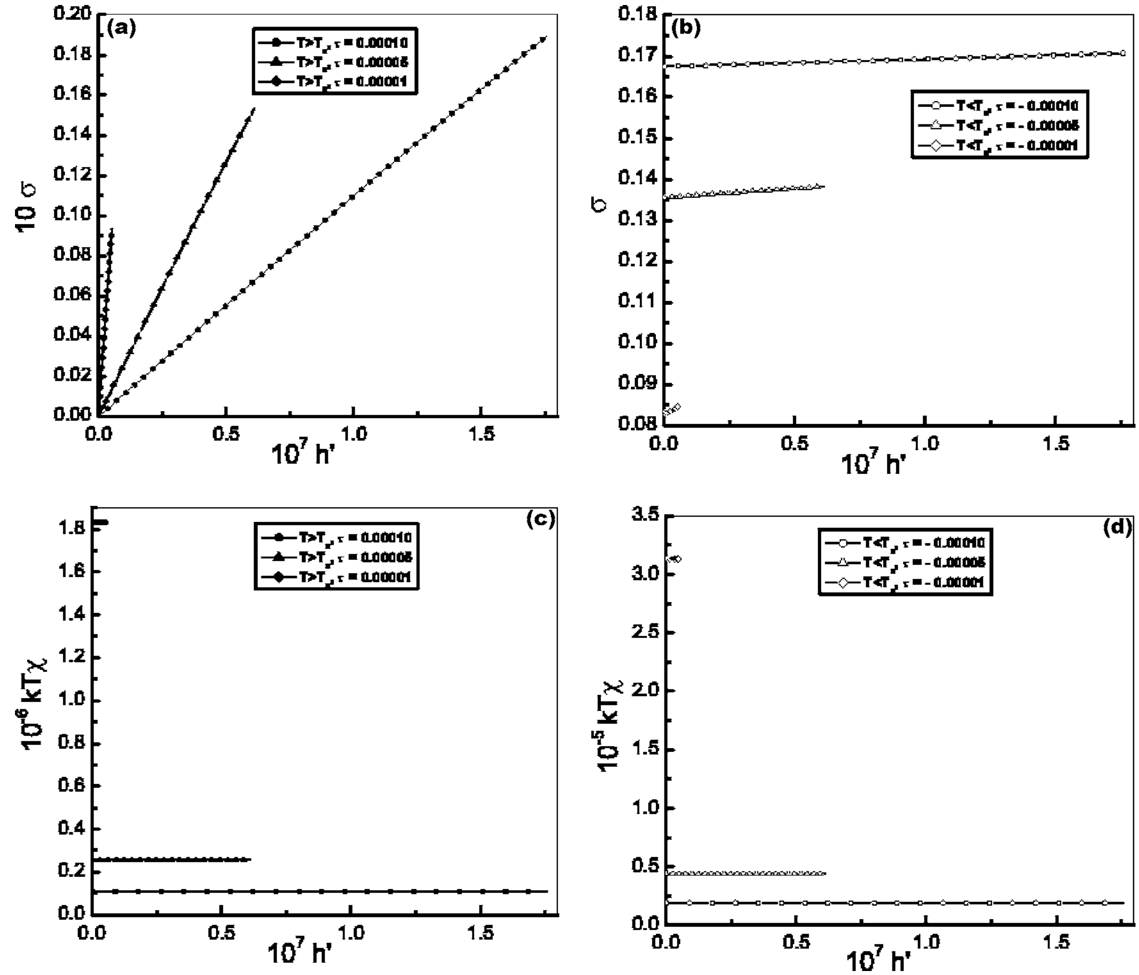

FIG. 2. The case of weak fields $\tilde{h} \leqslant 0.1\tilde{h}_c$ [or $\tilde{h} \leqslant 0.1|\tilde{\tau}|^{5\nu/2}$, see Eq. (7)]. Field dependences of the average spin moment and susceptibility of the system for $\tau = \pm 0.000\,01$, $\pm 0.000\,05$, and $\pm 0.000\,10$. Parts (a) and (b) of the figure display the average spin moment at temperatures above and below T_c, respectively. Parts (c) and (d) of the figure show the susceptibility at $T > T_c$ and $T < T_c$, respectively. Here $h' = h/(kT) = f_0\tilde{h}$ is the dimensionless field, f_0 characterizes one of the fixed-point coordinates.

the probability distributions of the order parameter. The behavior of the field dependence of the average spin moment $\sigma_h^{(T_c)}$ at $T = T_c$ is shown in Fig. 4. The curve 1 demonstrates our results. The dashed curve 2 corresponds to the results obtained in Ref. 19, where the 3D Ising model in an external magnetic field near the critical point is also studied by Monte Carlo simulations.

