Coulomb drag between two spin-incoherent Luttinger liquids

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In a one-dimensional electron gas at low enough density, the magnetic (spin) exchange energy J between neighboring electrons is exponentially suppressed relative to the characteristic charge energy, the Fermi energy E_F. At nonzero temperature T, the energy hierarchy $J \ll T \ll E_F$ can be reached, and we refer to this as the spin-incoherent Luttinger liquid state. We discuss the Coulomb drag between two parallel quantum wires in the spin-incoherent regime, as well as the crossover to this state from the low-temperature regime by using a model of a fluctuating Wigner solid. As the temperature increases from zero to above J for a fixed electron density, the $2k_F$ oscillations in the density-density correlations are lost. As a result, the temperature dependence of the Coulomb drag is dramatically altered and nonmonotonic dependence may result. Drag between wires of equal and unequal density are discussed, as well as the effects of weak disorder in the wires. We speculate that weak disorder may play an important role in extracting information about quantum wires in real drag experiments.

I. INTRODUCTION

In recent years correlated electron systems at the nanoscale and in reduced dimensions have attracted much attention.[1–3] In one spatial dimension electron correlations are expected to be enhanced, leading to the so-called Luttinger liquid (LL) state.[4,5] The existence of the LL state in a one-dimensional electron system is now an established experimental fact,[6–8] with direct measurements of the distinct spin and charge velocities in momentum resolved tunneling (as predicted by the theory) providing compelling evidence.[9,10]

Another way to explore the correlations in one-dimensional systems is in a drag experiment between two parallel quantum wires or nanotubes.[11–13] (Recall that in most cases the bare conductance of a quantum wire is $2e^2/h$ per transverse channel regardless of the electron interactions,[14,15] and so does not reveal information about electron correlations. An exception to this quantization condition is the situation discussed by Matveev in Refs. 16 and 17.) The typical drag setup involves a current driven in a "active" wire while a voltage drop is measured in a "passive" wire. See Fig. 1.

The quantity often taken to describe the drag effect is the "drag resistivity" (drag per unit length) defined as

$$r_D = - \lim_{I_1 \to 0} \frac{e^2}{h} \frac{1}{L} \frac{dV_2}{dI_1}, \tag{1}$$

where V_2 is a voltage induced in wire 2 (the "passive" wire) due to a current I_1 in wire 1 (the "active" wire). Here e is the charge of the electron, h is Planck's constant, and L is the length of the wire. The sign of the drag can be either positive or negative, but it is generally positive (note minus sign in formula) for repulsive interactions between like carriers in the wires (either electrons or holes).

Typically, one is interested in the dependence of r_D on the temperature, the interwire spacing, the electron density and Fermi wave vector in each wire, the disorder, the wire length,

and possibly on an external magnetic field. Physically drag is the result of collisions (momentum transfer) from electrons in the active wire which tend to "push" or "pull" the electrons in the passive wire. Electrons in the passive wire move under these collisions until an electric field is built up in the passive wire (due to a nonuniform density of electrons there) which just cancels the force of the momentum transfer of the electrons in the active wire. This is the physics behind the well-known drag formulas of Zheng and MacDonald[18] and Pustilnik *et al.*[19] [See Eq. (7).]

While the experimental data on drag between quantum wires are limited,[11–13] a fair amount of theoretical work has been published. Various studies have made use of LL theory,[20–24] Fermi liquid (FL) theory with[25] and without[19] multiple subbands, effects of interwire tunneling,[26] effects of disorder,[27] shot noise correlations,[28,29] mesoscopic fluctuations,[30,31] and the effects of different signs of electron exchange interactions in the wires.[32] Additionally, phonon-mediated drag has been studied in one dimension.[33,34] The main qualitative difference that is found between the LL and FL approaches is whether r_D tends to increase (LL) or decrease (FL) as the temperature is reduced to the lowest values. For the case of two infinitely long identical clean wires the following results are obtained: In a LL, electrons tend to "lock" into a commensurate state at the lowest temperatures, giving rise to a diverging drag, while in a FL the phase space available for scattering tends to zero as the temperature is
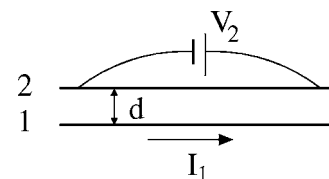

FIG. 1. Coulomb drag schematic. Two quantum wires are arranged parallel to one another. A dc current I_1 flows in the "active" wire 1 and a voltage bias V_2 is measured in the "passive" wire 2 when $I_2 = 0$.

lowered, implying a vanishing drag. For two nonidentical wires the drag is usually significantly suppressed at low temperatures relative to the drag in the identical case.[19,20]

In spite of the theoretical effort, a number of open questions remain. In particular, the drag effect is known to be strongest when the electron density is low,[11–13] which typically implies that electron interactions are strong, or equivalently that $r_s \equiv a/(2a_B)$ is large with $a=n^{-1}$ and a_B the Bohr radius. For very strong interactions, there is an exponential separation of the spin exchange energy, J, and the characteristic charge energy, E_F, which at finite temperatures can lead to incoherent (thermally excited) spin degrees of freedom while the charge degrees of freedom remain approximately coherent and close to their ground state.[16,17] This energy hierarchy at finite temperature T, $J \ll T \ll E_F$, we refer to as the spin-incoherent Luttinger liquid regime. Already there is mounting understanding of how such spin-incoherent Luttinger liquids behave in the Green's function,[35–37] in the momentum distribution function,[38] in momentum resolved tunneling,[39] and in transport.[16,17,40,41]

Our goal in this work is to explore some of the implications of spin incoherence on the drag between two quantum wires. We consider only the simplest case of a single channel wire. We attempt to elucidate what qualitative and quantitative changes one can expect for the Coulomb drag when the temperature is much smaller or larger than J. Since J/E_F is exponentially small,[16,17] a small change in the temperature can induce a dramatic change in the temperature dependence of the drag. Based on an earlier work of the authors,[40] we are able to discuss the drag deep in the spin incoherent regime in terms of spinless electrons using a simple mapping between the charge variables of the charge sector of a LL with spin and the variables of a spinless LL.

The crossover to the spin-incoherent regime is discussed using a model of a fluctuating Wigner solid with an antiferromagnetic Heisenberg spin chain in the spin sector. Distortions of the solid couple the spin and charge degrees of freedom. The model allows us to quantitatively address the crossover. The main result is that as the temperature increases from zero to above J for a fixed electron density, the (already weak) $2k_F$ oscillations in the density-density correlations are rapidly lost. As a result, the drag is dramatically suppressed (when $k_F d \gtrsim 1$) since forward scattering contributions vanish in the LL model and the Wigner solid model, and $4k_F$ contributions are suppressed relative to $2k_F$ contributions by $\tilde{U}(4k_F)/\tilde{U}(2k_F) \sim e^{-2k_F d}$ for $k_F d \gtrsim 1$. Here $\tilde{U}(kd)$ is the Fourier transform of the interwire electron-electron interaction, d is the interwire separation, and $k_F \equiv \pi/(2a)$ is the Fermi wave vector. See Fig. 2.

In addition to a possible dramatic suppression (depending on various physical parameters) of the drag, a nonmonotonic dependence on temperature may also occur, as illustrated schematically in Fig. 2. Specifically, when $T>T^*$, where T^* is the "locking" temperature (see Sec. II below), we find that the temperature dependence of the drag is given by

$$r_D = C_1 T^{\alpha_{2k_F}} f(T/J) + C_2 T^{\alpha_{4k_F}}, \qquad (2)$$

where $f(X \to 0) \to 1$ and $f(X \gtrsim 1) \sim e^{-cX}$, with c a constant of order one. The coefficients $C_1 \propto \tilde{U}(2kF)$ and $C_2 \propto \tilde{U}(4kF)$.
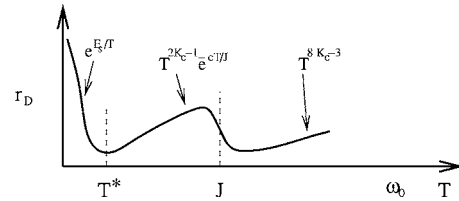

FIG. 2. Schematic of the possible temperature dependence of the Coulomb drag in a wire of sufficiently low electron density that $J \ll E_F \lesssim \hbar\omega_0$, where ω_0 is the frequency of oscillations as defined in Eq. (12). The nonmonotonic temperature dependence shown can be obtained for $K_c > 1/2$ and $\tilde{U}(4k_F) \ll \tilde{U}(2k_F)$, which may be realized for widely space wires. The temperature T^* is the "locking" temperature of two identical wires, below which the drag exhibits activated behavior and E_s is an energy gap associated with the "locking." In this paper we consider only temperatures $T>T^*$ and investigate the limits $T<J$ and $T>J$, as well as the crossover between the two. When $J \ll E_F$ a sharp drop in the drag resistance should be observable for $T \sim J$.

The exponents $\alpha_{2k_F/4k_F}$ depend on the interactions in the wires and on the presence or absence of disorder. We find

$$\alpha_{2k_F} = 2K_c - 1 \quad \text{clean,}$$

$$= 2K_c \quad \text{disordered,} \qquad (3)$$

and

$$\alpha_{4k_F} = 8K_c - 3 \quad \text{clean,}$$

$$= 8K_c - 2 \quad \text{disordered.} \qquad (4)$$

While it is interesting that disorder changes the temperature dependence of the drag, it may play an even more important role in the measurement of the drag effect itself in actual experiments. The reason is that the drag effect is maximal for identical wires; any k_F mismatch results in a drag that is generally strongly diminished relative to this case, indeed, *exponentially* so at low temperature. Disorder tends to "smear" the momentum structure of the density response that determines the drag, eliminating this exponential suppression. Hence, since in a real experiment one will never have truly identical wires, some residual disorder may actually play a key role in experimental studies of drag between one-dimensional systems.

This paper is organized in the following way. In Sec. II we discuss general considerations for drag between two quantum wires with an emphasis on features relevant to the effects of being in the spin-incoherent regime. In Sec. III we discuss a model of a fluctuating Wigner solid with a Heisenberg spin chain to describe the magnetic (spin) degrees of freedom. We derive expressions for the density fluctuations of the electron gas by including magnetoelastic coupling that induces $2k_F$ density modulations at low temperatures ($T \ll J$) and results in an instability towards a local lattice distortion favoring a spin-Peierls-like state. In Sec. IV we discuss the temperature dependence of the Fourier transform of the density-density correlation function in detail. In Sec. V we discuss the drag itself in detail by considering different

temperature regimes, the effects of disorder, and the case of wires with mismatched density. In Sec. VI we summarize the main results of our paper and in the appendicies we give some exact formulas for the density-density correlation function relevant to the spin coherent-incoherent crossover as well as other useful formulas.

II. GENERAL CONSIDERATIONS

We assume the Hamiltonian of our system is of the form

$$H = H_1 + H_2 + H_{12}, \qquad (5)$$

where H_i is the Hamiltonian in the ith wire and H_{12} describes the interactions between electrons in different wires. A proper drag situation is one in which the tunneling between wires can be neglected. We thus assume that H_{12} allows only interactions which forbid electrons to tunnel between the wires.[42] The Hamiltonian H_i in principle describes arbitrary interactions between electrons within the ith wire; depending on the particular situation of interest, a number of different models have been proposed from Fermi liquid[13,19,25] to Luttinger liquid.[20–24]

The Hamiltonian H_{12} is a function of the interwire electron interaction which is often taken to be of the form

$$U_{12}(x) = \frac{e^2}{\epsilon} \frac{1}{\sqrt{d^2 + x^2}}, \qquad (6)$$

where e is the charge of the electron, ϵ is the dielectric constant of the material, and d is the separation between the two wires. The Fourier transform, $\tilde{U}(k) = 2(e^2/\epsilon)K_0(kd)$, depends on the dimensionless parameter kd and for $kd \gg 1$, $\tilde{U}(k) \sim \frac{1}{\sqrt{k}} e^{-kd}$. This exponential dependence of the interwire interaction leads to an exponential dependence of the drag resistivity on wire separation (for large enough d) when the drag is dominated by large momentum transfer as it is in the LL model and the model we will discuss in this paper. The following drag formula (or its equivalent) has been derived by Zheng and MacDonald[18] for a disordered FL,[43] by Klesse and Stern[21] for a LL, and by Pustilnik et al.[19] for a clean FL:

$$r_D = \int_0^\infty dk \int_0^\infty d\omega \frac{k^2 \tilde{U}_{12}^2(k)}{4\pi^2 n_1 n_2 T} \frac{\text{Im}\,\chi_1^R(k,\omega)\text{Im}\,\chi_2^R(k,\omega)}{\sinh^2(\omega/2T)}, \qquad (7)$$

where $\text{Im}\,\chi_i^R(k,\omega)$ is the imaginary part of the Fourier-transformed retarded density-density correlation function, Eqs. (33) and (34), and n_i is the density of the electrons in the ith wire. Knowing the general features of the interwire interaction U_{12} (see Fig. 3), it is clear that to determine the drag one must determine the $\text{Im}\,\chi_i^R(k,\omega)$.

