

Kondo physics in transport through a quantum dot with Luttinger-liquid leads

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We study the gate voltage dependence of the linear conductance through a quantum dot coupled to one-dimensional leads. For interacting dot electrons but noninteracting leads Kondo physics implies broad plateau-like resonances. In the opposite case Luttinger-liquid behavior leads to sharp resonances. In the presence of Kondo as well as Luttinger-liquid physics and for experimentally relevant parameters, we find a line shape that resembles the one of the Kondo case.

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Electron correlations can strongly alter the low-energy physics of many-electron systems. The Kondo effect and Luttinger-liquid (LL) behavior are two of the most prominent examples, both affecting electron transport through a quantum dot coupled to one-dimensional (1D) leads.

For noninteracting leads the appearance of Kondo physics was investigated theoretically for the (two-lead) single impurity Anderson model (SIAM).¹ At small temperatures T and for sufficiently large U/Γ the Kondo effect leads to a resonance in the linear conductance $G(V_g)$ with an unusual *broad plateau-like* line shape replacing the Lorentzian resonance (of width 2Γ) known from tunneling at $U=0$. Here U denotes the local interaction on the dot, $\Gamma=\Gamma_L+\Gamma_R$ measures the hybridization of the dot and (left and right) lead states, and V_g is the gate voltage applied to the dot region. On resonance the number of dot electrons is odd, implying a local spin-1/2 degree of freedom on the dot that is responsible for the Kondo effect.² For $T\rightarrow 0$ the resonance height approaches $2(e^2/h)4\Gamma_L\Gamma_R/(\Gamma_L+\Gamma_R)^2$ —i.e., the unitary limit for symmetric dot-lead couplings—and its width is of order U .^{3,4} For the SIAM at $T=0$, G is proportional to the one-particle spectral weight of the dot at the chemical potential μ .⁴ Varying V_g within an energy range of order U the Kondo resonance of the spectral function is pinned at (close to) μ and has a fixed height² which explains the broad plateau-like resonance in $G(V_g)$. A series of transport experiments on quantum dots was interpreted in the light of these results.⁵

The line shape of $G(V_g)$ is equally strongly affected by the correlations in the 1D leads if the Kondo effect is suppressed. This can be achieved considering one of three cases: no spin degeneracy on the dot, no interaction on the dot, and spinless fermions. The low-energy physics of 1D wires of interacting electrons is characterized by a vanishing quasiparticle weight and power-law scaling of correlation functions known as LL behavior.⁶ In the case of spin-rotation-invariant interactions (and spinless fermions) all exponents can be expressed in terms of a single LL parameter K_ρ that depends on the interaction, the filling factor n , and other details of the model considered. For repulsive interactions, $K_\rho < 1$. At $T=0$ and for finite LL leads the resonances at V_g^r have approximately Lorentzian shape. For a spinful model and symmetric dot-lead couplings, $G(V_g^r)=2e^2/h$. At *asymptotically* large N , where N is the length of the LL leads, the

width w *vanishes* as a power law, in strong contrast to the broad resonances induced by the Kondo effect. At asymmetric couplings there are still resonances but $\lim_{N\rightarrow\infty} G=0$ also at V_g^r .⁷

The problem of a single spin 1/2 coupled to a LL was investigated generalizing the Kondo model.⁸ However, the very interesting question of the resonance line shape resulting from the *competition* between the two correlation effects has not been addressed so far. Here we will investigate this fundamental issue. We study three cases using an approximate method that is based on the functional renormalization group (fRG) (Ref. 9): (a) *noninteracting leads, interacting dot*; (b) *LL leads, noninteracting dot*; (c) *LL leads, interacting dot*. We focus on $T=0$. Unless otherwise stated we consider symmetric dot-lead couplings. For case (a) we reproduce the pinning of spectral weight at μ and thus the plateau-like resonance. For case (b) we confirm the expected LL line shape of the resonances. Some emphasis is put on the two-particle backscattering that for spinful particles plays an important role on intermediate length scales. The results show that the aspects of Kondo and LL physics essential to answer the above question are captured by our method. For case (c) we show that for LL leads of experimentally accessible length the line shape resembles the one of case (a). Our results indicate that the plateaus vanish for $N\rightarrow\infty$.