Our approach to obtaining explicit expressions for the thermodynamic characteristics of 3D Ising-like systems in an external field is approximate. Some discrepancy in Tables I and II, and Fig. 4 between Monte Carlo results and our overestimated values can be connected with our approximate calculation of the partition function confined to the asymmetric ρ^4 model. Indeed, the Ising model corresponds to the ρ^{2m} model approximation, where the order of the model is $2m \geqslant 4$ (see, for example, Ref. 22). We suppose that the results for the more complicated asymmetric ρ^6 model will be agreed more closely with the presented Monte Carlo data than the corresponding estimates for the asymmetric ρ^4 model. The previous study[22] shows that the sextic measure density with even powers of the variable in the exponent only (the ρ^6 model) provides a better quantitative description of critical properties of a 3D Ising-like system as compared with the quartic measure density. The average spin moment for the ρ^6 model[20,22,33,34] is smaller than that for the ρ^4 model and hence is in better agreement with Monte Carlo data.

Our expressions for the thermodynamic characteristics in the form of a power series in $\tilde{h}/\tilde{h}_c$ (the region of weak fields) and $\varphi_h = |\tilde{\tau}|\tilde{h}^{-1/p_0} = (\tilde{h}_c/\tilde{h})^{1/p_0}$ (the region of strong fields) are valid for $\tilde{h} \ll \tilde{h}_c$ and $\tilde{h} \gg \tilde{h}_c$, respectively. Therefore, for comparison of the results, the estimates in Tables I and II are given for the smallest field values and for the largest field values from Refs. 31 and 32. At the weak field $\tilde{h} = 0.205\tilde{h}_c$, our value for $\sigma^{(+)}$ is in good agreement with the corresponding estimate from Ref. 31 (see Table I). But the difference between the limiting field $\tilde{h}_c$ and the fields $\tilde{h} = 4.251\tilde{h}_c$ (Table I), $\tilde{h} = 0.746\tilde{h}_c$, $\tilde{h} = 4.337\tilde{h}_c$ (Table II) can be still deficient for the satisfactory application of our expansions. This is also the possible reason of some discrepancy between our data for the order parameter and numerical results from Refs. 31 and 32. In the region of $\tilde{h} \approx \tilde{h}_c$, where the scaling variable is of the order of unity and a power series of the scaling functions are not effective, the influences of the temperature and field