We will discuss the behavior of $\text{Im}\,\chi_i^R(k,\omega)$ and its temperature dependence in detail in Sec. IV but for now it is sufficient to point out that most of the weight occurs near momenta of $k \approx 0$, $k \approx 2k_F$, and $k \approx 4k_F$ so that one may write

$$\text{Im}\,\chi_i^R(k,\omega) \approx \text{Im}\,\chi_i^{R,0}(k,\omega) + \text{Im}\,\chi_i^{R,2k_F}(k,\omega)$$

$$+ \text{Im}\,\chi_i^{R,4k_F}(k,\omega). \qquad (8)$$

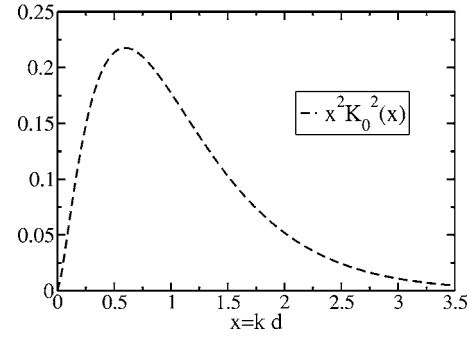As we will see, the $2k_F$ components of $\text{Im}\,\chi_i^R(k,\omega)$ present at

FIG. 3. Momentum dependence of the quantity $k^2 \tilde{U}_{12}^2(k) = \left(\frac{2e^2}{\epsilon d}\right)^2 [xK_0(x)]^2$ appearing in the drag formula Eq. (7) as a function of the dimensionless variable $x = kd$ where we have assumed the real space interwire electron interaction (6). Plotted is the quantity $[xK_0(x)]^2$ which has a maximum for $x \approx 0.6$. For $x \ll 1$, $[xK_0(x)]^2 \sim [-\gamma + \ln(2) - \ln(x)]^2 x^2$ where $\gamma \approx 0.5772$ is Euler's constant, while for $x \gg 1$, $[xK_0(x)]^2 \sim e^{-2x}$.

low temperatures, $T \ll J$, will disappear when $T \gg J$. Moreover, once it is known that $\text{Im}\,\chi_i^{R,0}(k,\omega) \propto \delta(\omega - v_{c,i}k)$ (see Sec. IV) where $v_{c,i}$ is the charge velocity of the ith wire, it is readily seen from Eq. (7) that for the LL model (or any model possessing a harmonic bosonized theory) the $k = 0$ contribution to the drag resistivity vanishes (in the absence of disorder), leaving only contributions from the higher $k = 2k_F, 4k_F$ values of $\text{Im}\,\chi_i^R(k,\omega)$. The higher $k = 2k_F, 4k_F$ values can be strongly suppressed by the interwire interaction because $\tilde{U}_{12}^2(k) \sim e^{-2kd}$ when $kd \gg 1$. Hence, in the limit $k_F d \gtrsim 1$ we expect a dramatic reduction in the drag and a change in the temperature dependence of the drag (see below) when $2k_F$ oscillations are lost and only $4k_F$ modulations remain. However, before we address these details, it is useful to go over what we can say about drag deep in the spin-incoherent regime itself.

Based on the results of earlier work,[20,21] we can immediately discuss the drag deep within the spin-incoherent LL regime, $J \ll T \ll E_F$. It has earlier been argued[40] that the spin-incoherent LL behaves essentially as a spinless LL by noting that the Hamiltonian is diagonal in spin to lowest order in $J/T \ll 1$, and by demanding the equivalence of the physical charge density in both cases. The equivalence can be summarized by the simple equation $K = 2K_c$ (in the notation of Klesse and Stern[21]) relating the interaction parameter of the spinless LL theory and the interaction parameter of the charge sector of the LL theory with spin. (Strictly speaking, for drag the relevant interaction parameter is $K = K_-$, the interaction parameter in the odd channel of the relative density of the two wires,[20,21] but when the interwire interactions are sufficiently weak K_- is essentially equal to the corresponding parameter of the isolated wire.) The relation $K = 2K_c$ is valid for any particle conserving operator;[40] the drag resistivity is derived from such an operator. Hence, those general results apply here.

As discussed in Refs. 20 and 21, drag in the spinless LL model comes from the backscattering of electrons. As long as the interwire interactions are sufficiently small the effects of backscattering can be treated perturbatively. Based on such a perturbative treatment, Klesse and Stern found[21] that

for identical wires there is a temperature scale T^* that separates high- and low-temperature drag regimes. [It is assumed throughout this paper that T^* is greater than the temperature $T_L = \hbar v_c/L$ (with v_c the charge velocity and L the length of the wire) so that the Fermi liquid leads attached to the quantum wire are not felt.] For temperatures $T \gg T^*, T_L$, the drag varies as a power of the temperature, while for temperatures $T^* \gg T \gg T_L$, the drag resistivity shows activated behavior with a gap of order T^* itself. The physics is similar to that of a pinned charge density wave. This result follows from an analysis of a sine-Gordon model in the odd channel of the coupled wire problem. Applying the equivalence rule $K = 2K_c$ discussed above in the spin incoherent regime, we have

$$\rho_D \sim T^{8K_c - 3}, \quad T \gg T^*, T_L, J, \tag{9}$$

$$\rho_D \sim e^{E_s/T}, \quad T^* \gg T \gg T_L, J, \tag{10}$$

where $E_s \sim T^*$.[21] Note that for $3/8 < K_c < 3/4$ the temperature dependence of the spinless (fully polarized) electron gas and the spin-incoherent electron gas exhibit very different drag resistivity behavior with temperature when $T \gg T^*$. (The spinless case has a diverging drag resistivity as T is lowered, while the spin-incoherent case has a suppressed drag resistivity as T is lowered.) This is qualitatively similar to the transport results found in Ref. 40 for $1/2 < g_c < 1$.

The results (9) and (10) above were derived from a perturbative analysis of the sine-Gordon equation which results from treating the backscattering in LL theory. For most realistic parameter values, the backscattering strength flows to strong coupling and the resulting state is that of the two quantum wires locked into a "zig-zag" charge pattern. The value of T^* depends on details of the quantum wire system such as the density, wire widths, and separation d,[21] but for most realistic situations $T^*/E_F \leq 0.01$.

It is interesting to consider how spin incoherence affects the "zig-zag" locking pattern of the electrons in the two wires. The relative size of J and T^* will determine what the periodicity of the "zig-zag" pattern will be for $T \ll T^*$. For $J \ll T \ll T^*$, there is a "$4k_F$" locking (seen easily from the $K = 2K_c$ mapping[40]) since $T \gg J$ ensures $2k_F$ pieces of the density are washed out, while for $T \ll J, T^*$, there is a "$2k_F$" locking. Of course, for $T^* \ll J \ll T$ the locking phase is not obtained. Throughout this paper we will assume $T \gg T^*$ so that we need not be concerned with "locking" from here forward.

Similar arguments to those given in Ref. 20 can also be used to describe the incommensurate-commensurate transition deep in the spin incoherent regime for wires of different electron densities. We now leave generalities behind and turn to a detailed calculation of the drag itself in the regime of very strongly interacting one-dimensional electrons.

III. THE FLUCTUATING WIGNER SOLID MODEL

We assume from the outset that the interactions between the electrons are very strong, which typically means the density is low enough that we can treat the electrons in each wire as a harmonic chain[44] in the charge sector and a nearest

neighbor Heisenberg anti-ferromagnet in the spin sector:[17,39]

$$H_{\text{wire}} = H_c + H_s, \tag{11}$$

where

$$H_c = \sum_{l=1}^{N} \frac{p_l^2}{2m} + \frac{m\omega_0^2}{2}(u_{l+1} - u_l)^2 \tag{12}$$

is the Hamiltonian in the charge sector with p_l the momentum of the lth electron, u_l the displacement from equilibrium of the lth electron, m the electron mass, and ω_0 the frequency of local electron displacements (this will depend on the electron density, the width of the wires, the dielectric constant of the material, and other parameters such as the distance to a nearby gate[45,46]). The position of the electrons along the chain are given by

$$x_l = la + u_l, \tag{13}$$

where a is the mean spacing of the electrons. The Hamiltonian of the spin sector takes the form

$$H_s = \sum_l J_l \mathbf{S}_{l+1} \cdot \mathbf{S}_l. \tag{14}$$

Note that in (14) the coupling J_l between spins depends on the distance between them. Assuming that the fluctuations from the equilibrium positions are small compared to the mean particle spacing, we can expand the exchange energy as

$$J_l = J_0 + J_1(u_{l+1} - u_l) + \mathcal{O}((u_{l+1} - u_l)^2). \tag{15}$$

In this case the full Hamiltonian takes the form

$$H = H_c + H_s + H_{s\text{-}c}, \tag{16}$$

where

$$H_{s\text{-}c} = J_1 \sum_l (u_{l+1} - u_l) \mathbf{S}_{l+1} \cdot \mathbf{S}_l. \tag{17}$$

Here H_{s-c} represents a magneto-elastic coupling as it couples the magnetic modes to the elastic distortions of the lattice that constitute the charge modes.

Our goal is to evaluate the Fourier transform of the retarded density-density correlation function $-i\theta(t-t')\langle[\rho(x,t),\rho(x',t')]\rangle$ [which appears in the drag formula (7)] up to second order in J_1 for T, $J_0 \ll \hbar\omega_0$, for both $T \ll J_0$ and $T \gg J_0$. We use the following definition of the electron density: $\rho(x,t) = \Sigma_l \delta(x - al - u_l(t))$. An exact calculation within this model is presented in Appendix A. Here we will pursue an approximate calculation that captures all of the essential features of the more exact perturbative results.

A. Low-energy approach to charge fluctuations

In this work we are concerned only with energies (temperatures) small compared to the characteristic lattice energy, i.e., $T \ll \hbar\omega_0$, but still large compared to the "locking" temperature T^*. When $T \ll J_0$ further approximations can be made, but for now our only restriction will be that $T \ll \hbar\omega_0$. We begin by expanding the displacement of the electron den-
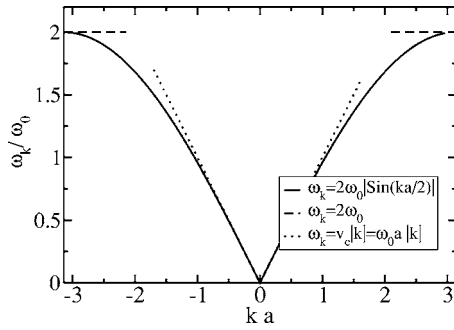
FIG. 4. Approximate form for the phonon spectrum. There are two relevant parts of the spectrum that enter the action S_c, the part near $k \approx 0$ and the part near $k \approx \pi/a$. The phonons with $k \approx 0$ are described by a Debye model while those phonons with $k \approx \pi/a$ are described by an Einstein model.

sity in a Fourier series. For low-energy distortions the $k \approx 0$ component is most important, while the magneto-elastic term (17) couples the $k \approx \pi/a$ component to the spin operator $\mathbf{S}_{l+1} \cdot \mathbf{S}_l$. Thus, the displacement, u_l, of the lth electron in the harmonic chain (12) is approximately given by

$$u_l = u_0(la) + u_\pi(la)(-1)^l, \qquad (18)$$

where u_0 refers to the $k \approx 0$ component of the displacement and u_π refers to the $k \approx \pi/a$ displacement. Both u_0 and u_π are assumed to be slowly varying functions of position, and we expect $u_\pi \ll u_0$.

1. Low-energy form of the action

The action for the low-density electron gas is

$$S = S_c + S_s + S_{s\text{-}c}. \qquad (19)$$

Using the expression (18) for the particle displacement fields, and noting that from the phonon dispersion of (12), $\omega_k = 2\omega_0 |\sin(ka/2)|$, one has for $k \approx 0$ the dispersion $\omega_k = v_c |k|$ with $v_c = \omega_0 a$, while for $k \approx \pi/a$ the dispersion $\omega_k = 2\omega_0$ is independent of k. (See Fig. 4.) In the charge sector we then have the following action of the form $S_c = S_c^0 + S_c^\pi$,

$$S_c = \int \frac{dx d\tau}{2\pi} \left[\frac{1}{K_c v_c} \{ [\partial_\tau \theta_c(x,\tau)]^2 + v_c^2 [\partial_x \theta_c(x,\tau)]^2 \} \right.$$
$$\left. + m\pi \{ [\partial_\tau u_\pi(x,\tau)]^2 + (2\omega_0)^2 u_\pi(x,\tau)^2 \} \right], \qquad (20)$$

where $K_c = \pi \hbar / (2amv_c)$. Note the lack of spatial derivative in the u_π piece of the action which results in the absence of any k dependence in ω_k near $k \approx \pi/a$. In effect, we have described the small k oscillations (phonons) with a Debye-type model and the large k oscillations as an Einstein model. Figure 4 illustrates the approximations to the full phonon spectrum.

In the standard LL model for weakly interacting electrons $K_c = v_F / v_c \lesssim 1$, where v_F is the Fermi velocity of non-interacting electrons. In the present case of strongly interacting electrons we have $K_c = \frac{\hbar^2 \pi}{2ma^2} \frac{1}{\hbar \omega_0} \sim \frac{E_F}{\hbar \omega_0} \sim \frac{\text{Kin}}{\text{Pot}} \sim \frac{1}{r_s}$, where E_F is the Fermi energy of non-interacting electrons and r_s

$\equiv a/(2a_B)$ where $a_B = \epsilon \hbar^2 / me^2$ is the Bohr radius of a material of dielectric constant ϵ. Thus, in the strongly interacting limit K_c scales roughly as the ratio of the kinetic energy to the potential energy which itself roughly scales as r_s^{-1}, implying that very strong (long-range) interactions can lead to small K_c. In practice, however, K_c rarely appears to be smaller than 0.2 or so.

By making the identification $\mathcal{E}_l \equiv (-1)^l \mathbf{S}_{l+1} \cdot \mathbf{S}_l \to \mathcal{E}(x,\tau)$ in the continuum limit, we have spin-charge coupling in the action

$$S_{s\text{-}c} = \int \frac{dx d\tau}{2\pi} 2J_1 u_\pi(x,\tau) \mathcal{E}(x,\tau). \qquad (21)$$

In the special situation where $T \ll J_0$, the action for the spin sector (14) can be bosonized as[47,48]

$$S_s = \int \frac{dx d\tau}{2\pi} \left[\frac{1}{K_s v_s} \{ [\partial_\tau \theta_s(x,\tau)]^2 + v_s^2 [\partial_x \theta_s(x,\tau)]^2 \} \right], \qquad (22)$$

where the $SU(2)$ symmetry of the Heisenberg model implies that $K_s = 1$ and the spin velocity is $v_s \sim J_0 a / \hbar$. (Here intrawire backscattering effects in the spin sector have been neglected and a sine-Gordon term dropped. Since we are ultimately interested in energies/temperatures much larger than J_0 or much smaller than J_0, this will not affect any of our conclusions.) However, when $T \gtrsim J_0$ the action for the spin sector must describe more accurately the short-distance physics of the Heisenberg chain. Nevertheless, the action (22) will prove useful in understanding the approach to the spin-incoherent regime in the limit $T \to J_0$ from below.

