

## Disorder effects in fluctuating one-dimensional interacting systems

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(Received 16 December 2005; published 10 April 2006)

The zero-temperature localization of interacting electrons coupled to a two-dimensional quenched random potential, and constrained to move on a fluctuating one-dimensional string embedded in the disordered plane, is studied using a perturbative renormalization group approach. In the reference frame of the electrons the impurities are dynamical and their localizing effect is expected to decrease. We consider several models for the string dynamics and find that while the extent of the delocalized regime indeed grows with the degree of string fluctuations, the critical interaction strength, which determines the localization-delocalization transition for infinitesimal disorder, does not change unless the fluctuations are softer than those of a simple elastic string.

DOI: [10.1103/PhysRevB.73.134201](https://doi.org/10.1103/PhysRevB.73.134201)

PACS number(s): 71.10.Pm, 72.15.Rn, 74.40.+k

### I. INTRODUCTION

Quenched disorder is a relevant perturbation in one- and two-dimensional noninteracting fermionic systems.<sup>1</sup> In these low dimensions the resulting fully localized state can be averted at zero temperature only under the influence of interactions. Such interaction-induced delocalization has been demonstrated in one-dimensional systems with strong attraction,<sup>2,3</sup> and evidence for its potential existence in two dimensions, even in the presence of repulsive forces, has also been put forward.<sup>4-6</sup> Most of our knowledge of the interplay between disorder and interactions is for static disorder, and much less is known about the interplay between interactions and time-dependent disorder.<sup>7,8</sup>

Although for quantum systems time-independent disorder is the standard case, one can encounter situations where the motion of the electrons is constrained to fluctuating geometries which are embedded in a static disordered space. In such a case, to the electrons, the disorder appears to change in time with temporal correlations which are inherited from the dynamics of the fluctuating geometry. Relevant realizations of this scenario include excitations in the core of vortex states<sup>9</sup> and the cuprate high-temperature superconductors, in which the importance of self-organized, fluctuating quasi-one-dimensional electronic structures within the (disordered) copper-oxide planes has been pointed out.<sup>10</sup> On the technical side, introducing the time dependence of the scattering potential in such a way has the advantage of enabling us to treat the problem using standard methods of equilibrium quantum statistical mechanics.

We therefore devote the present paper to the study of the consequences of coupling interacting electrons, constrained to move along a fluctuating string embedded in a plane, to static disorder within the plane. We consider several string Hamiltonians including that of a rigid string inside a harmonic well, a stretchable elastic string, and a floppy string of fixed length. These models form a hierarchy in terms of an increasing degree of string fluctuations. Assuming weak disorder and long wave-length fluctuations we investigate the relations between the electron-electron interactions, disorder, and string dynamics using a renormalization group analysis. Our one-loop treatment of the problem captures the mutual

renormalization of these three elements. It ignores subtle interference effects due to coherent scattering from many impurities, but in one dimension (unlike in two or more dimensions) where the localization length is known to be of the order of the mean free path, such effects are not expected to play an essential role. As anticipated, we find that the extent of the delocalized phase increases when the string fluctuations become more pronounced, as is demonstrated, for example, by Fig. 2. However, in the cases of the rigid and elastic strings, the critical point that separates the localized and delocalized phases for infinitesimal disorder remains at the same critical interaction strength as for a static string. Only when the softer fluctuations produced by the floppy string dynamics are considered does one find that the critical point shifts towards smaller values of interactions.

The main body of the paper is composed of three sections, each dealing with one of the realizations of the string dynamics as indicated above. Every one of these sections introduces the model for the coupled electronic and string degrees of freedom, in the presence of disorder. The renormalization equations are then derived and the resulting zero-temperature phase diagram discussed. The effects of the forward-scattering part of the disorder on the charge-density-wave (CDW) and spin-density-wave (SDW) correlations are also given. Some of the details pertaining to the derivation of the renormalization equations are relegated to the Appendixes.

### II. RIGID STRING IN A PARABOLIC WELL

#### A. The model

We begin by considering interacting electrons which are constrained to move on a straight rigid string of fixed length  $L$ , embedded in a two-dimensional disordered plane. The string is assumed to oscillate inside a parabolic potential, and its state is characterized by its deviation  $u(\tau)$  from the classical equilibrium configuration at the bottom of the well (which we define in the following as the  $x$  axis). The string Lagrangian in imaginary time is therefore

$$L_s = \frac{M}{2}(\partial_t u)^2 + \frac{M\omega_0^2}{2}u^2, \quad (1)$$

where  $M$  is the mass of the string and  $\omega_0$  its oscillation frequency.