The model we investigate is given by the Hamiltonian

$$H = H_{\text{kin}} + H_{\text{int}} + H_{\text{bar}} + H_{\text{gate}}, \quad (1)$$

with the kinetic energy

$$H_{\text{kin}} = -t \sum_{\sigma=\uparrow,\downarrow} \sum_{j=-\infty}^{\infty} (c_{j,\sigma}^\dagger c_{j+1,\sigma} + \text{H.c.}),$$

the interaction

$$H_{\text{int}} = \sum_{j=1}^N U_j \bar{n}_{j,\uparrow} \bar{n}_{j,\downarrow} + \sum_{j=1}^{N-1} U'_j \bar{n}_j \bar{n}_{j+1},$$

where $\bar{n}_{j,\sigma} = n_{j,\sigma} - \nu$ (see below) and $\bar{n}_j = \bar{n}_{j,\uparrow} + \bar{n}_{j,\downarrow}$, the tunnel barriers

$$H_{\text{bar}} = (t - t') \sum_{\sigma=\uparrow,\downarrow} (c_{j_l-1,\sigma}^\dagger c_{j_l,\sigma} + c_{j_r,\sigma}^\dagger c_{j_r+1,\sigma} + \text{H.c.}),$$

and the part containing the gate voltage

$$H_{\text{gate}} = V_g \sum_{\sigma=\uparrow,\downarrow} \sum_{j=j_l}^{j_r} n_{j,\sigma}.$$

Standard second-quantized notation is used. The interaction is restricted to a finite part of the wire ($j \in [1, N]$) corresponding to an experimental setup where the LL wires are connected to (higher-dimensional) Fermi-liquid (FL) leads. The model with $H_{\text{kin}} + H_{\text{int}}$ and interaction on all sites is known as the extended Hubbard model. Away from half-filling—that is, for $n \neq 1$ —it is a LL at least for sufficiently weak repulsive interactions. Very accurate results for K_ρ were recently obtained numerically.¹⁰ We allow the on-site $U_j \geq 0$ and nearest-neighbor $U'_j \geq 0$ interactions to depend on position. By turning on the interaction adiabatically over a few tens of lattice sites we can suppress any electron backscattering at the FL-LL contacts, which we are here not interested in.¹¹ The constant bulk values of U_j and U'_j are denoted by U and U' , respectively. We can choose different interactions in the leads and on the dot sites ($j \in [j_l, j_r]$) with $N_D = j_r - j_l + 1$ and thus model cases (a)–(c). As long as j_l and j_r are sufficiently far away from the contacts at sites 1 and N the position of the dot does not play a role. In H_{int} we shifted $n_{j,\sigma}$ by the parameter $\nu = \nu(n, U, U')$ which is chosen such that on sites 1 to N , but excluding the dot region, the average density n acquires the desired value.¹¹ This is important as K_ρ depends on n . The tunnel barriers are modeled by reduced hoppings $t' < t$ across the bonds linking the sites $j_l - 1$, j_l and j_r , $j_r + 1$.

At temperature $T=0$ the linear conductance of the system described by Eq. (1) can be written as¹²

$$G(V_g, N) = \frac{2e^2}{h} |t(0, V_g, N)|^2, \quad (2)$$

with the effective transmission $|t(\varepsilon, V_g, N)|^2 = (4t^2 - [\varepsilon + \mu]^2) |\mathcal{G}_{1,N}(\varepsilon, V_g, N)|^2$. The (spin-independent) one-particle Green function \mathcal{G} has to be computed in the presence of an interaction and in contrast to the noninteracting case acquires an N dependence. In the notation used in the following we suppress the argument N in G . To determine \mathcal{G} we use a recently developed fRG scheme. The starting point is an exact hierarchy of differential flow equations for the self-energy matrix Σ^Λ and higher-order vertex functions, where $\Lambda \in (\infty, 0]$ denotes an infrared energy cutoff which is the flow parameter. We truncate the hierarchy by only considering the one- and two-particle vertices. The two-particle vertex is projected onto the Fermi points and parametrized by a static effective interaction with local and nearest-neighbor parts. This implies a frequency-independent Σ^Λ . A detailed account of our method was given in Refs. 11 and 14. Using the Dyson equation an approximate expression for $\mathcal{G}_{1,N}$ is obtained from Σ^Λ taken at the end of the fRG flow at $\Lambda=0$. Generically the order- N coupled differential equations can only be integrated numerically. For a variety of transport problems through inhomogeneous LL's it was shown earlier

