Shape and size effects on the energy absorption by small metallic particles

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The electric and magnetic light absorption by small metallic particles of an ellipsoidal shape has been studied theoretically in frequency regions that are higher and lower than the characteristic frequency of an electron free path between walls of the particle. The assumption of diffuse reflection of electrons from an inner surface of the particle is taken as boundary conditions of the problem. Analytical expressions for the absorbed power are obtained provided that the thickness of a skin layer is large enough compared with a particle size. Size and shape effects on the absorption are investigated for three polarizations of an incident radiation. The particles may be either larger or smaller than the electron free path. The dependence of the relative contribution of the electric and magnetic absorption on the degree of particle asymmetry is discussed in detail for different wave polarizations. Similar dependence for the ratio of longitudinal and transverse components of the conductivity tensor are studied as well.

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I. INTRODUCTION

Optical properties of small metallic particles (SMPs) having diameters much less than the electromagnetic (EM) wavelength begin to depend on their shape. Thus, the study of the light scattering by individual nanometer-sized gold particles has experimentally demonstrated that the tuning of the particle's size and shape plays a crucial role in the light scattering process.[1] Moreover, for elliptical structures, the orientation of the polarization of the incident field, as well as the relative ratio of the ellipse dimensions along its main axes, govern the position of optical resonances and would therefore lead to a high-performance probe for scanning near-field optical microscopy.[1]

The theoretical estimations show that if the ratio between the major and minor axes of the ellipsoidal particle is of an amount of several units, then the absorption cross section of that particle will exceed by an order of magnitude or even more the absorption cross section of the spherical particle with the same volume.[2]

If, apart from the light wavelength, the characteristic size of the SMP is smaller than the bulk mean free path of the electron, then the dependence of absorption on the particle shape becomes more essential. In particular, in this case, when the particle shape differs from the spherical, the optical conductivity becomes a tensor variable. This means that the absorption cross section of such SMP will be determined now not only by components of a depolarization tensor, but also by the components of an optical conductivity tensor.[2]

It is well known that both the electric component of the EM wave (the electric absorption) and magnetic component (the magnetic absorption) contribute to the optical absorption by SMPs. The electric absorption arises from the polarization of the particle in a spatially uniform electric field, while the magnetic absorption arises from eddy currents induced by a time-varying uniform magnetic field. Which one of these two components dominates in a particular situation is depending on the light frequency, size, and shape of the SMP.[2]

Existent theoretical investigations were mainly devoted either to the electric or to the magnetic absorption of radiation by particles of a particular shape in the case when the electron free path is much smaller than the particle size. However, so far, the theory was absent which describes from a unified position the electric and magnetic absorption for the particles of different shapes and for arbitrary electron free paths.

Since the optical properties of SMPs depend on their shapes, the theory of the light absorption was built separately for the particles with the spherical,[3–8] cylindrical,[9,10] and ellipsoidal shape,[2,11] as well as for the particle of the parallelepipedic shape,[12] or mixtures of irregularly shaped particles.[13] So, in Refs. 3, 4, 9, 12, and 13 only the electric mechanism of absorption was treated, and in Refs. 5–7, 11, and 14 only the magnetic mechanism (an eddy current contribution) was considered, and in Refs. 2 and 15–17 both mechanisms were taken into account. Microwave spectroscopy allows one to separate the electric and magnetic absorption.[18] The experimental method exists for a precise and instantaneous analysis of the optical absorption of even a single microparticle.[19]

The experimental investigations of the optical properties of SMP were devoted mainly to the study of the specificity of an electron-phonon relaxation, size effects (see, for example, Refs. 20–23) or to the experimental test of the Mie theory.[24,25] Recently, a number of papers dedicated to examining the optical properties of metallic nanoshells of various shapes and the particles of different shapes (spherical,[26–29] cylindrical,[30–32] ellipsoidal[33]) has increased. In Refs. 29, 31, and 33 the problem of the manifestation of electron energy quantization in the optical spectra of thin shells was investigated.

The present work is dedicated to the comprehensive study of the particle shape- and size-induced effects on the optical properties of the SMP. We proceed in this work to a kinetic treatment of the problem focusing our attention to the properties of a single, isolated metallic particle.

The SMP of an ellipsoidal shape is treated as a model. This model has some advantages over other models considered earlier in literature. First, if we have formulas for the absorption cross section for the ellipsoidal shape particles, we can get similar formulas for a wide set of real particle shapes (from disklike to antennalike). Secondly, this particle shape is unique (unlike the spherical case), for which expressions for the electric and magnetic absorption can be derived in a general approach. And finally, for this shape there are no problems with edge effects, which can arise in the case of particles with sharp forms of borders.

The model of an ellipsoidal SMP was employed by us in Refs. 2, 11, and 15. In these works, the main attention was paid to the IR frequency range only. But, the problem of how the shape of the SMP affects the line form of plasma resonances was not treated at all.

It is necessary to notice that if the characteristic size of the SMP becomes smaller than the bulk mean free path of the electrons, then not only the momentum relaxation time is altered, but also the electron energy relaxation time. Usually, the momentum relaxation time is associated with the optical conductivity and so with the optical absorption of the SMP. The energy relaxation time characterizes the electron-lattice energy exchange and hence defines the electron temperature (in the case of the presence of hot electrons in SMP). Requirements necessary for the manifestation of hot electrons in SMP were considered, for instance, in Ref. 34.

Both the electron collisions with the bulk lattice vibrations and with the surface lattice vibrations contribute to the electron-lattice energy exchange in SMPs.[35–37] Hovever, one cannot be sure, as it is often assumed in experiment interpretations, that in SMPs the bulk contribution remains invariable with the particle resizing and the surface contribution merely add to it. As a matter of fact, if the size of the particle becomes smaller than the bulk mean free path of electrons, the bulk contribution to the electron-lattice energy exchange sharply diminishes.[35,38] The surface contribution to this exchange increases with reduction of the particle size, but for particles with a diameter comparable to the electron free path is still by two orders of magnitude less compared to the bulk contribution (see, e.g., estimations in Ref. 35). Therefore, the total energy exchange at first sharply falls with the reduction of the particle size (starting with the size comparable with the electron free path) and only afterwards begins to go up. It is necessary to take into consideration all these facts in both the absorption and the energy exchange in SMPs for correct interpretation of experimental results.

In our consideration we will neglect the quantum size effects (which are essential only for particles with radii of $\lesssim 20$ Å) on the optical properties of the SMP.

The rest of this paper is arranged as follows: Sec. II contains the formulation of the problem. Section III is devoted to the local fields. The electron distribution function is obtained in Sec. IV. In Sec. V the electric absorption by the SMP is considered. The plasma resonances are treated in Sec. VI. The magnetic absorption is studied in the limit of low and high light frequencies in Sec. VII. Obtained results and conclusions are presented in Sec. VIII.

II. FORMULATION OF THE PROBLEM

Assume that a metallic particle is in an externally applied field of an EM wave,

$$\begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} = \begin{pmatrix} \mathbf{E}_0 \\ \mathbf{H}_0 \end{pmatrix} e^{i(\mathbf{kr}-\omega t)}. \tag{1}$$

Here $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic components of the EM wave, ω, $\mathbf{k}$ are the wave frequency and the wave vector, $\mathbf{r}$, t describe the spatial coordinates and the time, respectively. We will suppose that the wavelength $\lambda = 2\pi/|\mathbf{k}|$ is far above the characteristic size of the particle. This allows us to treat the metallic particle as being immersed in a spatial uniform, but oscillating in time electric and magnetic fields. This implies that only dipole modes are excited and multipole scattering may be neglected. The electric field $\mathbf{E}^{(0)}\exp(-i\omega t)$ induces a local potential electric field $\mathbf{E}_{\text{loc}}$ inside the particle, which in its turn contributes to the electric current (with a current density $\mathbf{j}_e$). The magnetic field $\mathbf{H}^{(0)}\exp(-i\omega t)$ induces an eddy electric field $\mathbf{E}_{\text{ed}}$ in the particle, which gives rise to an eddy electric current $\mathbf{j}_m$ (Foucault current); here $\mathbf{j}_m$ is the eddy current density.

As the result, the total power absorbed by particle from the EM wave is

$$W = W_e + W_m = \frac{1}{2}\,\text{Re}\int_V d\mathbf{r}[\mathbf{j}_e(\mathbf{r})\cdot\mathbf{E}^*_{\text{loc}}(\mathbf{r}) + \mathbf{j}_m(\mathbf{r})\cdot\mathbf{E}^*_{\text{ed}}(\mathbf{r})], \tag{2}$$

where the integral extends over the entire particle volume V and an asterisk designates the complex conjugation. The first term on the right-hand side of Eq. (2) corresponds to the electric dipole contribution and the second one to the magnetic contribution, respectively. Hence, in order to determine the total absorption, we must know the potential electric field $\mathbf{E}_{\text{loc}}$, the eddy electric field $\mathbf{E}_{\text{ed}}$, and the corresponding current densities $\mathbf{j}_e$ and $\mathbf{j}_m$. Expression (2) for the total energy absorbed by a spherical particle is well known and takes the ordinary form (see, e.g., Ref. 39),

$$W = \frac{9}{8}\frac{V}{\pi}\omega\varepsilon''\left[\frac{1}{(2+\varepsilon')^2+\varepsilon''^2} + \frac{\omega^2 R^2}{90c^2}\right]|\mathbf{E}^{(0)}|^2, \tag{3}$$

where ε' and ε'' are, respectively, the real and imaginary parts of the dielectric permittivity of the particle, and R is the particle radius. The first term on the right-hand side of Eq. (3) indicates the electric absorption and the second one—the magnetic absorption.

If the particle size is larger than the mean free path, i.e., bulk scattering dominates, the expression for the dielectric constant of the metal has a known standard form

$$\varepsilon = \varepsilon' + i\varepsilon'' = 1 - \frac{\omega_{\text{pl}}^2}{\nu^2+\omega^2} + i\frac{\nu}{\omega}\frac{\omega_{\text{pl}}^2}{\nu^2+\omega^2}, \tag{4}$$

where ω_{pl} is the bulk plasma frequency of the metal, and ν is the collision rate of conducting electrons (for example, due to occasional scattering by phonons or impurities). The imaginary part of the dielectric permittivity ε'' in Eq. (4) is related to the optical conductivity σ of free electrons, that in its turn connects the electric current density $\mathbf{j}_e$ with the local electric field $\mathbf{E}_{\rm loc}$ in the Ohm law approximation, by

$$\mathbf{j}_e = \sigma \mathbf{E}_{\rm loc} = \frac{\omega}{4\pi}\varepsilon'' \mathbf{E}_{\rm loc}. \tag{5}$$

In the case when the particle size is smaller than the electron mean free path, Eq. (4) remains the same, but for the formal replacement $\nu \to \frac{3}{4}\frac{v_F}{R}$; here v_F is the electron Fermi velocity. A proof for this will be given below.

One can use Eq. (3) to estimate the relative contribution of the electric and magnetic absorption to the total absorption by the SPM of a spherical form,

$$\frac{W_m}{W_e} = \frac{1}{90}\left(\frac{\omega R}{c}\right)^2 [(2+\varepsilon')^2 + \varepsilon''^2]. \tag{6}$$

From Eq. (6), taking into account Eq. (4), it is easy to see that, depending on the light frequency and the particle diameter, either the electric or magnetic absorption can dominate. In the case of a particle with an asymmetrical shape, the contribution rate of these two mechanisms will depend on both the particle shape and the light polarization.

Going to the study of the optical properties of the asymmetrical SMP, let us start with the definition of the local fields.

III. LOCAL FIELDS

The metallic particles examined below are assumed to be ellipsoidal in shape. Such an assumption has certain advantages. First, considering ellipsoids of different oblateness and elongation, we can treat a variety of particle shapes (from a "pancake" to an antennalike). Second, finding the potential $\mathbf{E}_{\rm loc}$ and eddy $\mathbf{E}_{\rm ed}$ local fields for such particles is easy.