Finally, for $T \ll J_0$ the coupling term in the action can be expressed as[48]

$$S_{s\text{-}c} = \int \frac{dx d\tau}{2\pi} \frac{2J_1}{2\pi \alpha a} u_\pi(x,\tau) \sin[\sqrt{2}\theta_s(x,\tau)], \qquad (23)$$

where we have used the low-energy bosonized form $(-1)^l \mathbf{S}_{l+1} \cdot \mathbf{S}_l \approx \frac{1}{2\pi \alpha} \sin[\sqrt{2}\theta_s(x)]$. Here $\alpha = \mathcal{O}(a)$ is a short distance cutoff of the order of the lattice spacing. At low energies this term will lead to the spin-Peierls state indicated in Fig. 5.

2. Expressing the density

Expanding the density and making use of (18) then gives (we have suppressed the time dependence of u_0 and u_π immediately below for clarity of presentation)

$$\rho(x) = \sum_l \delta(x - la - u_l) = \sum_l \delta(x - la - u_0(la) + u_\pi(la)(-1)^l)$$

$$\approx \sum_l \left[\delta(x - la - u_0(la)) \right.$$

$$\left. - \delta'(x - la - u_0(la)) u_\pi(la)(-1)^l \right]. \qquad (24)$$

Multiplying by $\delta(x' - la)$, integrating over x', and using the Poisson summation identity,

$$\sum_l \delta(x' - la) = \frac{1}{a} \sum_{m=-\infty}^{\infty} \exp\left(i \frac{2\pi}{a} m x' \right), \qquad (25)$$
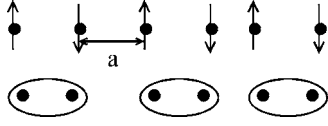
the expression for the density becomes

FIG. 5. Classical Wigner solid model. The electrons are indicated by solid black dots and are separated by a mean spacing distance a. Top: A "cartoon" of the antiferromagnetic spin arrangement is indicated by the arrows. The lattice has density oscillations of smallest wave vector $4k_F$. Bottom: Magneto-elastic coupling allows the system to lower its energy by slightly distorting the lattice in order to gain magnetic energy. Stronger spin correlations are indicated by the ovals which pair neighboring electrons at spacing less than a. The lattice has density oscillations of smallest wave vector $2k_F$, indicating twice the period of the undistorted lattice. When $T \gg J_0$ the (small) lattice distortion is thermally washed out, leaving only the $4k_F$ periodicity of the underlying Wigner solid.

$$\rho(x) \approx \int dx' \frac{1}{a} \delta(x - x' + u_0(x')) \sum_{m=-\infty}^{\infty} \exp\left(i\frac{2\pi}{a}mx'\right)$$
$$\times \left\{ 1 + \cos\left(\frac{\pi}{a}x'\right)\partial_{x'}u_\pi(x') \right.$$
$$+ i\frac{\pi}{2a}\left[(2m+1)\exp\left(i\frac{\pi}{a}x'\right) \right.$$
$$\left. \left. + (2m-1)\exp\left(-i\frac{\pi}{a}x'\right) \right] u_\pi(x') \right\}, \tag{26}$$

where we have integrated by parts in the second term of (24). Performing the integration over x', making use of the approximate relation $\delta(x - x' - u_0(x')) \approx \delta(x' - x - u_0(x))/|1 + \partial_x u_0(x)|$, and assuming $a\partial_x u_\pi(x) \ll u_\pi(x)$, we find the most important terms up to $4k_F$ are

$$\rho(x) \approx \frac{1}{a}[1 - \partial_x u_0(x)]\left(1 - \frac{2\pi}{a}\sin\{2k_F[x + u_0(x)]\}u_\pi(x) \right.$$
$$\left. + \cos\{4k_F[x + u_0(x)]\} \right)$$
$$\approx \rho_0 - \frac{\sqrt{2}}{\pi}\partial_x \theta_c(x) - \rho_0 \frac{2\pi}{a}\sin[2k_F x + \sqrt{2}\theta_c(x)]u_\pi(x)$$
$$+ \rho_0 \cos[4k_F x + \sqrt{8}\theta_c(x)], \tag{27}$$

where $\rho_0 \equiv 1/a$, $k_F \equiv \pi/(2a)$, and we have made the identification $u_0(x)/a = \sqrt{2}\theta_c(x)/\pi$. Recall the field θ_c is governed by the action (20). This formula resembles the standard bosonized expression for the density of a Luttinger liquid (except for the term u_π multiplying the $2k_F$ part of the density instead of a term involving the spin fields). As we will now see, a formula very similar to that obtained from the standard Luttinger liquid treatment results from integrating out the high-energy u_π phonon modes in favor of the low-energy spin modes. To see this, consider only the $2k_F$ part of the density and compute $\langle \rho_{2k_F}(x, \tau) \rangle$ to lowest order in the action S_{s-c} and integrate out the u_π fields to obtain a new $\rho_{2k_F}^{\text{eff}}(x, \tau)$ independent of u_π. At lowest order we find

$$\langle u_\pi(x, \tau) \rangle = -\frac{2J_1}{\hbar} \int \frac{dx' d\tau'}{2\pi} \mathcal{E}(x', \tau')$$
$$\times \langle T_\tau u_\pi(x, \tau)u_\pi(x', \tau') \rangle_{S_c^\pi}, \tag{28}$$

where T_τ is the τ-ordering operator and the τ-ordered product is evaluated in the action S_c^π given in (20). The τ-ordered product is readily evaluated as

$$\langle u_\pi(x, \tau)u_\pi(x', \tau') \rangle_{S_c^\pi}$$
$$= \frac{\hbar \delta(x - x')}{4m\omega_0}\left(\frac{e^{2\omega_0|\tau-\tau'|}}{e^{\beta 2\omega_0} - 1} - \frac{e^{-2\omega_0|\tau-\tau'|}}{e^{-\beta 2\omega_0} - 1} \right)$$
$$\rightarrow \frac{\hbar \delta(x - x')}{4m\omega_0}e^{-2\omega_0|\tau-\tau'|} \text{ as } \beta\omega_0 \rightarrow \infty. \tag{29}$$

Recall that we are interested only in temperatures low compared to the phonon energy ω_0 so the limit $\beta\omega_0 \rightarrow \infty$ is the appropriate one. Here $\beta = (k_B T)^{-1}$ where k_B is Boltzmann's constant. The integral over position in (28) is immediately evaluated with the delta function in (29) and the remaining integral over τ' can be approximately evaluated under the assumption $\beta\omega_0 \rightarrow \infty$, which produces the dominant contribution at $\tau' \approx \tau$ with a width in τ' of order $1/(2\omega_0)$ resulting in

$$\langle u_\pi(x, \tau) \rangle \approx -\frac{2J_1}{16\pi^2 m\omega_0^2}\mathcal{E}(x, \tau), \tag{30}$$

which yields

$$\rho_{2k_F}^{\text{eff}}(x, \tau) = \frac{1}{8\pi}\left(\frac{J_1}{m\omega_0^2 a^2} \right)\sin[2k_F x + \sqrt{2}\theta_c(x, \tau)] \times \mathcal{E}(x, \tau). \tag{31}$$

The result (31) is general, and valid whenever $T \ll \hbar\omega_0$. However, when $T \ll J_0$ one may use the expression $\mathcal{E}(x, \tau) = \frac{1}{2\pi\alpha}\sin[\sqrt{2}\theta_s(x, \tau)]$ which leads to the familiar looking density

$$\rho^{\text{eff}}(x, \tau) = \rho_0 - \frac{\sqrt{2}}{\pi}\partial_x \theta_c(x, \tau) - \frac{\rho_0}{16\pi}\left(\frac{J_1}{m\omega_0^2 a^2} \right)$$
$$\times \sin[2k_F x + \sqrt{2}\theta_c(x, \tau)]\sin[\sqrt{2}\theta_s(x, \tau)]$$
$$+ \rho_0 \cos[4k_F x + \sqrt{8}\theta_c(x, \tau)]. \tag{32}$$

The expression for the effective density (32) with the high-energy u_π modes integrated out in favor of the spin variables is valid for $T \ll J_0$ and may be compared directly with the LL result obtained for weakly interacting electrons.[5] At temperatures $T \gg J_0$ (31) must be used for the $2k_F$ density variations. The only material difference between (32) and the standard LL result is the dimensionless ratio of spin and charge energies $\left(\frac{J_1}{m\omega_0^2 a^2} \right) \sim \frac{v_s}{v_c}$ which is absent (since it is of order 1) in the familiar LL case. When the interactions are strong as we have assumed them to be here, then $\left(\frac{J_1}{m\omega_0^2 a^2} \right) \sim \frac{v_s}{v_c} \ll 1$, since J_1 diminishes and ω_0 increases with increasing strength of the interactions. However, starting from the strongly interacting limit and decreasing the interaction strength the ratio $\left(\frac{J_1}{m\omega_0^2 a^2} \right) \rightarrow 1$. It is worth emphasizing, then,

that when the temperature is low compared to both spin and charge energies a 1-D electron gas always behaves as a LL in the sense of the various power laws that will appear in the correlation functions, although the overall prefactors of the $2k_F$ pieces will be down by the ratio $\left(\frac{J_1}{m\omega_0^2 a^2}\right)$. If, on the other hand, the system is in the spin-incoherent regime $J_0 \ll T \ll \hbar\omega_0$, the $2k_F$ parts of the correlations will be washed out from thermal effects. We now turn to an investigation of how this happens in detail for the case of the density-density correlation function and then discuss the implications for the Coulomb drag between two quantum wires.

IV. EVALUATION OF Im $\chi_i^R(k,\omega)$

Here we consider two limits of the double wire system shown in Fig. 1: (*i*) clean wires without disorder and (*ii*) wires with weak disorder. The case of strong disorder is uninteresting as the electrons are all localized over the relevant energy/length scales of the experiment. As we discussed in Sec. II, the drag formula (7) generically contains contributions at $k\approx 0$, $k\approx 2k_F$, and $k\approx 4k_F$ so that Im $\chi_i^R(k,\omega) \approx$ Im $\chi_i^{R,0}(k,\omega) +$ Im $\chi_i^{R,2k_F}(k,\omega) +$ Im $\chi_i^{R,4k_F}(k,\omega)$. We now turn to an evaluation of each of these pieces. We have used the two standard (equivalent) definitions

$$\chi_i^R(k,\omega) = -i \int_{-\infty}^{\infty} dx \int_0^{\infty} dt\, e^{i((\omega+i\eta)t - kx)}$$
$$\times \langle [(\rho_i(x,t) - \rho_{i,0}), (\rho_i(0,0) - \rho_{i,0})] \rangle, \quad (33)$$

and

$$\chi_i(k,\omega_n) = -\int_{-\infty}^{\infty} dx \int_0^{\beta} d\tau\, e^{i(\omega_n\tau - kx)}$$
$$\times \langle (\rho_i(x,\tau) - \rho_{i,0})(\rho_i(0,0) - \rho_{i,0}) \rangle, \quad (34)$$

where $\rho_{i,0}$ is the average density of the ith wire, and η is a small infinitesimal that ensures convergence of the time integral in (33). The retarded correlation function is obtained from (34) via the substitution $i\omega_n \rightarrow \omega + i\eta$. We will use both of the formulas above in the subsections that follow.

A. Clean wires

We first consider wires with no disorder. We will also assume initially that $T \ll J_0$ so that we may use the form of the density (32). [This is only an issue for the evaluation of Im $\chi_i^{R,2k_F}(k,\omega)$ since Im $\chi_i^{R,0}(k,\omega)$ and Im $\chi_i^{R,4k_F}(k,\omega)$ do not involve the spin sector of the Hamiltonian.] As the authors discussed in Ref. 40 the approach to the spin incoherent regime from temperatures well below the spin energy can be understood in this way. In all calculations below, recall that we have assumed the temperature is low, $T \ll \hbar\omega_0$, so that the charge sector is always in the LL regime and described by the action S_c^0 in (20).