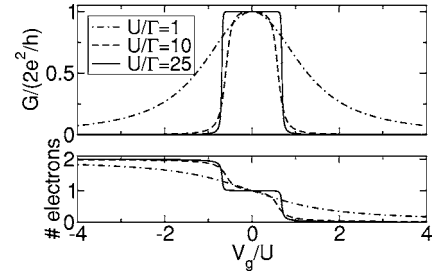


FIG. 1. *Upper panel*: conductance as a function of V_g for the SIAM at different U/Γ . *Lower panel*: average number of electrons on the dot.

that our method leads to reliable results for weak to intermediate interactions.^{11,14–16}

Case (a): noninteracting leads, interacting dot. We focus on $N_D = 1$ for which our model reduces to the SIAM and here (for $N=1$) approximate the two-particle vertex by the bare interaction. Without LL leads, ν only shifts the position of the resonance. We chose $\nu = 1/2$ for which $G(V_g)$ is symmetric around 0. The set of flow equations reduces to a single one for the effective on-site energy $V^\Lambda = V_g + \sum_{j_D, j_D}^\Lambda$ on the dot site j_D . It reads

$$\frac{\partial}{\partial \Lambda} V^\Lambda = -\frac{U}{\pi} \text{Re} \mathcal{G}_{j_D, j_D}^\Lambda(i\Lambda) = \frac{UV^\Lambda/\pi}{(\Lambda + \Gamma)^2 + (V^\Lambda)^2}, \quad (3)$$

with the initial condition $V^{\Lambda=\infty} = V_g$ and the hybridization $\Gamma = 2\pi t'^2 \rho$, where ρ denotes the spectral weight at the end of the leads. As usual we here assume an energy-independent ρ (infinite-bandwidth limit).² Note that in this case U can be taken as the unit of energy. The upper panel of Fig. 1 shows $G(V_g)$ for different U/Γ . For $U \gg \Gamma$ we recover the plateau-like resonance of unitary height.³ Also for asymmetric dot-lead couplings we reproduce the exact height of the plateau. The occupation of the dot can be computed from the Green function and is shown in the lower panel. In the plateau region it turns out to be close to 1 while it sharply rises (drops) to 2 (0) to the left (right) of the plateau. Our dot self-energy is frequency independent which leads to a Lorentzian dot spectral function of width 2Γ and height $1/(\pi\Gamma)$ centered around $V = V^{\Lambda=0}$. This implies that the spectral weight at μ and thus $G(V_g)$ is determined by V .⁴ The solution of the differential equation (3) at $\Lambda=0$ is obtained in implicit form

$$\frac{vJ_1(v) - \delta J_0(v)}{vY_1(v) - \delta Y_0(v)} = \frac{J_0(v_g)}{Y_0(v_g)}, \quad (4)$$

with $v = V\pi/U$, $v_g = V_g\pi/U$, $\delta = \Gamma\pi/U$, and Bessel functions J_n , Y_n . For $|V_g| < V_c$, with $v_c = V_c\pi/U$ being the first zero of J_0 —i.e., $V_c = 0.7655U$ —this equation has a solution with a small $|V|$. For $U \gg \Gamma$ the crossover to a solution with $|V|$ being of order U (for $|V_g| > V_c$) is fairly sharp. Expanding both sides of Eq. (4) for small $|v|$ and $|v_g|$ gives $V = V_g \times \exp[-U/(\pi\Gamma)]$. The exponential pinning of the spectral weight at $\mu=0$ for small $|V_g|$ and the sharp crossover to a V of order U when $|V_g| > V_c$ leads to the observed resonance line shape. For $U \gg \Gamma$ the width of the plateau is $2V_c$

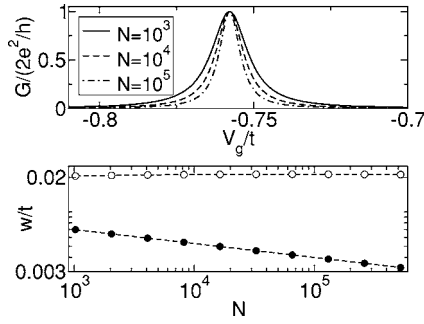


FIG. 2. *Upper panel:* $G(V_g)$ for a noninteracting dot with LL leads at different N . The parameters are $N_D=1$, $t'=0.1t$, $n=3/4$, $U=t$, and $U'=0.65t$ (small two-particle backscattering). *Lower panel:* scaling of the resonance width. Solid circles: the same parameters as in the upper panel. Open circles: $N_D=1$, $t'=0.1t$, $n=1/2$, $U=t$, and $U'=0$ (large two-particle backscattering).

