Dynamics of a spin-polarized electron liquid: Spin oscillations with a low decay

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We propose spintronic systems which spin dynamics is determined by the frequent normal electron collisions. Spin oscillation induced by the electrical field is predicted. The hydrodynamic equations for the electron spin-polarized liquid have been obtained and analyzed. We found that in the case of a conducting magnetic-inhomogeneous ring, both the spin polarization and the drift current may oscillate simultaneously with a large decay time ("spin pendulum"). We demonstrate also that the spin polarization of the electron density may be revealed via the voltage between the ends of the open circuit with an inhomogeneous spin polarization. The effect may be observed both in the hydrodynamic and the diffusive regimes.

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I. INTRODUCTION

In recent years, the generation and control of nonequilibrium spin-polarization in nonmagnetic conductors have been a focus of attention (see, e.g., Ref. 1). A number of promising methods and schemes have been proposed and discussed. However, the effects of electron-electron scattering are not to be used in these schemes. Our goal is to demonstrate that the normal electron scattering offers new possibilities for the spintronic systems. The effects of electron-electron scattering in the spin dynamics were considered in Refs. 2–4, but not in the case of hydrodynamic electron flow,[5] when the electron-electron scattering dominates and the electron system may be considered as a liquid with its inherent effects.

Meanwhile, a hydrodynamic flow regime is quite real in a low-dimensional electron gas in heterostructures (as well as in electron systems over the liquid helium surface[6]). The main condition is the following. The transport electron mean free path should be determined by the "normal" collisions that conserve the total momentum of the system of interacting particles. They may be electron-electron collisions (when the umklapp processes are absent due to small sizes of the Fermi surfaces of low-dimensional conductors) or electron-phonon collisions[7] (when phonons are tightly coupled to the electron system). In other words, the condition $l_N \ll l_V$ should be satisfied, where l_N is the mean free path (m.f.p.) for the normal collisions and l_V is the m.f.p. for collisions that do not conserve the quasimomentum.

A hydrodynamic electron flow was observed experi mentally in a high-mobility electron gas in (Al,Ga)As heterostructures[8] (see also Ref. 9) at 1.5–20 K. For that range of the electron temperatures, the electron-electron m.f.p., l_{ee}, is much less than the electron m.f.p. determined by the collisions with imperfections of the heterostructure, l_i. Besides that, in the experiment,[8] the phonon temperature was much lower than the electron temperature and collisions with phonons were inessential. The hydrodynamic regime breaks under an increase of the temperature of the sample, because the electron-phonon m.f.p., l_{ep}, becomes less than l_{ee}, and phonons can remove effectively a momentum from the low-dimensional electron system being in good acoustic contact with the surrounding media. If, experimentally, it is possible to provide a weak contact with the surrounding media (that results in the conservation of the total momentum of the electron-phonon system), then the increasing of the temperature favors the hydrodynamic regime due to the decreasing of the normal m.f.p., $l_N \approx (l_{ee}^{-1}+l_{ep}^{-1})^{-1}$.

In the hydrodynamic flow regime, the state of the electron liquid is characterized by the velocity $\mathbf{u}(\mathbf{r})$ and its density $\rho(\mathbf{r})$. Naturally, when our electron system is spin-polarized, the densities of the spin components differ from each other. In this case, the electron liquid is a two-component mixture and the density variables have the spin indexes, i.e., $\rho_\sigma(\mathbf{r})$. Meanwhile, in the leading approximation, the velocity $\mathbf{u}(\mathbf{r})$ is the same for all the spin components. The reason is that frequent collisions between electrons with different spins form a common drift of the electron system. Moreover, we have to regard the electron liquid as incompressible when the geometric size of a conductor is larger than the electron screening radius (which is comparatively small in metals and heterostructures) and the characteristic frequencies of the considered processes are less than the plasma frequency. We would like to emphasize that the incompressibility means here that the nonequilibrium addition to the electron density is vanishing (nevertheless, the equilibrium density may be inhomogeneous; see discussion below). Obviously, the incompressibility and the equal velocities of the spin components lead to the following simple fact. The total current through the channel cross section, I, does not depend on the coordinate along the channel and it is distributed between the spin components in proportion to their densities. Thus, the current characteristics of the system are determined by the value of I.

