Transport in chaotic quantum dots: Effects of spatial symmetries which interchange the leads

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We investigate the effect of spatial symmetries on phase coherent electronic transport through chaotic quantum dots. For systems which have a spatial symmetry that interchanges the source and drain leads, we find in the framework of random matrix theory that the density of the transmission eigenvalues is independent of the number of channels N in the leads. As a consequence, the weak localization correction to the conductance vanishes in these systems, and the shot noise suppression factor F is independent of N. We confirm this prediction by means of numerical calculations for stadium billiards with various lead geometries. These calculations also uncover transport signatures of partially preserved symmetries.

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I. INTRODUCTION

Over the past years transport experiments on phase coherent mesoscopic systems have attained unprecedented levels of sophistication.[1–3] It is now possible to precisely design and control the geometries of quantum dots while reducing the effects of impurity scattering to a level where the transport can be considered as purely ballistic (with scattering only off the confining boundaries).[1–5] For geometries which give rise to chaotic classical motion, universal system properties are expected.[2,3] Prototypical chaotic cavities (such as the stadium and the Sinai billiard) do, however, feature geometrical symmetries that leave signatures on the transport properties,[6–9] which we investigate in the present communication. In particular, we consider systems with a *lead-transposing* reflection symmetry which interchanges the source and drain leads that couple the dot to the electronic reservoirs [see, e.g., Figs. 1(a) and 1(b)], and contrast them with systems that do not possess such a symmetry [see, e.g., Figs. 1(c) and 1(d)].

Our investigation is based on random-matrix theory (RMT) for the scattering matrix.[10,11] In fact, the present RMT problem is long standing, dating back to almost 10 years ago when first theoretical and numerical results were presented in Refs. 6 and 7, respectively. These earlier works identified the correct invariant measure of the scattering matrix and they studied several statistical properties of the conductance. Here we give the complete solution of this problem by deriving the joint probability density of transmission eigenvalues, which determine all stationary transport properties, for an arbitrary number of open channels supported by the leads attached to the quantum dot. The density of the transmission eigenvalues turns out to take a particularly simple form. A striking signature of the solution is the absence of any nontrivial dependence of the ensemble-averaged conductance and shot noise on the number of open transport channels. This prediction is confirmed numerically for stadium billiards with different geometries. Our theory also explains the deviations from standard random-matrix theory observed in earlier numerical investigations.[12]

The random-matrix theory of transport[10,11] is based on Landauer's scattering approach, which describes the transport properties by the scattering matrix

$$S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}, \tag{1}$$

composed of amplitudes for transmission (t,t') and reflection (r,r') between the channels in the entrance and exit lead,

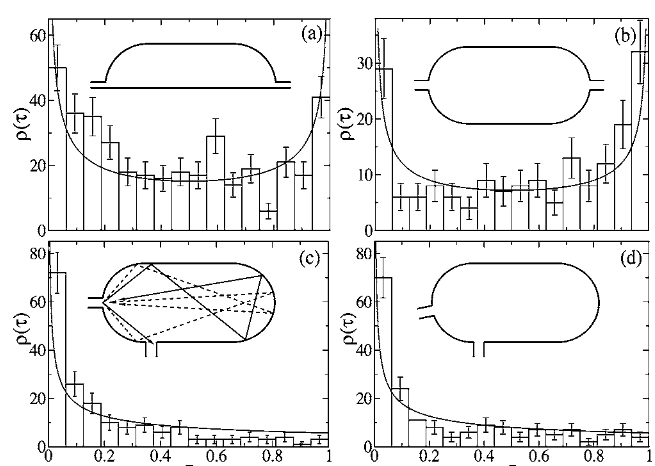

FIG. 1. Distribution of transmission eigenvalues $\rho(\tau)$ (normalized to the sample size), for $N=1$, in stadium geometries with lead-transposing left-right symmetry [(a), (b)] and without global symmetry [(c), (d)]. The numerical data (histogram) are compared with the corresponding RMT predictions (solid lines). Due to the horizontal alignment of the entrance lead in (c) a fraction of back-reflected paths have an up-down reflected partner trajectory of equal length (see, e.g., pair of dashed lines). Although for a class of trajectories the corresponding partner is cut short by the exit lead (see, e.g., pair of solid lines) transport is strongly influenced by this partial symmetry. Cavity area (a) $A=(4+\pi)/2$, (b), (c), (d) $A=4+\pi$, lead widths (a) $d=0.125$, (b), (c), (d) $d=0.25$, and entrance lead tilt angle in (d) $\delta=11.8°$.