The low-energy physics of the one-dimensional electron gas (1DEG) is captured by the Tomonaga-Luttinger model<sup>11</sup> in which the spectrum is linearized around the two Fermi points at  $\pm k_F$  and the electronic operator  $\psi_{\sigma=\{\uparrow,\downarrow\}}$  is decomposed in terms of slowly varying left- ( $\nu=-$ ) and right- ( $\nu=+$ ) moving components:  $\psi_{\sigma}(x) = e^{-ik_F x} \psi_{-, \sigma}(x) + e^{ik_F x} \psi_{+, \sigma}(x)$ . We follow the standard notation and parametrize the interactions between the electrons according to

$$\begin{aligned} & \frac{1}{2} \sum_{\nu, \sigma, \sigma'} [g_4 \rho_{\nu, \sigma} \rho_{\nu, \sigma'} + g_2 \rho_{\nu, \sigma} \rho_{-\nu, \sigma'} - g_{1\parallel} \delta_{\sigma, \sigma'} \rho_{\nu, \sigma} \rho_{-\nu, \sigma'} \\ & + g_{1\perp} \delta_{\sigma, -\sigma'} \psi_{\nu, \sigma}^\dagger \psi_{-\nu, \sigma'}^\dagger \psi_{\nu, \sigma'} \psi_{-\nu, \sigma'}], \end{aligned}$$

where  $\rho_{\nu, \sigma} = \psi_{\nu, \sigma}^\dagger \psi_{\nu, \sigma}$ . The model can also be expressed in terms of charge and spin bosonic fields  $\phi_{\alpha=\{c, s\}}$  and their conjugated momenta  $\partial_x \theta_\alpha$ , in terms of which the electronic Lagrangian is given by (here and throughout  $\hbar=1$ )

$$\begin{aligned} L_e = & \int_0^L dx \sum_{\alpha=c, s} \left[ -i \partial_\tau \phi_\alpha \partial_x \theta_\alpha + \frac{v_\alpha K_\alpha}{2} (\partial_x \theta_\alpha)^2 + \frac{v_\alpha}{2K_\alpha} (\partial_x \phi_\alpha)^2 \right] \\ & + \frac{g_{1\perp}}{2(\pi a)^2} \cos(\sqrt{8\pi} \phi_s), \end{aligned} \quad (2)$$

where  $a$  is a short-distance cutoff of the order of the lattice constant and the velocities and Luttinger parameters are related to the Fermi velocity  $v_F$  and the interaction couplings according to

$$\begin{aligned} v_c &= \frac{1}{2\pi} \sqrt{(2\pi v_F + 2g_4)^2 - (2g_2 - g_{1\parallel})^2}, \\ v_s &= \frac{1}{2\pi} \sqrt{(2\pi v_F)^2 - g_{1\parallel}^2}, \\ K_c &= \sqrt{\frac{2\pi v_F + 2g_4 - 2g_2 + g_{1\parallel}}{2\pi v_F + 2g_4 + 2g_2 - g_{1\parallel}}}, \\ K_s &= \sqrt{\frac{2\pi v_F + g_{1\parallel}}{2\pi v_F - g_{1\parallel}}}. \end{aligned} \quad (3)$$

As can be seen from our analysis of the more general problem of a fluctuating string, as presented in Sec. III, in the present model (which is equivalent to a 1DEG coupled to an optical phonon in the transverse direction) the only coupling between the electrons and string (in the absence of disorder) is via a renormalization of the string mass, due to the fact that the electrons are dragged with the string as it moves. We assume that this renormalization  $M \rightarrow M + mN_e$ , where  $m$  is the electronic mass and  $N_e$  is the number of electrons on the string, has been incorporated into the definition of the parameters which appear in the string Lagrangian, Eq. (1).

The plane containing the string and electrons is taken to include a weak random potential which couples to the electrons, but not directly to the string. This choice is motivated primarily by the situation in the cuprates where the ‘‘strings’’ are actually defined as the loci of points occupied by holes in the plane and, as such, couple to the disorder only indirectly via the electronic degrees of freedom. The forward-scattering  $k \sim 0$  and backward-scattering  $k \sim 2k_F$  components of the impurities potential are assumed to be uncorrelated Gaussian random fields  $\eta$  and  $\xi$  with  $\overline{\eta(x, y) \eta(x', y')} = D_f \delta(x-x') \delta(y-y')$  and  $\overline{\xi(x, y) \xi(x', y')} = D_b \delta(x-x') \delta(y-y')$ , respectively.<sup>3</sup> The coupling of the electrons to the impurities is given by

$$\begin{aligned} L_{dis}^f &= - \sum_{\nu, \sigma} \int_0^L dx \eta[x, u(\tau)] \rho_{\nu, \sigma}(x, \tau) \\ &= - \int_0^L dx \sqrt{\frac{2}{\pi}} \eta[x, u(\tau)] \partial_x \phi_c(x, \tau) \end{aligned} \quad (4)$$

and

$$\begin{aligned} L_{dis}^b &= - \sum_{\sigma} \int_0^L dx \xi[x, u(\tau)] \psi_{-, \sigma}^\dagger(x, \tau) \psi_{+, \sigma}(x, \tau) + \text{H.c.} \\ &= - \int_0^L dx \frac{dx}{\pi a} \xi[x, u(\tau)] e^{i\sqrt{2\pi} \phi_c(x, \tau)} \cos[\sqrt{2\pi} \phi_s(x, \tau)] + \text{H.c.} \end{aligned} \quad (5)$$

Here the constraint which confines the electrons to the string manifests itself in the position at which the disorder potential is evaluated. This, in turn, induces a coupling between the electronic and string degrees of freedom, and leads, in the reference frame of the electrons, to an effective time-dependent disorder. This time dependence precludes the possibility of gauging out the forward-scattering disorder component from the action by an appropriate shift of  $\phi_c$ , as was done in Ref. 3. Instead, both the forward-scattering and backward-scattering components will be treated on equal footing within a one-loop renormalization group analysis.