As known[39] for ellipsoidal particles, the potential local electric field $\mathbf{E}_{\rm loc}$ induced by the uniform external electric field $\mathbf{E}^{(0)}$ is coordinate independent. The field $\mathbf{E}_{\rm loc}$ can be linearly expressed in terms of $\mathbf{E}^{(0)}$ by employing the depolarization tensor. In terms of the principal axes of the depolarization tensor, which coincide with the principal axes of the ellipsoid, we have[40]

$$(E_{\rm loc})_j = E_j^{(0)} - L_j(\varepsilon - 1)(E_{\rm loc})_j, \tag{7}$$

where L_j are the principal values of the components of the depolarization tensor.

For asymmetric particles smaller than the mean free path, light-induced conductivity becomes a tensor, as we will soon see, and Eq. (7) needs to be modified. This will be done a little bit later. We also note that in the case of many particles, the polarization vector in a given particle is induced not only by the field of the external wave, but also by the dipoles induced by this wave in other particles.[5] Here we ignore such effects, but they can easily be incorporated into the picture.[37]

Finding $(\mathbf{E}_{\rm loc})_j$ from Eq. (7), we find that

$$|(\mathbf{E}_{\rm loc})_j|^2 = \frac{(\mathbf{E}_j^{(0)})^2}{[1 + L_j(\varepsilon' - 1)]^2 + [L_j \varepsilon'']^2}. \tag{8}$$

When the light-induced conductivity of small particles becomes a tensor (we will return to this later), we must substitute $4\pi\sigma_{jj}/\omega$ instead of ε'' in Eq. (8) (σ_{jj} is the jth diagonal component of the light-induced conductivity tensor).

Let us now find the eddy local field $\mathbf{E}_{\rm ed}$. This field must obey Maxwell's equations

$$\nabla \times \mathbf{E}_{\rm ed} = i\frac{\omega}{c}\mathbf{H}^{(0)},$$

$$\nabla \cdot \mathbf{E}_{\rm ed} = 0. \tag{9}$$

In the right-hand side of the first equation of (9), we take the external uniform magnetic field $\mathbf{H}^{(0)}$ as the magnetic field inside the particle. Such an approximation is justified since the skin depth δ_H is large enough compared with the particle size R,

$$\delta_H \equiv \left(\frac{\omega}{c}\,\mathrm{Im}\,\sqrt{\varepsilon}\right)^{-1} \gg R. \tag{10}$$

For an ellipsoidal particle, R is the principal semiaxis of the ellipsoid.

Notice that the case (10) is the most interesting, since the contribution of eddy currents to the absorption is then at its maximum.

Let us integrate the second equation of (9) over the volume of the particle. We will have

$$\int d^3r(\nabla \cdot \mathbf{E}_{\rm ed}) = \oint ds(\mathbf{E}_{\rm ed} \cdot \mathbf{n}_s) = 0, \tag{11}$$

where $\mathbf{n}_s$ is a unit vector normal to the surface S. The criterion of Eq. (11) will be satisfied automatically provided that we take for the field $\mathbf{E}_{\rm ed}$ the boundary condition in the form

$$\mathbf{E}_{\rm ed} \cdot \mathbf{n}|_s = 0. \tag{12}$$

Equations (9) with the boundary condition (12) completely define the eddy field $\mathbf{E}_{\rm ed}$.

As in virtue of the assumed inequality (10), the right-hand side of the first of Eqs. (9) is constant, and the required eddy field $\mathbf{E}_{\rm ed}$ should be a linear function of coordinates. This means that in a general form, we can write down

$$(E_{\rm ed})_j = \sum_{k=1}^{3} \alpha_{jk} x_k. \tag{13}$$

To be short in Eq. (13) we set $x_1 = x$, $x_2 = y$, $x_3 = z$; $\alpha_{12} = \alpha_{xy}$, $\alpha_{23} = \alpha_{yz}$, etc. If now we substitute Eq. (13) into Eq. (9) and separate into components, we will obtain

$$\alpha_{zy} - \alpha_{yz} = i\frac{\omega}{c}H_x^{(0)},$$

$$\alpha_{xz} - \alpha_{zx} = i\frac{\omega}{c}H_y^{(0)},$$

$$\alpha_{yx} - \alpha_{xy} = i\frac{\omega}{c}H_z^{(0)}. \quad (14)$$

Other equations necessary for the complete definition of α_{ij}, we obtain from the boundary conditions (12). Let us assume that the particle has an ellipsoidal shape

$$\sum_{i=1}^{3}\left(\frac{x_i}{R_i}\right)^2 = 1, \quad (R_1 = R_x,\ R_2 = R_y,\ R_3 = R_z). \quad (15)$$

The normal to the ellipsoidal surface (15) looks like

$$\mathbf{n}_s = (n_1, n_2, n_3); \quad n_i = \frac{x_i}{R_i^2}\left[\sum_{j=1}^{3}\left(\frac{x_j}{R_j^2}\right)^2\right]^{-1/2}. \quad (16)$$

If now we substitute Eqs. (13) and (16) into Eq. (12), and equate to the zero term after term the coefficients at $x_i x_j$, we get

$$\alpha_{xx} = \alpha_{yy} = \alpha_{zz} = 0,$$

$$\alpha_{xy}/R_x^2 + \alpha_{yx}/R_y^2 = 0,$$

$$\alpha_{xz}/R_x^2 + \alpha_{zx}/R_z^2 = 0,$$

$$\alpha_{yz}/R_y^2 + \alpha_{zy}/R_z^2 = 0. \quad (17)$$

Equations (14) and (17) completely define all α_{ij}. As the result, we find that

$$\alpha_{xy} = -i\frac{\omega}{c}\frac{R_x^2}{R_x^2 + R_y^2}H_z^{(0)}, \quad \alpha_{yx} = i\frac{\omega}{c}\frac{R_y^2}{R_y^2 + R_z^2}H_z^{(0)},$$

$$\alpha_{xz} = i\frac{\omega}{c}\frac{R_x^2}{R_x^2 + R_z^2}H_y^{(0)}, \quad \alpha_{zx} = -i\frac{\omega}{c}\frac{R_z^2}{R_z^2 + R_x^2}H_y^{(0)},$$

$$\alpha_{zy} = i\frac{\omega}{c}\frac{R_z^2}{R_z^2 + R_y^2}H_x^{(0)}, \quad \alpha_{yz} = -i\frac{\omega}{c}\frac{R_y^2}{R_y^2 + R_z^2}H_x^{(0)}, \quad (18)$$

where R_x, R_y, R_z are the ellipsoid semiaxes along the x, y, z axes. Substituting Eq. (18) in Eq. (13), we find the eddy field vector components explicitly. In particular, we derive

$$E_{\text{ed}}^x = i\frac{\omega}{c}\left(\frac{zH_y^{(0)}}{R_z^2 + R_x^2} - \frac{yH_z^{(0)}}{R_x^2 + R_y^2}\right)R_x^2. \quad (19)$$

The other two components can be easily obtained via cyclic permutations of indexes in Eq. (19).

From Eqs. (13) and (18), it is easily seen that in the case of a spherical particle

$$\mathbf{E}_{\text{ed}} = \frac{\omega}{2ic}[\mathbf{r}, \mathbf{H}^{(0)}]. \quad (20)$$

In the case of the ellipsoidal particle, the potential and eddy electric fields inside the particle are defined by formulas (7) and (19) accordingly. In order to find the energy absorbed by a particle in unit time, it is also necessary to determine the current density induced by these fields. When the size of the particle considerably exceeds the bulk mean free path of the electron, the problem of a current calculation becomes essentially simpler. In this case, the electric current density is given by Eq. (5) and similarly the eddy current density is equal to

$$\mathbf{j}_m = \sigma\mathbf{E}_{\text{ed}} = \frac{\omega}{4\pi}\varepsilon''\mathbf{E}_{\text{ed}}. \quad (21)$$

Substituting Eqs. (5), (21), and (8), (19) in Eq. (2) and integrating over the volume of the particle, we have

$$W = V\frac{\omega\varepsilon''}{8\pi}\left\{\sum\frac{(E_j^{(0)})^2}{[1 + L_j(\varepsilon' - 1)]^2 + (L_j\varepsilon'')^2} + \frac{\omega^2 R_\perp^2}{10c^2}(\mathbf{H}_\parallel^{(0)})^2 + \frac{\omega^2}{5c^2}\frac{R_\perp^2 R_\parallel^2}{R_\perp^2 + R_\parallel^2}(\mathbf{H}_\perp^{(0)})^2\right\}. \quad (22)$$

In Eq. (22), for the sake of simplicity, we restrict ourselves to the ellipsoid of revolution, i.e., we set $R_x = R_y = R_\perp$, $R_z = R_\parallel$. Besides, the following notations are used: $\mathbf{H}_\parallel^{(0)} = \mathbf{H}_z^{(0)}$, $(\mathbf{H}_\perp^{(0)})^2 = (\mathbf{H}_x^{(0)})^2 + (\mathbf{H}_y^{(0)})^2$.

Equation (22) generalizes Eq. (3) to the case of ellipsoidal particles. Note that in Eq. (22), $\mathbf{E}^{(0)} = \mathbf{H}^{(0)}$. In Eq. (22) we do not express $\mathbf{H}^{(0)}$ in terms of $\mathbf{E}^{(0)}$ [as we did in Eq. (3)] in order to make more visible the dependence of the absorbed power not only on the polarization of electric components of the wave, but also on the polarization of its magnetic component. One can use Eq. (22) to explain the reason of a strong dependence of the absorption on the particle shape (especially in the IR frequency range). The dependence of the electric absorption on the shape [the first term in Eq. (22)] contains in denominator the diagonal components of depolarization tensor L_j. The principal values of these components for particles whose shape is that of ellipsoids of revolution are[39]

$$L_x(e_p) = L_y(e_p) = \frac{1}{2}[1 - L_z(e_p)],$$

$$L_z(e_p) = \begin{cases} \dfrac{1 - e_p^2}{2e_p^3}\left(\ln\dfrac{1 + e_p}{1 - e_p} - 2e_p\right), & R_\perp < R_\parallel \\ \dfrac{1 + e_p^2}{e_p^3}(e_p - \arctan e_p), & R_\perp > R_\parallel. \end{cases} \quad (23)$$

where

$$e_p^2 = \begin{cases} 1 - R_\perp^2/R_\parallel^2, & R_\perp < R_\parallel \\ R_\perp^2/R_\parallel^2 - 1, & R_\perp > R_\parallel. \end{cases} \quad (24)$$

It is seen from Eq. (23) that by changing the ellipsoid semiaxes ratio, the value of L_j can vary between zero and unity. Note that $\Sigma_{j=1}^3 L_j = 1$.

For the further reasoning, we shall make some estimations. For a gold particle $\omega_{\text{pl}} \approx 5\times10^{15}$ s^{-1}, $\nu \approx 10^{13}$ s^{-1}. Besides, we take for $\omega \approx 2\times10^{14}$ s^{-1} (the frequency of a carbon dioxide laser). For these values of parameters, we obtain from Eq. (4), $\varepsilon' \approx -620$, $\varepsilon'' \approx 30$. Now, as is easily seen, the denominator of the first term of Eq. (22) can vary from 1 at

$L_j \approx 0$ to 4×10^5 at $L_j \approx 1$. This estimation explains the high sensitivity of the electric absorption to the particle shape and to the light polarization.

Above, we have treated the reason of sensitivity of the electric absorption to the particle shape. As for the particle shape effect on the magnetic absorption, it is obvious from Eq. (22). Therefore, we shall not muse on it.

IV. ELECTRON DISTRIBUTION FUNCTION

In Sec. III, we have determined the local field and, at the same time, derived a general expression for the power absorbed by an ellipsoidal metallic particle in the case of bulk scattering (i.e., for particle size larger than the electron mean free path). Below, we will focus our attention on the particles smaller than the mean free path. Incidently, the method we now develop can also be applied to the particles larger than the mean free path.