respectively. We assume that both leads support the same number of open channels N. We investigate the transport properties in terms of the eigenvalues τ_n of $t^\dagger t$ which determine the conductance $G_N=\bar{G}\Sigma_{n=1}^N \tau_n$ (where $\bar{G}=2e^2/h$ is the conductance quantum) and the shot noise power[13] $P_N=(2e\bar{G}V)\Sigma_{n=1}^N \tau_n(1-\tau_n)$ (where V is the applied voltage). For uncorrelated electrons the shot noise power is given by the Poisson value $\bar{P}_N=2eI$, where I is the time averaged current. Fermi statistics, however, induces electronic correlations and deviations from the Poisson value which are customarily quantified by the Fano factor

$$F_N=\frac{\langle P_N\rangle}{\langle \bar{P}_N\rangle}=\frac{\left\langle \sum_{n=1}^N \tau_n(1-\tau_n)\right\rangle}{\left\langle \sum_{n=1}^N \tau_n\right\rangle}, \tag{2}$$

where $\langle\ldots\rangle$ stands for ensemble or energy average.

II. JOINT PROBABILITY DISTRIBUTION OF TRANSMISSION EIGENVALUES

Throughout this paper we assume the absence of magnetic fields and spin-orbit scattering, hence, we consider time-reversal symmetric systems without Kramers degeneracy.

A. Chaotic quantum dots without spatial symmetries

As a benchmark for our results we use the well-established random-matrix theory for such systems in absence of any spatial symmetries[10,11] (see the geometries in the bottom row of Fig. 1). The scattering matrix is then a member of Dyson's circular orthogonal ensemble (COE),[14] and the joint probability distribution of the transmission eigenvalues τ_n is given by

$$W(\{\tau\})=\prod_{n<m}|\tau_m-\tau_n|\prod_i \tau_i^{-1/2}. \tag{3}$$

For a large number of channels $N\to\infty$, the density of transmission eigenvalues $\rho_N(\tau)=\langle\Sigma_n^N\delta(\tau-\tau_n)\rangle$ approaches the bimodal distribution[15,16]

$$\rho_N(\tau)\approx\rho_\infty(\tau)=N\left(\pi\sqrt{\tau(1-\tau)}\right)^{-1}. \tag{4}$$

This gives an ensemble-averaged conductance $\langle G_N\rangle=G_\infty+O(N^0)$, where $G_\infty=N\bar{G}/2$, and a Fano factor $F_N=F_\infty+O(N^{-1})$, where $F_\infty=1/4$.

The bimodal distribution Eq. (4) is only valid asymptotically for large N. The leading finite-N correction for the conductance is the well-known weak-localization correction $\langle G_N\rangle-G_\infty=-\bar{G}/4+O(N^{-1})$.[10] Similar corrections exist also for other transport properties such as the Fano factor. Generally, they can be related to deviations of $\rho_N(\tau)$ from $\rho_\infty(\tau)$, and are most pronounced for a small number of channels.

For a single channel ($N=1$), the density of transmission eigenvalues is given by[15,16]

$$\rho_1(\tau)=\frac{1}{2\sqrt{\tau}}; \tag{5}$$

hence the conductance and shot noise power is given by $\langle G_1\rangle/\bar{G}=1/3$, $\langle P_1\rangle/(2e\bar{G}V)=2/15$, respectively. The Fano factor is then $F_1=2/5$.

For two channels, from Eq. (3), we find

$$\rho_2(\tau)=4\tau-3\sqrt{\tau}+\frac{1}{\sqrt{\tau}}. \tag{6}$$

Thus $\langle G_2\rangle/\bar{G}=4/5$, $\langle P_2\rangle/(2e\bar{G}V)=9/35$, and $F_2=9/28$. These results show that for a small number of channels the corrections to the large-N asymptotics are clearly noticeable.

B. Chaotic quantum dots with spatial symmetries

We now turn to systems with a spatial reflection symmetry which interchanges the leads, such as the geometries of Figs. 1(a) and 1(b).