To this end, we use the replica trick and average the replicated action over the random fields  $\eta$  and  $\xi$ . We will obtain the renormalization equations to first order in  $D_b$  and  $D_f$  and to second order in  $g_{1\perp}$ , which is also assumed small. To this order, the replica indices play no role and consequently they will be omitted in the following. The averaged action is therefore

$$\begin{aligned} S &= \int d\tau [L_s + L_e] - \frac{D_f}{2} \int d\tau d\tau' dx \delta[u(\tau) - u(\tau')] \\ &\times \sum_{\nu, \nu', \sigma, \sigma'} \rho_{\nu, \sigma}(x, \tau) \rho_{\nu', \sigma'}(x, \tau') - \frac{D_b}{2} \int d\tau d\tau' dx \\ &\times \delta[u(\tau) - u(\tau')] \sum_{\sigma, \sigma'} [\psi_{+, \sigma}^\dagger(x, \tau) \psi_{-, \sigma}(x, \tau) \psi_{-, \sigma'}^\dagger(x, \tau') \\ &\times \psi_{+, \sigma'}(x, \tau') + \text{H.c.}]. \end{aligned} \quad (6)$$

### B. Renormalization and phase diagram

We derive the renormalization flow equations for the model by requiring invariance of the long-wavelength low-frequency behavior of the correlation functions under a change of the cutoff.<sup>3,12</sup> We begin with the electronic correlations and integrate out the string degrees of freedom in order to obtain an effective electronic action. The result is an action which is derived from Eq. (6) by replacing  $\delta[u(\tau) - u(\tau')]$  by its average over the string dynamics:

$$\begin{aligned} F(\tau - \tau') &\equiv \langle \delta[u(\tau) - u(\tau')] \rangle \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} d\lambda \langle e^{i\lambda[u(\tau) - u(\tau')]} \rangle \\ &= \frac{1}{\sqrt{2\pi}} \langle [u(\tau) - u(\tau')]^2 \rangle^{-1/2}. \end{aligned} \quad (7)$$

As we demonstrate below, it is this function which determines the modification in the renormalization flow of the model as a result of the string fluctuations. In the present model one finds

$$F(\tau) = \frac{1}{2\sqrt{\pi\lambda}} \frac{1}{\sqrt{1 - e^{-\omega_0|\tau|}}}, \quad (8)$$

where the length

$$\lambda = \frac{1}{\sqrt{2M\omega_0}} \quad (9)$$

is a measure of the amplitude of the string fluctuations inside the harmonic well.

Our renormalization procedure is akin to real-space renormalization in the sense that electronic degrees of freedom at points  $(x, \tau)$  and  $(x, \tau')$  are identified inside the interval  $|\tau - \tau'| < a/v_s$ . Upon such identification, the  $|\tau - \tau'| < a/v_s$  part of the forward-scattering term in Eq. (6) is equivalent to  $g_2$  and  $g_4$  interaction processes, while the corresponding part of the backward-scattering term is equivalent to  $g_{1\parallel}$  and  $g_{1\perp}$  processes. As a result one can absorb the contributions of these parts via a redefinition of the interaction parameters according to

$$\begin{aligned} \bar{g}_{2,4} &= g_{2,4} - 2D_f \int_0^{av_s} d\tau F(\tau), \\ \bar{g}_{1\parallel,1\perp} &= g_{1\parallel,1\perp} - 2D_b \int_0^{av_s} d\tau F(\tau). \end{aligned} \quad (10)$$

The effective electronic action then reads

$$\begin{aligned} \bar{S}_e &= \int d\tau \bar{L}_e - \frac{D_f}{\pi} \int_{|\tau-\tau'|>av_s} d\tau d\tau' dx F(\tau - \tau') \partial_x \phi_c(x, \tau) \\ &\quad \times \partial_x \phi_c(x, \tau') - \frac{D_b}{(\pi a)^2} \int_{|\tau-\tau'|>av_s} d\tau d\tau' dx F(\tau - \tau') \\ &\quad \times \cos[\sqrt{2\pi}\phi_s(x, \tau)] \cos[\sqrt{2\pi}\phi_s(x, \tau')] \\ &\quad \times \cos[\sqrt{2\pi}\phi_c(x, \tau) - \sqrt{2\pi}\phi_c(x, \tau')], \end{aligned} \quad (11)$$

where  $\bar{L}_e$  has the same form as Eq. (2), but with modified velocities and Luttinger parameters, which to first order in the disorder strength are given by

$$\begin{aligned} \bar{K}_c &= K_c + \left[ \frac{2K_c^2 D_f}{\pi v_c} - \frac{1}{2\pi v_c} (1 + K_c^2) D_b \right] \int_0^{av_s} d\tau F(\tau), \\ \bar{K}_s &= K_s - \frac{1}{2\pi v_s} (1 + K_s^2) D_b \int_0^{av_s} d\tau F(\tau), \\ \bar{v}_c &= v_c - \left[ \frac{2K_c D_f}{\pi} + \frac{1}{2\pi} \left( \frac{1}{K_c} - K_c \right) D_b \right] \int_0^{av_s} d\tau F(\tau), \\ \bar{v}_s &= v_s - \frac{1}{2\pi} \left( \frac{1}{K_s} - K_s \right) D_b \int_0^{av_s} d\tau F(\tau), \\ \bar{y} &= y + \left[ \frac{y}{2\pi v_s} \left( \frac{1}{K_s} - K_s \right) - \frac{2}{\pi v_s} \right] D_b \int_0^{av_s} d\tau F(\tau), \end{aligned} \quad (12)$$