To obtain a formula for the absorbed power, besides knowing the potential and local eddy electric fields in the particle, we must also derive expressions for the high-frequency currents induced by these fields.

The current density is defined by the expression

$$\mathbf{j}(\mathbf{r}) = 2e\left(\frac{m}{2\pi\hbar}\right)^3 \int\int\int \mathbf{v}(\mathbf{r}) f(\mathbf{r},\mathbf{v}) d^3 v, \qquad (25)$$

where $f(\mathbf{r},\mathbf{v})$ is the electron distribution function over the coordinates $\mathbf{r}$ and velocities $\mathbf{v}$, e and m are the electron charge and mass. The integrations in Eq. (25) are fulfilled over all possible velocities.

The fields $\mathbf{E}_{\rm loc}$ and $\mathbf{E}_{\rm ed}$ cause the deviation from the equilibrium Fermi electron distribution $f_0(\varepsilon)$, which initially depends only on the kinetic electron energy $\varepsilon = mv^2/2$. The total distribution function $f(\mathbf{r},\mathbf{v})$ can be found as the sum of the equilibrium $f_0(\varepsilon)$ and the nonequilibrium parts of $f_1(\mathbf{r},\mathbf{v})$

$$f(\mathbf{r},\mathbf{v}) = f_0(\varepsilon) + f_1(\mathbf{r},\mathbf{v}). \qquad (26)$$

In other words, $f_1(\mathbf{r},\mathbf{v})$ is a correction generated by the fields. We can find $f_1(\mathbf{r},\mathbf{v})$ by solving the appropriate kinetic equation. In the linear approximation in an external EM field, the kinetic Boltzmann equation for the charge carriers distribution function takes the form

$$(\nu - i\omega) f_1(\mathbf{r},\mathbf{v}) + \mathbf{v}\frac{\partial f_1(\mathbf{r},\mathbf{v})}{\partial \mathbf{r}} + e(\mathbf{E}_{\rm loc} + \mathbf{E}_{\rm ed})\mathbf{v}\frac{\partial f_0(\varepsilon)}{\partial \varepsilon} = 0. \qquad (27)$$

Here the steady-state time dependence $f_1 \propto e^{-i\omega\tau}$ is assumed and the collision integral is evaluated in the relaxation time approximation $(\partial f_1/\partial t)_{\rm col} = -f_1/\tau$, $\tau = 1/\nu$.

Equation (27) must be completed with boundary conditions for $f_1(\mathbf{r},\mathbf{v})$ at the ellipsoidal surface. For this purpose we take, as is commonly done, the assumption of the diffuse electron scattering at the boundary, i.e.,

$$f_1(\mathbf{r},\mathbf{v})|_s = 0, \quad v_n < 0, \qquad (28)$$

where v_n is the velocity component normal to the surface S.

In the literature, both diffuse and mirror boundary conditions at the surface of the particle are examined for similar problems (see for example, Ref. 7). The degree of the reflectivity of the border depends on its smoothness in the atomic scale. For nonplanar border such smoothness is extremely difficult to achieve. Therefore, we consider only more realistic diffusive boundary conditions (28).

In order to solve Eq. (27) with the boundary conditions (28), we employ the method of characteristic curves. Its effectiveness has been demonstrated in Ref. 6 where the magnetic absorption by SMP of spherical shape was investigated. But for ellipsoidal particles, the method developed in Ref. 6 needs to be modified. The essence of this modification will be clarified later.

Thus, we transform to a deformed system of coordinates in which the original ellipsoidal particle,

$$\sum_{i=1}^{3} \frac{x_i^2}{R_i^2} = 1 \qquad (29)$$

becomes a sphere (of radius R). In other words, we assume that

$$x_i = \frac{x_i'}{\gamma_i}, \quad \gamma_i = \frac{R}{R_i}, \quad R = (R_1 R_2 R_3)^{1/3}, \quad \gamma_1\gamma_2\gamma_3 = 1. \qquad (30)$$

Under such deformation only the shape of the particle alters, but not its volume. This means that the conduction electron density remains invariable, and so does the normalization of the function $f(\mathbf{r}',\mathbf{v}')$.

In the deformed coordinate system, Eq. (26) and the boundary conditions (28) can be presented as

$$(\nu - i\omega) f_1(\mathbf{r}',\mathbf{v}') + \mathbf{v}'\frac{\partial f_1(\mathbf{r}',\mathbf{v}')}{\partial \mathbf{r}'} + e[\mathbf{E}_{\rm loc}(\mathbf{r}') + \mathbf{E}_{\rm ed}(\mathbf{r}')]\mathbf{v}\frac{\partial f_0(\varepsilon)}{\partial \varepsilon} = 0, \qquad (31)$$

$$f_1(\mathbf{r}',\mathbf{v}')|_{\mathbf{r}'=\mathbf{R}} = 0, \quad \mathbf{r}'\mathbf{v}' < 0. \qquad (32)$$

The case $\mathbf{r}'\mathbf{v}' < 0$ corresponds to the motion of electrons from the outer surface of the particle (the origin of coordinates is chosen to be at the center of the particle). In Eqs. (31) and (32) we have also introduced the new "deformed" velocity components,

$$v_i' = \gamma_i v_i. \qquad (33)$$

Equation (31) for the characteristic curves is given by

$$\frac{dx_i'}{v_i'} = -\frac{df_1}{\tilde{\nu} f_1} = dt'; \quad \tilde{\nu} = \nu - i\omega, \qquad (34)$$

which implies that

$$\mathbf{r}' = \mathbf{v}'t' + \mathbf{R}, \qquad (35)$$

where $\mathbf{R}$ is the radius vector whose tip is at a given point of the sphere from which a trajectory begins. Here the parameter t' can be formally considered as the "time" of the electron motion along the trajectory.

If in Eq. (35) we transfer $\mathbf{v}'t'$ to the left-hand side and square the resulting equation, the solution of this new scalar equation will have the form

$$t' = \frac{1}{v'^2}[\mathbf{r}'\mathbf{v}' + \sqrt{(R^2 - r'^2)v'^2 + (\mathbf{r}'\mathbf{v}')^2}]. \quad (36)$$

The characteristic curve of Eq. (36) depends only on the absolute value of $\mathbf{R}$ and does not on the orientation of $\mathbf{R}$. Such independence of the characteristic curve from the position of a point on the surface was achieved by transforming to the coordinates (30).

From Eq. (36) we can also see that $t'=0$ at $r'=R$. Bearing this in mind, we can employ Eq. (34) to find f_1 that satisfies Eq. (31) and the boundary conditions (32),

$$f_1(\mathbf{r},\mathbf{v}) = -\frac{\partial f_0(\varepsilon)}{\partial \varepsilon}\int_0^{t'} d\tau \exp[-\tilde{\nu}(t'-\tau)]e\mathbf{v}\cdot\mathbf{F}[\mathbf{r}' - \mathbf{v}'(t'-\tau)], \quad (37)$$

with $\mathbf{F}=\mathbf{E}_{\text{loc}}+\mathbf{E}_{\text{ed}}$. Taking into account the coordinate dependence of $\mathbf{F}$ [see Eqs. (31) and (19)], from Eq. (37) we thus obtain

$$f_1(\mathbf{r},\mathbf{v}) = -e\frac{\partial f_0(\varepsilon)}{\partial \varepsilon}\left[\mathbf{v}\mathbf{E}_{\text{loc}} + \sum_{i,j=1}^{3}\alpha_{ij}v_i\left(\frac{x_j'}{\gamma_j} + v_j\frac{\partial}{\partial(\nu - i\omega)}\right)\right] \times \left(\frac{1-e^{-(\nu-i\omega)t'}}{\nu - i\omega}\right). \quad (38)$$

If, initially, the particle is spherical, then $\alpha_{ij}=-\alpha_{ji}$, and the last term in Eq. (38) vanishes.

V. ELECTRIC ABSORPTION

In this section, we concentrate on the electric absorption. Combining Eqs. (38), (25), and (2), we obtain the following expression for the electric absorption:

$$W_e = e^2\left(\frac{m}{2\pi\hbar}\right)^3 \times \text{Re}\left[\frac{1}{\tilde{\nu}}\int d^3r' \int |\mathbf{v}\cdot\mathbf{E}_{\text{loc}}|^2\delta(\varepsilon-\mu)(1-e^{-\tilde{\nu}t'})d^3v\right], \quad (39)$$

where μ is the Fermi energy, and where we assume that

$$\frac{\partial f_0(\varepsilon)}{\partial \varepsilon} \approx -\delta(\varepsilon - \mu).$$

With account of the form of t' [according to Eq. (36)], it is convenient to fulfill the integration with respect to $\mathbf{r}'$ in Eq. (39) by directing the z' axis along the vector $\mathbf{v}'$ and introducing new variables,

$$\zeta = \frac{r'}{R}, \quad \xi = \frac{v'}{R}t'. \quad (40)$$

As a result we have

$$\int(1-e^{-\tilde{\nu}t'})d^3r' = 2\pi\int_0^R dr' r'^2\int_0^\pi d\theta\sin\theta\ (1-e^{-\tilde{\nu}t'})$$
$$= 2\pi R^3\int_0^1 d\zeta\zeta^2\int_{1-\zeta}^{1+\zeta}d\xi\left(\frac{\xi^2-\zeta^2+1}{2\xi^2\zeta}\right)\left(1-e^{-(\tilde{\nu}R/v')\xi}\right)$$
$$= \pi R^3\int_0^2\frac{d\xi}{\xi^2}\left(1-e^{-(\tilde{\nu}R/v')\xi}\right)\int_{|\xi-1|}^1 d\zeta\zeta(\xi^2-\zeta^2+1).$$

Further calculation of the integral is easy, and, as a result, we obtain from Eq. (39),

$$W_e = \pi e^2\left(\frac{mR}{2\pi\hbar}\right)^3\text{Re}\left\{\frac{1}{\tilde{\nu}}\int d^3v|\mathbf{v}\cdot\mathbf{E}_{\text{loc}}|^2\delta(\varepsilon-\mu)\psi(q)\right\}, \quad (41)$$

where we have introduced

$$\psi(q) = \frac{4}{3} - \frac{2}{q} + \frac{4}{q^3} - \frac{4}{q^2}\left(1+\frac{1}{q}\right)e^{-q}, \quad (42)$$

$$q \equiv q_1 - iq_2 = \frac{2\tilde{\nu}}{v'}R = \frac{2\nu}{v'}R - i\frac{2\omega}{v'}R. \quad (43)$$

Equation (41) determines a general form of the electric absorption by an ellipsoidal metallic particle for an arbitrary bulk-to-surface scattering ratio.

The integrations over velocities can be conducted without any difficulties only for the spherical particle. In this case, we obtain

$$W_e = \frac{3V}{8\pi}\nu_s|\mathbf{E}_{\text{loc}}|^2\frac{\omega_{\text{pl}}^2}{\nu^2+\omega^2}\left\{\frac{\nu}{3\nu_s} - \frac{1}{2}\frac{\nu^2-\omega^2}{\nu^2+\omega^2} + \frac{\nu_s^2}{(\nu^2+\omega^2)^2}\left[\frac{(\nu^2-\omega^2)^2-4\nu^2\omega^2}{\nu^2+\omega^2} + e^{-\nu/\nu_s} \times\left[\sin\frac{\omega}{\nu_s}\left(\frac{1}{\nu_s}(3\nu^2-\omega^2) + 4\nu\frac{\nu^2-\omega^2}{\nu^2+\omega^2}\right)\omega - \cos\frac{\omega}{\nu_s}\left(\frac{\nu}{\nu_s}(\nu^2-3\omega^2) + \frac{(\nu^2-\omega^2)^2-4\nu^2\omega^2}{\nu^2+\omega^2}\right)\right]\right]\right\}, \quad (44)$$

where $\nu_s=v_F/(2R)$ is the collision frequency of an electron with the surface, and $\omega_{\text{pl}}^2=4\pi ne^2/m$, n being the density of conduction electrons, which can be expressed in terms of the Fermi velocity v_F or the Fermi energy μ,

$$n = \frac{8\pi}{3}\left(\frac{mv_F}{2\pi\hbar}\right)^3, \quad v_F = \sqrt{\frac{2\mu}{m}}. \quad (45)$$

The result (44) is quite accurate for a particle with radii both smaller or larger than the electron free path.

