Effect of spontaneous spin depopulation on the ground-state energy of a two-dimensional spintronic system

Godfrey Gumbs
Department of Physics and Astronomy, Hunter College of the City University of New York, 695 Park Avenue, New York, New York 10021, USA

(Received 17 October 2005; revised manuscript received 8 February 2006; published 17 April 2006)

We calculate the r_s expansion of the ground-state energy per particle of a two-dimensional electron gas with spin-orbit interaction induced by the Rashba coupling. At high areal electron density, we obtain the energy for noninteracting electrons, their exchange energy, and the lowest-order approximation for the correlation energy. A closed-form expression is obtained for the energy of noninteracting electrons. However, we must calculate the exchange and correlation energy numerically. As the density is increased and r_s decreases, the ground-state energy changes rapidly at some value $r_s=r_s^*$ and the Fermi energy E_F changes from negative to positive. When $E_F<0$, only the lower spin subband is occupied. An interesting effect occurs in the presence of electron-electron interaction as E_F increases through $E=0$ and the upper spin subband suddenly gets a finite population rather than increasing gradually.

DOI: 10.1103/PhysRevB.73.165315 PACS number(s): 71.10.Pm, 71.70.Ej, 73.20.Mf

I. INTRODUCTION

In recent years, both theoreticians and experimentalists have been investigating the effect of spin-orbit interaction (SOI) due to the linear Rashba and Dresselhaus splitting on the physical properties of quantum systems.[1–16] These include both spectral and transport properties of low-dimensional semiconductor structures such as the two-dimensional electron system (2DES), quantum dots, and carbon nanotubes.[17–32] The SOI can be reliably controlled in experiments on these systems and made relatively strong. It has been demonstrated that the SOI may give rise to interesting behavior in both the noninteracting and interacting properties of low-dimensional semiconductor structures. The mechanism for the Rashba and Dresselhaus SOI is a unique feature of the spatial asymmetry.[1,2] For the Dresselhaus effect, the crystal symmetry is broken at the heterointerface, resulting in a significant modification of the mechanism of SOI by the crystal field. Although the Dresselhaus spin splitting may compete with the Rashba splitting, there seems to be more control over the latter which could be made to dominate in systems such as InGaAs/InAlAs heterostructures, InAs/GaSb and AlSb/InAs/AlSb quantum wells, and Si metal-oxide-semiconductor field-effect transistors. Furthermore, because the Rashba SOI seems more promising for potential device applications, we will disregard the Dresselhaus mechanism in this paper.

All the potential profiles that can be produced by various flexible means of band engineering have electrostatic origin and give rise to a local electric field. For some confining potentials, the average electric field within the quantum well is different from zero and electrons in the quantum well experience a finite electric field directed along the normal to the plane of the 2DES. This electric field may be strong enough to give a SO coupling as large as $\sim$150 meV Å in experimentally achievable heterostructures.[15] We use a large value for the SO coupling to demonstrate the effect it can have on the ground-state energy. Strong SO coupling may also be examined for the dependence of the spin susceptibility on electron density.[33,34]

The paper by Dharma-wardana[35] does not include SOI in the calculations. Consequently, Ref. 35 does not yield the effects we report here. The triangular quantum well is an obvious and typical example of an asymmetric potential. The presence of this *interface* electric field suggests that for such systems, there should be an additional (compared with the bulk three-dimensional crystal) mechanism of SOI associated with this interface field. As a matter of fact, the average electric field couples to the in-plane momentum **p** to give rise to SOI. This mechanism was introduced by Bychkov and Rashba[1] in 1984 and is usually referred to as the Rashba or quantum well coupling.

There is SOI in the 2DES in the GaAs/AlGaAs sample used in the experiments of Zhu *et al.*[22] and Tan *et al.*[34] Neither Ref. 33 nor Ref. 35 includes the effect due to SOI in their calculations. However, it seems possible to explain the experimental data with the theory of De Palo *et al.*[33] and of Dhrama-wardana.[35] One possible explanation for this could be that the spin-orbit coupling parameter for the sample used in the experiments is small. As a matter of fact, we do not see the effect displayed in the results of our calculations presented below for small Rashba parameter ($\alpha_R \sim 1.0$ meV Å). Therefore, additional experiments on heterojunction-insulated gate field-effect transistor samples having large SOI such as InAs/InGaAs should be conducted over a wide range of electron densities to explore the predictions we are making in this paper.