As we demonstrate below, in the conducting ring with inhomogeneous magnetic properties the value of the total current undergoes low-decay harmonic oscillations which frequency depends on the characteristics of the inhomogeneity. The nature of these oscillations is the following. The electron drift in the magnetic-inhomogeneous ring causes the appearance of a nonequilibrium spin polarization, i.e., an accumulation of the nonequilibrium densities of the spin-up and spin-down components occurs (while the total density is conserved). The accumulation exists until the moment when the drift is stopped due to the interaction of the nonequilibrium spin density with a field that induces the inhomogeneity of the electron spectrum. However, electrons have inertial masses and the process will evolve back. We call this oscillation process a "spin pendulum."

Note, the existence of the well-known hydrodynamic waves, which frequencies are less than the plasma frequency, is impossible here due to the Coulomb interaction. Consequently, "spin-pendulum-like" oscillations are the only possible oscillations of the system in this case.

The interesting spin-electrical effect related to the nonequilibrium spin polarization may be observed not only in the ring but in the open circuit as well. The skewed spin polarization causes a voltage between the ends of the open circuit and it allows easy registering of the spin polarization. Note, the spin-electrical effect in magnetoelectric materials in the equilibrium inhomogeneous state was discussed early in Ref. 10.

Note that all the aforementioned effects are possible both in the magnetic conductors, when densities of the spin components differ from each other initially (e.g., due to the exchange interaction), and in nonmagnetic materials, when either spin separation is due to the Rashba effect[11] or is caused by an external magnetic field. It is important that nonmagnetic materials get magnetic properties due to the appearance of the induced nonequilibrium spin distribution. Consequently, the aforementioned steady state and dynamic effects also exist here, but there are second-order effects. In the case of a two-dimensional electron gas in heterostructures, the variation of magnetic properties can be induced by a nonuniform gate voltage applied, which affects the Rashba effect,[12] by a variation of the external magnetic field or by a space-dependent spin injection.

II. SPIN HYDRODYNAMICS

A time-dependent distribution function $f(\mathbf{r},\mathbf{p},t)$ for electrons at the position $\mathbf{r}$ and with the momentum $\mathbf{p}$ obeys the Boltzmann transport equation

$$\frac{\partial f}{\partial t}+\mathbf{v}\frac{\partial f}{\partial \mathbf{r}}=\frac{\partial(\varepsilon+e\varphi)}{\partial \mathbf{r}}\frac{\partial f}{\partial \mathbf{p}}+J(f). \tag{1}$$

The hydrodynamic equations can be derived from the Boltzmann transport equation by the method used in Refs. 13 and 14, taking into account that the function $f(\mathbf{r},\mathbf{p},t)$ depends also on the spin index σ, which corresponds to the different spin components. It is assumed in Eq. (1) that the energy spectrum of the current carriers depends on the coordinates, momentum, and spin index: $\varepsilon=\varepsilon(\mathbf{r},\mathbf{p},\sigma)$. The electrical potential φ appears due to the space-dependent nonequilibrium spin density. $J(f)$ is the scattering term and $\mathbf{v}=\partial\varepsilon/\partial\mathbf{p}$ is the electron velocity.