The above case of bulk scattering (the Drude case) follows from Eq. (41) when $q \gg 1$, that gives for $\psi(q)\approx 4/3$. The Drude case results also from Eq. (44) by neglecting the effects arising from the kinetic approach to the problem (ν_s

$\rightarrow 0$). Then, in accordance with Eqs. (41) and (44), we obtain for the electric absorption

$$W_e \approx \frac{V}{8\pi}\nu\frac{\omega_{\rm pl}^2}{\nu^2+\omega^2}|\mathbf{E}_{\rm loc}|^2 \equiv \frac{V}{2}\sigma(\omega)|\mathbf{E}_{\rm loc}|^2. \tag{46}$$

It is clearly seen that Eq. (46) corresponds to the first term in Eq. (22).

We now analyze the situation in which the particle is smaller than the mean free path, and hence the surface scattering is dominant. This corresponds to

$$q_1 = \frac{2\nu}{v'}R \ll 1. \tag{47}$$

As for the parameter $q_2 \equiv 2\omega R/v'$, when surface scattering is dominant, this parameter can be either larger or smaller than unity. We will study the both possible limiting cases

$$q_2 \ll 1, \tag{48}$$

and

$$q_2 = \frac{2\omega}{v'}R \gg 1. \tag{49}$$

The case (48) corresponds to the low-frequency surface scattering, and that of Eq. (49) to the high-frequency surface scattering.

If we ignore the bulk scattering ($q_1 \rightarrow 0$) and assume that q_2 is arbitrary, then Eq. (42) yields

$$\mathrm{Re}\left[\frac{1}{\tilde{\nu}}\psi(q)\right] \approx \frac{1}{\omega}\left[\frac{2}{q_2} - \frac{4}{q_2^2}\sin q_2 + \frac{4}{q_2^3}(1-\cos q_2)\right]. \tag{50}$$

Then this approximate expression must be inserted into Eq. (41). We see that the terms appear oscillating as functions of the particle size. For a spherical particle we get them from Eq. (44) in an analytic form. Such oscillation effects were studied numerically in spherical particles earlier by Austin and Wilkinson[41] for the electric absorption, and by Lesskis *et al.*[6] for the magnetic absorption.

These effects, which are moderate by themselves, are even less essential for asymmetric particles. The reason is that the "deformed" velocity v', which enters into the expression for q_2, is angle dependent. In view of this, the integration over angles smooths out the oscillation effects. Furthermore, Eq. (50) suggests that these oscillations can exist only when $q_2 = 2R\omega/v' \approx 1$, i.e., when the electron travel time from wall to wall, $2R/v'$, coincides with the period of the EM wave. Within the limits given by Eqs. (48) and (49), these effects are negligible.

A. Low-frequency absorption

Let us start with the low-frequency surface scattering (48), for which

$$q_1 \ll q_2 \ll 1. \tag{51}$$

Here the frequency of the EM wave is much higher than the bulk collision rate but is much lower than the frequency of the electron travel from wall to wall. If the criterion (51) is met, we have $\psi(q) \approx q/2$ and obtain from Eq. (41),

$$W_e \approx \frac{\pi e^2 m^3 R^4}{(2\pi\hbar)^3}\int \frac{d^3v}{v'}|\mathbf{v}\cdot\mathbf{E}_{\rm loc}|^2\delta(\varepsilon-\mu). \tag{52}$$

In order to study the dependence of the absorption on the particle shape, we need only to consider an ellipsoid of revolution (spheroid). In this case

$$v' = R\sqrt{\frac{v_\perp^2}{R_\perp^2} + \frac{v_\parallel^2}{R_\parallel^2}}, \tag{53}$$

where $v_\perp$ and $v_\parallel$ are the electron velocity components perpendicular and parallel to the axis of revolution. With an account for Eq. (53), the integral in Eq. (52) can be easily calculated,

$$W_e = \frac{9}{16}V\frac{ne^2}{m}\frac{R_\perp}{v_F}[\rho_L(e_p)|E_{\perp,\rm loc}|^2 + \eta_e^L(e_p)|E_{\parallel,\rm loc}|^2]. \tag{54}$$

Here ρ_L and η_e^L are functions of the ellipsoid eccentricity (we note once more that $e_p^2 = |1 - R_\perp^2/R_\parallel^2|$),

$$\rho_L(e_p) = \begin{cases} \frac{1}{2e_p^2}\sqrt{1-e_p^2} - \frac{1}{2e_p^3}(1-2e_p^2)\arcsin e_p, & R_\perp < R_\parallel \\ -\frac{1}{2e_p^2}\sqrt{1+e_p^2} + \frac{1}{2e_p^3}(1+2e_p^2)\ln(e_p+\sqrt{1+e_p^2}), & R_\perp > R_\parallel, \end{cases} \tag{55}$$

$$\eta_e^L(e_p) = \begin{cases} -\frac{1}{e_p^2}\sqrt{1-e_p^2} + \frac{1}{e_p^3}\arcsin e_p, & R_\perp < R_\parallel \\ \frac{1}{e_p^2}\sqrt{1+e_p^2} - \frac{1}{e_p^3}\ln(e_p+\sqrt{1+e_p^2}), & R_\perp > R_\parallel. \end{cases} \tag{56}$$

The subscript e in the left-hand side of Eq. (56) indicates that this function associates only with the electric field, and the symbol L refers to the low-frequency case. The limiting expression for the functions mentioned above are

$$\rho_L = \begin{cases} \dfrac{\pi}{4}, & R_\perp \ll R_\parallel \\ \dfrac{2}{3}, & R_\perp = R_\parallel \\ \dfrac{R_\parallel}{R_\perp}\left(\ln 2\dfrac{R_\perp}{R_\parallel} - \dfrac{1}{2}\right), & R_\perp \gg R_\parallel, \end{cases}$$

$$\eta_e^L = \begin{cases} \dfrac{\pi}{2}, & R_\perp \ll R_\parallel \\ \dfrac{2}{3}, & R_\perp = R_\parallel \\ \dfrac{R_\parallel}{R_\perp}, & R_\perp \gg R_\parallel. \end{cases} \quad (57)$$

They are independent of e_p.

Having the general formula (54) for the electric absorption of an ellipsoidal metallic particle in the case of low-frequency scattering, we may easily find the components of the light-induced conductivity tensor. To this end, we write the expression for the electric absorption in terms of the principal values of the conductivity tensor σ_{jj},

$$W_e = \frac{1}{2} V \sum_{j=1}^{3} \sigma_{jj} |E_{j,\text{loc}}|^2 \quad (58)$$

and compare it with Eq. (54). As a result we have

$$\sigma_{xx} = \sigma_{yy} \equiv \sigma_{e,\perp} = \frac{9}{8}\frac{ne^2}{m}\frac{R_\perp}{v_F}\rho_L(e_p);$$

$$\sigma_{zz} \equiv \sigma_{e,\parallel} = \frac{9}{8}\frac{ne^2}{m}\frac{R_\perp}{v_F}\eta_L^E(e_p). \quad (59)$$

The case of a spherical particle results from Eq. (59) in the limit $e_p \to 0$. Applying Eq. (57), we find that

$$\sigma_{e,\perp} = \sigma_{e,\parallel} = \frac{3}{4}\frac{ne^2}{m}\frac{R}{v_F}. \quad (60)$$

Comparing Eq. (59) with Eq. (60), we conclude that the light-induced conductivity of metallic particles smaller than the mean free path is a scalar quantity only for spherical particles.

In the general case of asymmetric particles, the light-induced conductivity becomes a tensor whose components depend on the particle shape. Figure 1 (curve 1) depicts the result of the calculation for the dependence $\sigma_\perp/\sigma_\parallel$ on the ellipsoid semiaxes ratio $R_\perp/R_\parallel$ in the low-frequency limit. To build the curves, Eq. (59) was used.

Expression (54) for the absorbed power takes a simple analytic form in the limits of highly flattened and highly elongated ellipsoids

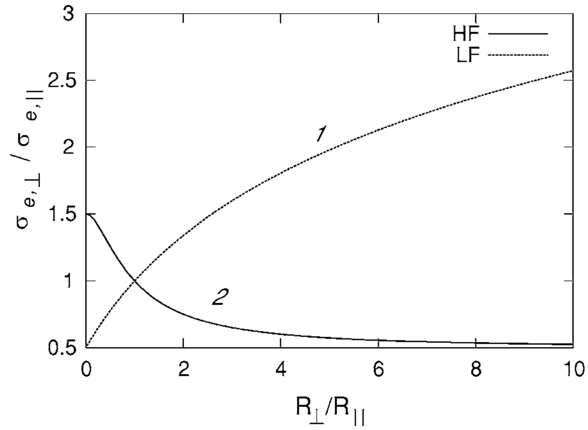

FIG. 1. Ratio between transverse $\sigma_\perp$ and longitudinal $\sigma_\parallel$ components of the electric conductivity of the spheroidal SMP as a function of the ratio between minor and major spheroid's semiaxes, at low $\omega \ll v_F/(2R)$ (1) and high $\omega \gg v_F/(2R)$ (2) light frequencies.

$$W_e \approx \frac{9}{32}\pi V \frac{ne^2}{m}\frac{R_\perp}{v_F}\left(\frac{1}{2}|E_{\perp,\text{loc}}|^2 + |E_{\parallel,\text{loc}}|^2\right), \quad R_\perp \ll R_\parallel, \quad (61)$$

$$W_e \approx \frac{9}{16} V \frac{ne^2}{m}\frac{R_\parallel}{v_F}\left\{\left[\ln\left(2\frac{R_\perp}{R_\parallel}\right) - \frac{1}{2}\right]\cdot |E_{\perp,\text{loc}}|^2 + |E_{\parallel,\text{loc}}|^2\right\},$$

$$R_\perp \gg R_\parallel.$$

B. High-frequency absorption

Let us now examine the case of the high-frequency scattering using Eq. (49). For $q_2 \gg 1$ and $q_1 \ll 1$, we have

$$\text{Re}\left[\frac{1}{\tilde{\nu}}\psi(q)\right] \approx \frac{2}{\omega q_2},$$

which, in accordance with Eq. (41), yields

$$W_e \approx \frac{\pi e^2 m^3 R^2}{(2\pi\hbar)^3\omega^2}\int d^3v v' |\mathbf{v}\cdot\mathbf{E}_{\text{loc}}|^2 \delta(\varepsilon - \mu). \quad (63)$$

After evaluation of the integral in Eq. (63), we may reduce the result to the form of Eq. (54),

$$W_e = \frac{9}{16} V \frac{ne^2}{m\omega^2}\frac{v_F}{R_\perp}[\rho_H(e_p)|E_{\perp,\text{loc}}|^2 + \eta_e^H(e_p)|E_{\parallel,\text{loc}}|^2], \quad (64)$$

where $\rho_H(e_p)$ and $\eta_e^H(e_p)$ are

$$\rho_H(e_p) = \begin{cases} \frac{1}{8e_p^2}(1+2e_p^2)\sqrt{1-e_p^2} - \frac{1}{8e_p^3}(1-4e_p^2)\arcsin e_p, & R_\perp < R_\parallel \\ -\frac{1}{8e_p^2}(1-2e_p^2)\sqrt{1+e_p^2} + \frac{1}{8e_p^3}(1+4e_p^2)\ln(e_p+\sqrt{1+e_p^2}), & R_\perp > R_\parallel, \end{cases} \quad (65)$$

$$\eta_e^H(e_p) = \begin{cases} -\frac{1}{4e_p^2}(1-2e_p^2)\sqrt{1-e_p^2} + \frac{1}{4e_p^3}\arcsin e_p, & R_\perp < R_\parallel \\ \frac{1}{4e_p^2}(1+2e_p^2)\sqrt{1+e_p^2} - \frac{1}{4e_p^3}\ln(e_p+\sqrt{1+e_p^2}), & R_\perp > R_\parallel. \end{cases} \quad (66)$$