II. THE MODEL HAMILTONIAN

The spin-orbit Hamiltonian can be obtained from the Dirac equation in an external electromagnetic field, described by a vector potential **A** and a scalar potential Φ, by taking the nonrelativistic limit up to terms quadratic in v/c inclusive. This limit can be obtained in two different ways: either by direct expansion of the Dirac equation in powers of

v/c or by using the asymptotically exact Foldy-Wouthuysen formula. It may be shown that the spin-orbit Hamiltonian arising from the electrostatic confinement is given by

$$H_{SO} = \frac{\hbar}{4m^{*2}c^2}[\nabla V(\mathbf{r}) \times \mathbf{p}] \cdot \vec{\sigma} \tag{1}$$

where $V(\mathbf{r}) = -e\Phi(\mathbf{r})$ and $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is the vector of Pauli spin matrices. This Hamiltonian includes mechanisms arising from both the electric dipole moment and Thomas precession. In general, Eq. (1) consists of three terms arising from the spatial confinement. The z component leads to the Rashba term for the quantum well. If, in addition, there is lateral confinement, then this may lead to additional terms in the calculation. Taking into account the electric field within the quantum well as an average $\bar{\mathbf{E}}$ whose direction is perpendicular to the interface of the heterojunction, the spin-orbit Hamiltonian (1) can be rewritten for the Rashba coupling as

$$H_{SO} = \frac{\Delta_R}{\hbar}(\vec{\sigma} \times \mathbf{p})_z, \tag{2}$$

where the z component of the momentum does not contribute to (2) since in the stationary state there is no transfer of electrons across the interface. The constant Δ_R contains all the universal constants from (1) and it is proportional to the interface electric field. Therefore, the contribution to the total electron Hamiltonian from the Rashba SOI is controlled by the value of Δ_R. For different systems, Δ_R takes on values in the range $10 \lesssim \Delta_R \lesssim 150$ meV Å.[15] In our numerical calculations, we use the upper limit of this range for Δ_R for illustrative purposes.

Before proceeding to the presentation of our calculations, we note that the form of the Hamiltonian in Eq. (1) may be applied to the Dresselhaus term. In a bulk 3D semiconductor, $V(\mathbf{r})$ arises from the periodic crystal potential. All multicomponent III-V semiconductors lack inversion symmetry. Dresselhaus[2] has shown that this leads to a SO-induced splitting of the conduction band into two subbands and that the magnitude of this splitting is proportional to the cube of the wave number k. For a heterostructure, the crystal symmetry is broken at the interface where 2D electrons or holes are confined in a quantum well. Consequently, the host 3D crystal cannot be considered as ideal. In fact, symmetry of the underlying crystal is lowered by the reduction of the dimensionality and gives rise to an additional term in the energy which is linear in the in-plane wave vector $\mathbf{k}_{\|}$ for the Dresselhaus splitting. The linear term is dominant for a sufficiently narrow quantum well.[36–39] Thus, the form given in Eq. (1) is not restricted to a particular model of the potential $V(\mathbf{r})$.

III. THE GROUND-STATE ENERGY IN THE PRESENCE OF SOI

The total Hamiltonian for free electrons in the 2DES is a sum of the kinetic energy and H_{SO}. Since it is independent of coordinates, the wave function may be sought in the form of plane waves (for simplicity, we denote the in-plane wave vector simply as $\mathbf{k}$) $\Psi_k(\mathbf{r}) = \chi_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}/\sqrt{A}$. Here, $\mathbf{r}$ and $\mathbf{k}$ are the in-plane spatial coordinate and wave vectors, A is a normalization area, the spinor χ_k satisfies the equation $H_k\chi_{\mathbf{k}} = \epsilon_k\chi_{\mathbf{k}}$, and the Hamiltonian has the following explicit representation in spin space:

$$H_k = \begin{pmatrix} \hbar^2k^2/2m^* & i\Delta_R k \exp[-i\phi(\mathbf{k})] \\ -i\Delta_R k \exp[i\phi(\mathbf{k})] & \hbar^2k^2/2m^* \end{pmatrix}. \tag{3}$$