1. Im $\chi_i^{R,0}(k,\omega)$

We first evaluate the $k \approx 0$ piece of the retarded density-density correlation function. From the expression (32), we have

$$\rho_0^{\text{eff}}(x,t) = \rho_0 - \frac{\sqrt{2}}{\pi} \partial_x \theta_c(x,t), \quad (35)$$

whose correlation function is readily computed (see Appendix B) to yield

$$\text{Im } \chi_i^{R,0}(k,\omega) = \frac{k^2}{a^2} \frac{\hbar L}{2m\omega_k} [\pi\delta(\omega - \omega_k) - \pi\delta(\omega + \omega_k)]. \quad (36)$$

The equation above, (36), is the central result of this subsection and it is worth pausing to emphasize some of its features. Most notably, while the calculation was done at finite temperature, there is no temperature dependence of Im $\chi_i^{R,0}(k,\omega)$. Thus, the finite temperature $k \approx 0$ response is identical to the zero temperature response. This means that temperature does not "broaden" the zero temperature δ-function reponse. Moreover, for the model at hand, at small $|k|$, $\omega_k = v_{i,c}|k|$ so that for a given k there is a unique value of ω_k. This means, then, when the result (36) is substituted into the drag formula (7) the drag is identically zero. (An exception is the measure zero point where the wires are identical, i.e., $v_{1,c} = v_{2,c}$, and the drag response is infinite. For real wires this precise matching is not possible and the $k = 0$ part of the drag generically vanishes. In fact, the unphysical divergence can be traced back to the wrong order of the limits of vanishing curvature in the spectrum and vanishing velocity difference between the two wires.[19])

In our work here, we have assumed from the outset that the electron interactions are very strong and a direct bosonization of the electron operator is not valid. Instead, the approximation we have made to obtain the action (20), which is formally identical to that obtained for weakly interacting electrons with a linear dispersion (aside from the u_π terms), is to treat the displacements of electrons to lowest order in the Taylor series: $(u_{l+1} - u_l)/a \approx \sqrt{2}\partial_x\theta_c(x)/\pi$. Including higher derivatives would result in an interacting bosonic theory and would likewise broaden the delta functions in (36) by an amount inversely proportional to the lifetime and would yield a finite $k \approx 0$ drag. The precise nature of this contribution to the drag is still a subject of ongoing research.[19,49] It is therefore difficult to compare it quantitatively in theoretical calculations to the $2k_F$ and $4k_F$ contributions. However, we expect that it may be larger or smaller than the latter depending upon circumstances. For instance, the $k \approx 0$ drag is clearly *subdominant* for drag between identical, clean wires, with repulsive interactions at the lowest temperatures. Fortunately, for our purposes of discerning the spin coherent to incoherent crossover at $T \approx J$, we may satisfy ourselves with the observation that the $k \approx 0$ drag is in any case *featureless* at this temperature. Hence, it can easily be "subtracted" by looking for strong temperature-dependent changes in the drag in this temperature window. The "subtraction" procedure of course assumes that the $2k_F$ contribution is non-negligible compared to the $k \approx 0$ part which requires that the exchange energy J is smaller than a crossover temperature T_{cross} below which $2k_F$ contributions dominate the $k \approx 0$ drag contributions. This temperature can be roughly estimated[19] as $T_{\text{cross}} \sim \omega_0 \left(\frac{J_1}{m\omega_0^2 a^2} \sqrt{\frac{l_0}{l_{2k_F}}}\right)^{1/(3/2 - K_c)}$, where l_0 and

l_{2k_F} are the zero momentum and $2k_F$ scattering lengths, respectively.

2. Im $\chi_i^{R,2k_F}(k,\omega)$

The $2k_F$ component of the density response and its temperature dependence are the central issue in this paper and we now turn to it in detail. We have already discussed general features of the spin-incoherent limit $T \gg J_0$ in Sec. II, and we will discuss other more detailed and quantitative features of that regime in the next subsection where we consider Im $\chi_i^{R,4k_F}(k,\omega)$. Here, we will initially assume that the temperature is low compared to the spin energies, $T \ll J_0$, and use the low-energy density expression (32). Starting from the low-temperature limit we show that as the temperature becomes of the order of the spin energy, the temperature dependence of the $2k_F$ part of the drag changes and rapidly vanishes as $J_0 \rightarrow 0$ for fixed $T \gg J_0$. We also show that in the low-temperature limit we recover the temperature dependence of the drag obtained by Klesse and Stern[21] for electrons with spin. When $k_F d \gtrsim 1$ the loss of $2k_F$ contributions to the drag (when $T \approx J_0$) implies [via Eq. (7) and Fig. 3] that there is expected to be a dramatic reduction in the drag over a very small temperature window when only the $4k_F$ contribution remains, as $\tilde{U}(4k_F)/\tilde{U}(2k_F) \sim e^{-2k_F d}$ for $k_F d \gtrsim 1$.

The $2k_F$ part of the low-energy density operator (32) is

$$\rho_{2k_F}^{\text{eff}}(x,t) = -\frac{\rho_0}{16\pi}\left(\frac{J_1}{m\omega_0^2 a^2}\right)\sin[2k_F x + \sqrt{2}\theta_c(x,\tau)]$$
$$\times \sin[\sqrt{2}\theta_s(x,\tau)], \tag{37}$$

which leads to the following finite temperature result for the $2k_F$ part of the density-density correlation function computed from (20) and (22):

$$-i\langle[\rho_{2k_F}^{\text{eff}}(x,t),\rho_{2k_F}^{\text{eff}}(0,0)]\rangle$$
$$= \left(\frac{\rho_0}{16\pi}\right)^2\left(\frac{J_1}{m\omega_0^2 a^2}\right)^2\cos(2k_F x)$$
$$\times \text{Im}\left(\frac{(\pi T\alpha/v_c)^{K_c}}{\left[\sinh\left(\frac{\pi T}{v_c}(x-v_c t)\right)\sinh\left(\frac{\pi T}{v_c}(x+v_c t)\right)\right]^{K_c/2}}\right.$$
$$\left.\times \frac{(\pi T\alpha/v_s)^{K_s}}{\left[\sinh\left(\frac{\pi T}{v_s}(x-v_s t)\right)\sinh\left(\frac{\pi T}{v_s}(x+v_s t)\right)\right]^{K_s/2}}\right). \tag{38}$$

Here α is a short-distance cutoff of the order of the lattice spacing. We note that in Eq. (38)—and in subsequent similar formulas—singularities at $x = \pm v_c t$, $\pm v_s t$ are regularized by infinitesimal imaginary parts to the time t, which for ease of presentation are not shown. It is worth pointing out that because of the hyperbolic nature of the correlation function at finite temperature, a temperature-dependent "coherence length" naturally appears in both the spin and charge sectors. From inspection, the charge coherence length $\xi_c(T) = v_c/(K_c\pi T) \sim a\hbar\omega_0/(k_B T)$, and the spin coherence length $\xi_s(T) = v_s/(K_s\pi T) \sim aJ_0/(k_B T)$. Strong interactions imply

$v_s/v_c \ll 1$ $(J_0 \ll \hbar\omega_0)$ so that $\xi_s(T) \ll \xi_c(T)$. Note that $\xi_s(T) \approx a$ when $T \approx J_0$.

Our task is now to substitute (38) into the integral in (33) and evaluate the integrals over position and time. Unfortunately, this integral does not appear to have a closed, analytical form. Nevertheless, its general structure is apparent. At zero temperature the structure in the (k,ω) plane is very similar to that of the Green's function already computed by Voit[50] and by Meden and Schönhammer.[51] Depending on the value of K_c there are singularities or thresholds at $\omega = v_s k_{\pm}$ and $\omega = v_c k_{\pm}$, where $k_{\pm} = k \pm 2k_F$. With small but finite temperature these features are smoothed out. However, as the temperature increases towards J_0, the overall weight in $\chi_i^{R,2k_F}(k_{\pm},\omega)$ begins to rapidly diminish. To see this, consider the limit $T \rightarrow J_0$ from below. Then $\chi_i^{R,2k_F}(k_{\pm},\omega)$ can be bounded as

$$\chi_i^{R,2k_F}(k_{\pm},\omega)$$
$$< \left(\frac{k_B T}{J_0}\right)^{K_s}\exp\left(-c\frac{k_B T}{J_0}\right)\int_{-\infty}^{\infty}dx\int_0^{\infty}dt\, e^{i(\omega t - k_{\pm}x)}$$
$$\times \left(\frac{\rho_0}{16\pi}\right)^2\left(\frac{J_1}{m\omega_0^2 a^2}\right)^2$$
$$\times \text{Im}\left(\frac{(\pi\alpha T/v_c)^{K_c}}{\left[\sinh\left(\frac{\pi T}{v_c}(x-v_c t)\right)\sinh\left(\frac{\pi T}{v_c}(x+v_c t)\right)\right]^{K_c/2}}\right), \tag{39}$$

where c is a constant of order unity. For fixed T, $\chi_i^{R,2k_F}(k_{\pm},\omega) \rightarrow 0$ as $J_0 \rightarrow 0$. This conclusion is independent of the particular form of the operator used in the spin sector. For example, using the more general expression (31) will lead to the same conclusion for any $\mathcal{E}(x,t)$. Thus, the already weak [because $\left(\frac{J_1}{m\omega_0^2 a^2}\right)^2 \ll 1$] $2k_F$ density oscillations are rapidly suppressed with temperatures once $T \sim J_0$. See Fig. 5 for an illustration.

Having emphasized how "fragile" $\chi_i^{R,2k_F}(k_{\pm},\omega)$ is for $T \sim J_0$, let us now return to the low-temperature limit $T \ll J_0$. In this limit, the temperature dependence of Im $\chi_i^{R,2k_F}(k,\omega) = \chi_i^{R,2k_F}(k_+,\omega) + \chi_i^{R,2k_F}(k_-,\omega)$ can be readily extracted by making the substitutions $\tilde{x} = \pi Tx/v_c$ and $\tilde{t} = \pi Tt$ and then computing the Fourier transform. With this substitution, we find

$$r_D^{2k_F} \propto \left(\frac{J_1}{m\omega_0^2 a^2}\right)^2(2k_F)^2\tilde{U}_{12}^2(2k_F)T^{2(K_c+K_s)-3}f(T/J_0), \tag{40}$$

where we have used the result that at low enough temperatures, $\int_0^{\infty}dk k^2\tilde{U}_{12}^2(k)\text{Im}\,\chi_1^R(k,\omega)\text{Im}\,\chi_2^R(k,\omega) \approx (2k_F)^2\tilde{U}_{12}^2(2k_F)\text{Im}\,\chi_1^R(2k_F,\omega)\text{Im}\,\chi_2^R(2k_F,\omega)\Delta k$, where $\Delta k \sim T$, and used the result that the ω integration in (7) for (38) does not contribute to any temperature dependence of the drag. The function $f(X \rightarrow 0) \rightarrow 1$ and $f(X \gtrsim 1) \sim e^{-cX}$, where c is a constant of order unity.

The result (40) is identical to the result obtained by Stern and Klesse[21] in the weakly interacting limit of the 1-D

electron gas when $T \ll J_0$. Note that while the temperature dependence is the same in the low-temperature limit, the overall result is still down by a factor $\sim \left(\frac{J_1}{m\omega_0^2 a^2}\right)^2 \ll 1$ when the interactions are strong.

$$-i\langle[\rho_{2k_F}^{\rm eff}(x,t),\rho_{2k_F}^{\rm eff}(0,0)]\rangle = \frac{1}{16}\left(\frac{J_1}{m\omega_0^2 a^2}\right)^2 \cos(2k_F x) {\rm Im}\left(\frac{(\pi T\alpha/v_c)^{K_c}}{\left[\sinh\left(\frac{\pi T}{v_c}(x-v_c t)\right)\sinh\left(\frac{\pi T}{v_c}(x+v_c t)\right)\right]^{K_c/2}}\langle[\mathcal{E}(x,t),\mathcal{E}(0,0)]\rangle\right).$$

(41)

In the high-temperature regime (41) will not behave much differently from (38) when $T \approx J_0$. In particular, we expect

$$\langle[\mathcal{E}(x,0),\mathcal{E}(0,0)]\rangle \sim \exp(-|x|/\xi_s),$$ (42)

where $\xi_s \lesssim a$ and so in the high-temperature limit the results will be qualitatively similar to what we discussed earlier. Of course, the detailed structure of ${\rm Im}\,\chi_i^{R,2k_F}(k,\omega)$ for $T \approx J_0$ requires that (41) be used. This in turn requires that the dimer-dimer correlation function $\langle[\mathcal{E}(x,t),\mathcal{E}(0,0)]\rangle$ be evaluated by a more general (perhaps numerical) method than the effective low-energy theory given in (22).

3. ${\rm Im}\,\chi_i^{R,4k_F}(k,\omega)$

In the previous subsection we saw that when $v_s/v_c \ll 1$ and temperature $T \gg J_0$, the $2k_F$ contributions to the drag are dramatically suppressed and only the $4k_F$ contributions remain. In contrast to the case of the $2k_F$ density fluctuations, the present model (20) allows for a closed analytic expression for ${\rm Im}\,\chi_i^{R,4k_F}(k,\omega)$. We begin with the $4k_F$ part of the density operator (32),

$$\rho_{4k_F}^{\rm eff}(x,t) = \rho_0 \cos[4k_F x + \sqrt{8}\,\theta_c(x,t)],$$ (43)

which leads, after evaluating the correlators at finite temperature, to

$$-i\langle[\rho_{4k_F}^{\rm eff}(x,t),\rho_{4k_F}^{\rm eff}(0,0)]\rangle$$
$$= \rho_0^2 \cos(4k_F x)$$
$$\times {\rm Im}\left(\frac{(\pi T\alpha/v_c)^{4K_c}}{\left[\sinh\left(\frac{\pi T}{v_c}(x-v_c t)\right)\sinh\left(\frac{\pi T}{v_c}(x+v_c t)\right)\right]^{2K_c}}\right).$$

(44)

As in the case of ${\rm Im}\,\chi_i^{R,2k_F}(k,\omega)$ the temperature dependence at low enough temperatures can be extracted by making the substitutions $\tilde{x}=\pi Tx/v_c$ and $\tilde{t}=\pi Tt$. This then leads us to $\chi_i^{R,4k_F}(k,\omega)=\chi_i^{R,4k_F}(k_+,\omega)+\chi_i^{R,4k_F}(k_-,\omega)$ where

For completeness, it is worth emphasizing that in the high-temperature regime $(T \gg J_0)$ the expression (31) must be used for the $2k_F$ part of the density. In this case, one must compute the Fourier transform of the correlator

$$\chi_i^{R,4k_F}(k_\pm,\omega) = T^{4K_c-2}v_c \int_{-\infty}^{\infty} d\tilde{x} \int_0^{\infty} d\tilde{t}\,\exp i\left(\frac{\omega\tilde{t}-v_c k_\pm\tilde{x}}{\pi T}\right)$$
$$\times \frac{\rho_0^2}{\pi^2}{\rm Im}\left(\frac{(\pi\alpha/v_c)^{4K_c}}{[\sinh(\tilde{x}-\tilde{t})\sinh(\tilde{x}+\tilde{t})]^{2K_c}}\right),$$

(45)

and $k_\pm = k \pm 4k_F$. By the same arguments made in the previous subsection [that the form (45) substituted into (7) leads to no temperature dependence of the drag from the ω integration, and that the dominant contribution from the k integral comes from $k_\pm \approx 0$ with $\Delta k \sim T$], the temperature dependence of the $4k_F$ contribution to the drag is

$$r_D^{4k_F} \propto (4k_F)^2 \tilde{U}_{12}^2(4k_F) T^{8K_c-3},$$ (46)

which is identical to the result (9) obtained in Sec. II by applying the general arguments of Ref. 40 for the mapping of a spin incoherent LL to a spinless LL.