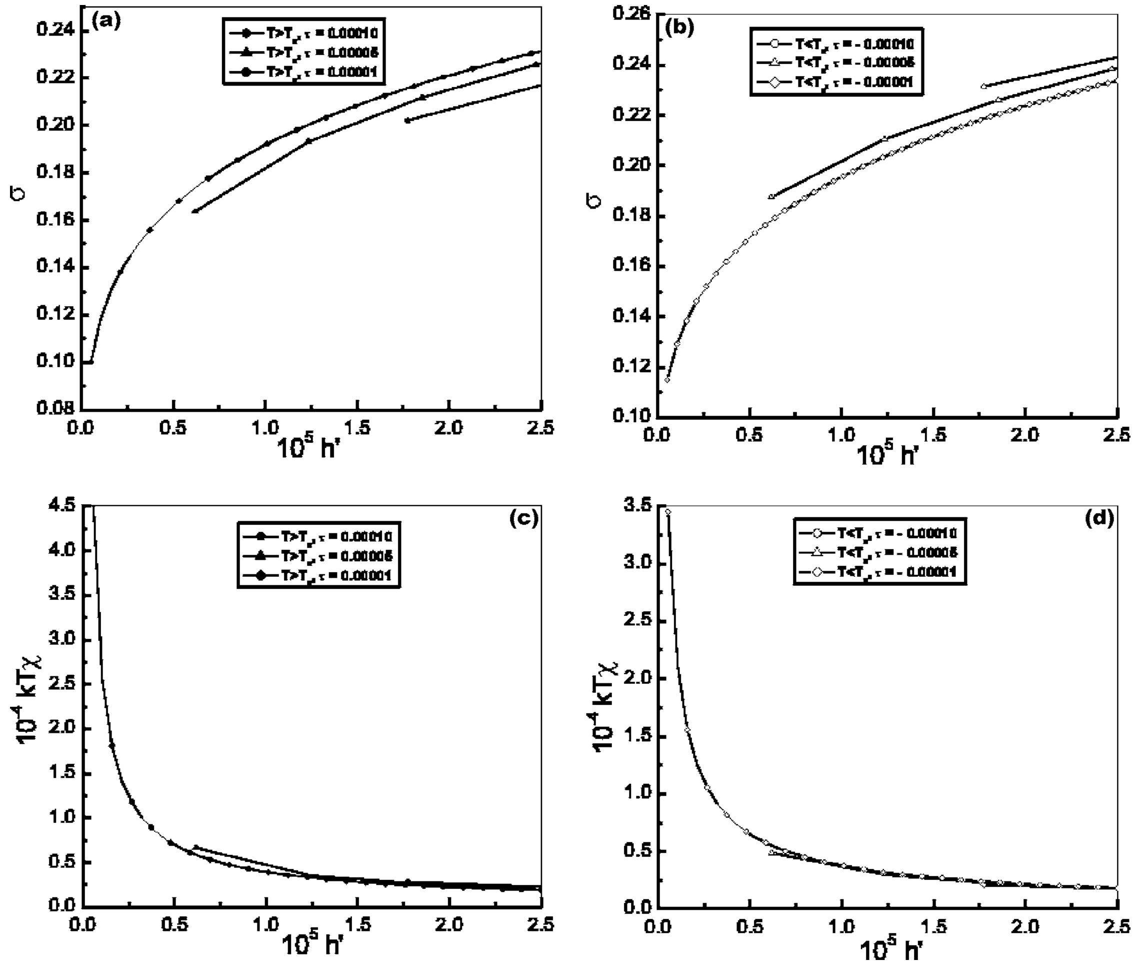

FIG. 3. The case of strong fields $\tilde{h} \geqslant 10\tilde{h}_c$ [or $\tilde{h} \geqslant 10|\tilde{\tau}|^{5\nu/2}$, see Eq. (7)]. Behavior of the average spin moment and susceptibility of the system with increasing field h' for various values of τ. Notation is the same as in Fig. 2.

on the critical behavior of the system are equally important. Then, the point of exit of the system from the critical regime is a function of both the temperature and field variables. However, in this case, we cannot single out the temperature and field explicitly in the complete expressions for the thermodynamic characteristics. Our main task is to obtain explicit expressions for the total free energy and other thermodynamic characteristics as functions of the temperature, field, and microscopic parameters of the system. This task was performed by using the approximation that the exit point was only a function of one of the temperature and field variables. Such an approximation is valid in the weak-field ($\tilde{h} \ll \tilde{h}_c$) and strong-field ($\tilde{h} \gg \tilde{h}_c$) regions. One chooses the variable, which has a stronger effect on the critical behavior than the other one.

It should be noted that the present paper supplements the earlier works,[33,34] in which the effect of an external field on the critical behavior of a 3D Ising magnet was studied using the above-mentioned sextic measure density with even pow-

TABLE I. Estimates for the order parameter of a 3D Ising-like system in the presence of an external field for the temperature from the region above T_c (τ=0.0050). The values of $\sigma^{(+)}$ and $\sigma_h^{(+)}$ refer to fields $\tilde{h}$=0.205$\tilde{h}_c$ (or h'=0.00013) and $\tilde{h}$=4.251$\tilde{h}_c$ (or h'=0.0027), respectively.

Average spin moment $T>T_c$	Present work, the collective variables method	Ref. 31, the Monte Carlo method, $L^3=58^3$
$\sigma^{(+)}$	0.110	0.100
$\sigma_h^{(+)}$	0.500	0.359

TABLE II. Numerical estimates of the order parameter of the system for the temperature from the region below T_c (τ=−0.0047). The case of a zero external field is presented by $\sigma_{h=0}^{(-)}$. The values of $\sigma^{(-)}$ and $\sigma_h^{(-)}$ correspond to fields $\tilde{h}$=0.746$\tilde{h}_c$ (or h'=0.00043) and $\tilde{h}$=4.337$\tilde{h}_c$ (or h'=0.0025), respectively.