Here, $\phi(\mathbf{k})$ is the polar angle of the wave vector $\mathbf{k}$. Diagonalizing the matrix in Eq.(3), we obtain the energy eigenvalues $\epsilon_{k\pm} = \hbar^2k^2/2m^* \pm \Delta_R k$ and eigenspinors

$$\chi_{\mathbf{k}}^{\pm} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm\exp[i\phi(\mathbf{k})] \end{pmatrix}. \tag{4}$$

The dependence of the spinor (4) on the angle $\phi(\mathbf{k})$ means that the two states, denoted by + and −, are polarized along the directions $+(\mathbf{k}\times\hat{\mathbf{z}})$ and $-(\mathbf{k}\times\hat{\mathbf{z}})$, respectively, where $\hat{\mathbf{z}}$ is a unit vector in the z direction. This polarization lifts the spin degeneracy within the xy plane of the free-electron Hamiltonian and the system has a + and a − branch. These results show that the effect of the Rashba SOI manifests itself through a mutual shift of the spin branches, resulting in an energy gap between the + and − spin branches. The presence of this Rashba spin splitting has been verified experimentally through Raman scattering and measurements of Shubnikov–de Haas oscillations.[40,41]

In this paper, we demonstrate that as the electron density is reduced, there is a threshold density when $r_s = r_s^*$ where the ground-state energy varies rapidly. This occurs when one of the spin bands (+ branch) is depleted of electrons. This result will be demonstrated for both an interacting and a noninteracting 2DES.

For a total areal electron density n_{2D}, there will be n_+ + spins and n_- − spins per unit area with $n_{2D} = n_- + n_+$. At $T = 0$ K, these are determined by the following equation, with $\lambda = \pm$:

$$\frac{n_\lambda}{n_{2D}} - \frac{1}{2} + \lambda A_R\left[\left(\frac{n_\lambda}{n_{2D}}\right)^{1/2} + \left(1 - \frac{n_\lambda}{n_{2D}}\right)^{1/2}\right] = 0, \tag{5}$$

where $A_R = k_R/k_F$ with $k_R = m^*\Delta_R/\sqrt{2}\hbar^2$ and $k_F = (2\pi n_{2D})^{1/2}$. For $A_R < 1/2$, both bands are occupied. When $A_R \geq 1/2$, $n_+ = 0$ and all spins are in the − spin branch.

In the ground state, the high-density expansion for the total energy of the system is given by

$$E_{GS} = E^{(0)} + E^{(1)} + E^{(2)} + \cdots \tag{6}$$

where the first term is the energy for noninteracting electrons and is given by

$$\frac{E^{(0)}}{N} = \frac{1}{N}\sum_{k,\lambda} \epsilon_{k\lambda}\eta_+(k_\lambda - k) = \left(\frac{e^2}{a_0^*\varepsilon_s}\right)\frac{1}{r_s}\sum_{\lambda=\pm1}\left[\frac{1}{r_{s\lambda}} + \left(\frac{2\lambda}{3}\right)\frac{\Delta_R/a_0^*}{(1\ \mathrm{Ry})}\right]\left(\frac{r_s}{r_{s\lambda}}\right)^3. \tag{7}$$

Here, $\varepsilon_s = 4\pi\varepsilon_0\epsilon_b$ where ϵ_b is the dielectric constant of the background medium, $1\ \mathrm{Ry} = e^2/2a_0^*\varepsilon_s$, $\eta_+(x)$ is the Heaviside

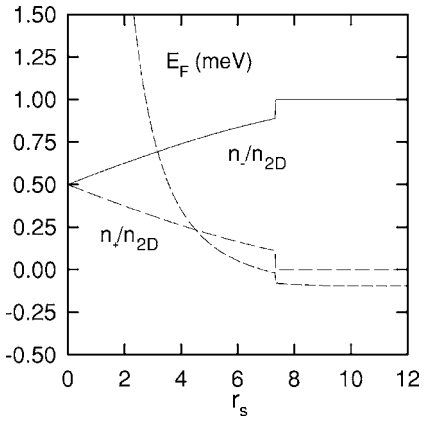

FIG. 1. Plots of n_+, n_- as well as E_F as functions of r_s for a Rashba parameter $\Delta_R = 150$ meV Å. We chose $\epsilon_b = 13.0$ and $m^* = 0.065 m_e$ where m_e is the free-electron mass, as appropriate parameters for GaAs. The value of r_s when the subband occupation changes suddenly is denoted by r_s^*.