Fortunately, the Fourier transform (45) can be computed exactly.[52,53] This is done by making the change of variables $s_1 = \tilde{x}-\tilde{t}$ and $s_2 = \tilde{x}+\tilde{t}$, and using the integral result[54]

$$\int_0^{\infty} ds\,\frac{e^{izs}}{[\sinh(s)]^g} = 2^{g-1}\frac{\Gamma(g/2-iz/2)\Gamma(1-g)}{\Gamma(1-g/2)}$$ (47)

to obtain

$$\chi_i^{R,4k_F}(k_\pm,\omega)$$
$$= -\frac{\rho_0^2\alpha^2}{2\pi v_c}\left(\frac{2\pi T\alpha}{v_c}\right)^{4K_c-2}\frac{\Gamma(1-2K_c)}{\Gamma(2K_c)}$$
$$\times \frac{\Gamma\left(K_c-i\frac{\omega+v_c k_\pm}{4\pi T}\right)\Gamma\left(K_c-i\frac{\omega-v_c k_\pm}{4\pi T}\right)}{\Gamma\left(1-K_c-i\frac{\omega+v_c k_\pm}{4\pi T}\right)\Gamma\left(1-K_c-i\frac{\omega-v_c k_\pm}{4\pi T}\right)}.$$
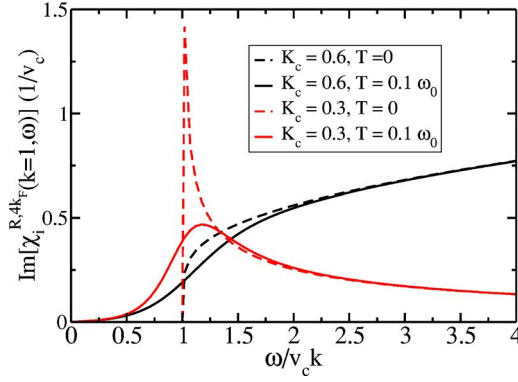
(48)

FIG. 6. (Color online) Frequency dependence of $\text{Im}[\chi_i^{R,4k_F}(k_\pm =1,\omega)]$, Eq. (48), for various interaction strengths K_c and temperatures T. The charge velocity is fixed at $v_c=1$. At $K_c=0.5$ there is a crossover from a sharp peak for $K_c<0.5$ at $\omega=k_\pm$ to monotonically increasing (with ω) threshold-type behavior for $K_c>0.5$. Finite temperature acts to smear the $T=0$ $\omega=k_\pm$ singularity and adds weight to $\text{Im}[\chi_i^{R,4k_F}(k_\pm=1,\omega)]$ for $\omega<k_\pm$.

The temperature dependence of $\text{Im}\,\chi_i^{R,4k_F}(k_\pm=1,\omega)$ for different interaction values K_c is shown in Fig. 6. At $K_c=0.5$ there is a crossover from a divergence to a threshold-type behavior. The main effect of the temperature is to smooth the sharper features near $\omega=v_c k$.

B. Weakly disordered wires

1. Slowly varying background potential

Modulation doping in quantum wire systems gives rise to a smoothly varying background potential. Such disorder has an important effect on the drag as it impacts the nature of the electronic states that participate in drag.[55] Since the coupling of the density to the potential depends crucially on the Fourier components k, there is an important difference in how the charge density couples to disorder in the spin-coherent and spin-incoherent regimes. Consider the following coupling of the density to background potential modulations:

$$H_{\text{dis}} = \int dx V(x)\rho(x)$$

$$\approx \int dx (V_0(x)\rho_0(x) + V_{2k_F}(x)\rho_{2k_F}(x) + V_{4k_F}(x)\rho_{4k_F}(x)). \tag{49}$$

Here $V_{2k_F}(x)=\text{Re}(V_{2k_F}e^{i2k_Fx})$ and $V_{4k_F}(x)=\text{Re}(V_{4k_F}e^{i4k_Fx})$. We can study the scaling dimensions of V_{2k_F}, V_{4k_F} using the expression for the density, Eq. (32), after integrating out the high-energy field u_π in favor of the lower energy spin fields. The scaling dimensions of the different scattering terms can then be determined from the action (where the integration over x has already been caried out)

$$S_{\text{dis}}^{2k_F} \sim \int d\tau V_{2k_F} e^{i\sqrt{2}\theta_c} e^{i\sqrt{2}\theta_s} + \text{h.c.,} \tag{50}$$

and

$$S_{\text{dis}}^{4k_F} \sim \int d\tau V_{4k_F} e^{i\sqrt{8}\theta_c} + \text{h.c.,} \tag{51}$$

which gives

$$\dim(V_{2k_F}) = 1 - \frac{K_c}{2} - \frac{K_s}{2}, \tag{52}$$

$$\dim(V_{4k_F}) = 1 - 2K_c. \tag{53}$$

In these units $SU(2)$ invariance implies $K_s=1$, so that the $2k_F$ piece is more relevant than the $4k_F$ piece of the potential whenever $K_c>1/3$. Thus, we expect to see strong temperature dependence of the pinning of the density whenever $K_c>1/3$ as the more relevant $2k_F$ piece will be lost for $T\gg J_0$. Moreover, if $1/2<K_c<1$, the $2k_F$ piece is relevant while the $4k_F$ piece is irrelevant. In this case, the effect should be most dramatic. The regime $1/2<K_c<1$ can be reached for large but finite U in a one-dimensional Hubbard model.

2. Effects of random correlations in forward scattering on the correlation functions

As the $k\approx 0$ parts of the background potential fluctuations are often the most important at low energies, it is worthwhile to review[5] their influence on the density-density correlation function. The forward scattering part of the background potential is

$$H_{\text{dis}}^f = \int dx V_0(x)\rho_0(x) = -\frac{\sqrt{2}}{\pi}\int dx V_0(x)\partial_x\theta_c(x), \tag{54}$$

where we have used the result (32) and assumed $\int dx V_0(x)\rho_0=0$. Forward scattering can be completely eliminated from the Hamiltonian (action) by making the change of variables

$$\tilde{\theta}_c(x) = \theta_c(x) - \frac{\sqrt{2}K_c}{v_c}\int^x dz V_0(z) \tag{55}$$

and completing the square in Eq. (20). Assuming that the disorder has white noise correlations given by the distribution $P_{V_0}=\exp[-D^{-1}\int dz|V_0(z)|^2]$,

$$\overline{V_0(x)V_0(x')} = \frac{D}{2}\delta(x-x'), \tag{56}$$

where the overbar indicates a disorder average, and the constant $D\sim v_c/\tau_{\text{scatt}}$ with τ_{scatt} the typical scattering time for the electrons. The disorder-averaged parts of the density-density correlation function can then be readily be determined:

$$\overline{\langle[\rho_0^{\text{eff}}(x,t),\rho_0^{\text{eff}}(0,0)]\rangle} = \langle[\rho_0^{\text{eff}}(x,t),\rho_0^{\text{eff}}(0,0)]\rangle|_{V_0=0}, \tag{57}$$

$$\overline{\langle[\rho_{2k_F}^{\text{eff}}(x,t),\rho_{2k_F}^{\text{eff}}(0,0)]\rangle} = \exp\left[-D\left(\frac{K_c}{v_c}\right)^2|x|\right]$$
$$\times|\langle[\rho_{2k_F}^{\text{eff}}(x,t),\rho_{2k_F}^{\text{eff}}(0,0)]\rangle|_{V_0=0}, \tag{58}$$
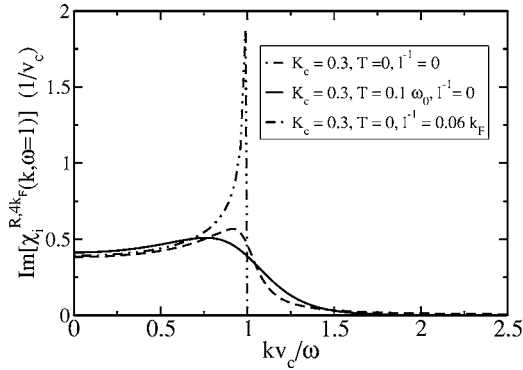
FIG. 7. Disorder and thermal broadening of $\mathrm{Im}[\chi_i^{R,4k_F}(k,\omega=1)]$. The charge velocity is fixed at $v_c=1$. The disorder broadening is computed using Eq. (60). The effects of weak forward scattering on the $4k_F$ density fluctuations are qualitatively similar to that of thermal broadening—see the case of $l=0.06k_F$. Analogous results are also obtained for the $2k_F$ density fluctuations.

$$\overline{\langle[\rho_{4k_F}^{\mathrm{eff}}(x,t),\rho_{4k_F}^{\mathrm{eff}}(0,0)]\rangle}=\exp\left[-4D\left(\frac{K_c}{v_c}\right)^2|x|\right]$$
$$\times|\langle[\rho_{4k_F}^{\mathrm{eff}}(x,t),\rho_{4k_F}^{\mathrm{eff}}(0,0)]\rangle|_{V_0=0}. \quad (59)$$

It is evident that larger wave vectors are suppressed more by the forward scattering with no suppression at all for the $k\approx0$ part of the density. Treating the $2k_F$ and $4k_F$ backscattering contributions with white noise correlations analogous to (56) is more involved and requires studying renormalization group flows.[56] However, we reiterate that if the disorder is slowly varying (as expected from donor potential modulations in modulation doped structures), the $2k_F$ and $4k_F$ potentials are relatively weak and probably negligible, at least in the sort of structures optimized for the drag measurements we envision here!

3. Fourier transform of disorder-averaged correlation functions

Once the Fourier transforms of the $2k_F$ and $4k_F$ density-density correlation functions (38) and (44) are known, the Fourier transforms of the disorder-averaged correlation functions are readily computed from the convolution theorem

$$\overline{\mathrm{Im}\,\chi_i^R(k,\omega)}=\int_{-\infty}^{\infty}\frac{dq}{2\pi}\frac{2l^{-1}}{l^{-2}+(k-q)^2}\,\mathrm{Im}\,\chi_i^R(q,\omega), \quad (60)$$

where $l^{-1}=D(K_c/v_c)^2$ for the $2k_F$ pieces and $l^{-1}=4D(K_c/v_c)^2$ for the $4k_F$ pieces. The main effect of the disorder is thus to broaden the singularities in $\chi_i^R(k,\omega)$ by an amount of order l^{-1} in momentum space. Hence, the $4k_F$ singularities are broadened four times as much as the $2k_F$ singularities.

Figure 7 illustrates the effect of weak disorder on $\mathrm{Im}\,\chi_i^{R,4k_F}(k,\omega)$. Qualitatively the effects of disorder are very similar to finite temperature—there is a broadening of the sharpest features in the response. This has implications for the temperature dependence of the drag as we discuss in the next section.

V. STUDY OF THE COULOMB DRAG

We have already discussed several features of the drag in the previous sections of this paper, including the temperature dependence (in certain limits) of the $k\approx2k_F$ and $k\approx4k_F$ contributions to the drag. Implicit in those discussions was that the wires were identical. In this section we will present numerical calculations of the Coulomb drag as a function of temperature for identical and nonidentical wires and attempt to illuminate the crossover to the spin-incoherent regime with semi-quantitative estimates.

A. Drag at low temperatures and in the crossover to the spin-incoherent regime

1. The low-temperature Luttinger liquid regime

At the lowest temperatures where $T\ll J_0$ we showed that the low-energy theory (20) and (22) results in the following temperature dependence of the drag resistivity (7),

$$r_D\approx A(K_c,v_c)(4k_F)^2\widetilde{U}_{12}^2(4k_F)\left(\frac{T}{T_{4k_F}}\right)^{8K_c-3}+B(K_c,K_s,v_s/v_c)$$
$$\times(2k_F)^2\widetilde{U}_{12}^2(2k_F)\left(\frac{T}{T_{2k_F}}\right)^{2(K_c+K_s)-3}, \quad (61)$$

where $A(K_c,v_c)$ and $B(K_c,K_s,v_s/v_c)$ are functions that depend on the variables indicated and $T_{2k_F}^{-1}=\pi\alpha/\sqrt{v_cv_s}$ and $T_{4k_F}^{-1}=\pi\alpha/v_c$ are effective temperatures in the respective sectors. It is evident from (61) that the temperature dependence of the $k\approx2k_F$ and $k\approx4k_F$ contributions are different so there is a temperature at which the two balance out:

$$\frac{T^{**}}{T_{2k_F}}=\left[\left(\frac{T_{4k_F}}{T_{2k_F}}\right)^{8K_c-3}\frac{B(K_c,K_s,v_s/v_c)\widetilde{U}_{12}^2(2k_F)}{4A(K_c,v_c)\widetilde{U}_{12}^2(4k_F)}\right]^{1/(6K_c-2K_s)}. \quad (62)$$

From here on, we will assume $SU(2)$ symmetry which implies $K_s=1$. When $K_s=1$ it is clear that the $2k_F$ contribution to the drag is an increasing function of temperature whenever $K_c>1/2$ and a decreasing function otherwise. For the $4k_F$ contribution the boundary between increasing and decreasing contributions is $K_c=3/8$. Finally, by comparing the exponents of the $2k_F$ and $4k_F$ terms, one finds that the $4k_F$ pieces dominate the drag for $K_c>1/3$ when $T>T^{**}$, while the $2k_F$ pieces dominate the drag for $K_c<1/3$ in the same temperature regime. Note that this implies that the $2k_F$ density oscillations are more important for the drag at *higher* temperatures when the interactions are strong enough that $K_c<1/3$. This requires, of course, that the system is still at low enough temperatures that the spin degrees of freedom can be described by the effective low-energy theory (22). In order to obtain $T^{**}<J_0$ and for the analysis above to be reasonable, we expect that we must have $\widetilde{U}(2k_F)\gg\widetilde{U}(4k_F)$.