as a consequence of Eqs. (3) and (10). Here and in the following we have defined the dimensionless spin-flipping backward-scattering coupling

$$y = \frac{g_{1\perp}}{\pi v_s}. \quad (13)$$

The derivation of the renormalization equations for the parameters which appear in the effective electronic action, Eq. (11), follows closely Ref. 3. We give some details concerning the contribution of the forward-scattering disorder to these equations in Appendix A, where the latter are also listed. We then use them, together with Eqs. (8) and (12), to obtain the renormalization equations of the electronic and disorder parameters in the original disordered averaged action, Eq. (6). The result is

$$\begin{aligned} \frac{dK_c}{d\ell} &= -\frac{1}{2} \frac{v_c}{v_s} \frac{D_b}{\sqrt{1 - e^{-\varpi}}} \left[ K_c^2 - \frac{1}{2} \left( \frac{v_c}{v_s} \right)^{K_c-2} (1 + K_c^2) \right] \\ &\quad + \frac{D_b}{4} \left( \frac{v_c}{v_s} \right)^{K_c-1} (2 - K_c - K_s - y) (1 + K_c^2) \\ &\quad \times \left[ 1 + \frac{2}{\varpi} \ln(1 + \sqrt{1 - e^{-\varpi}}) \right], \end{aligned}$$

$$\begin{aligned} \frac{dv_c}{d\ell} &= -\frac{1}{2} \frac{v_c^2}{v_s} \frac{D_b}{\sqrt{1 - e^{-\varpi}}} \left[ K_c - \frac{1}{2} \left( \frac{v_c}{v_s} \right)^{K_c-2} \left( \frac{1}{K_c} - K_c \right) \right] \\ &\quad + \frac{D_b}{4} \left( \frac{v_c}{v_s} \right)^{K_c-1} (2 - K_c - K_s - y) \left( \frac{1}{K_c} - K_c \right) \\ &\quad \times \left[ 1 + \frac{2}{\varpi} \ln(1 + \sqrt{1 - e^{-\varpi}}) \right], \end{aligned}$$

$$\begin{aligned} \frac{dK_s}{d\ell} = & -\frac{1}{2}y^2K_s^2 - \frac{1}{2}\frac{D_b}{\sqrt{1-e^{-\varpi}}}\left[K_s^2 - \frac{1}{2}\left(\frac{v_c}{v_s}\right)^{K_c}(1+K_s^2)\right] \\ & + \frac{D_b}{4}\left(\frac{v_c}{v_s}\right)^{K_c}\left[(2-K_c-K_s-y)(1+K_s^2) + 4K_s^2y\right] \\ & \times \left[1 + \frac{2}{\varpi}\ln(1 + \sqrt{1-e^{-\varpi}})\right], \end{aligned}$$

$$\begin{aligned} \frac{dv_s}{d\ell} = & -\frac{1}{2}\frac{D_b}{\sqrt{1-e^{-\varpi}}}\left[K_s - \frac{1}{2}\left(\frac{v_c}{v_s}\right)^{K_c}\left(\frac{1}{K_s} - K_s\right)\right] \\ & + \frac{D_b}{4}\left(\frac{v_c}{v_s}\right)^{K_c}(2-K_c-K_s-y)\left(\frac{1}{K_s} - K_s\right) \\ & \times \left[1 + \frac{2}{\varpi}\ln(1 + \sqrt{1-e^{-\varpi}})\right], \end{aligned}$$

$$\begin{aligned} \frac{dy}{d\ell} = & (2-2K_s)y - \frac{D_b}{\sqrt{1-e^{-\varpi}}}\left\{1 - \left(\frac{v_c}{v_s}\right)^{K_c}\left[1 - \frac{y}{4}\left(\frac{1}{K_s} - K_s\right)\right]\right\} \\ & + \frac{D_b}{4}\left(\frac{v_c}{v_s}\right)^{K_c}\left\{4(K_s - K_c)\right. \\ & \left. - y\left[1 - K_s^2 + (2-K_c)\left(\frac{1}{K_s} - K_s\right)\right]\right\} \\ & \times \left[1 + \frac{2}{\varpi}\ln(1 + \sqrt{1-e^{-\varpi}})\right], \end{aligned}$$

$$\frac{dD_f}{d\ell} = D_f, \quad \frac{dD_b}{d\ell} = (3-K_c-K_s-y)D_b, \quad (14)$$

where the running cutoff is given by  $a = a_0 e^\ell$  and where we have defined the dimensionless quantities

$$D_b = \frac{D_b a}{\pi^{3/2} v_s^2 \lambda} \left(\frac{v_s}{v_c}\right)^{K_c}, \quad D_f = \frac{D_f a}{\pi^{3/2} v_c^2 \lambda}, \quad \varpi = \frac{a \omega_0}{v_s}. \quad (15)$$