Average spin moment $T<T_c$	Present work, the collective variables method	Ref. 32, the Monte Carlo method, $L^3=74^3$
$\sigma_{h=0}^{(-)}$	0.516	0.289
$\sigma^{(-)}$	0.591	0.335
$\sigma_h^{(-)}$	0.624	0.420

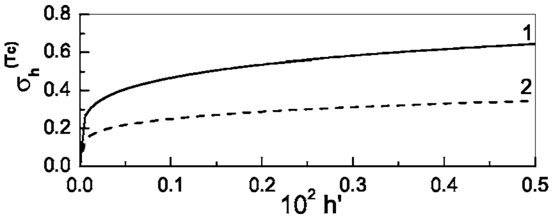

FIG. 4. The average spin moment of the system at $T=T_c$ (curve 1) as a function of the field h'. Curve 2 corresponds to the results obtained in Ref. 19.

ers. In the absence of an external field, maxima points of the non-Gaussian (quartic) distribution are investigated on the basis of Euler equations in Ref. 12. The effective distributions, which permit deriving the equation of state and plotting the temperature (for various values of the field) and field (for various values of the temperature) dependences of the order parameter in the ρ^6 model approximation, are analyzed in Refs. 33 and 34. These papers contain the comparison with the case of a zero external field. In the presence of any external field, the average spin moment of the system becomes different from zero at every temperature. The discrete phase-transition point disappears; the transition is smeared. The temperature dependence of the order parameter presented in Refs. 33 and 34 graphically makes it possible to see and to estimate numerically the smear of the phase transition for some given values of the field, while the field dependence of the order parameter demonstrates the first-order phase-transition pattern at $\tau<0$ when the field passes through zero value. Plots[33,34] of the mentioned dependences allow one to obtain the information about the location of the phase boundary, metastable states, and thermodynamically stable states of the system.

IV. CONCLUSIONS

Considerable progress in the study of phase transitions and critical phenomena is made by using the perturbation theory and numerical methods. But the perturbative framework is generally not well adapted to the variety of the systems.[35] Numerical methods are also not quite appropriated for the complete description of some models, especially of more complex ones. Since the simulations have to be performed for systems of relatively small sizes, the system is not very deep into scaling region. Therefore, the nonperturbative RG theory is some alternative way for the following progress in the research of the large systems with the collective behavior. Such investigations are performed using the Wilson-Polchinski exact RG equation (see, for example, Refs. 35–38). Our method, which employs the nonperturbative RG theory, is similar to the Wilson-Polchinski RG method (integration on fast modes and construction of an effective theory for slow modes).

The analytic method proposed for describing a 3D uniaxial magnet near the critical point by using the asymmetric ρ^4 model takes into account the simultaneous effect of the temperature and field on the behavior of the system. Explicit expressions for the total free energy and other thermodynamic characteristics of the system are presented as functions of the temperature and field in the regions of the weak and strong fields for temperatures above and below T_c. The description is based on the first principles of statistical physics and is naturally realized without any general assumptions in terms of the variables, which coincide with the accepted choice of the arguments for scaling functions in accordance with the scaling theory. As is seen from relations (65) and (66) as well as from Figs. 2 and 3, the system behavior in the weak fields is described in general by the temperature variable, but the role of the temperature variable in the case of the strong fields is not dominant. For the strong fields, the leading terms of the thermodynamic characteristics are defined by the field variable.

The CV method permits to calculate the partition function of the system and to obtain not only the universal quantities (critical exponents) but also analytic expressions for the thermodynamic characteristics. The methods existing at present make it possible to calculate universal quantities to quite a high degree of accuracy. The advantage of the proposed method lies in the possibility to perform the calculations on the microscopic level without any adjustable parameters that makes this method useful in describing the critical behavior of a wide class of 3D systems.[11,15] The obtained expressions for the thermodynamic functions allow us to analyze their dependence on microscopic parameters of the system (the lattice constant and parameters of the interaction potential). The main benefit of these expressions is the presence of relations connecting their coefficients with microscopic parameters and the coordinates of the fixed point.

We hope that the proposed method as well as our explicit representations and plots may provide useful benchmarks in studying the dependence of the thermodynamic functions of 3D Ising-like systems on the temperature, field, parameters of the interaction potential, and characteristics of the crystal lattice.

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