unit step function, $k_\lambda = (4\pi n_\lambda)^{1/2}$, $\pi r_{0\lambda}^2 = A/N_\lambda = 1/n_\lambda$ in terms of the areal density per particle for + or − spins, and $r_{s\lambda} = r_{0\lambda}/a_0^*$ is the scaled interparticle spacing in terms of an effective Bohr radius $a_0^* = \hbar^2 \epsilon_s / m^* e^2$. Also, $r_s = r_0/a_0^*$, where $r_s^{-2} = r_{s+}^{-2} + r_{s-}^{-2}$. Clearly, $E^{(0)}$ could be negative if $r_{s-} \gg \frac{2}{3}\Delta_R/(1\ \mathrm{Ry})$. Also, when $\Delta_R = 0$, Eq. (7) reduces to the well-known result for a noninteracting 2D electron gas, i.e., $E^{(0)}/N = 1/r_s^2$ Ry, since in this case we have $r_{s\lambda} = \sqrt{2} r_s$, for $\lambda = \pm$.[42] Thus, for finite Δ_R, $E^{(0)}/N$ does not scale only with r_s but depends separately on the density parameters r_{s+} and r_{s-} for each subband.

The second term in Eq. (6) is the first-order correction to $E^{(0)}$. The evaluation of $E^{(1)}$ involves calculation of the expectation value of the electron-electron Hamiltonian for the noninteracting system in the ground state.[42] This is given by

$$\frac{E^{(1)}}{N} = -\left(\frac{e^2}{a_0^* \epsilon_s}\right)\left(\frac{a_0^*}{4n_{2D}}\right)\frac{1}{A^2}\sum_{\mathbf{k},\mathbf{q}}{}' \sum_{\lambda_1,\lambda_2=\pm 1} \frac{2\pi}{q}\eta_+\left(k_{\lambda_2} - k\right) \times \eta_+\left(k_{\lambda_1} - |\mathbf{k}+\mathbf{q}|\right)\left(1 + \lambda_1\lambda_2 \frac{k + q\cos\theta}{|\mathbf{k}+\mathbf{q}|}\right), \quad (8)$$

where the prime on the summation means that $q = 0$ is forbidden, in order to ensure global charge neutrality of the system. The k integration is done over the area of intersection of two circles of radii k_{λ_1} and k_{λ_2} separated by a distance q, and is finite only if $|k_{\lambda_1} - k_{\lambda_2}| \leq q \leq k_{\lambda_1} + k_{\lambda_2}$. This always negative contribution to the total energy of the electron liquid is the *exchange energy* arising from the antisymmetry of the wave function. When $\Delta_R = 0$, the exchange energy can easily be evaluated and we have the well-known result $E^{(1)}/N = -8\sqrt{2}/3\pi r_s \mathrm{Ry} \approx -1.2/r_s$ Ry.[42] The calculation is a bit more involved so as to obtain an analytical result when $\Delta_R \neq 0$ due to the presence of the form factor arising from the amplitude of the wave function in Eq. (4). A numerical calculation of $E^{(0)}$ and $E^{(1)}$ is presented below. However, to understand these results, in Fig. 1 we plot the electron density for each subband as a function of r_s. In the high-density limit ($r_s \to 0$), both subbands are equally occupied. As the density is reduced and r_s increases, the spins are distributed in the + and − subbands unequally. As a matter of fact, the + subband is completely depopulated at some density r_s^*. This depopulation of the subbands at r_s^* is manifested in both $E^{(0)}$ and $E^{(1)}$, as we show in Fig. 2(a). Consequently, we see that when $E_F < 0$, only the lower spin subband is occupied for noninteracting electrons. We also note that in the presence of electron-electron interactions, there is still a notable effect occurring as E_F increases through $E = 0$ and the upper + spin subband suddenly gets a finite population rather than increasing gradually. The value for r_s^* in Fig. 1 is 7.38. For GaAs, with background dielectric constant 13.0 and electron effective mass $m^* = 0.067 m_e$, where m_e is the free-electron mass, the 2D electron density for this r_s^* is $n_{2D} \approx 5.84 \times 10^9$ cm^{-2}. The reason for the abrupt change in the Fermi energy at r_s^* is due to the fact that as the electron density decreases continuously, the − spin subband below $E = 0$ has to accommodate the spins which were in the + spin subband just below r_s^*. However, as shown in Ref. 43, the DOS for − spins increases rapidly below $E = 0$.