Expressing the flow equations in terms of the original velocities and Luttinger parameters has the advantage that these, unlike the barred quantities of Eq. (12), are related to the electronic interaction couplings in a familiar manner. As a result, it is straightforward to check that if the system is initially at the noninteracting point  $K_c = K_s = 1$ ,  $v_c = v_s = v_F$ , and  $y = 0$ ; then, it stays there in the course of the renormalization—i.e.,  $dK_c/d\ell = dK_s/d\ell = dy/d\ell = 0$ . In other words, a system of independent electrons remains so even in the presence of (time-dependent) disorder. Second, the equations preserve spin-rotation symmetry. If one starts from a spin-rotation-invariant Hamiltonian ( $g_{\parallel} = g_{\perp}$ ), it continues giving respect to this symmetry during the renormalization process. This fact can be easily checked for small  $g_1$ , in which case  $K_s \approx 1 + y/2$ . The flow maintains this relation<sup>13</sup> since it satisfies  $dK_s/d\ell = 1/2(dy/d\ell)$ .

Note that the renormalization equations for the electronic parameters do not include the impurity forward-scattering amplitude  $D_f$ . It does appear in the flow equations (A4) for

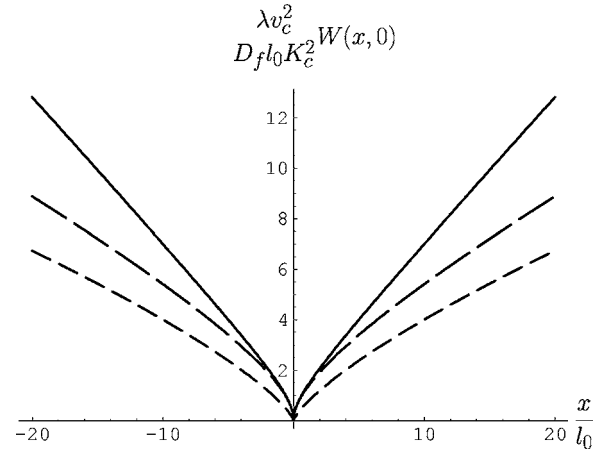


FIG. 1. The function  $W(x,0)$  for the different models: a rigid string in a parabolic well (solid line), an elastic string (long-dashed line), and a floppy string (dashed line). Both axes are scaled by  $l_0 = v_c / \omega_0$ ,  $(v_c / v_u) a$ , and  $4\pi(v_c / \gamma)\lambda$ , in the three cases, respectively.

the parameters in the effective action  $\bar{S}_e$ , but cancels out for the original parameters as a consequence of the relations given in Eq. (12). This fact has been demonstrated in the case of a static string,<sup>3</sup> where it is possible to completely absorb the forward scattering due to impurities by a redefinition of the field  $\phi_c$ . Here we show how it also arises when one treats the scattering perturbatively and find that it extends also to cases where the string is dynamical.

The forward scattering does influence, however, various correlation functions, most notably those of the charge-density-wave  $R_{CDW}$  and spin-density-wave  $R_{SDW}$  order parameters, which are both proportional to the function  $R_c$  defined in Appendix A. Consequently one finds  $R_{CDW,SDW}(x, \tau) \propto e^{-W(x, \tau)}$ , with the exact result<sup>3</sup> for the static string  $W(x, \tau) = 2D_f^{1D}(K_c/v_c)^2|x|$  [here  $D_f^{1D}$  measures the disorder correlations along one dimension and as such differs by a factor of  $(2\sqrt{\pi}\lambda)^{-1}$  from  $D_f$  which appears in our analysis]. When fluctuations are included we are able to calculate, using Eq. (A3), the asymptotic behavior of  $W$  to first order in the disorder and obtain

$$W(x,0) = \begin{cases} \frac{3\pi v_c D_f}{\sqrt{2} v_s \varpi} K_c^2 \sqrt{\frac{\omega_0 x}{v_c}} + \mathcal{O}(x^2), & \omega_0 |x| \ll v_c, \\ \pi D_f K_c^2 \frac{|x|}{a} + \mathcal{O}(\ln|x|), & \omega_0 |x| \gg v_c, \end{cases} \quad (16)$$

and

$$W(0, \tau) = \begin{cases} \frac{v_c D_f}{v_s \varpi} K_c^2 \sqrt{\omega_0 |\tau|} + \mathcal{O}(\tau^2), & \omega_0 |\tau| \ll 1, \\ \ln 4 \frac{v_c D_f}{v_s \varpi} K_c^2 \ln(\omega_0 |\tau|) + \mathcal{O}(1), & \omega_0 |\tau| \gg 1. \end{cases} \quad (17)$$

The full behavior of  $W(x,0)$  is presented in Fig. 1. The exponential suppression, at large distances, of the charge-

and spin-density-wave correlations due to forward scattering is similar to that in the case of a static string. Note that the static limit result is obtained by taking  $\omega_0 \rightarrow \infty$  in order to keep the string in its ground state, at least as far as the low-energy electronic physics is concerned. Since  $\mathcal{D}_f \propto \lambda^{-1}$ , the latter limit implies the need to scale  $D_f$  with  $\lambda$  in order to remain in the perturbative regime, where our results are valid. In contrast to the case of the static string the fluctuations also increase the algebraic decay of  $R_{\text{CDW}}$  and  $R_{\text{SDW}}$  in the time domain and lead to a decrease in the conductivity and pairing correlations. We will present results pertaining to this latter issue elsewhere.