From the results of Appendix C we can express the ratio

$$\frac{B(K_c, K_s, v_s/v_c)}{A(K_c, v_c)} = \left(\frac{J_1}{16\pi m\omega_0^2 a^2}\right)^4 \frac{I_{2k_F}(K_c, K_s, v_s/v_c)}{I_{4k_F}(K_c)},$$

(63)

where $I_{2k_F}(K_c, K_s, v_s/v_c)$ and $I_{4k_F}(K_c)$ are given by Eqs. (C1) and (C2). As $v_s/v_c \to 0$ the crossover temperature (62) becomes very small because both $I_{2k_F}(K_c, K_s, v_s/v_c) \ll I_{4k_F}(K_c)$ and $\left(\frac{J_1}{16\pi m\omega_0^2 a^2}\right)^4 \sim \left(\frac{v_s}{v_c}\right)^4 \ll 1$ in that limit. Of course, as v_s shrinks for fixed v_c, the temperature range over which the LL theory itself is valid is also shrinking and the spin-incoherent regime (where only the $4k_F$ density modulations remain) is approached.

2. Crossover to the spin-incoherent regime

The hallmark of the spin-incoherent regime is the equivalence of the real electron system with spin to a spinless system[40] with the exception of the Green's function[37] and other non-particle-conserving operators. In the case of drag, spin incoherence manifests itself as a thermal washing out of the $2k_F$ oscillations in the density-density correlation function (8). When the interactions are as strong as they are here, the weight of the $2k_F$ oscillations are already down by a factor $\sim \left(\frac{J_1}{16\pi m\omega_0^2 a^2}\right)^2$ even at zero temperature.

As we have discussed before,[37] the spin-incoherent regime can be understood by starting with $T \gg J_0$, taking $J_0 \to 0$, for fixed T and then finally taking $T \to 0$. In the present formulation this is equivalent to fixing a finite but low temperature, applying the low-energy theory (20) and (22), and then taking the limit $v_s/v_c \to 0$ as we did in the previous section. The approach to the spin-incoherent drag regime can be directly obtained via this procedure. One expects that as v_s is lowered, the power law (40) will first breakdown (at temperatures it once held for larger v_s) before the contribution vanishes altogether from $f(T/J_0)$.

B. Drag in the spin-incoherent regime

In this subsection we present some numerical results justifying earlier analytical arguments for the temperature dependence of the drag. We first consider identical wires and then we study nonidentical wires.

1. Identical wires

When the wires are identical we expect the temperature dependence of the Coulomb drag given in Eq. (61) to be obtained at the lowest energies. However, as we have seen in the previous sections, the $2k_F$ oscillations are rapidly washed out in the limit $v_s/v_c \to 0$ and only the $4k_F$ oscillations remain. In this subsection, we provide numerical evidence that the manipulations leading to the $4k_F$ temperature dependence of (61) are justified. Since these are also the same arguments leading to the $2k_F$ temperature dependence of r_D at the lowest temperature, these are implicitly justified as well. Figure 8 illustrates the comparison between the exact result from (48) substituted into (7), and the approximate power law (46). A disorder value of $l^{-1} = 0.13k_F$ was used in Eq. (60) to compute the drag of the disordered system from Eq. (7). The
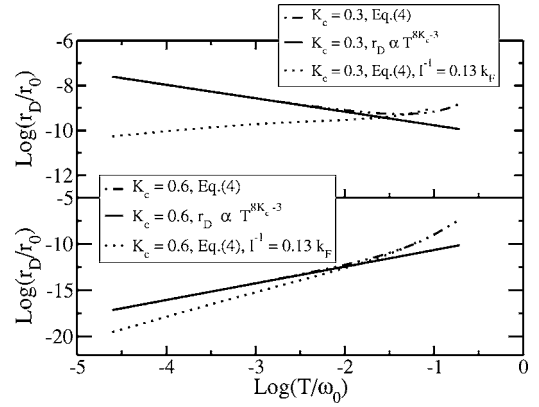

FIG. 8. Temperature dependence of the Coulomb drag in the spin incoherent regime where only the $4k_F$ components of the density fluctuations contribute. The drag was computed over a temperature range $0.01\omega_0 \le T \le 0.5\omega_0$ and the logarithms are the natural logs. The scale of the drag resistivity is $r_0 \sim \frac{\hbar\alpha}{e^2}(4k_F)^2 \tilde{U}(4k_F)^2$. Shown is a comparison between the drag formula (7) with (48) substituted in and the approximate power law (46). Two values of the interaction parameter K_c are shown. A disorder value of the interaction parameter K_c are shown. A disorder value of $l^{-1} = 0.13k_F$ was used in Eq. (60) to compute the drag from Eq. (7). For $k_BT \gg \hbar v_c/l$ the drags for the clean and disordered systems are indistinguishable, while for $k_BT \ll \hbar v_c/l$ there is a crossover of the temperature dependence to another power of the temperature. Empirically, we found the power law $r_D^{4k_F, \text{disorder}} \propto T^{8K_c - 2}$ to be a very good fit for any value of $0 < K_c < 1$.

drag was computed over a temperature range $0.01\omega_0 \le T \le 0.5\omega_0$. For $k_BT \gg \hbar v_c/l$ the drags of the clean and disordered systems are indistinguishable, while for $k_BT \ll \hbar v_c/l$ there is a crossover of the temperature dependence to another power of the temperature. Empirically, we found the power law

$$r_D^{4k_F, \text{disorder}} \propto T^{8K_c - 2}$$

(64)

to be a very good fit for any value of $0 < K_c < 1$. This temperature dependence can actually be inferred from (45), (7), and Fig. 6. As we have argued several times earlier, the ω integration in (7) does not contribute any temperature dependence beyond the $(T^{4K_c - 2})^2$ factors in front of (45) [with the square coming from the drag formula (7)]. When $k_BT \ll \hbar v_c/l$, the k integration picks up a contribution proportional to T^2 rather than T. Adding up the exponents leads to $r_D^{4k_F, \text{disorder}} \propto T^{8K_c - 2}$.

2. Drag for nonidentical wires

Coulomb drag for nonidentical wires and the incommensurate-commensurate transition has been discussed for fully coherent clean wires in Ref. 20. Via the mapping detailed in Ref. 40 the incommensurate-commensurate transition discussed can be ready deep in the spin-incoherent regime. In Fig. 9 we present some numerical results for the dependence of the drag for small density mismatches between the two wires. Note that weaker interactions and higher temperatures lead to a more robust drag effect between two wires of slightly different densities. Note also that, with only a few percent change in the relative
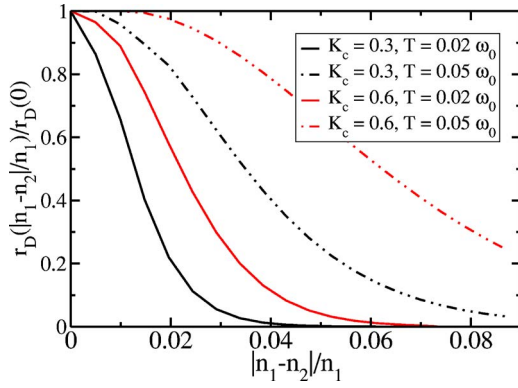
FIG. 9. (Color online) Coulomb drag dependence on relative wire density at fixed temperature. We have set $v_c=1$. The Coulomb drag between two wires drops rapidly as a function of density mismatch. Only a few percent change in the relative densities of the wires results in a dramatic suppression of the drag. At higher temperatures and weaker interactions the drag response is more robust to small density differences between the wires.

densities of the wires, the drag effect is substantially reduced.

The effects of disorder on the drag for density mismatched wires is shown in Fig. 10. When $\hbar v_c/l \gtrsim k_B T$ the disorder has a significant effect—making the drag more robust for non-identical wires. While for small $|n_1-n_2|$ the drag is reduced relative to the clean limit, for larger $|n_1-n_2|$ there can be appreciable enhancement.

Finally, we note that, in the case of clean wires of different charge velocities but the same Fermi wave vector (in which case the K_c are diffferent), the temperature dependence of the drag is

$$r_D^{4k_F} \sim T^{4(K_{c,1}+K_{c,2})-3}, \tag{65}$$

and if the temperature is also much less than J_0,

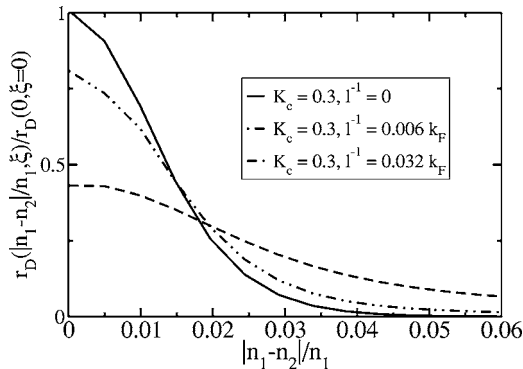$$r_D^{2k_F} \sim T^{K_{c,1}+K_{c,2}+K_{s,1}+K_{s,2}-3}. \tag{66}$$

FIG. 10. Coulomb drag dependence on relative wire density at fixed temperature for different values of disorder. Here $T=0.02\omega_0$ and $v_c=1$. While for clean wires the Coulomb drag drops rapidly as a function of density mismatch, some amount of residual disorder allows for a more robust drag effect between quantum wires that may have slightly different values of k_F.

VI. SUMMARY OF RESULTS

We have discussed the Coulomb drag between two quantum wires in the limit of low electron density where at finite temperatures the energy hierarchy $J \ll T \ll E_F$ can be obtained.[57] In this limit, the spin degrees of freedom are completely incoherent and we have shown this implies the loss of $2k_F$ oscillations in the density-density correlation function. As a result, a nonmonotonic temperature dependence of the drag on temperature may result. In the spin-incoherent Luttinger liquid regime, the drag problem maps onto the identical problem for a spinless Luttinger liquid only with $K_{c-} \rightarrow 2K_{c-}$ so that for a clean wire the drag resistivity goes as $\rho_D \propto T^{8K_{c-}-3}$, where K_{c-} is the coupling parameter of the antisymmetric charge mode of a Luttinger liquid theory with spin. We have shown this temperature dependence explicitly with approximate analytical calculations and confirmed those approximations with numerics.

Our results are based on a fluctuating Wigner solid model appropriate to quantum wires in a very strongly interacting regime, which typically implies low electron density. The spin sector is modeled as a nearest-neighbor antiferromagnetic Heisenberg spin chain. Without any coupling of the spin and charge sectors, no $2k_F$ density modulations appear. However, including a magneto-elastic coupling term that allows for a linear change in the nearest-neighbor spin coupling for small distortions induces $2k_F$ oscillations in the density. The coupling is weak, however, and the $2k_F$ oscillations are easily washed out by temperatures $T \gtrsim J$.

The fluctuating Wigner solid model is studied by deriving effective expressions for the density operator when the highest energy phonon modes are integrated out in favor of spin operators. At the lowest energies (including the spin energy) an expression is obtained equivalent to that known well in Luttinger liquid theory except the $2k_F$ terms contain a prefactor of the order of the ratio of kinetic to potential energy. Nevertheless, in this limit all correlation functions exhibit power law decay with the familiar exponents of the spin and charge sectors.

These density operators are then used to compute density-density correlation functions which are Fourier transformed into frequency and momentum space and used in previously derived drag formulas. Since the density operator has contributions at momenta $k \approx 0$, $k \approx 2k_F$, and $k \approx 4k_F$, the Coulomb drag will generically contain contributions from each of these pieces. We show explicitly that the $k \approx 0$ piece vanishes due to the harmonic approximation to the Wigner solid. This is equivalent to linearizing the electron dispersion in the standard Luttinger liquid treatment for weakly interacting electrons. Generically, the $k \approx 2k_F$ and $k \approx 4k_F$ contributions are nonvanishing and we explicitly compute their temperature dependence, $T^{2K_c-1}e^{-cT/J_0}$ and T^{8K_c-3}, in the low-temperature regimes.

We have also considered the case of nonideal wires in which weak disorder is present. We find that white-noise-correlated forward scattering disorder does not affect the $k \approx 0$ result, while it tends to broaden the sharp k-space features of the $k \approx 2k_F$ and $k \approx 4k_F$ density-density correlation function in a manner similar to temperature. As a result, the drag resistivity crosses over to a different power law, T^{2K_c}

and T^{8K_c-2}, which is increased by one power of the temperature relative to the clean result. Finally, we have also studied the reduction of the drag due to a density mismatch between the two wires and show the drag may be substantially reduced with only a few percent change in the relative densities of the wires. When disorder is present the drag is more robust to density mismatches between the two wires and this fact is likely to play an important role in real drag experiments.

We hope that this work will help to inspire further experimental studies on one-dimensional drag, which to date are quite limited.

ACKNOWLEDGMENTS

APPENDIX A: EXACT EXPRESSIONS FOR $\langle \rho(x,\tau)\rho(x',\tau') \rangle$ UP TO SECOND ORDER IN J_1

The low-energy description given in Sec. III can be treated more accurately, but in a less physically transparent way, by applying the results of this appendix.