These functions are independent of e_p in the limits of $e_p \to 1$, $e_p \to 0$, $e_p \to R_\perp^2/R_\parallel^2$, and can be simplified to the forms,

$$\rho_H = \begin{cases} \frac{3}{16}\pi, & R_\perp \ll R_\parallel \\ \frac{2}{3}, & R_\perp = R_\parallel \\ \frac{1}{4}\frac{R_\perp}{R_\parallel}, & R_\perp \gg R_\parallel; \end{cases} \qquad \eta_e^H = \begin{cases} \frac{\pi}{8}, & R_\perp \ll R_\parallel \\ \frac{2}{3}, & R_\perp = R_\parallel \\ \frac{1}{2}\frac{R_\perp}{R_\parallel}, & R_\perp \gg R_\parallel. \end{cases} \quad (67)$$

In this case, the high-frequency components of the conductivity tensors look like

$$\sigma_{e,\perp} = \frac{9}{8}\frac{ne^2}{m\omega^2}\frac{v_F}{R_\perp}\rho_H(e_p); \quad \sigma_{e,\parallel} = \frac{9}{8}\frac{ne^2}{m\omega^2}\frac{v_F}{R_\perp}\eta_e^H(e_p). \quad (68)$$

At $R_\perp = R_\parallel$, Eq. (68) yields the well-known result for a spherical particle,

$$\sigma_e = \frac{3}{4}\frac{ne^2}{m\omega^2}\frac{v_F}{R}. \quad (69)$$

Comparing Eq. (69) with the one obtained for the conductivity [which is believed to result from Eq. (46) for $\omega \gg \nu$], one can see that in the case of a spherical particle, the expression for the electric conductivity with domination of the surface scattering can be obtained from a similar expression for the Drude case by formally substituting $3v_F/(4R)$ for ν. This method is widely employed in the literature. However, in the case of asymmetric particles, this procedure leads to incorrect results. The appearance of the factor $v_F/R_\perp$ in Eq. (64), which has the formal meaning of the "wall to wall travel frequency," is caused by the detachment of the particle volume V as a separate multiplier. But, when the surface scattering dominates, the absorbed power is proportional to the surface area of the particle. This easily can be seen by using Eqs. (64)–(67) to derive simple analytic formulas for highly elongated and highly flattened ellipsoids,

$$W_e \approx \frac{9\pi}{128}V\frac{ne^2}{m\omega^2}\frac{v_F}{R_\perp}\left(\frac{3}{2}|(E_{\rm loc})_\perp|^2 + |(E_{\rm loc})_\parallel|^2\right), \quad R_\perp \ll R_\parallel, \quad (70)$$

$$W_e \approx \frac{9}{32}V\frac{ne^2}{m\omega^2}\frac{v_F}{R_\parallel}\left(\frac{1}{2}|(E_{\rm loc})_\perp|^2 + |(E_{\rm loc})_\parallel|^2\right), \quad R_\perp \gg R_\parallel. \quad (71)$$

The quantities $V/R_\perp \approx R_\parallel R_\perp$ in Eq. (70) and $V/R_\parallel \approx R_\perp^2$ in Eq. (71) are merely the surface areas of the proper ellipsoids in the specified limits.

The dependence of $\sigma_\perp/\sigma_\parallel$ on the ellipsoid semiaxes ratio $R_\perp/R_\parallel$ is plotted in Fig. 1 (curve 2) using Eqs. (65), (66), and (68). We see that the components of the electric conductivity tensor differ considerably, depending on the degree of the particle asymmetry. Comparing curves 1 and 2, one can see that the effect of the particle asymmetry on the ratio between the components of the conductivity tensor differs in the high- and low-frequency limits not only quantitatively but also qualitatively (provided that surface scattering is dominant).

VI. PLASMA RESONANCES

So far we did not analyze how absorption power looks like at the light frequencies close to the plasma resonances. Let us consider this question now.

When the particle size is much smaller than the bulk mean free path of the electrons, the absorbed capacity is defined by Eqs. (58) and (8), and is equal to

$$W_e = \frac{V}{2}\sum_{j=1}^{3}\sigma_{jj}|E_{j,\rm loc}|^2 = \frac{V}{2}\sum_{j=1}^{3}\frac{\sigma_{jj}|E_j^{(0)}|^2}{[1+L_j(\varepsilon'-1)]^2 + \left(L_j\frac{4\pi}{\omega}\sigma_{jj}\right)^2}. \quad (72)$$

It should be remembered that so far we dealt with the electric absorption. At high light frequencies compared to ν, in accordance with Eq. (4),

$$\varepsilon' = 1 - \frac{\omega_{\rm pl}^2}{\omega^2}. \quad (73)$$

Therefore, the denominator in expression (72) becomes

$$[1+L_j(\varepsilon'-1)]^2+\left(L_j\frac{4\pi}{\omega}\sigma_{jj}\right)^2 = \left(1-L_j\frac{\omega_{\rm pl}^2}{\omega^2}\right)^2+\left(L_j\frac{4\pi}{\omega}\sigma_{jj}\right)^2. \quad (74)$$

With the account of Eq. (74), the form of Eq. (72) reduces explicitly to

$$W_e=\frac{V}{2}\sum_{j=1}^{3}\sigma_{jj}\frac{\omega^4|E_j^{(0)}|^2}{(\omega^2-\omega_j^2)^2+(4\pi L_j\sigma_{jj})^2\omega^2}. \quad (75)$$

In Eq. (75) we have adopted the notation

$$\omega_j=\sqrt{L_j}\omega_{\rm pl}. \quad (76)$$

In Eq. (76), ω_j has a meaning of the frequency of plasma vibrations. In the case of spherical particles $L_x=L_y=L_z=1/3$, and, consequently, there is only one frequency of the surface plasma vibrations equal to $\omega_{\rm pl}/\sqrt{3}$. It should be remembered that $\omega_{\rm pl}$ is the bulk plasma frequency in the common terminology.

For ellipsoidal SMPs there are three frequencies of surface plasma vibrations (three resonances).

As follows from Eq. (75), a half-width of plasma resonances depends on the shape of a particle, both through components of the depolarization tensor, and via components of the optical conductivity tensor. For particles, the characteristic size of which is much greater than the bulk mean free path of the electrons, the optical conductivity becomes a scalar and the dependence of the half-width of the plasma resonance on the shape of a particle contains only in the depolarization factor.

Up to now, we have examined a metallic particle in vacuum. If such a particle is put into a medium with dielectric permeability ε_m, the connection between the local field components and the components of an external electric field of the wave E_j, one finds

$$(E_{\rm loc})_j=\frac{E_j^0}{1+L_j(\varepsilon/\varepsilon_m-1)}. \quad (77)$$

In this case for energy absorbed by a particle in unit time, we obtain instead of Eq. (72),

$$W_e=\frac{V}{2}\sum_{j=1}^{3}\frac{\sigma_{jj}\varepsilon_m^2|E_j^{(0)}|^2}{[\varepsilon_m+L_j(\varepsilon'-\varepsilon_m)]^2+(4\pi L_j\sigma_{jj}/\omega)^2}. \quad (78)$$

If we divide the absorbed capacity (78) by the energy flux

$$S=\frac{c}{8\pi}\sqrt{\varepsilon_m}|E^{(0)}|^2, \quad (79)$$

and if we suppose, for the sake of generality, that there are N identical and equally oriented with respect to each other particles in unit volume, we derive the absorption coefficient in the form,

$$\alpha(\omega)=\frac{4\pi\varepsilon_m^{3/2}NV}{c}\sum_{j=1}^{3}\frac{\sigma_{jj}\cos^2\theta_j}{[\varepsilon_m+L_j(\varepsilon'-\varepsilon_m)]^2+(4\pi L_j\sigma_{jj}/\omega)^2}. \quad (80)$$

In Eq. (80), θ_j is the angle between the direction of the jth semiaxis of an ellipsoid and the direction of the electric field $\mathbf{E}^{(0)}$.

When the size of the particle is large compared to the electron free path, the optical conductivity becomes a scalar, and after the replacement $\sigma_{jj}\to\sigma=\omega\varepsilon''/(4\pi)$, Eq. (80) is reduced to the well-known Gans formula.[42]

If the particles have chaotic orientation, usually an appropriate averaging is used. As far as an electric field, the EM wave induces the electric dipole moments in all metallic particles, which results in the dipole-dipole interaction. The mentioned effects can essentially affect the magnitude of local fields, but in Eq. (80) they are not taken into account.

The procedure of averaging with respect to orientations of particles as well as the account of influence of the dipole-dipole interaction were discussed in the literature repeatedly, since Refs. 43–45. There is extensive literature on the physics of plasma resonances (see, for example, Refs. 46 and 47, and the works quoted there). Therefore, we shall not dwell on this question. We would like to pay attention only to the fact, that for metallic particles of diameters smaller than the free path of electrons, the optical conductivity assumes tensorial form. And this circumstance essentially influences the shape of a plasma resonance line.

In the case of a metallic particle in a vacuum, this influence is illustrated by Eq. (75). If the metallic particle is in a dielectric environment with a dielectric permeability ε_m, the power absorbed by the particle is governed by the formula (78). This formula can be brought to the form similar to Eq. (75), if we note that the denominator in Eq. (78) satisfies the identity

$$[\varepsilon_m+L_j(\varepsilon'-\varepsilon_m)]^2+(4\pi L_j\sigma_{jj}/\omega)^2 = g_i^2(1-\omega_j^2/\omega^2)+(4\pi L_j\sigma_{jj}/\omega)^2. \quad (81)$$

In Eq. (81), the following notations have been used:

$$g_j=\varepsilon_m+L_j(1-\varepsilon_m);\quad \omega_j^2=\frac{L_j}{\varepsilon_m+L_j(1-\varepsilon_m)}\omega_{\rm pl}^2. \quad (82)$$

With the account of Eq. (81), the expression (78) is found to read

$$W_e=\frac{V}{2}\sum_{j=1}^{3}\frac{\sigma_{jj}(\varepsilon_m\omega^2/g_i)^2|E_j^{(0)}|^2}{(\omega^2-\omega_j^2)^2+(4\pi L_j\sigma_{jj}/g_i)^2\omega^2}. \quad (83)$$

In Eq. (82), ω_j is the frequency of surface plasma vibrations of a metallic particle in a dielectric matrix. At $\varepsilon_m=1$ this frequency reduces to Eq. (76). It is clearly seen from Eq. (83) that the line shape of a plasma resonance depends on appropriate components of the optical conductivity tensor. As for the dependence of the optical conductivity tensor on the shape of a metallic particle, we have already considered it in detail in the previous section.