Let us assume that momentum dissipation is vanishing and the normal scattering processes dominate, and the following conditions are fulfilled: $l_N/L\ll 1$ and $\omega\tau_N\ll 1$. Here L is the characteristic geometric size of the system, ω is the characteristic frequency of the oscillation processes in the system, $\tau_N=l_N/v_F$ is the relaxation time corresponding to the normal collisions, and v_F is the Fermi velocity. In this case, we may expand Eq. (1) in series in these small parameters. In the leading approximation, we obtain the following equation:

$$J_N(f^{(0)})=0. \tag{2}$$

Here J_N is the part of the collision term that corresponds to the normal collisions. The solution of Eq. (2) is the quasiequilibrium drift distribution

$$f^{(0)}=n(\varepsilon_\sigma-\delta\mu_\sigma-\mathbf{up}),\quad n(z)=(e^{(z-\mu)/T}+1)^{-1}, \tag{3}$$

where the drift velocity $\mathbf{u}$ and nonequilibrium additions to the chemical potential $\delta\mu_\sigma$ depend on the coordinates $\mathbf{r}$ and time t (we neglect here the temperature variations). Following the method used in Refs. 13 and 14, we analyzed the conditions of solvability up to the next two orders of series expansions and have obtained the following equations for $\delta\mu_\sigma,\mathbf{u}$, and φ:

$$\frac{\partial\delta\rho_\sigma}{\partial t}+\mathrm{div}\rho_{0\sigma}\mathbf{u}=(\hat{D}+\hat{F})\delta\rho_\sigma, \tag{4}$$

$$m\rho\frac{\partial\mathbf{u}}{\partial t}+\sum_\sigma\rho_\sigma\nabla(\delta\mu_\sigma+e\varphi)=(\hat{V}+\hat{U})\mathbf{u}, \tag{5}$$

$$\sum_\sigma\delta\rho_\sigma=0. \tag{6}$$

Here $\rho_\sigma=\rho_{0\sigma}+\delta\rho_\sigma$ is the spin-dependent density and $\rho_{0\sigma}$ is the equilibrium spin density. (Note that $\delta\rho_\sigma$ is the function on $\delta\mu_\sigma$, and, in the linear approximation, $\delta\rho_\sigma=\Pi_\sigma\delta\mu_\sigma$, where Π_σ is the spin-dependent density of states on the Fermi surface.) $m\rho=(\frac{1}{h})^r r^{-1}\Sigma_\sigma\int p^2(-\frac{\partial n}{\partial\varepsilon})d^rp$, $\rho=\Sigma_\sigma\rho_\sigma$, where r is the dimensionality of the electron system.$\hat{D}$ is the diffusion operator, $\hat{F}$ is the spin-flip operator, the operator $\hat{V}$ relates to the viscosity, and the operator $\hat{U}$ corresponds to the scattering with momentum dissipation (see, e.g., Refs. 13 and 14). Note that the specific forms of these operators are not important for us: the corresponding terms in Eqs. (4) and (5) are small enough and we use these terms only for the estimates. Note, both diffusion processes in the mixture of liquids and the viscous braking of flow, see Ref. 15, are too slow to compare with dynamic processes, which we discuss below. This is valid when the hydrodynamic parameter, l_N/L, is small, $l_N/L\ll 1$; additionally, we suppose that both the spin-flip and impurity scattering are not very effective, see Eq. (13).

Meanwhile, Eqs. (4)–(6) have a clear physical meaning. Equation (4) describes the law of conservation of the number of colliding particles, while Eq. (5) corresponds to the conservation of the total momentum. Equation (6) is the result of incompressibility of the electron liquid with the Coulomb interaction. [To derive Eqs. (4) and (5) in the form presented, we assume that electrons have a spherical Fermi surface.] Equations (4) and (5) are written in the linear approximation in the drift velocity $\mathbf{u}$, but the terms that contain $\delta\mu_\sigma,\delta\rho_\sigma$ are given in the explicit form under the assumption $\mathbf{u}\to\mathbf{0}$. This approximation is sufficient here since the nonlinear effects will be considered for the case when the common drift is absent.