$=1.531U$, which is larger than the width U found with Bethe ansatz.³ The agreement has been improved significantly using a more elaborate fRG scheme, as will be explained in an upcoming paper (see also Ref. 17). In the present context, however, this difference is a minor issue. It is remarkable that our technically fairly simple approximation reproduces the pinning of spectral weight, which is an important feature of Kondo physics. This has to be contrasted with other simple approximations as, e.g., low-order perturbation theory or the self-consistent Hartree approximation, which do not give the correct line shape of $G(V_g)$.

Case (b): LL leads, noninteracting dot. Using the fRG-based approximation, we have earlier studied tunneling through a quantum dot embedded in a spinless LL. We showed that for $N \rightarrow \infty$ the resonance width w vanishes following a power law with a K_ρ -dependent exponent. Also the case of asymmetric dot-lead coupling was investigated.¹¹ Including the spin degree of freedom the physics becomes more complex due to the possibility of two-particle scattering of electrons with opposite spin at opposite Fermi points. This limits power laws to exponentially large length scales. To clearly observe LL behavior at experimentally accessible scales one has to consider a situation in which this backscattering process is small. In our model for fixed n and U this can be achieved by fine-tuning U' .¹⁴ In the upper panel of Fig. 2 we show the N dependence of $G(V_g)$ for a single-site dot computed for a small backscattering amplitude. At resonance voltage V_g^r the conductance is $2e^2/h$ independent of N . In the lower panel the extracted w (solid circles) is shown as a function of N . It follows the power law $N^{(K_\rho-1)/2}$ (Ref. 7), with an fRG approximation to the LL parameter K_ρ , which is correct to leading order in the interaction.^{9,11,14,16} E.g., for $n=3/4$, $U=t$, and $U'=0.65t$ we find $K_\rho=0.76$, in excellent agreement with the numerical result $K_\rho=0.7490$.¹⁰ Off resonance G asymptotically vanishes $\propto N^{1-K_\rho}$.^{7,11} For V_g close to V_g^r such that $1-G/(2e^2/h) \ll 1$, the deviation from the unitary limit scales as N^{1-K_ρ} , characteristic for a weak single impurity. Further increasing N this difference increases and the behavior eventually crosses over to the off-resonance power-law suppression of G discussed above. Due to an exponentially large crossover scale, even for the very

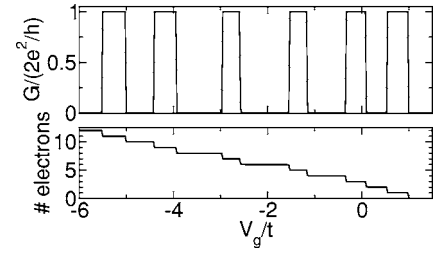


FIG. 3. *Upper panel:* $G(V_g)$ for an interacting dot with LL leads. The parameters are $n=1/2$, $U=t$, $U'=0.5t$, $N=10^4$, $N_D=6$, and $t'=0.1t$. *Lower panel:* average number of electrons on the dot.

large N accessible with our method the complete crossover from one to the other power law cannot be shown for a single fixed V_g but follows from one-parameter scaling.^{7,11} At sizable backscattering the off-resonance conductance and thus the width w first slightly increase for increasing N —becoming larger than the noninteracting width—as shown by the open circles in the lower panel of Fig. 2. For larger N both quantities start to decrease and eventually go to zero for exponentially large N . The backscattering process scales to zero (only) logarithmically in the low-energy limit, and power-law behavior cannot be observed even for fairly long LL leads.¹⁴ An upper bound of the length of one-dimensional wires realized in experiments is of the order of μm , roughly corresponding to 10^4 lattice sites.¹⁸