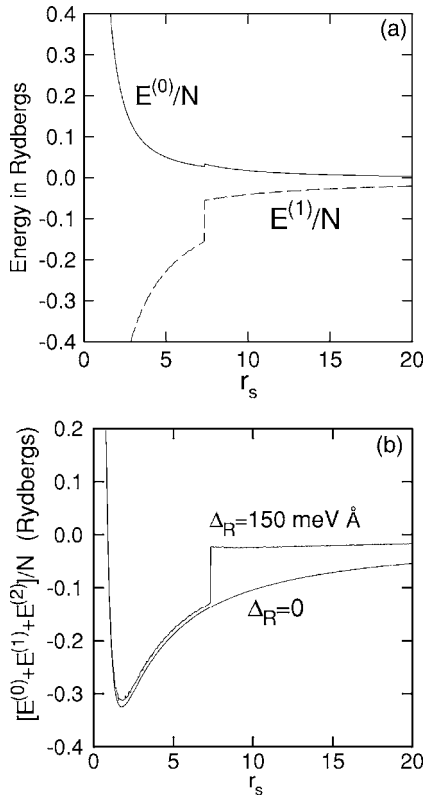

FIG. 2. (a) Plot of $E^{(0)}/N$ and $E^{(1)}/N$ as functions of r_s at intermediate densities for the same values of Δ_R, ϵ_b, and m^* as in Fig. 1. (b) Plot of the total energy per particle $(E^{(0)} + E^{(1)} + E^{(2)})/N$ using the results in (a) as a function of r_s. For comparison, we plot this total energy when $\Delta_R = 0$. The unit of energy is the effective Rydberg $e^2/2a^*\epsilon_s$.

The third term $E^{(2)}$ in Eq. (6) is the leading-order contribution to the correlation energy of the 2D electron gas.[44–46]

Calculation shows that in the random-phase approximation (RPA) we have

$$E^{(2)} = \frac{\hbar}{2}\sum_{\mathbf{q}} \int_0^\infty \frac{d\omega}{\pi}\{\ln[1+\alpha(q,i\omega)] - \alpha(q,i\omega)\}. \quad (9)$$

In this notation, $\alpha(q,\omega)$ is the polarization function defined by

$$\alpha(q,\omega) = \frac{\pi e^2}{2A\epsilon_s q}\sum_{\mathbf{k}} \sum_{\lambda,\lambda'=\pm 1} \frac{f_0(\epsilon_{\mathbf{k}},\lambda) - f_0(\epsilon_{\mathbf{k}-\mathbf{q},\lambda'})}{\hbar\omega + \epsilon\mathbf{k} - \mathbf{q},\lambda' - E_{\mathbf{k},\lambda} + i0^+} \times\left(1 + \lambda\lambda' \frac{|\mathbf{k}| - |\mathbf{q}|\cos\theta}{|\mathbf{k}-\mathbf{q}|}\right), \quad (10)$$

and θ is the angle between the wave vectors $\mathbf{k}$ and $\mathbf{q}$. Also, when $\Delta_R = 0$, Eq. (9) reduces to the well-known RPA result for a 2D electron gas, i.e., $E^{(2)}/N = -(0.38\pm 0.04) - 0.1726 r_s \ln r_s + \mathcal{O}(r_s)$ Ry (see Chapters 1 and 5 of Ref. 42 for a list of references).