In order to derive the flow equations of the string parameters we first average the action, Eq. (6), over the electronic degrees of freedom. In accordance with the procedure we have utilized previously, here, too, we distinguish between electronic degrees of freedom only as long as their time separation is larger than  $a/v_s$ . Consequently, for shorter separations the electronic operators which appear in the forward- and backward-scattering parts of the action, Eq. (6), become  $(\partial_x \phi_c)^2$  and  $[(\partial_x \theta_c)^2 - (\partial_x \phi_c)^2 + (\partial_x \theta_s)^2 - (\partial_x \phi_s)^2]/4\pi + 1/(2\pi^2 a^2) \cos(\sqrt{8\pi} \phi_s)$ , respectively. The averages of these operators diverge quadratically and need to be regularized using an appropriate cutoff (except for the cosine operator which vanishes whenever the system is in a spin-gapless phase). To do so, we note that in the course of the renormalization procedure the short time piece of the integral is being constructed by adding infinitesimal slices from the initial cutoff  $a_0$  to the running value  $a$ . We therefore evaluate the electronic averages within each slice using the value of the cutoff at the time the slice was added along the renormalization process. The outcome is the following effective action for the string degrees of freedom:

$$\begin{aligned} \bar{S}_s = \int d\tau S_s - L \left\{ \frac{D_f K_c}{2(\pi v_c)^2} + \frac{D_b}{8\pi^2} \left[ \frac{1}{v_c^2} \left( \frac{1}{K_c} - K_c \right) \right. \right. \\ \left. \left. + \frac{1}{v_s^2} \left( \frac{1}{K_s} - K_s \right) \right] \right\} \int_{a/v_s > |\tau - \tau'| > a_0/v_s} d\tau d\tau' \frac{\delta[u(\tau) - u(\tau')]}{(\tau - \tau')^2} \\ - L \int_{|\tau - \tau'| > a/v_s} d\tau d\tau' \delta[u(\tau) - u(\tau')] \left[ \frac{D_f}{2\pi^2 v_c^2} \frac{K_c}{(\tau - \tau')^2} \right. \\ \left. + \frac{D_b}{2(\pi a)^2} \left( \frac{a}{v_c |\tau - \tau'|} \right)^{K_c} \left( \frac{a}{v_s |\tau - \tau'|} \right)^{K_s} \right]. \end{aligned} \quad (18)$$

In Appendix B we use this action to derive the flow equations for  $\varpi$  and  $\lambda$ . Using the dimensionless string length

$$\mathcal{L} = \frac{L}{2\pi a}, \quad (19)$$

the result is

$$\begin{aligned} \frac{d\varpi}{d\ell} &= \varpi + \mathcal{D}_b \mathcal{L} \varpi \left[ [1 - f(v, K)] \frac{1 - \cosh \varpi}{\varpi (1 - e^{-\varpi})^{3/2}} \right. \\ &\quad \left. - f(v, K) (2 - K_c - K_s - y) \int_{e^{-\ell/\varpi}}^{\varpi} d\eta \frac{1 - \cosh \eta}{\eta^2 (1 - e^{-\eta})^{3/2}} \right], \\ \frac{d\lambda}{d\ell} &= - \frac{\mathcal{D}_b \mathcal{L} \lambda}{2} \left[ [1 - f(v, K)] \frac{1 - \cosh \varpi + \varpi \sinh \varpi}{\varpi (1 - e^{-\varpi})^{3/2}} - f(v, K) \right. \\ &\quad \left. \times (2 - K_c - K_s - y) \int_{e^{-\ell/\varpi}}^{\varpi} d\eta \frac{1 - \cosh \eta + \eta \sinh \eta}{\eta^2 (1 - e^{-\eta})^{3/2}} \right], \\ \frac{d\mathcal{L}}{d\ell} &= -\mathcal{L}, \end{aligned} \quad (20)$$

where we have introduced the compact notation

$$f(v, K) \equiv \frac{1}{4} \left( \frac{v_c}{v_s} \right)^{K_c} \left[ \left( \frac{v_s}{v_c} \right)^2 \left( \frac{1}{K_c} - K_c \right) + \frac{1}{K_s} - K_s \right]. \quad (21)$$

Equations (14) and (20) describe the interplay between the electronic interactions, disorder, and string dynamics. In the present model the critical point which separates the regimes of relevant and irrelevant disorder, in the limit of extremely weak bare impurity potential  $D_b \rightarrow 0$ , is insensitive to the string fluctuations and is given by the condition

$$3 - K_c - K_s^* = 0, \quad (22)$$

where  $K_s^*$  is the renormalized value of  $K_s$  due to the coupled flow with  $y$ . However, for finite disorder, fluctuation-induced delocalizing effects are present and are manifested in two ways. First, since  $\mathcal{D}_b \propto \lambda^{-1}$  [see Eq. (15)], large-amplitude oscillations tend to decrease the initial value of the effective disorder strength. Second, the oscillations frequency defines a crossover scale  $\varpi \sim 1$ , below which the disorder is more effective in renormalizing the interactions towards more repulsive values, as can be seen from the flow equation for  $K_c$ . Since strong attractive interactions are necessary to drive the system into a delocalized phase, decreasing the oscillation frequency makes this eventuality less likely to occur. The combined outcome of these two mechanisms is an increase in the extent of the delocalized region as one increases the oscillation amplitude and frequency. The effect of varying the latter is presented in Fig. 2 where we plot the separatrix between the regimes of relevant and irrelevant disorder for several oscillation frequencies while keeping  $\lambda$  fixed.