Neglecting both the viscosity of the electron liquid and momentum dissipation, we rewrite Eq. (4) in the following form $[\delta n=n(\varepsilon-\delta\mu_\sigma)-n(\varepsilon)]$:

$$m\rho\frac{\partial\mathbf{u}}{\partial t}+\nabla P+e\rho\nabla\varphi+\sum_\sigma\int\frac{\nabla\varepsilon_\sigma\delta n}{h^r}d^rp=0, \tag{7}$$

$$P=\sum_\sigma\int\frac{\mathbf{pv}\delta n}{rh^r}d^rp,\quad\frac{\partial P}{\partial\delta\mu_\sigma}=\rho_\sigma. \tag{8}$$

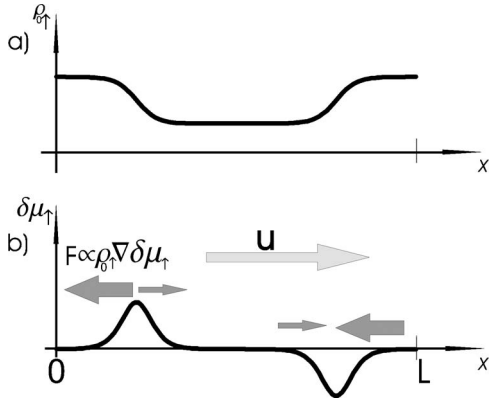

FIG. 1. Equilibrium distribution of the spin density $\rho_{0\uparrow}$ along the ring of length L (a). When electrons drift into the right [(b)], the nonequilibrium spin-up potential $\delta\mu_\uparrow$ appears in the regions where the spin-up density $\rho_{0\uparrow}$ varies. The total force is a sum of all forces acting on electrons and it is directed to the left. Consequently, it leads to the change of the drift direction.

Note that, integrating Eq. (7) by parts in the momentum space, we obtain Eq. (5). Equation (7) is a natural generalization of the Euler equation[15] for the mixture of liquids interacting with an external electrical field and with the field that induces the inhomogeneity of the electron spectrum. $P(\delta\mu_\sigma)$ is a nonequilibrium addition to the electron pressure.

In the case of a conducting ring, the electron pressure does not cause the net effect on the electrons because $\oint \nabla P dx = 0$ (if our conductor is homogeneous, pressure can be compensated by an electrical field, $\nabla\varphi$). Meanwhile, the force $\nabla\varepsilon_\sigma$ may cause a common drift in the ring as it acts on the electrons which are nonequilibrium on the spin. Figure 1 illustrates the physical picture arising in the conducting ring with the inhomogeneity of the equilibrium spin density and explains the origin of the total force which acts on the spin-nonequilibrium distribution: this force is caused by the electron drift along the inhomogeneous ring.

III. SPIN PENDULUM

Let us discuss the oscillation process described by Eqs. (4)–(6), which is related to the current $I = I_0 e^{i\omega t}$, i.e., a spin pendulum. In the linear approximation, we may solve the problem completely and find the frequency of the oscillations,

$$\omega^2 = \oint \left(\frac{\rho_\uparrow}{\rho}\right)'^2 \frac{1}{\Pi^*} dx \left(\oint \frac{m}{\rho} dx\right)^{-1}, \quad \frac{1}{\Pi^*} = \sum_\sigma \frac{1}{\Pi_\sigma}, \tag{9}$$

$$u = \frac{I}{es\rho}, \quad \delta\mu_\sigma = i \frac{I}{es\omega\Pi_\sigma}\left(\frac{\rho_\sigma}{\rho}\right)',$$

$$e\varphi' = -\frac{i}{\omega es}\left\{\frac{m\omega^2}{\rho} + \sum_\sigma \frac{\rho_\sigma}{\rho}\left[\frac{1}{\Pi_\sigma}\left(\frac{\rho_\sigma}{\rho}\right)'\right]'\right\} I. \tag{10}$$

Here x is the coordinate along the conducting channel, the "up arrow" corresponds to the one of spin components, s is the cross section of the channel, which is assumed x-independent, a prime denotes differentiation with respect to x, and L is the length of the conducting channel (i.e., the ring length in our case). Note, both $\delta\mu_\sigma$ and φ' are written here as imaginary values, which means that they oscillate with a phase shift in the quarter of the period so as to compare with the oscillation of the current I.