Case (c): LL leads, interacting dot. We here focus on the case in which the interactions on the dot and in the LL leads are taken to be equal. In the upper panel of Fig. 3, $G(V_g)$ is shown for a parameter set with sizable two-particle backscattering and LL lead length $N=10^4$ typical for experiments. For interactions large compared to the hybridization we find the broad plateaulike resonances induced by the Kondo effect, at least for *finite* LL leads. The same holds for other N_D , in particular for $N_D=1$. The width of the plateaus is proportional to the local component of the effective interaction at the end of the fRG flow and to $1/N_D$. Here we are interested in the interplay of Kondo and LL physics and thus focus on tunnel barriers with small transmission. In the plateau regions the number of electrons on the dot (lower panel) is odd while it is even for gate voltages where G is small. The upper

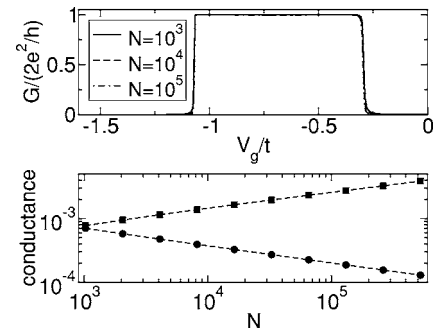


FIG. 4. *Upper panel:* $G(V_g)$ for an interacting dot with LL leads at different N . The parameters are as in the upper panel of Fig. 2, but with interaction on the dot. *Lower panel:* scaling of $G/(2e^2/h)$ at $V_g=0$ outside the plateau (circles) and of $1-G/(2e^2/h)$ on the plateau at $V_g=-0.685t$ (squares).

panel of Fig. 4 shows the N dependence of $G(V_g)$ computed for the same parameters as in the upper panel of Fig. 2 (small backscattering), but including the interaction on the dot. Note the different V_g -axis scales of Figs. 2 and 4. In Fig. 4 the differences between the curves for different N are barely visible; in particular, the changes of the resonance width are marginal. For parameters with sizable backscattering, as in Fig. 3, the difference between curves computed for different N are even smaller. To analyze the N dependence at small backscattering in more detail in the lower panel of Fig. 4 we show the scaling of G for a gate voltage outside the plateau (circles) and of $2e^2/h - G$ for a gate voltage on the plateau-like resonance (squares). For V_g outside the plateau G follows a power law with the exponent $1 - K_\rho^{-1}$ and G vanishes for $N \rightarrow \infty$. Within every plateau we find a V_g^r at which $G = 2e^2/h$ independent of N . For $V_g \neq V_g^r$, still within the plateau, the deviation of G from the unitary limit scales as N^{1-K_ρ} —i.e., with the weak single impurity exponent. This shows that any deviation from V_g^r acts as an impurity. By analogy with the single impurity behavior^{7,14} we conclude that in the asymptotic low-energy limit the impurity will effectively grow and for $N \rightarrow \infty$ the plateaus will vanish. For infinitely long LL leads the resonances are infinitely sharp even in the presence of Kondo physics. However, for $t' \ll U$ the plateaus at finite N are well developed and the length scale on which they start to deteriorate is extremely large.

Also for asymmetric dot-lead couplings we find (almost)

plateaulike resonances. To discuss this in more detail we focus on typical parameters with $N \approx 10^4$ and an asymmetry $\Gamma_L/\Gamma_R \approx 2$. Then the width is almost unaffected by the asymmetry. For the interaction and filling as in Fig. 4 (small backscattering) the height within the plateaus varies by a few percent (with maxima at the left and right boundaries) and has an average value of $\approx 0.85(2e^2/h)$. For sizable backscattering the difference to the symmetric case is even smaller. With increasing N the variation of the conductance on the plateaus increases and the average value decreases. We expect that for $N \rightarrow \infty$ the resonances disappear.

In summary, we studied how the linear conductance through a quantum dot is modified in the presence of both Kondo physics and LL leads. Using an approximate method that is based on the fRG we investigated the dependence of G on the gate voltage as well as the length N of the LL leads. We found that for all experimentally accessible length scales and for typical left-right asymmetries of the dot-lead hybridizations the plateaulike resonances characteristic for Kondo physics will also be present if the leads are LL's. The plateaus are more pronounced if the two-particle backscattering is sizable, although they disappear for $N \rightarrow \infty$.

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