The analysis of the various flow regimes, as determined by the renormalization equations, is similar to the one presented in Ref. 3, and we comment on it briefly. When both  $\mathcal{D}_b$  and  $y$  flow to zero the system approaches a fixed line parametrized by  $K_c^* \geq 2$  (in the spin rotationally invariant case). This line corresponds to a delocalized phase with dominant triplet pairing correlations. When  $y$  is initially small or negative it flows to large negative values and pins the spin field  $\phi_s$ . At the same time  $\mathcal{D}_b$  may still be irrelevant provided its bare value is small and  $K_c$  is large. For infinitesimal disorder the latter requirement becomes  $K_c > 3$ , implying that the resulting delocalized phase is dominated by

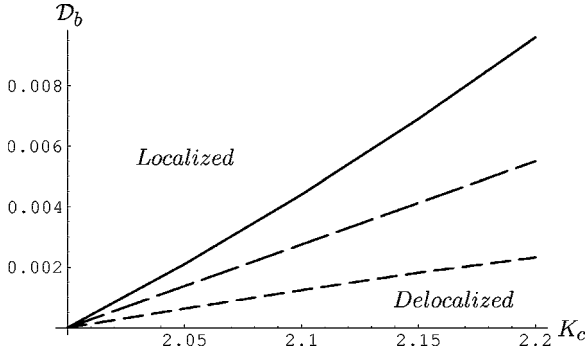


FIG. 2. The separatrix between the localized and delocalized phases in the  $K_c$ - $\mathcal{D}_b$  plane for different values of the string fluctuation frequency:  $\varpi=1$  (solid line),  $\varpi=0.1$  (long-dashed line), and  $\varpi=0.01$  (dashed line). Here  $v_c=v_F=1$ ,  $y=0.1$ ,  $K_s=\sqrt{(1+y/2)/(1-y/2)}$ ,  $v_s=\sqrt{1-(y/2)^2}$ , and  $L=10$ . For infinitesimal disorder the critical point separating the localized and delocalized regimes is identical to the static one. However, for finite disorder the size of the delocalized region is increased by the string fluctuations.

singlet pairing fluctuations. In both cases the string parameters  $\omega_0$  and  $\lambda$  decrease but attain finite fixed values corresponding to a still fluctuating string.

When  $\mathcal{D}_b$  scales to infinity the perturbative renormalization equations can no longer be trusted. This strong-coupling regime contains the noninteracting system, which in the static case is known to be localized, based on exact calculations. As long as one assumes that there is no additional fixed point at intermediate coupling this fact implies that the entire strong-coupling phase is localized. For the fluctuating string no such exact solutions are available. However, the noninteracting problem is equivalent to that of a particle (with anisotropic mass) moving inside a two-dimensional disordered potential and a harmonic well in the  $y$  direction. Its wave function  $\Psi(x, u)$  is believed to be localized for any strength of the disorder, implying localization of the electrons and pinning of the string. The latter effect is also reflected in the renormalization equations for the string parameters, which in the case of relevant backward electronic scattering, flow towards  $\omega_0 \rightarrow 0$  and  $\lambda \rightarrow 0$ . The nature of the localized state depends on the sign of the renormalized coupling  $g_1^*$ . The system is expected<sup>3</sup> to consist of localized pairs of spins for  $g_1^* < 0$  and of isolated spins interacting via antiferromagnetic superexchange when  $g_1^* > 0$ .

### III. ELASTIC STRING

#### A. The model

Here we consider an elastic string whose projection on the  $x$  axis is of fixed length  $L_x$  and which obeys periodic boundary conditions in this direction. We consider the limit of a stiff and massive string such that we can ignore overhangs and describe its state by the deviation  $u(x, \tau)$  from the classical equilibrium configuration. The string dynamics is governed by the Lagrangian

$$L_s = \int_0^{L_x} dx \left[ \frac{\rho}{2} (\partial_\tau u)^2 + \frac{\sigma}{2} (\partial_x u)^2 \right], \quad (23)$$

in which it is characterized by its linear mass density  $\rho$  and tension  $\sigma$ . Alternatively, the string can be described by the velocity of the elastic waves which it carries,

$$v_u = \sqrt{\frac{\sigma}{\rho}}, \quad (24)$$

and by the length

$$\lambda = \sqrt{\frac{v_u}{2\pi\sigma}}. \quad (25)$$

The latter is related to the average slope of the string, relative to its equilibrium configuration, according to

$$\sqrt{\langle (\partial_x u)^2 \rangle} = \lambda/a. \quad (26)$$

The condition of a stiff and massive string implies  $\lambda \ll a$ .