As is evident from Eqs. (9) and (10), the oscillations exist only when the relative concentration of the spin components varies along the channel. The frequency of oscillations is proportional to the level of the magnetic inhomogeneity, $\alpha = (\rho_{\sigma\max} - \rho_{\sigma\min})/\rho$, viz.,

$$\omega \approx \alpha v_F (L L_1)^{-1/2}, \tag{11}$$

where L_1 is the characteristic scale of the spectrum inhomogeneity.

Initially, the oscillations of the spin pendulum could be excited by the "magnetic push" when an external magnetic field is switched on (much faster than ω^{-1}) and an initial current $I = es\Phi/cm$ is induced (where Φ is the magnetic flux through the ring). Naturally, there are dissipation processes in the hydrodynamic flow, which we did not take into account. They lead to the damping of the spin-pendulum oscillations. It is easy to estimate the decay time,

$$\tau_d \approx \left(\frac{e^2 \rho s R}{mL} + \frac{D}{L_1^2} + \frac{1}{\tau_{sf}}\right)^{-1}, \tag{12}$$

where R is the electrical resistance of the homogeneous ring. The second term on the right-hand side of Eq. (12) is related to the diffusion of the nonequilibrium spins through the inhomogeneous region. In the hydrodynamic regime, the diffusion coefficient is determined by the normal collisions, i.e., $D \approx l_N v_F$. The third term on the right-hand side of Eq. (12) corresponds to the spin relaxation, which is due to the spin-flip with the characteristic time τ_{sf}. Note, as it follows from Eq. (12), the "space-sharp" inhomogeneity (e.g., a sharp interface boundary between different magnetic materials) causes a very fast decay of the oscillations.

Thus, we predict the appearance of low-decay oscillations of the spin polarization with the frequency ω under the conditions

$$\tau_N \ll \omega^{-1} \ll \tau_d. \tag{13}$$

This brings up the following question: is it possible to satisfy these inequalities for the existing experimental objects? It seems the second inequality may bring experimental problems, and the inequality $\rho s R/mL\omega \ll 1$ is the most difficult condition: when $\alpha \simeq 1$, $L_1 \simeq L$, it means that the transport m.f.p. of electrons, $l_{\rm tr}$, should be larger than the length of the sample L. Nevertheless, the hydrodynamic regime has been observed experimentally in high-mobility GaAs hereostructures,[8] and l_N was much less than the width of the 2D conducting channel (~ 4 μm) and the electron-impurity m.f.p., $l_i \simeq 20$ μm. Thus, conditions (13) could be realized for such structures when the electron transport m.f.p. is determined by the electron-impurity scattering. To reach this case, one should decrease the diffusive boundary scattering (in Ref. 8, the probability of diffusive reflection of electrons from the boundary $q \simeq 0,2$). In the hydrodynamic regime, the corresponding contribution to the transport relaxation rate, $l_{\rm tr}^{-1}$, is of the order $q l_N/d^2$ and it may be decreased by increasing d.

It seems interesting that there exists a possibility to keep the amplitude of the oscillations constant (as in an ordinary pendulum clock) owing to the magnetic connection of the ring with an energy-pumping cell.