IV. NUMERICAL RESULTS AND DISCUSSION

We now present and discuss our numerical results. In Fig. 1, we plot n_-, n_+, and E_F as functions of r_s for a Rashba parameter $\Delta_R = 150$ meV Å. In the limit $r_s \to 0$, the + and − spin bands are almost equally populated. As the density is decreased, there are more electrons in the − spin band compared to the + spin band. As a matter of fact, the single-particle or thermodynamic density of states (DOS) for each subband ($\lambda = \pm$) is given by $\rho_\lambda^{(0)}(E) = -\frac{1}{\pi A}\Sigma_{\mathbf{k}} G_\lambda^{(0)}(k,E) = \frac{1}{A}\Sigma_{\mathbf{k}}\delta(E - E_{k,\lambda})$, where $G_\lambda^{(0)}(k,E)$ is the single-particle Green's function. These have been calculated and the results are given by[43]

$$\rho_+^{(0)}(E) = \eta_+(E)\left(\frac{m^*}{2\pi\hbar^2}\right)\left(1 - \sqrt{\frac{E_\Delta}{E+E_\Delta}}\right), \quad (11)$$

$$\rho_-^{(0)}(E) = \left(\frac{m^*}{2\pi\hbar^2}\right)\left[\eta_+(E)\left(1 + \sqrt{\frac{E_\Delta}{E+E_\Delta}}\right) + 2\eta_+(-E)\eta_+(E+E_\Delta)\sqrt{\frac{E_\Delta}{E+E_\Delta}}\right], \quad (12)$$

where $E_\Delta = k_R\Delta_R/\sqrt{2}$ is a measure of the spin gap in the DOS. Equations (11) and (12) show that the total DOS $\rho_{2D}^{(0)} = \rho_+^{(0)}(E) + \rho_-^{(0)}(E)$ is $m^*/\pi\hbar^2$ for $E \geq 0$. This is equal to the DOS for a spin-degenerate 2D electron system. However, the DOS for $-E_\Delta < E < 0$ is increased as E is reduced from zero and even becomes infinite at the bottom of the − spin band where $E = -E_\Delta$. For each subband, the DOS is not independent of energy and $\rho_-^{(0)}(E) > \rho_+^{(0)}(E)$ for all energies satisfying $-E_\Delta < E < \infty$. As the Fermi energy passes through the bottom of the + spin band (ϵ_{k+}), this corresponds to $r_s = r_s^*$ in Fig. 1 when the + spin band is depleted of electrons.

In Fig. 2(a), we plot $E^{(0)}/N$ and $E^{(1)}/N$ separately as functions of r_s for the same Rashba parameter used in Fig. 1. In Fig. 2(b), we plot the total energy per particle $(E^{(0)}+E^{(1)}+E^{(2)})/N$ also as a function of r_s. In Fig. 2(b), we compare the results for finite Rashba parameter with the values obtained when there is no SOI. The total energy per particle in Fig. 2(b) is not changed by the SOI in the high-density limit. However, as r_s increases, we find that the effect of SOI becomes important as $r_s \to r_s^*$. At this density, the spontaneous spin depopulation of the + spin subband leads to the rapid change in both $E^{(0)}$ and $E^{(1)}$ as $r_s \to r_s^*$. The results of our calculations to obtain the correlation energy per particle, i.e., $E^{(2)}/N$, in the RPA show that in the high-density limit, finite Δ_R has a negligible effect on the correlation energy. This was obtained for the range of densities corresponding to $0 \lesssim r_s \lesssim 20$, as shown in Fig. 2(b). Over the years, several authors have calculated the correlation energy for a 2D electron gas, showing that the RPA is not that reliable as the electron density is reduced. A more accurate calculation including SOI should be carried out to complement the results in this paper.

V. CONCLUDING REMARKS

In this paper, we calculated the ground-state energy $E^{(0)}/N$ per particle for noninteracting electrons and the exchange energy per particle $E^{(1)}/N$ for a 2DES in the presence of SOI. These results clearly show that spontaneous spin depopulation of one of the energy bands leads to a rapid change in the total energy of the system. We also calculated the correction to this total energy due to correlation effects in the RPA, i.e., $E^{(2)}/N$. In the high-density limit where the RPA is a good approximation, the effect due to Δ_R on $E^{(2)}/N$ is negligible. Our results indicate a phase change of the 2D electron liquid from a paramagnetic-to a ferromagneticlike behavior as the density is decreased. These two phases are stable at intermediate densities and are not beyond the reach of experiment. Therefore, they should be explored. The effects of impurities at intermediate densities are not expected to destroy this behavior of the 2DES. Only at low densities, where a Wigner crystal is formed, should a system of electrons bound to randomly distributed impurities completely modify our results. Accurate calculations employing the variational Monte Carlo and diffusion Monte Carlo methods have been used previously to obtain the ground-state energy of a 2DES in the absence of SOI.[47,48] The effect due to SOI using these methods will be carried out in a future publication.