VII. MAGNETIC ABSORPTION

Magnetic absorption is given by the second term in Eq. (2). Combining Eqs. (13), (25), and (38), we obtain an expression for the magnetic absorption

$$W_m = e^2\left(\frac{m}{2\pi\hbar}\right)^3 \mathrm{Re} \int d^3r' d^3v\, \delta(\varepsilon-\mu) \times \sum_{(1)}^{(3)} \frac{\alpha_{lk}^*\alpha_{ij}}{\gamma_l\gamma_k\gamma_i\gamma_j} v_l' x_k' v_i' x_j' \left(\frac{1-e^{-\tilde{\nu}t'}}{\tilde{\nu}}\right), \quad (84)$$

where summation over all indices is from 1 to 3. In order to calculate the integral with respect to r', we direct the z' axis along the vector $\mathbf{v}$. Then, according to Eq. (36), the value t' becomes uniform, whatever the angle φ' (in the plane perpendicular to $\mathbf{v}'$). Hence, we can first integrate in Eq. (84) with respect to φ'. Thus,

$$\int d\varphi' x_j' x_k' = \int d\varphi' (\mathbf{e}_j\cdot\mathbf{r}')(\mathbf{e}_k\cdot\mathbf{r}'). \quad (85)$$

Here $\mathbf{e}_j$ and $\mathbf{e}_k$ are directing orts in an initial coordinate system connected with semiaxes of the ellipsoid.

Let us write down the scalar product $\mathbf{e}_j\cdot\mathbf{r}'$ in the new coordinate system with the axis z' directed along the vector $\mathbf{v}'$,

$$\mathbf{e}_j\cdot\mathbf{r}' = |\mathbf{r}'|[\cos\theta' \cos\psi_j + \sin\theta' \sin\psi_j \cos(\varphi'-\varphi_j)]. \quad (86)$$

Here θ' is an angle between the vectors $\mathbf{r}'$ and $\mathbf{v}'$, and φ' is an angle in a plane perpendicular to $\mathbf{v}'$. Similarly, ψ_j is an angle between the vectors $\mathbf{e}_j$ and $\mathbf{v}'$, and φ_j is a polar angle in the plane perpendicular to the vector $\mathbf{v}'$. Substituting Eq. (86) and the similar expressions for $\mathbf{e}_k\cdot\mathbf{r}'$ in Eq. (85), it is easy to carry out the integration over φ'. Taking advantage of the equality

$$\cos\psi_j \cos\psi_k + \sin\psi_j \sin\psi_k \cos(\varphi_j-\varphi_k) = \mathbf{e}_j\mathbf{e}_k = \delta_{jk},$$

then, as a result from Eq. (85), we find

$$\int_0^{2\pi} d\varphi' x_j' x_k' = 2\pi r'^2\left[\frac{v_j'v_k'}{v'^2} + \frac{\sin^2\theta'}{2}\left(\delta_{jk} - 3\frac{v_j'v_k'}{v'^2}\right)\right]. \quad (87)$$

Therefore, $\cos\psi_j=\mathbf{e}_j\cdot\mathbf{v}'/v'$, and $\cos\psi_k=\mathbf{e}_k\cdot\mathbf{v}'/v'$.

Substituting Eq. (87) into Eq. (84), we can conduct the further calculation in the way similar to that we used above in Eq. (39) for the electric absorption. As a result, Eq. (84) is found to be

$$W_m = \frac{\pi}{2}e^2\left(\frac{mR}{2\pi\hbar}\right)^3 \mathrm{Re}\left[\frac{1}{\bar{\nu}}\int d^3v\,\delta(\varepsilon-\mu) \times\left(\psi_1(v')\sum_{i,j=1}^{3}|\alpha_{ij}|^2R_j^2v_i^2 + 2\psi_2(v')R^2 \times\sum_{i,j=1}^{3}|\alpha_{ij}+\alpha_{ji}|^2\frac{v_i^2v_j^2}{v'^2}\right)\right], \quad (88)$$

where

$$\psi_1(v') = \frac{8}{15} - \frac{1}{q} + \frac{4}{q^3} - \frac{24}{q^5} + \frac{8}{q^3}\left(1+\frac{3}{q}+\frac{3}{q^2}\right)e^{-q}, \quad (89)$$

$$\psi_2(v') = \frac{2}{5} - \frac{1}{q} + \frac{8}{3q^2} - \frac{6}{q^3} + \frac{32}{q^5} - \frac{2}{q^2}\left(1+\frac{5}{q}+\frac{16}{q^2}+\frac{16}{q^3}\right)e^{-q} - \frac{3}{4}\psi_1(v'). \quad (90)$$

Expression (88) determines the magnetic absorption of the particle in a general form for an arbitrary ratio between the bulk and the surface contributions. For spherical particles, the last term on the right-hand side of Eq. (88) vanishes, since, in this case, $\alpha_{ij}=-\alpha_{ji}$ and after integration over all velocities one has

$$W_m = \frac{3V}{16\pi}\nu_s\left(\frac{\omega}{c}R\right)^2|\mathbf{H}^{(0)}|^2\frac{\omega_{\rm pl}^2}{\nu^2+\omega^2}\left\{\frac{\nu}{15\nu_s} - \frac{1}{8}\frac{\nu^2-\omega^2}{\nu^2+\omega^2} + \frac{\nu_s^2}{(\nu^2+\omega^2)^3}\left[\frac{(\nu^2-\omega^2)^2-4\nu^2\omega^2}{2} - 3\nu_s^2(\nu^2-\omega^2)\frac{(\nu^2-\omega^2)^2-12\nu^2\omega^2}{(\nu^2+\omega^2)^2} - e^{-\nu/\nu_s}\left[\sin\frac{\omega}{\nu_s}\left(4\nu(\nu^2-\omega^2) + \frac{3\nu_s}{\nu^2+\omega^2}((\nu^2-\omega^2)^2+4\nu^2(\nu^2-2\omega^2)) + 6\nu\nu_s^2\frac{3(\nu^2-\omega^2)^2-4\nu^2\omega^2}{(\nu^2+\omega^2)^2}\right)\omega - \cos\frac{\omega}{\nu_s}\left((\nu^2-\omega^2)^2-4\nu^2\omega^2 + \frac{3\nu\nu_s}{\nu^2+\omega^2}((\nu^2-\omega^2)^2-4\omega^2(2\nu^2-\omega^2)) + \frac{3\nu_s^2(\nu^2-\omega^2)}{(\nu^2+\omega^2)^2}((\nu^2-\omega^2)^2-12\nu^2\omega^2)\right)\right]\right]\right\}. \quad (91)$$

Using Eq. (88) we can derive ordinary analytic expressions in the limit of the pure bulk scattering and the pure surface scattering. In the first case, $|q|\gg 1$, we arrive at the known formula for the magnetic absorption determined by the second and third terms on the right-hand side of Eq. (22). We can derive the same results from Eq. (91) by neglecting kinetic effects, i.e., $\nu_s\to 0$.

The information about the scattering mechanism is contained in the parameters $q=2R\tilde{\nu}/v'$ and $\tilde{\nu}\equiv\nu-i\omega$. For example, assuming $|q|\gg 1$, we get $\psi_1\approx 8/15$ and $\psi_2\approx 0$. Then it is easy to verify that the magnetic absorption is governed by an expression exactly the same as the one obtained for magnetic terms in the formula (22) for the limit of bulk scattering. On the contrary, if the parameter $|q|\ll 1$, then the governing role in the absorption of the energy from the wave is played by the electron scattering on the surface of the SMP. Let us discuss this in detail. For computational purposes it will be convenient to separate the analysis into the low-frequency ($\nu\ll\omega\ll\nu_s$) and high-frequency ($\omega\gg\nu_s\gg\nu$) absorption, as we had done before, where ν_s is defined below Eq. (44).

A. Low-frequency absorption

As one can readily verify, in the case of the low-frequency absorption,

$$\mathrm{Re}\left[\frac{1}{\bar{\nu}}\psi_1(v)\right]\approx\frac{1}{3}\frac{R}{v'},\quad \mathrm{Re}\left[\frac{1}{\bar{\nu}}\psi_2(v)\right]\approx\frac{1}{36}\frac{R}{v'},\tag{92}$$

and Eq. (88) can be rewritten as

$$W_m=\frac{\pi e^2m^2R^4}{6(2\pi\hbar)^3}\int\frac{d^3v}{v'}\delta(v^2-v_F^2)\times\left(\sum_{i,j=1}^{3}|\alpha_{ij}|^2R_j^2v_i^2+\frac{R^2}{6}\sum_{i,j=1}^{3}|\alpha_{ij}+\alpha_{ji}|^2\frac{v_i^2v_j^2}{v'^2}\right).\tag{93}$$

For a particle of a spheroidal form the sum in Eq. (93), with the help of Eq. (18), becomes

$$\sum_{i,j=1}^{3}|\alpha_{ij}|^2R_j^2v_i^2=\left(\frac{\omega}{c}R_\perp\right)^2\left\{\mathbf{H}_\parallel^2\frac{v_\perp^2}{4}+\frac{R_\parallel^4}{(R_\parallel^2+R_\perp^2)^2}\times\left[\mathbf{H}_\perp^2v_\parallel^2+\left(\frac{R_\perp}{R_\parallel}\right)^2(\mathbf{H}_x^2v_x^2+\mathbf{H}_y^2v_y^2)\right]\right\},\tag{94}$$

$$\sum_{i,j=1}^{3}|\alpha_{ij}+\alpha_{ji}|^2v_i^2v_j^2=2\left(\frac{\omega}{c}\right)^2\frac{|R_\parallel^2-R_\perp^2|^2}{(R_\parallel^2+R_\perp^2)^2}v_\parallel^2(\mathbf{H}_x^2v_x^2+\mathbf{H}_y^2v_y^2),\tag{95}$$

where it should be kept in mind that $v_x^2+v_y^2=v_\perp^2$ and $\mathbf{H}_\parallel$, $\mathbf{H}_\perp$ are the magnetic field components along and transverse the axis of revolution of the ellipsoid. For convenience, here and below the superscript (0) has been omitted from the notation for the components of the external magnetic and electric fields.

Taking into account Eqs. (94) and (95), we may easily evaluate the integrals over velocities, and finally, one can express the absorbed power as

$$W_m=\frac{3}{64}V\frac{ne^2}{mv_F}\left(\frac{\omega}{c}\right)^2\times R_\perp^3\left[\rho_L(e_p)\mathbf{H}_\parallel^2+\eta_m^L(e_p)\left(\frac{R_\parallel^2}{R_\parallel^2+R_\perp^2}\right)^2\mathbf{H}_\perp^2\right],\tag{96}$$

where

$$\eta_m^L(e_p)=\begin{cases}-\left(2+\dfrac{1}{e_p^2}\right)\sqrt{1-e_p^2}+\dfrac{1}{e_p}\left(4+\dfrac{1}{e_p^2}-\dfrac{8}{3}e_p^2\right)\arcsin e_p, & R_\perp<R_\parallel\\ \left(-2+\dfrac{1}{e_p^2}\right)\sqrt{1+e_p^2}+\dfrac{1}{e_p}\left(4-\dfrac{1}{e_p^2}+\dfrac{8}{3}e_p^2\right)\ln(e_p+\sqrt{1+e_p^2}), & R_\perp>R_\parallel,\end{cases}\tag{97}$$

and $\rho_L(e_p)$ was introduced earlier by Eq. (55) for the prolate ($R_\perp<R_\parallel$) or oblate ($R_\perp>R_\parallel$) ellipsoids. The subscript m in the left-hand side of Eq. (97) indicates that this function relates only to magnetic fields. In the limits of an antennalike shape, a sphere and a disklike shape ($e_p\to 1, e_p\to 0, e_p\to R_\perp^2/R_\parallel^2$), this function behaves as follows:

$$\eta_m^L=\begin{cases}\pi\left(1+\dfrac{1}{6}\right), & R_\perp\ll R_\parallel\\ \dfrac{8}{3}, & R_\perp=R_\parallel\\ \dfrac{R_\perp}{R_\parallel}\left(\dfrac{8}{3}\ln\left(2\dfrac{R_\perp}{R_\parallel}\right)-2\right), & R_\perp\gg R_\parallel.\end{cases}\tag{98}$$

This allows us to rewrite the expression for the absorbed power (96) for the cases of highly prolate or highly oblate ellipsoids, respectively, as

$$W_m\approx\frac{3\pi}{4\cdot 64}V\frac{ne^2}{mv_F}\left(\frac{\omega}{c}\right)^2R_\perp^3(\mathbf{H}_\parallel^2+5\mathbf{H}_\perp^2),\quad R_\perp\ll R_\parallel,\tag{99}$$

$$W_m \approx \frac{3}{64} V \frac{ne^2}{mv_F}\left(\frac{\omega}{c}\right)^2 R_\parallel^3 \left[\ln\left(2\frac{R_\perp}{R_\parallel}\right) - \frac{1}{2}\right] \times \left[\left(\frac{R_\perp}{R_\parallel}\right)^2 \mathbf{H}_\parallel^2 + 4\mathbf{H}_\perp^2\right], \quad R_\perp \gg R_\parallel. \tag{100}$$

In particular, it results from Eq. (99) that for SMPs in the form of highly prolate ellipsoids, the magnetic absorption in the case when the magnetic field direction is perpendicular to the axis of revolution, under otherwise equal conditions, is five times greater than in the case when the field is oriented along this axis.