1. Diagonalization of H_c

The charge Hamiltonian (12) is diagonalized by the transformation

$$u_l = \frac{1}{\sqrt{N}}\sum_k e^{ikal}u_k, \tag{A1}$$

$$p_l = \frac{1}{\sqrt{N}}\sum_k e^{-ikal}p_k \tag{A2}$$

(assuming periodic boundary conditions) where the theory is quantized by imposing $[u_l,p_m]=i\hbar\delta_{lm}$ and $[u_k,p_{k'}]=i\hbar\delta_{kk'}$. Making these substitutions we find

$$H_c = \sum_{k=k_1}^{k_N} \frac{p_k p_{-k}}{2m} + \frac{m\omega_k^2}{2}u_k u_{-k}, \tag{A3}$$

where $\omega_k = 2\omega_0|\sin(\frac{ka}{2})| \approx v_c|k|$ for small k with $v_c = \omega_0 a$ the sound velocity of the charge modes. In momentum space the harmonic chain is just a sum of harmonic oscillators with frequencies that depend on the wave number k.

The Hamiltonian (A3) can be brought into a particularly simple form via the transformation

$$a_k = \sqrt{\frac{m\omega_k}{2\hbar}}\left(u_k + \frac{i}{m\omega_k}p_{-k}\right), \tag{A4}$$

$$a_k^\dagger = \sqrt{\frac{m\omega_k}{2\hbar}}\left(u_{-k} - \frac{i}{m\omega_k}p_k\right), \tag{A5}$$

which brings H_c to

$$H_c = \sum_k \hbar\omega_k\left(n_k + \frac{1}{2}\right). \tag{A6}$$

For later reference it is useful to note that

$$u_l(t) = \sum_k M_k(l)(a_k e^{-\omega_k\tau} + a_{-k}^\dagger e^{\omega_k\tau}), \tag{A7}$$

where $M_k(l) \equiv \sqrt{\frac{\hbar}{2Nm\omega_k}}e^{ikal}$ and we have used $\omega_k = \omega_{-k}$.

2. Notation for perturbation theory

The general expression for the density-density correlation function at finite temperature $T = 1/\beta$ is

$$\langle \rho(x,\tau)\rho(x',\tau') \rangle = \frac{\langle U(\beta)\rho(x,\tau)\rho(x',\tau')\rangle_0}{\langle U(\beta)\rangle_0}, \tag{A8}$$

where

$$U(\beta) = \sum_{n=0}^\infty (-1)^n \int_0^\beta d\tau_1 \cdots \int_0^{\tau_{n-1}} \hat{H}'(\tau_1) \cdots \hat{H}'(\tau_n), \tag{A9}$$

with the operators taken in the interaction representation. The averages $\langle\cdots\rangle_0 \equiv \mathrm{Tr}(e^{-\beta H_0}\cdots)$ where the zeroth-order Hamiltonian is (16) taken with $J_1 \equiv 0$:

$$H_0 = H_c + J_0\sum_l \mathbf{S}_{l+1}\cdot\mathbf{S}_l, \tag{A10}$$

and H' is the correction to this to be treated in perturbation theory

$$H' = J_1\sum_l (u_{l+1} - u_l)\mathbf{S}_{l+1}\cdot\mathbf{S}_l. \tag{A11}$$

3. Evaluation of the density-density correlation function

a. Zeroth order

At zeroth order we have $J_1 \equiv 0$ and there is no coupling between the charge and the spin degrees of freedom. Therefore we can completely ignore the spin sector since it traces out trivially. Thus,

$$\langle \rho(x,\tau)\rho(x',\tau') \rangle^{(0)} = Z_c^{-1}\mathrm{Tr}[e^{-\beta H_c}\rho(x,\tau)\rho(x',\tau')], \tag{A12}$$

where $Z_c = \mathrm{Tr}(e^{-\beta H_c})$ is the partition function of the charge sector and the density is expressed as $\rho(x,\tau) = \sum_{l=1}^N \delta(x-al-u_l(\tau))$. From the Hamiltonian (A6) Z_c can be readily evaluated as

$$Z_c = \prod_k \sum_{n_k=0}^\infty \exp - \beta\hbar\omega_k(n_k + 1/2)$$

$$= \prod_k \exp(-\beta\hbar\omega_k/2)\frac{1}{1-e^{-\beta\hbar\omega_k}}, \tag{A13}$$

which we will make use of later. Thus,

$$\langle \rho(x,\tau)\rho(x',\tau')\rangle^{(0)} = Z_c^{-1} \sum_{l,l'} \int \frac{d\eta}{2\pi} \frac{d\xi}{2\pi} e^{i\eta(x-al)} e^{i\xi(x'-al')}$$

$$\times \text{Tr}(e^{-\beta H_c} e^{-i\eta u_l(\tau)} e^{-i\xi u_{l'}(\tau')}), \quad (A14)$$

where the quantities u_l appearing in the exponent of the trace can be expressed using Eq. (A7). Using $e^{A+B} = e^A e^B e^{-[A,B]/2}$ (where $[A,B]$ commutes with both A and B separately), we focus on the trace and obtain

$$\text{Tr}(e^{-\beta H_c} e^{-i\eta u_l(\tau)} e^{-i\xi u_{l'}(\tau')})$$

$$= \exp\left(-\frac{1}{2}\sum_q |M_q|^2(\eta^2 + \xi^2)\right)$$

$$\times \exp\left(-\eta\xi\sum_q |M_q|^2 e^{iqa(l-l')-\omega_q(\tau-\tau')}\right)$$

$$\times \prod_k \text{Tr}^{(k)}(\exp[-\beta\hbar\omega_k(a_k^\dagger a_k + 1/2)]$$

$$\times \exp[-iC^*(k)a_k^\dagger e^{-iC(k)a_k}]), \quad (A15)$$

where $C(k) \equiv |M_k|(\eta e^{ikal-\omega_k\tau} + \xi e^{ikal'-\omega_k\tau'})$. Evaluating the trace for each k independently and using the definition of a Laguerre polynomial of order n_k,

$$L_{n_k}(-|C(k)|^2) \equiv \langle n_k|e^{-iC^*(k)a_k^\dagger}e^{-iC(k)a_k}|n_k\rangle, \quad (A16)$$

and then applying the important formula

$$\sum_{n=0}^{\infty} L_n(|C|^2)z^n = \frac{1}{1-z}\exp\left(|C|^2\frac{z}{z-1}\right), \quad (A17)$$

we find

$$\langle \rho(x,\tau)\rho(x',\tau')\rangle^{(0)} = \sum_{l,l'} \int \frac{d\eta}{2\pi} \frac{d\xi}{2\pi} e^{i\eta(x-al)} e^{i\xi(x'-al')}$$

$$\times \exp\left(-\frac{1}{2}F(\eta^2 + \xi^2)\right)e^{-\eta\xi G(l,l')}, \quad (A18)$$

where

$$F \equiv \sum_q |M_q|^2\left(1 + \frac{2e^{-\beta\hbar\omega_q}}{1-e^{-\beta\hbar\omega_q}}\right), \quad (A19)$$

and

$$G(l,l') \equiv \sum_q |M_q|^2\left(e^{i\theta_q(l,l';t,t')} + \frac{2\cos[\theta_q(l,l';t,t')]e^{-\beta\hbar\omega_q}}{1-e^{-\beta\hbar\omega_q}}\right), \quad (A20)$$

and $\theta_q(l,l';t,t') = qa(l-l') - \omega_q(t-t')$, where $t = -i\tau$. Finally, shifting the summation variables $\tilde{l} = l-l'$ and performing the integrations we obtain

$$\langle \rho(x,t)\rho(x',t')\rangle^{(0)} = \frac{1}{a}\sum_l \frac{1}{2\pi}\sqrt{\frac{\pi}{F-G(l)}}$$

$$\times \exp\left(-\frac{(x-x'-al)^2}{4(F-G(l))}\right), \quad (A21)$$

where

$$F - G(l)$$

$$= \frac{a\hbar}{mv_c 2\pi}\left[\ln\left(\frac{\alpha^2 + (al)^2 + [v_c(\tau-\tau')]^2}{\alpha^2}\right) + 2\sum_{n=0}^{\infty}\right.$$

$$\times \ln\left(\frac{[\alpha + (n+1)\beta\hbar v_c]^2 + (al)^2 + [v_c(\tau-\tau')]^2}{[\alpha + (n+1)\beta\hbar v_c]^2}\right)\bigg]. \quad (A22)$$

Here α is a short distance cutoff of the order of the lattice spacing a. Note that for finite temperatures the second sum is cut off when the argument of the ln becomes $\mathcal{O}(1)$ which occurs when $n = n_l \sim \sqrt{(al)^2 + [v_c(\tau-\tau')]^2}/(\beta\hbar v_c)$. In the limit of $T \to 0$, $\beta \to \infty$ and the second terms drops out all together. In this paper we are interested in the limit $T \ll E_F$, so the second term can be ignored altogether. We will not explicitly consider finite temperatures in the first- and second-order expressions.

b. First order

The manipulations needed here are identical to those used to compute the zeroth-order result, so we simply quote the result:

$$\langle \rho(x,\tau)\rho(x',\tau')\rangle^{(1)} = J_1 N \sum_n \int_0^{\beta} d\tilde{\tau}\langle \mathbf{S}_{l+1}\cdot\mathbf{S}_l(\tilde{\tau})\rangle^{(0)}$$

$$\times \frac{I(n)}{2\pi a}\prod_{k>0}^{k_{max}} |M(k)|^4 4[1-\cos(ka)]$$

$$\times \{1 - \cos[\theta_k(n,0;\tau,\tau')]\}e^{-2\tilde{\tau}\hbar\omega_k}, \quad (A23)$$

where l is arbitrary, N is the number of electrons in the system, and

$$I(n) = \exp\frac{B^2}{4A}\sum_{j=0}^{N} \frac{\sqrt{\pi}(-1)^j(2N)!}{(2j)!(2N-2j)!}\left(\frac{B}{2A}\right)^{2(N-j)}$$

$$\times \frac{(2j-1)!!}{2^j}A^{-1/2-j}, \quad (A24)$$

where the n dependence enters through $B \equiv i(x - x' - an)$ and $A \equiv F - G(n)$. It is worth noting that neither $\langle \rho(x,\tau)\rho(x',\tau') \rangle^{(0)}$ nor $\langle \rho(x,\tau)\rho(x',\tau') \rangle^{(1)}$ contain a $2k_F$ component. This component will only appear in the second-order term, as we now discuss.

c. Second order

The second-order corrections are [where $\mathbf{x} = (x, \tau)$]

$$\langle \rho(\mathbf{x})\rho(\mathbf{x}') \rangle^{(2)} = \int_0^\beta d\tau_1 \int_0^{\tau_1} d\tau_2 \frac{\langle \hat{H}'(\tau_1)\hat{H}'(\tau_2)\rho(\mathbf{x})\rho(\mathbf{x}') \rangle_0}{Z_c Z_s} - \langle \rho(\mathbf{x})\rho(\mathbf{x}') \rangle^{(0)} \int_0^\beta d\tau_1 \int_0^{\tau_1} d\tau_2 \frac{\langle \hat{H}'(\tau_1)\hat{H}'(\tau_2) \rangle_0}{Z_c Z_s}$$

$$= (J_1)^2 \sum_{l,l'} \int_0^\beta d\tau_1 \int_0^{\tau_1} d\tau_2 \langle \mathbf{S}_{l+1} \cdot \mathbf{S}_l(\tau_1)\mathbf{S}_{l'+1} \cdot \mathbf{S}_{l'}(\tau_2) \rangle^{(0)}$$

$$\times \left[\frac{\langle \hat{H}'_c(l,\tau_1)\hat{H}'_c(l',\tau_2)\rho(\mathbf{x})\rho(\mathbf{x}') \rangle_0}{Z_c} - \langle \rho(\mathbf{x})\rho(\mathbf{x}') \rangle^{(0)} \frac{\langle \hat{H}'_c(l,\tau_1)\hat{H}'_c(l',\tau_2) \rangle_0}{Z_c} \right], \quad \text{(A25)}$$

where $\hat{H}'_c(l,\tau) = u_{l+1}(\tau) - u_l(\tau)$ is the charge part of H'. From (A25) it is clear that when dimer-dimer correlations $\langle \mathbf{S}_{l+1} \cdot \mathbf{S}_l(\tau_1)\mathbf{S}_{l'+1} \cdot \mathbf{S}_{l'}(\tau_2) \rangle^{(0)}$ are present (presumably when $T \lesssim J_0$), then a $2k_F$ component appears in $\langle \rho(x,\tau)\rho(x',\tau') \rangle$. After some algebra, we reach the final form

$$\langle \rho(x,\tau)\rho(x',\tau') \rangle^{(2)} = (J_1)^2 \sum_{l,l'} \int_0^\beta d\tau_1 \int_0^{\tau_1} d\tau_2 \langle \mathbf{S}_{l+1} \cdot \mathbf{S}_l(\tau_1)\mathbf{S}_{l'+1} \cdot \mathbf{S}_{l'}(\tau_2) \rangle^{(0)} \sum_{n,m} \int \frac{d\eta}{2\pi}\frac{d\xi}{2\pi} e^{i\eta(x-an)} e^{i\xi(x'-am)}$$

$$\times \exp\left(-\frac{1}{2}\sum_q |M_q|^2(\eta^2 + \xi^2)\right) \exp\left(-\eta\xi \sum_q |M_q|^2 e^{i\theta_q(n,m;\tau,\tau')}\right)$$

$$\times \prod_{k>0}^{k_{max}} |M_k|^4 4[1 - \cos(ka)]^2 e^{-2(\tau_1 - \tau_2)\hbar\omega_k} \times \left(-1 + \prod_{k'>0}^{k_{max}} [1 + h(k')]\right), \quad \text{(A26)}$$

where

$$h(k) = |M_k|^2 e^{-4\tau_2\hbar\omega_k}\left[\eta^2\left(1 - \frac{e^{2\tau_2\hbar\omega_k}R(2\tilde{n})}{1 - \cos(ka)}\right)\right.$$