For a left-right symmetric structure, Fig. 1(a), the scattering matrix has the structure

$$S=\begin{pmatrix} r & t\\ t & r\end{pmatrix}, \tag{7}$$

where r and t are symmetric $N\times N$ matrices. Matrices of this structure can be cast into a block diagonal form by using the rotation matrix[6]

$$R=\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1\\ -1 & 1\end{pmatrix}, \tag{8}$$

i.e.,

$$\tilde{S}=RSR^T=\begin{pmatrix} s_1 & 0\\ 0 & s_2\end{pmatrix}, \tag{9}$$

where $s_1=r+t$ and $s_2=r-t$ are symmetric unitary matrices. In terms of these matrices, the transmission matrix is given by $t=\frac{1}{2}(s_1-s_2)$ and the reflection matrix is given by $r=\frac{1}{2}(s_1+s_2)$.

The transmission eigenvalues are obtained from the matrix

$$tt^\dagger=\frac{1}{4}[2-s_1s_2^\dagger-(s_1s_2^\dagger)^\dagger], \tag{10}$$

which involves the unitary matrix $Q=s_1s_2^\dagger$. Note that Q and $Q^\dagger$ can be diagonalized simultaneously. Hence, the eigenvalues $\exp(i\theta_n)$ of Q determine the transmission eigenvalues by $\tau_n=\sin^2(\theta_n/2)$.

In random-matrix theory the matrices s_1 and s_2 are assumed to be independent members of the COE. We then can show that the joint probability distribution of the eigenvalues $\exp(i\theta_n)$ is also given by the COE.[17,18] The matrix Q can be symmetrized by the unitary transformation $Q'=s_2^{-1/2}Qs_2^{1/2}=s_2^{-1/2}s_1s_2^{-1/2}$, which leaves the eigenvalues invariant. Moreover, the COE is invariant under the automorphism $Q'\to U^TQ'U$, where U is an arbitrary unitary matrix which we identify with $U=s_2^{-1/2}$. Hence the eigenvalues of Q inherit the

COE statistics of s_1, and the joint distribution function of transmission eigenvalues takes the form

$$W(\{\tau\}) = \prod_k \left\{ [\tau_k(1-\tau_k)]^{-1/2} \sum_{\sigma_k=\pm 1} \right\} \times \prod_{n<m} |\sigma_m \sqrt{\tau_m(1-\tau_n)} - \sigma_n \sqrt{\tau_n(1-\tau_m)}|. \quad (11)$$

The probability density $\rho_N(\tau)$ can be obtained directly from the uniform distribution of eigenphases θ_n in the COE. This yields $\rho_N(\tau)=\rho_\infty(\tau)$ [given in Eq. (4)] *exactly, for any value of N*. Hence the ensemble-averaged conductance is given by $\langle G_N\rangle = G_\infty = N\bar{G}/2$, i.e., the weak-localization correction is absent. Moreover, the Fano factor is given by $F_N = F_\infty = 1/4$. The absence of any N-dependent corrections is in striking contrast to the previously discussed case of asymmetric systems.

C. Chaotic quantum dots with fourfold symmetries

The results above also apply to systems with a 180° *rotational* symmetry mapping the leads onto each other. They can also be extended to incorporate further spatial symmetries which may be present in addition to the lead-transposing symmetry. For systems with two leads, the only remaining case is the fourfold symmetry as indicated in Fig. 1(b). In this case, the S matrix can be written as

$$S = \begin{pmatrix} S_e & 0 \\ 0 & S_o \end{pmatrix}, \quad (12)$$

where S_e and S_o are the scattering matrices associated with the even and odd channels, respectively. Each matrix $S_{e(o)}$ has the structure of Eq. (7); thus each of them can be analyzed by the same procedure as in the previous lead-transposing symmetry case and the statistical transport properties are given by the superposition of the even and odd subsystems. For a cavity with a fourfold symmetry we therefore have $\langle G_N\rangle = \bar{G}N/2$ and a constant Fano factor $F_N = 1/4$.

III. NUMERICAL SIMULATIONS

We now compare our theoretical results to numerical simulations of stadium billiards which feature the four different setups depicted in Fig. 1. The numerical data are obtained by solving the Schrödinger equation using a modular recursive Green's function method which allows for an efficient calculation of the scattering matrix even for a large number of open channels.[19]

The most drastic effects of the lead-transposing reflection symmetry are expected for the case of a single channel in the leads ($N=1$). Figure 1 compares the numerical probability distribution $\rho_1(\tau)$ with the analytical predictions for the systems with and without a lead-transposing reflection symmetry (see upper and lower panel, respectively). Note that for each of the two pairs of geometries the numerical results (i) show a clear signature of the absence/presence of the symmetry and (ii) we can see a good agreement with the analytical predictions.