Having a general expression (96) for the magnetic absorption of SMPs in the limit of the low-frequency scattering, one can rewrite it (similar to the electric absorption) in terms of the σ tensor component (we denote it by the subscript m) as

$$W_m = \frac{V}{2}(\sigma_{m,\parallel}\mathbf{H}_\parallel^2 + \sigma_{m,\perp}\mathbf{H}_\perp^2), \tag{101}$$

where

$$\sigma_{m,\parallel} = \frac{3}{32}\frac{ne^2}{mv_F}\left(\frac{\omega}{c}\right)^2 R_\perp^3 \rho_L(e_p),$$

$$\sigma_{m,\perp} = \frac{3}{32}\frac{ne^2}{mv_F}\left(\frac{\omega}{c}\right)^2 R_\perp^3 \left(\frac{R_\parallel^2}{R_\parallel^2 + R_\perp^2}\right)^2 \eta_m^L(e_p) \tag{102}$$

are its longitudinal and transverse components. In the case of a spherical particle ($R_\parallel = R_\perp \equiv R$) it becomes a scalar quantity

$$\sigma_m = \frac{1}{16}\frac{ne^2}{mv_F}\left(\frac{\omega}{c}\right)^2 R^3. \tag{103}$$

In the general case of nonspherical particles the σ_m is a tensor that essentially depends on the shape of the particle. Figure 2 (curve 1) shows how the ratio between components of σ_m along mutually perpendicular directions depends on the shape of the ellipsoid, specified by the ratio between its major and minor semiaxes $R_\perp / R_\parallel$. The dependence is plotted in accordance with formulas (102). By making use of this figure, one can examine how the ratio between components of σ_m changes in an oblate SMP in comparison to a prolate particle.

B. High-frequency absorption

In the case of the high-frequency absorption, $\omega \gg \nu_s \gg \nu$. This means also that $\omega \gg \nu$. Hence

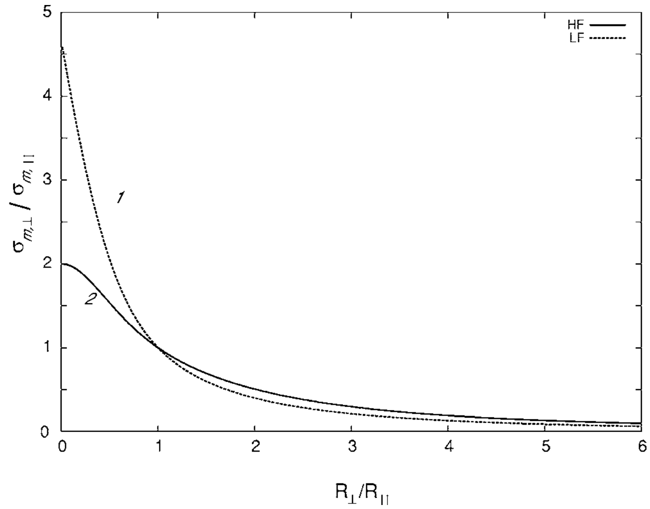

FIG. 2. Ratio between transverse and longitudinal components of tensor σ_m of the spheroidal SMP versus the ratio between minor and major axes of the spheroid, at low $\omega \ll v_F/(2R)$ (1) and high $\omega \gg v_F/(2R)$ (2) light frequencies.

$$\mathrm{Re}\left[\frac{1}{\nu}\psi_1(v)\right] \approx \frac{v'}{2R\omega^2}, \quad \mathrm{Re}\left[\frac{1}{\nu}\psi_2(v)\right] \approx \frac{v'}{8R\omega^2}, \tag{104}$$

and Eq. (88) can be rewritten as

$$W_m = \frac{\pi e^2 m^3 R^2}{4(2\pi\hbar)^3\omega^2}\int d\mathbf{v}\,\delta(\varepsilon - \mu) \times \left(\sum_{i,j=1}^{3}|\alpha_{ij}|^2 R_j^2 v_i^2 v' + \frac{R^2}{2}\sum_{i,j=1}^{3}|\alpha_{ij} + \alpha_{ji}|^2 \frac{v_i^2 v_j^2}{v'}\right). \tag{105}$$

After integration over $\mathbf{v}$ with account of Eqs. (94) and (95), we obtain the following expression for the magnetic absorption in the high-frequency case:

$$W_m = \frac{9}{128} V \frac{ne^2}{mc^2} v_F R_\perp \left[\rho_H(e_p)\mathbf{H}_\parallel^2 + \eta_m^H(e_p)\left(\frac{R_\parallel^2}{R_\parallel^2 + R_\perp^2}\right)^2 \mathbf{H}_\perp^2\right]. \tag{106}$$

Here the function $\rho_H(e)$ has been defined above by Eq. (65) and $\eta_m^H(e)$ is

$$\eta_m^H(e_p) = \begin{cases} -\frac{1}{4e_p^2}(1 - 8e_p^2 + 4e_p^4)\sqrt{1 - e_p^2} + \frac{1}{4e_p^3}(1 + 2e_p^2)\arcsin e_p, & R_\perp < R_\parallel \\ \frac{1}{4e_p^2}(1 + 8e_p^2 + 4e_p^4)\sqrt{1 + e_p^2} - \frac{1}{4e_p^3}(1 - 2e_p^2)\ln(e_p + \sqrt{1 + e_p^2}), & R_\perp > R_\parallel. \end{cases} \tag{107}$$

In above mentioned limits it behaves as follows:

$$\eta_m^H = \begin{cases} \frac{\pi}{4}\left(1+\frac{1}{2}\right), & R_\perp \ll R_\parallel \\ \frac{8}{3}, & R_\perp = R_\parallel \\ \left(\frac{R_\perp}{R_\parallel}\right)^3, & R_\perp \gg R_\parallel. \end{cases} \tag{108}$$

This allows us to determine the energy absorbed by a SMP having the form of a prolate ellipsoid, a sphere, or a disk, for which we find, respectively,

$$W_m \approx \frac{27\pi}{8\cdot 128} V \frac{ne^2}{mc^2} v_F R_\perp \left(\frac{1}{2}\mathbf{H}_\parallel^2 + \mathbf{H}_\perp^2\right), \quad R_\perp \ll R_\parallel, \tag{109}$$

$$W_m \approx \frac{3}{64} V \frac{ne^2}{mc^2} v_F R \mathbf{H}^2, \quad R_\perp = R_\parallel \equiv R, \tag{110}$$

$$W_m \approx \frac{9}{128} V \frac{ne^2}{mc^2} v_F R_\parallel \left[\left(\frac{R_\perp}{2R_\parallel}\right)^2 \mathbf{H}_\parallel^2 + \mathbf{H}_\perp^2\right], \quad R_\perp \gg R_\parallel. \tag{111}$$

Similarly to the previous case, we find from formula (109), in particular, that the magnitude of the magnetic absorption of a SMP in the form of a highly prolate ellipsoid with its axis of revolution oriented perpendicular to the external magnetic field is by factor about of two larger than the one that can be calculated for the case of its orientation along the field.

If, by analogy with Eq. (101), the absorbed energy is expressed in terms of σ_m, then we obtain for the tensor components in the high-frequency case

$$\sigma_{m,\parallel} = \frac{9}{64}\frac{ne^2}{mc^2} v_F R_\perp \rho_H(e_p),$$

$$\sigma_{m,\parallel} = \frac{9}{64}\frac{ne^2}{mc^2} v_F R_\perp \left(\frac{R_\parallel^2}{R_\parallel^2 + R_\perp^2}\right)^2 \eta_m^H(e_p). \tag{112}$$

Curve 2 in Fig. 2 illustrates the result of the calculation for the ratio between the tensor components σ_m in the high-frequency limit along mutually perpendicular directions as a function of the ratio between minor and major axes of the spheroid. The dependence is plotted in accordance with Eqs. (112). Comparing this result with those obtained in the low-frequency case (curve 1), we see that for oblate particles, in the interval of $1 < R_\perp/R_\parallel < 6$, the high-frequency transverse component is slightly higher than the low-frequency one. For the case of a prolate shape the dependence is just the opposite; the low-frequency transverse component becomes noticeably higher. For a spherical particle, the low-frequency and high-frequency components coincide.

By making use of the above results, namely, Eq. (54) for the electric absorption W_e and Eq. (96) valid for the magnetic absorption, one can compare their relative contributions to the low-frequency case. For this purpose it is necessary to choose one of the possible light polarizations. The polarization for which the electric wave vector is directed along the major axis of the ellipsoid ($\mathbf{E}\parallel\mathbf{E}_\parallel$, $\mathbf{E}_\perp=0$) or, accordingly, the magnetic wave vector $\mathbf{H}=\mathbf{H}_\perp$, $\mathbf{H}_\parallel=0$, we will call the EL-MT polarization. If, on the contrary, the vector of the electric wave is along the minor axis of the ellipsoid ($\mathbf{E}\perp\mathbf{E}_\parallel$, $\mathbf{E}_\parallel=0$), then there are two possible directions of the vector of the magnetic wave: along the major axis of the ellipsoid ($\mathbf{H}=\mathbf{H}_\parallel$, $\mathbf{H}_\perp=0$) or along the minor axis ($\mathbf{H}=\mathbf{H}_\perp$, $\mathbf{H}_\parallel=0$). The first of these possibilities we will call the ET-ML polarization, while the second one will be called the ET-MT polarization. The ratio between the magnetic and electric energy absorbed in the low-frequency case is given by the relations

$$\frac{W_m}{W_e} = \frac{1}{12}\left(\frac{\omega}{c}R_\perp\right)^2 \frac{\eta_m^L(e_p) L_\parallel(e_p,\omega)}{\eta_e^L(e_p)[1+(R_\perp/R_\parallel)^2]^2}, \tag{113}$$

for the EL-MT polarization,

$$\frac{W_m}{W_e} = \frac{1}{12}\left(\frac{\omega}{c}R_\perp\right)^2 L_\perp(e_p,\omega), \tag{114}$$

for ET-ML polarization, and

$$\frac{W_m}{W_e} = \frac{1}{12}\left(\frac{\omega}{c}R_\perp\right)^2 \frac{\eta_m^L(e_p) L_\perp(e_p,\omega)}{\rho_L(e_p)[1+(R_\perp/R_\parallel)^2]^2}, \tag{115}$$

for the ET-MT polarization, where quantities $\rho_L(e_p)$, $\eta_e^L(e_p)$, and $\eta_m^L(e_p)$ are defined by Eqs. (55), (56), and (97), correspondingly. In Eqs. (113)–(115), we set $\mathbf{H}_\perp^2=\mathbf{E}_\parallel^2=\mathbf{H}_\parallel^2=\mathbf{E}_\perp^2$. The internal electric field is expressed in terms of the external field using the factor

$$L_{\parallel,\perp}(e_p,\omega) = [1 + L_{\parallel,\perp}(e_p)(\varepsilon'(\omega)-1)]^2 + [L_{\parallel,\perp}(e_p)\varepsilon''(\omega)]^2, \tag{116}$$

and other notations are the same as those introduced previously.