IV. SPIN-ELECTRICAL EFFECT

Let us discuss now the spin-electrical effect in an open-ended homogeneous conductor. At $\mathbf{u}=\mathbf{0}$, we obtain from Eqs. (7)–(10) the following potential difference between the open ends:

$$\Delta\varphi \equiv \varphi(L) - \varphi(0) = \frac{e}{\rho}[P(0) - P(L)], \tag{14}$$

$$P(x) = \left(\rho_\uparrow - \frac{\rho_\downarrow \Pi_\uparrow}{\Pi_\downarrow}\right)\delta\mu_\uparrow + \left(\Pi_\uparrow - \frac{\rho_\uparrow}{\Pi_\uparrow}\frac{\partial\Pi_\uparrow}{\partial\mu_\uparrow}\right)\delta\mu_\uparrow^2 + \cdots . \tag{15}$$

Equation (15) yields the electron pressure as the expansion in series on the $\delta\mu$. The second-order term is written in the form of the case of nonmagnetic materials. All the quantities in the coefficients of $\delta\mu_\uparrow$ and $(\delta\mu_\uparrow)^2$ are understood as the equilibrium state quantities. It is obvious from Eq. (14) that the difference between the nonequilibrium spin pressures at the ends of the circuit induces an electrical voltage that can be measured experimentally. The voltage exists during the lifetime τ_φ, which is determined either by the spin relaxation due to the spin-flip processes or by the diffusion equalization of the spin concentration along the circuit, $\tau_\varphi \approx (\tau_{\mathrm{sf}}^{-1} + D/L^2)^{-1}$. The electrical charge $q \approx \Delta\varphi\tau_\varphi/R_c$ (where R_c is the electrical resistance of the circuit) flows through the meter during this time. Let us discuss the case in which the spin polarization is induced in a nonmagnetic conductor due to the current spin injection from the magnetic material. Then, a meter connected to the ends of the nonmagnetic conductor will fix the electrical charge q and indicates the nonequilibrium spin density existed in the nonmagnetic conductor.

We should note here that this "spin-electrical" effect is not a specifically hydrodynamic effect. In the diffusion regime, when collisions that do not conserve momenta are dominated over normal collisions, we may write the continuity equations in the following form:

$$\frac{\partial\delta\rho_\sigma}{\partial t} + \mathrm{div}\,\sigma_\sigma[\nabla(\delta\mu_\sigma + e\varphi)] = 0, \tag{16}$$

where σ_σ is the contribution to the conductivity of the corresponding spin component, $\sigma=\Sigma_\sigma\sigma_\sigma$. Consequently, taking into account Eq. (6) we obtain the following equation for the case of a homogeneous conductor:

$$e\varphi(x) = \left(\frac{\sigma_\downarrow\Pi_\uparrow}{\Pi_\downarrow} - \sigma_\uparrow\right)\frac{\delta\mu_\uparrow}{\sigma} + \left(\frac{\sigma_\uparrow}{\Pi_\uparrow}\frac{\partial\Pi_\uparrow}{\partial\mu_\uparrow} - \frac{\partial\sigma_\uparrow}{\partial\mu_\uparrow}\right)\frac{\delta\mu_\uparrow^2}{\sigma} + \cdots . \tag{17}$$

Here, the second term on the right-hand side of Eq. (19) is written for the case of nonmagnetic conductors only.

V. CONCLUSION

In summary, it is shown that different types of spin oscillations are possible when the normal electron scattering prevails over other scattering processes. The low-decay oscillations of the spin polarization with the frequency ω accompanied by the oscillations of the drift current may be induced in the conducting ring with inhomogeneous magnetic properties under the conditions Eq. (13). In the case of the heterostructures, the inhomogeneity may be induced by the inhomogeneous Rashba splitting due to the space-dependent electrical field. Assuming that the splitting Δp_F is small enough, i.e., $\Delta p_F/p_F \ll 1$, we obtain from Eq. (9) that $\omega = v_F\left[\frac{2}{L}\oint\left(\frac{\Delta p_F}{p_F}\right)'^2 dx\right]^{1/2}$. In the case of a completely nonmagnetic and inhomogeneous ring the excitation of local spin polarization also induces non-linear "spin-pendulum-like" oscillations, but the consideration of that goes beyond the framework of this paper. In the case of the open circuit, one may detect a non-equilibrium spin-polarization, measuring the voltage between the open ends of the circuit. It can be done both in the hydrodynamic and diffusion regimes.

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