$$+ 2\eta\xi\left(\cos[ka(l - l')] - \frac{e^{2\tau_2\hbar\omega_k}R(\tilde{n} + \tilde{m})}{1 - \cos(ka)}\right)$$

$$\left. + \xi^2\left(1 - \frac{e^{2\tau_2\hbar\omega_k}R(2\tilde{m})}{1 - \cos(ka)}\right)\right], \quad \text{(A27)}$$

with

$$\tilde{n} = n + i\omega_k\tau/(ka), \quad \text{(A28)}$$

$$\tilde{m} = m + i\omega_k\tau'/(ka), \quad \text{(A29)}$$

and

$$R(s) = \cos[ka(2l' + 2 - s)] + \cos[ka(2l' + 1 - s)]$$

$$+ \cos[ka(2l - s)]. \quad \text{(A30)}$$

APPENDIX B: COMPUTING $\text{Im}\,\chi_i^{R,0}(k,\omega)$

$\text{Im}\,\chi_i^{R,0}(k,\omega)$ is computed by making use of the Fourier expansion of $\rho_0^{eff}(x,\tau)$, Eq. (35), and the formula (34). Consider first the Fourier transform to momentum space:

$$\int_{-\infty}^{\infty} dx e^{-ikx} \langle \rho_0^{eff}(x,\tau)\rho_0^{eff}(0,0) \rangle = \langle \rho_0^{eff}(k,\tau)\rho_0^{eff}(-k,0) \rangle, \quad \text{(B1)}$$

where translational invariance was used. The Fourier decomposition of $\rho_0^{eff}(k,\tau)$ is readily obtained by making use of Eq. (35), the relation $u_0(x)/a = \sqrt{2}\theta_c(x)/\pi$, and the representation of $u_0(x)$ given in Eq. (A7):

$$\rho_0^{eff}(k,\tau) = -\frac{ik}{a}\sqrt{\frac{\hbar L}{2m\omega_k}}(a_k e^{-\omega_k\tau} + a_{-k}^\dagger e^{\omega_k\tau}), \quad \text{(B2)}$$

where we have implicitly converted the discrete k sums to integrals and L is the length of the system. It is easily verified that this has the right units to give $\langle \rho_0^{eff}(k,\tau)\rho_0^{eff}(-k,0) \rangle$ in (B1) the correct dimensions of inverse length. Only the expectation values of the cross terms $a_k a_k^\dagger$ and $a_{-k}^\dagger a_{-k}$ are non-zero, giving

$$\langle \rho_0^{eff}(k,\tau)\rho_0^{eff}(-k,0) \rangle = -\frac{k^2}{a^2}\frac{\hbar L}{2m\omega_k}$$

$$\times \{e^{-\omega_k\tau}[1 + n_B(k)] + e^{\omega_k\tau}n_B(-k)\}, \quad \text{(B3)}$$

where $n_B(k) = \langle a_k^\dagger a_k \rangle = (e^{\beta\omega_k} - 1)^{-1}$ is the boson occupation

factor. Returning to the expression (34) and evaluating the τ integral we find

$$\chi_i^0(k,\omega_n) = \frac{k^2}{a^2}\frac{\hbar L}{2m\omega_k}\left(\frac{-1}{i\omega_n-\omega_k}+\frac{1}{i\omega_n+\omega_k}\right), \quad \text{(B4)}$$

which upon the analytic continuation to real frequencies $i\omega_n \to \omega+i\eta$ leads directly to Eq. (36).

APPENDIX C: EXPRESSIONS FOR $I_{2k_F}(K_c,K_s,v_s/v_c)$ AND $I_{4k_F}(K_c)$

The functions $I_{2k_F}(K_c,K_s,v_s/v_c)$ and $I_{4k_F}(K_c)$ defined in Eq. (63) are given by

$$I_{2k_F}(K_c,K_s,v_s/v_c) = \int_0^\infty \frac{d\omega}{T}\frac{\Xi_{2k_F}(K_c,K_s,v_s/v_c,\omega/T)}{\sinh^2(\omega/2T)},$$
$$\text{(C1)}$$

and

$$I_{4k_F}(K_c) = \int_0^\infty \frac{d\omega}{T}\frac{\Xi_{4k_F}(K_c,\omega/T)}{\sinh^2(\omega/2T)}, \quad \text{(C2)}$$

where

$$\Xi_{2k_F}(K_c,K_s,v_s/v_c,\omega/T)$$
$$= \left[\text{Im}\int_{-\infty}^\infty d\tilde{x}\int_0^\infty d\tilde{t}\exp\left(i\frac{\omega}{\pi T}\tilde{t}\right)\right.$$
$$\times \text{Im}\left(\frac{1}{[\sinh(\tilde{x}-\tilde{t})\sinh(\tilde{x}+\tilde{t})]^{K_c/2}}\right.$$
$$\left.\left.\times \frac{1}{[\sinh(\tilde{x}v_c/v_s-\tilde{t})\sinh(\tilde{x}v_c/v_s+\tilde{t})]^{K_s/2}}\right)\right]^2,$$
$$\text{(C3)}$$

and

$$\Xi_{4k_F}(K_c,\omega/T) = \left[\text{Im}\int_{-\infty}^\infty d\tilde{x}\int_0^\infty d\tilde{t}\exp\left(i\frac{\omega}{\pi T}\tilde{t}\right)\right.$$
$$\left.\times \text{Im}\left(\frac{1}{[\sinh(\tilde{x}-\tilde{t})\sinh(\tilde{x}+\tilde{t})]^{2K_c}}\right)\right]^2.$$
$$\text{(C4)}$$

[1] H. C. Manoharan, C. P. Lutz, and D. M. Eigler, Nature (London) **403**, 512 (2000).

[2] J. Nygard, D. H. Cobden, and P. E. Lindelof, Nature (London) **408**, 342 (2000).

[3] P. Jarillo-Herrero, J. Kong, H. S. J. van der Zant, L. Kouwenhoven, and S. D. Francheschi, Nature (London) **434**, 484 (2005).

[4] F. Haldane, J. Phys. C **14**, 2585 (1981).

[5] J. Voit, Rep. Prog. Phys. **58**, 977 (1995).

[6] H. Ishii, H. Kataura, H. Shiozawa, H. Yoshioka, H. Otsubo, Y. Takayama, T. Miyahara, S. Suzuki, Y. Achiba, M. Nakatake *et al.*, Nature (London) **426**, 540 (2003).

[7] M. Bockrath, D. H. Cobden, J. Lu, A. G. Rinzler, R. E. Smalley, L. Balents, and P. L. McEuen, Nature (London) **397**, 598 (1999).

[8] Z. Yao, H. W. C. Postma, L. Balents, and C. Dekker, Nature (London) **402**, 273 (1999).

[9] O. M. Auslaender, A. Yacoby, R. de Picciotto, K. W. Baldwin, L. N. Pfeiffer, and K. W. West, Science **295**, 825 (2002).

[10] O. M. Auslaender, H. Steinberg, A. Yacoby, Y. Tserkovnyak, B. I. Halperin, K. W. Baldwin, L. N. Pfeiffer, and K. W. West, Science **308**, 88 (2005).

[11] M. Yamamoto, M. Stopa, Y. Tokura, Y. Hirayama, and S. Tarucha, Physica E (Amsterdam) **12**, 726 (2002).

[12] P. Debray, V. N. Zverev, O. Raichev, R. Klesse, P. Vasilopoulos, and R. S. Newrock, J. Phys.: Condens. Matter **13**, 3389 (2001).

[13] P. Debray, V. N. Zverev, V. Gurevich, R. Klesse, and R. S. Newrock, Semicond. Sci. Technol. **17**, R21 (2002).

[14] D. L. Maslov and M. Stone, Phys. Rev. B **52**, R5539 (1995).

[15] I. Safi and H. J. Schulz, Phys. Rev. B **59**, 3040 (1999).

[16] K. A. Matveev, Phys. Rev. Lett. **92**, 106801 (2004).

[17] K. A. Matveev, Phys. Rev. B **70**, 245319 (2004).

[18] L. Zheng and A. H. MacDonald, Phys. Rev. B **48**, 8203 (1993).

[19] M. Pustilnik, E. G. Mishchenko, L. I. Glazman, and A. V. Andreev, Phys. Rev. Lett. **91**, 126805 (2003).

[20] T. Fuchs, R. Klesse, and A. Stern, Phys. Rev. B **71**, 045321 (2005).

[21] R. Klesse and A. Stern, Phys. Rev. B **62**, 16912 (2000).

[22] Y. V. Nazarov and D. V. Averin, Phys. Rev. Lett. **81**, 653 (1998).

[23] P. Schlottmann, Phys. Rev. B **69**, 035110 (2004).

[24] K. Flensberg, Phys. Rev. Lett. **81**, 184 (1998).

[25] O. Raichev and P. Vasilopoulos, Phys. Rev. B **61**, 7511 (2000).

[26] O. E. Raichev and P. Vasilopoulos, Phys. Rev. Lett. **83**, 3697 (1999).

[27] V. V. Ponomarenko and D. V. Averin, Phys. Rev. Lett. **85**, 4928 (2000).

[28] V. L. Gurevich and M. I. Muradov, Phys. Rev. B **62**, 1576 (2000).

[29] B. Trauzettel, R. Egger, and H. Grabert, Phys. Rev. Lett. **88**, 116401 (2002).

[30] N. A. Mortensen, K. Flensberg, and A.-P. Jauho, Phys. Rev. B **65**, 085317 (2002).

[31] N. A. Mortensen, K. Flensberg, and A.-P. Jauho, Phys. Rev. Lett. **86**, 1841 (2001).

[32] P. Schlottmann, Phys. Rev. B **70**, 115306 (2004).

[33] M. I. Muradov, Phys. Rev. B **66**, 115417 (2002).

[34] O. E. Raichev, Phys. Rev. B **64**, 035324 (2001).

[35] V. V. Cheianov and M. B. Zvonarev, Phys. Rev. Lett. **92**, 176401 (2004).

[36] V. V. Cheianov and M. B. Zvonarev, J. Phys. A **37**, 2261 (2004).

[37] G. A. Fiete and L. Balents, Phys. Rev. Lett. **93**, 226401 (2004).

[38] V. V. Cheianov, H. Smith, and M. B. Zvonarev, Phys. Rev. A **71**, 033610 (2005).

[39] G. A. Fiete, J. Qian, Y. Tserkovnyak, and B. I. Halperin, Phys. Rev. B **72**, 045315 (2005).

[40] G. A. Fiete, K. Le Hur, and L. Balents, Phys. Rev. B **72**, 125416 (2005).

[41] M. Kindermann and P. W. Brouwer, cond-mat/0506455 (unpublished).

[42] The relevance (in the RG sense) of tunneling in drag experiments was first pointed out by Klesse and Stern,[21] but similar physics has also been discussed in other contexts by K. Le Hur, Phys. Rev. B **63**, 165110 (2001), by F. Kusmart, A. Luther, and A. Nersesyan, JETP Lett. **55**, 724 (1992), by V. Yakovenko, *ibid.* **56**, 523 (1992), and by A. O. Gogolin, A. A. Nersesyan, and A. M. Tsvelik, *Bosonization and Strongly Correlated Systems* (Cambridge University Press, Cambridge, England, 1998).

[43] There has been a significant amount of theoretical work done on drag in 2-D systems in recent years. See the following references and references therein: E. H. Hwang, S. Das Sarma, V. Braude, and Ady Stern, Phys. Rev. Lett. **90**, 086801 (2003); S. Das Sarma and E. H. Hwang, Phys. Rev. B **71**, 195322 (2005); Felix von Oppen, Steven H. Simon, and Ady Stern, Phys. Rev. Lett. **87**, 106803 (2001); I. V. Gornyi, A. D. Mirlin, and F. von Oppen, Phys. Rev. B **70**, 245302 (2004); G. Vignale, *ibid.* **71**, 125103 (2005); I. D'Amico and Giovanni Vignale, *ibid.* **68**, 045307 (2003).

[44] A complementary approach to Wigner crystals in one dimension has recently been discussed by D. S. Novikov, Phys. Rev. Lett. **95**, 066401 (2005); Phys. Rev. B **72**, 235428 (2005).

[45] L. I. Glazman, I. M. Ruzin, and B. I. Shklovskii, Phys. Rev. B **45**, 8454 (1992).

[46] W. Häusler, L. Kecke, and A. H. MacDonald, Phys. Rev. B **65**, 085104 (2002).

[47] S. Sachdev, *Quantum Phase Transitions* (Cambridge University Press, Cambridge, 1999).

[48] T. Giamarchi, *Quantum Physics in One Dimension* (Clarendon Press, Oxford, 2004).

[49] S. Teber, cond-mat/0511257 (unpublished).

[50] J. Voit, Phys. Rev. B **47**, 6740 (1993).

[51] V. Meden and K. Schönhammer, Phys. Rev. B **46**, 15753 (1992).

[52] S. Sachdev, T. Senthil, and R. Shankar, Phys. Rev. B **50**, 258 (1994).

[53] H. J. Schulz, Phys. Rev. B **34**, 6372 (1986).

[54] I. S. Gradshteyn and I. M. Ryzhik, *Tables of Integrals, Series, and Products* (Academic, New York, 1965), formula 3.312.1.

[55] I. V. Gornyi, A. D. Mirlin, and D. G. Polyakov, Phys. Rev. Lett. **95**, 046404 (2005).

[56] T. Giamarchi and H. J. Schulz, Phys. Rev. B **37**, 325 (1988).

[57] H. Steinberg, O. M. Auslaender, A. Yacoby, J. Qian, G. A. Fiete, Y. Tserkovnyak, B. I. Halperin, K. W. Baldwin, L. N. Pfeiffer, and K. W. West, Phys. Rev. B **73**, 113307 (2006).