Figure 3 shows how the ratio W_m/W_e depends on the shape of the ellipsoid for two light polarizations (EL-MT and ET-ML). It is seen that, while for a spherical particle of radius 50 Å the contributions of the electric and magnetic components into the absorption are approximately equal, for particles with $R=75$ Å the contribution of the magnetic component is already twice as large. In the high-frequency case (e.g., for $\omega=2\times10^{14}$ s^{-1}; Ref. 2) such increase becomes possible (independently of the light polarization) only for particles of the radius of 300 Å. The behavior of W_m/W_e as a function of the ratio $R_\perp/R_\parallel$ for this case is shown by crosses in Fig. 3. It exactly follows the dependence for particles with $R=75$ Å for the ET-ML polarization. As the shape of the particle changes from spherical to antennalike, the magnetic absorption decreases in the EL-MT and ET-ML polarizations, although when the particle assumes a disklike form, the magnetic absorption increases for the EL-MT polarization and falls for the ET-ML polarization. We note that the largest value of the ratio of W_m/W_e is reached in the EL-MT polarization for particles of an oblate shape at $R_\perp/R_\parallel\approx 4$ (independently of the size of the particle), while for the ET-ML polarization, W_m/W_e reaches a maximum value for

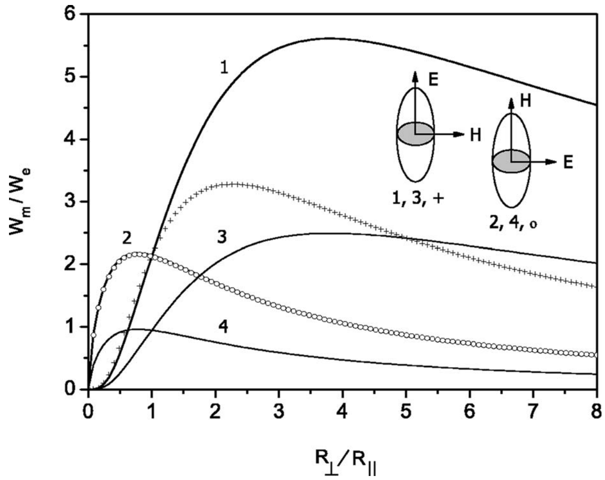

FIG. 3. Dependence of the ratio between magnetic and electric energy absorbed by SMP on the ratio between minor and major spheroid's semiaxes, at light frequencies $\omega \ll v_F/(2R)$ and for two polarizations of the incident wave: EL-MT (curves 1 and 3) and ET-ML (2 and 4). Curves (1) and (2) are for a particle with $R=75$ Å, and curves (3) and (4) for $R=50$ Å. The results for a particle with $R=300$ Å at frequencies $\omega \gg v_F/(2R)$ are shown for the EL-MT polarization by crosses (×) and for the ET-ML polarization by circles (○) [they are coincident with the curve (2)]. The insets show the polarization of the EM wave with regard to the spheroid's axes.

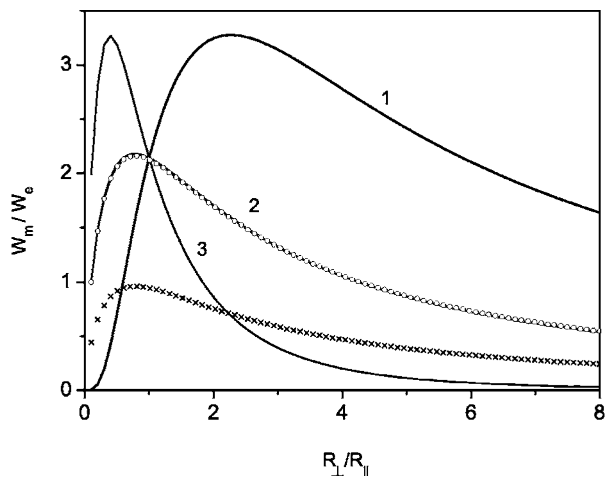

FIG. 5. Dependence of the ratio between magnetic and electric energy absorbed by SMP on the ratio between minor and major semiaxes of the spheroid, at light frequencies $\omega = 2\times10^{14}$ s^{-1} $\gg v_F/(2R)$ and for three polarizations of the incident wave: EL-MT (1), ET-ML (2), and ET-MT (3). Curves (1)–(3) are for a particle with $R=300$ Å. The results for particles with $R=75$ Å and =50 Å at frequencies $\omega=4\times10^{13}\text{s}^{-1} \ll v_F/(2R)$ are shown for the ET-ML polarization by circles (○) [they are coincident with curve (2)] and by crosses (×), accordingly.

prolate particles, being only slightly greater than the corresponding value for the spherical particle.

The same dependencies for the remaining EL-MT polarization of the EM wave are shown in Fig. 4. For the same ratio between the magnetic and electric energy absorbed by

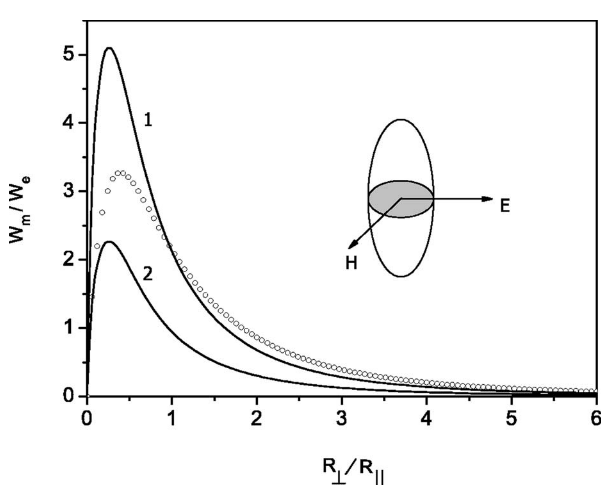

FIG. 4. Dependence of the ratio between magnetic and electric energy absorbed by SMP on the ratio between minor and major spheroid's semiaxes, for $\omega \ll v_F/(2R)$ and for the orientation of the electric field along the minor axes of the spheroid and the MT polarization of the magnetic vector. Curve (1) corresponds to the particle with $R=75$ Å and curve (2) to a particle with $R=50$ Å. The results for a particle with $R=300$ Å at light frequencies $\omega \gg v_F/(2R)$ in the same polarization of the electric and magnetic vectors are shown by circles (○).

the spherical particle, the effect observed in this polarization when its shape is altered, is opposite to that described above for the EL-MT polarization; namely, the magnetic absorption is enhanced in comparison with the electric absorption in the prolate SMP and falls in the oblate one. In this case, the growth of W_m/W_e reaches a maximum for a prolate shape (independently of the particle size) at a radius ratio $R_\perp/R_\parallel \approx 0.25$, and with decreasing this ratio the contribution of the magnetic absorption falls compared to that obtained for the electric absorption.

Consequently, it is seen from the calculations illustrated in Figs. 3 and 4 that if, for example, the magnetic absorption in a spherical particle exceeds the electric absorption by a factor of 2, then by deformation of such a particle by a factor of 4 (to an oblate or prolate shape) one can make the magnetic absorption more than five times larger for an appropriate light polarization in the low-frequency limit.

We have calculated the ratio of W_m/W_e in the high-frequency limit as well. Applying Eqs. (64) and (106), we have relations similar to Eqs. (113)–(115) in which the coefficient 1/12 must be changed to 1/8 and the following quantities must be replaced: $\rho_L(e_p) \to \rho_H(e_p)$, $\eta_e^L(e_p) \to \eta_e^H(e_p)$, $\eta_m^L(e_p) \to \eta_m^H(e_p)$, where $\rho_H(e_p)$, $\eta_e^H(e_p)$, $\eta_m^H(e_p)$ are given by Eqs. (65), (66), and (107), correspondingly.

As in the low-frequency limit for an asymmetric particle, the contribution to the absorption of the electric and magnetic components of the EM wave is strongly dependent on the degree of the particle asymmetry and the wave polarization. In Fig. 5 the relation W_m/W_e versus the ratio between ellipsoid's semiaxes at the frequency of a carbon dioxide laser $\omega \approx 2\times10^{14}$ s^{-1} is shown. Comparing the curves for three different light polarizations, one can see that if the shape of the particle is changed from a spherical to an antennalike, the magnetic absorption decreases in the EL-MT polarization and increases in both ET-ML and ET-MT polarizations. When the particle transforms to a disklike form, the magnetic absorption increases only in the EL-MT polarization and decreases in both the ET-ML and ET-MT polarizations. We note that the largest value of the ratio $W_m/W_e \approx 3.3$ is reached both in the EL-MT polarization for particles of oblate shape at $R_\perp/R_\parallel \approx 2.4$, and in the ET-MT polarization for the prolate shape of particles at $R_\perp/R_\parallel \approx 0.4$.

We see that at high frequencies, for the same ratio between magnetic and electric energy absorbed by the spherical particle, the effect observed when the shape is altered, is opposite of the ET-MT polarization to the one described above for the EL-MT polarization; the magnetic absorption is enhanced in comparison to the electric absorption for the prolate SMP and falls for the oblate one.

For comparison we have shown in Fig. 5 the calculation results for particle radii of 50 Å and 75 Å at the low-frequency in the ET-ML polarization. The plot of W_m/W_e as a function of the ratio $R_\perp/R_\parallel$ in the ET-ML polarization coincides for particles with $R=300$ Å and $R=75$ Å.

VIII. CONCLUSIONS

In this work we present the general theory of the electric and magnetic absorption by an ellipsoidal metallic particle for an arbitrary relation between the bulk and the surface scattering of conducting electrons. In conditions when either the bulk or surface scattering of electrons is dominant, we have obtained analytical expressions that can be employed to determine the dependence of the power absorbed by a particle on its shape and size.

For SMPs in the form of spheroids with mean sizes, which are much smaller than the wavelength of the incident EM wave and/or the skin depth, we have calculated the energy of the electric and magnetic absorption at light frequencies above and below the characteristic frequency of the electron travel between walls of the particle. We have considered cases when the electron free path is larger than the particle's size and when it is smaller. It was assumed that electrons reflect from the inner surface of the particle in a diffuse manner.

We have shown that the optical conductivity of nonspherical particles in the case when a surface scattering dominates (the size of a particle is less than the bulk mean free path of the electrons) becomes a tensor quantity in contrast to the classical Drude case. For particles having the spheroidal shape, we have found the components of this tensor and investigated their dependence on the deviation of the particle shape from a spherical one. The mentioned properties have an effect both on the polarization dependence of the absorption and on the line shape of plasma resonances. Simple analytical formulas were obtained for the energy absorbed by highly prolate or oblate particles.

The high sensitivity of the optical absorption at the IR frequency range to the shape of metallic particles is pointed out. It is shown that the ratio of the electric and magnetic field contributions to the absorption depends on the particle shape and its orientation with respect to the direction of the incident EM radiation, as well as on the wave frequency. In particular, the magnetic absorption for spherical particles of radii of 50 Å in three different light polarizations and for the frequencies below the electron "wall to wall travel frequency," is comparable in magnitude to the electric absorption, and as the radius of the particle is further increased, the magnetic absorption begins to dominate. We demonstrate that, when the shape of the particle changes from the spherical to the oblate ellipsoid, one can obtain the increase of the magnetic absorption, provided that $\mathbf{E}^{(0)}$ is directed along the major axis of the ellipsoid. If $\mathbf{E}^{(0)}$ is directed along the minor axis, then for particles of the prolate shape, the significant increase of the magnetic absorption (in comparison with the spherical particle) can be achieved in the MT light polarization and a slight increase in the ML polarization. For oblate particles, the electric absorption is dominant in these polarizations.

At last, we have also found that in the case of an asymmetrical particle the absorption depends not only on the orientation of the electric field of the EM wave, but also on the orientation of its magnetic component.

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