Kogan and Rosenstein[49] calculated the effect of electron-electron interaction on the "constant" single-particle DOS of a degenerate 2DES. They obtained a dip in the DOS at the Fermi energy due to the dynamical shielding of the electron-electron interaction. We shall examine the way in which $\rho_-^{(0)}(E)$ and $\rho_+^{(0)}(E)$ are modified in the RPA as well as with disorder.[50] It is straightforward to show that the corrections to the results in Eqs. (11) and (12) are given by

$$\delta\rho_\lambda(E) = -\frac{1}{\pi}\,\mathrm{Im}\int \frac{d^2\mathbf{q}}{(2\pi)^2}\,\delta G(q,E), \tag{13}$$

where $\delta G_\lambda(q,E) = G_\lambda(q,E) - G_\lambda^{(0)}(q,E)$ is the difference between the interacting and single-particle Green's functions. This involves an evaluation of the self-energy which can be obtained using diagrammatic methods.[49] The effect due to SOI on the tunneling DOS of an interacting 2DES will be calculated and will be the subject of a future investigation.

ACKNOWLEDGMENTS

The author acknowledges partial support from the National Science Foundation under Grant No. CREST 0206162 and PSC-CUNY Award No. 67172-00 36. He would also like to express his thanks to the U.S.-Israel Educational Foundation for additional financial support. Helpful discussions with Neepa Maitra are also gratefully acknowledged.