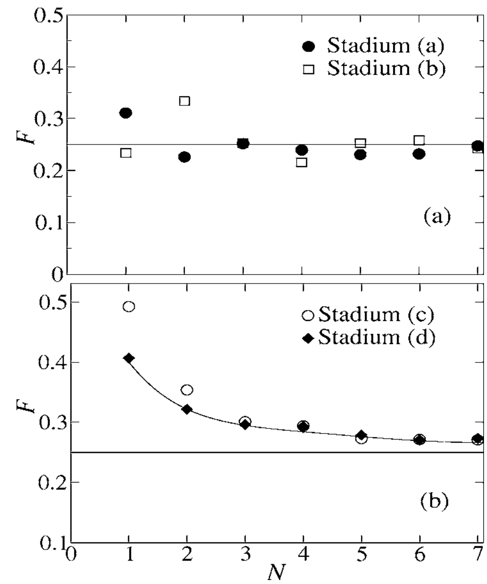

FIG. 2. Energy-averaged Fano factor in different mode intervals. Upper panel: Numerical data for systems with a lead-transposing symmetry [see Figs. 1(a) and 1(b)] compared with the corresponding mode-independent random-matrix prediction ($F=1/4$, solid line). Lower panel: Analogous plot for systems without global symmetry [see Figs. 1(c) and 1(d)]. The deviations to the RMT result [solid line, numerically obtained from Eq. (3) for $N>2$] for the geometry depicted in Fig. 1(c) can be explained in terms of the partial symmetry of short trajectories in the structure.

Several earlier studies have demonstrated that dynamical signatures of geometries can also be identified in terms of the Fano factor.[12,20–28] Figure 2(a) shows the Fano factor for the symmetric geometries as a function of the number of channels. For the two systems with lead-transposing reflection symmetry our numerical simulations show an overall constant behavior of the Fano factor with the number of channels, around $F=1/4$, in agreement with our modified random-matrix theory. The modes of the left-right symmetric cavity in Fig. 1(a) are identical to the modes with even index of the cavity in Fig. 1(b). Therefore, the presence of the odd channels in our fourfold symmetric cavity does not change the statistical properties of the transmission eigenvalues as predicted above. The flat behavior of the Fano factor in the symmetric cavities is nicely contrasted by the results for the two geometries without symmetry, Fig. 2(b): As predicted by the conventional random-matrix theory (absence of spatial symmetries) the Fano factor decreases as N increases and approaches its universal value $1/4$ for large N.[30]

Note, however, that in Fig. 2(b) the Fano factor of the geometry with the horizontally attached lead [Fig. 1(c)] consistently lies above the RMT prediction for small channel numbers. This feature can be attributed to the fact that in this geometry a significant fraction of back-reflected classical tractories come in pairs of two equally long paths which are related to each other by an up-down mirror reflection. This feature does, however, not apply to those trajectories the mirror images of which are cut short by the exit lead [see the two trajectory pairs in the inset of Fig. 1(c)]. Since this "partial" symmetry enhances the constructive interference of reflected paths but spares all transmitted trajectories, the weak localization contribution to the conductance and the shot noise power are increased. By tilting the entrance lead slightly [as in Fig. 1(d)] this effect should be destroyed, resulting in transport statistics which follow the RMT prediction. Checking with our numerical data in Fig. 2 we note that this assessment is indeed nicely corroborated. Given this excellent agreement, one might wonder why recently proposed corrections to RMT due to "noiseless scattering channels"[24–26,29] do not apply in the present cases. Due to the small ratio of lead width d to cavity area A in our geometries (see caption Fig. 1) the criterion for the emergence of such fully transmitted or reflected channels, $N \gtrsim (k_F\sqrt{A})^{1/2}$,[24] is, however, not fulfilled in the energy regimes studied above [for the highest energy considered here: $N=7$ and $(k_F\sqrt{A})^{1/2} \approx 15$].

IV. SUMMARY

We have studied analytically and numerically the effects of spatial symmetries on electronic transport properties of ballistic lateral quantum dots, modeled by a quantum chaotic cavity. Especially, we considered geometries with a symmetry which maps the two leads onto each other. For such systems, random-matrix theory can be solved exactly for an arbitrary number of channels N in each lead. We predict that finite-N corrections are absent for all transport quantities, such as the conductance and the shot noise. This is confirmed by our numerical simulations. We further explored the effects of partial symmetries on transport which, as we showed, can yield significant corrections to the random-matrix predictions.

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