[1] Yu. A. Bychkov and E. I. Rashba, J. Phys. C **17**, 6039 (1984).
[2] G. Dresselhaus, Phys. Rev. **100**, 580 (1955).
[3] G. Lommer, F. Malcher, and U. Rössler, Phys. Rev. Lett. **60**, 728 (1988).
[4] M. Johnson and R. H. Silsbee, Phys. Rev. B **37**, 5312 (1988).
[5] T. Ando, Phys. Rev. B **40**, 5325 (1989).
[6] B. Das, D. C. Miller, S. Datta, R. Reifenberger, W. P. Hong, P. K. Bhattacharya, J. Singh, and M. Jaffe, Phys. Rev. B **39**, 1411 (1989).
[7] J. Luo, H. Munekata, F. F. Fang, and P. J. Stiles, Phys. Rev. B **41**, 7685 (1990).
[8] B. Das, S. Datta, and R. Reifenberger, Phys. Rev. B **41**, 8278 (1990).
[9] P. D. Dresselhaus, C. M. A. Papavassiliou, R. G. Wheeler, and R. N. Sacks, Phys. Rev. Lett. **68**, 106 (1992).
[10] B. Jusserand, D. Richards, H. Peric, and B. Etienne, Phys. Rev. Lett. **69**, 848 (1992).
[11] G. L. Chen, J. Han, T. T. Huang, S. Datta, and D. B. Janes, Phys. Rev. B **47**, 4084 (1993).
[12] E. A. de Andrada e Silva, G. C. La Rocca, and F. Bassani, Phys. Rev. B **55**, 16293 (1997).
[13] J. Nitta, T. Akazaki, H. Takayanagi, and T. Enoki, Phys. Rev. Lett. **78**, 1335 (1997).
[14] P. R. Hammar and M. Johnson, Phys. Rev. B **61**, 7207 (2000).
[15] A. V. Moroz and C. H. W. Barnes, Phys. Rev. B **60**, 14 272 (1999); A. V. Moroz and C. H. W. Barnes, *ibid.* **61**, R2464 (2000); A. V. Moroz, K. V. Samokhin, and C. H. W. Barnes, Phys. Rev. Lett. **84**, 4164 (2000).
[16] M. Governale, Phys. Rev. Lett. **89**, 206802 (2002).
[17] *Semiconductor Spintronics and Quantum Computation*, edited by D. D. Awschalom, D. Loss, and N. Samarth, Series in Nanoscience and Technology (Springer, Berlin, 2002).
[18] C. A. Ullrich and M. E. Flatté, Phys. Rev. B **66**, 205305 (2002).
[19] C. A. Ullrich and M. E. Flatté, Phys. Rev. B **68**, 235310 (2003).
[20] J. Splettstoesser, M. Governale, and U. Zülicke, Phys. Rev. B **68**, 165341 (2003).
[21] M. W. C. Dharma-wardana and F. Perrot, Phys. Rev. Lett. **90**, 136601 (2003).
[22] J. Zhu, H. L. Stormer, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, Phys. Rev. Lett. **90**, 056805 (2003).
[23] A. Ghosh, C. J. B. Ford, M. Pepper, H. E. Beere, and D. A. Ritchie, Phys. Rev. Lett. **92**, 116601 (2004)
[24] Godfrey Gumbs, Appl. Phys. Lett. **85**, 2821 (2004).
[25] S. D. Ganichev, V. V. Bel'kov, L. E. Golub, E. L. Ivchenko, Petra Schneider, S. Giglberger, J. Eroms, J. De Boeck, G. Borghs, W. Wegscheider, D. Weiss, and W. Prettl, Phys. Rev. Lett. **92**, 256601 (2004).
[26] Godfrey Gumbs, Phys. Rev. B **70**, 235314 (2004).
[27] C. Koitzsch, C. Battaglia, F. Clerc, L. Despont, M. G. Garnier, and P. Aebi, Phys. Rev. Lett. **95**, 126401 (2005).
[28] D. W. Yuan, W. Xu, Z. Zeng, and F. Lu, Phys. Rev. B **72**, 033320 (2005).
[29] Qing-feng Sun and X. C. Xie, Phys. Rev. B **71**, 155321 (2005).
[30] Tapash Chakraborty and Pekka Pietilainen, Phys. Rev. B **71**, 113305 (2005).
[31] A. De Martino and R. Egger, J. Phys.: Condens. Matter **17**, 5523 (2005).
[32] Guang-Hong Chen and M. E. Raikh, Phys. Rev. B **60**, 4826 (1999).
[33] S. De Palo, M. Botti, S. Moroni, and Gaetano Senatore, Phys. Rev. Lett. **94**, 226405 (2005).
[34] Y.-W. Tan, J. Zhu, H. L. Stormer, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, Phys. Rev. Lett. **94**, 016405 (2005).
[35] M. W. C. Dharma-wardana, Phys. Rev. B **72**, 125339 (2005).
[36] R. Eppenga and M. F. H. Schuurmans, Phys. Rev. B **37**, 10923 (1988).
[37] J. Luo, H. Munekata, F. F. Fang, and P. J. Stiles, Phys. Rev. B **41**, 7685 (1990).
[38] B. Jusserand, D. Richards, H. Peric, and B. Etienne, Phys. Rev. Lett. **69**, 848 (1992).
[39] A. G. Mal'shukov, K. A. Chao, and M. Willander, Phys. Rev. Lett. **76**, 3794 (1996).
[40] S. J. Papadakis, E. P. De Poortere, H. C. Manoharan, M. Shayegan, and R. Winkler, Science **283**, 2056 (1999).
[41] Dirk Grundler, Phys. Rev. Lett. **84**, 6074 (2000).
[42] Gabrielle Giuliani and Giovanni Vignale, *Quantum Theory of the Electron Liquid* (Cambridge University Press, Cambridge, U.K., 2005), p. 31.
[43] Godfrey Gumbs, Phys. Rev. B **72**, 165351 (2005).
[44] E. Wigner, Phys. Rev. **46**, 1002 (1934).
[45] David Pines, Phys. Rev. **92**, 626 (1953).
[46] Murray Gell-Mann and Keith A. Brueckner, Phys. Rev. **106**, 364 (1957).
[47] B. Tanatar and D. M. Ceperley, Phys. Rev. B **39**, 5005 (1989).
[48] Claudio Attaccalite, Saverio Moroni, Paola Gori-Giorgi, and Giovanni B. Bachelet, Phys. Rev. Lett. **88**, 256601 (2002).
[49] E. Kogan and B. Rosenstein, Phys. Rev. B **69**, 113105 (2004).
[50] Richard Berkovits, Phys. Rev. B **60**, 26 (1999).