In a previous publication, Ref. 14, one of us has demonstrated that the constraint which forces the particles to remain on the string induces a nontrivial metric for the electrons and couples them to effective gauge potentials, which are functions of the string degrees of freedom. In the absence of external electromagnetic fields the metric and gauge potentials are given by

$$g(x, \tau) = 1 + (\partial_x u)^2,$$

$$A_0(x, \tau) = -\frac{m}{2e} (\partial_\tau u)^2, \quad A_1(x, \tau) = -\frac{imc}{e} \partial_x u \partial_\tau u. \quad (27)$$

Assuming a fixed number of electrons,  $N_e$ , on the string we require that their projected density  $n_x = N_e/L_x$  or, equivalently, their projected Fermi wave-vector  $k_{F_x} = \pi n_x/2$  obey  $k_{F_x} a \gtrsim 1$ , such that the  $2k_{F_x}$  components of the gauge potentials and the metric may be neglected. We also assume that the energy associated with the short-wavelength string waves obeys  $v_u/a \ll E_{F_x} = k_{F_x}^2/2m = v_{F_x} k_{F_x}/2$ . Consequently, we disregard backward scattering due to string fluctuations. Under these conditions the electronic Lagrangian is

$$L_e = \int_0^{L_x} dx \left\{ \sum_{\alpha=c,s} \left[ -i \partial_\tau \phi_\alpha \partial_x \theta_\alpha + \frac{\tilde{v}_\alpha \tilde{K}_\alpha}{2} (\partial_x \theta_\alpha)^2 + \frac{\tilde{v}_\alpha}{2\tilde{K}_\alpha} (\partial_x \phi_\alpha)^2 \right] + g^{-1/2} \frac{g_{1\perp}}{2(\pi a)^2} \cos(\sqrt{8}\pi\phi_s) - \left[ eA_0 - g^{-1}E_{F_x} - \frac{1}{32m} g^{-3} (\partial_x g)^2 \right] \left( n_x + \sqrt{\frac{2}{\pi}} \partial_x \phi_c \right) - \tilde{v}_c \tilde{K}_c \sqrt{\frac{2}{\pi}} \frac{e}{c} A_1 \partial_x \theta_c + \tilde{v}_c \tilde{K}_c \frac{e^2}{\pi c^2} A_1^2 \right\}. \quad (28)$$

Here  $\tilde{v}_\alpha$  and  $\tilde{K}_\alpha$  are derived from the previously defined quantities in Eq. (3) via the replacements  $v_F \rightarrow g^{-1}v_{F_x}$  and  $g_{1\parallel,2,4} \rightarrow g^{-1/2}g_{1\parallel,2,4}$  [this is a result of assuming short-range interactions of the form  $V(s-s') \propto \delta(s-s')$ , where  $s$  is the arclength parameter along the string].

The terms involving  $n_x$  in Eq. (28) induce a renormalization of the bare string parameters at the tree level. The first of them is identically  $mn_x \int_0^{L_x} dx (\partial_x u)^2/2$  and expresses the fact the electrons are dragged with the string as it moves, thereby increasing its effective mass density

$$\rho \rightarrow \rho + mn_x. \quad (29)$$

When expanded to second order in derivatives of  $u$ , the second term reads  $E_{Fx} n_x - 2E_{Fx} n_x \int_0^{L_x} dx (\partial_x u)^2/2$ . It encodes the fact that when the number of electrons is fixed, their total kinetic energy is lowered as the string fluctuates and becomes longer on the average. This gain in kinetic energy favors a more flexible string through the renormalization of its tension according to

$$\sigma \rightarrow \sigma - 2E_{Fx} n_x. \quad (30)$$

In the following we assume that these effects are small and have already been incorporated into the definitions of the string parameters which appear in Eq. (23).

### B. Renormalization and phase diagram

In order to derive the renormalization equations for the electronic parameters we integrate over the string configurations. Using a cumulant expansion to average the electronic part of the action, Eq. (28), we find that to second order in  $\lambda/a$ , the result is of the ordinary Luttinger type, Eq. (2). In the averaged action the velocities  $v_\alpha$  are related to the values  $v_{\alpha x}$  they would have in a static straight string of length  $L_x$  with Fermi velocity  $v_{Fx}$  and the same interactions, according to

$$v_\alpha = v_{\alpha x} - \left(\frac{\lambda}{a}\right)^2 \left[ \frac{v_{\alpha x}}{2} + \frac{v_{Fx}}{4} \left( \frac{1}{K_{\alpha x}} + K_{\alpha x} \right) \right], \quad (31)$$

while the Luttinger parameters and  $g_{1\perp}$  are renormalized from their values in the straight system as

$$K_\alpha = K_{\alpha x} - \left(\frac{\lambda}{a}\right)^2 \frac{v_{Fx}}{v_{\alpha x}} (1 - K_{\alpha x}^2),$$

$$g_{1\perp} = g_{1\perp x} - \frac{1}{2} \left(\frac{\lambda}{a}\right)^2 g_{1\perp x}. \quad (32)$$

This renormalization is geometric in origin, as explained in Ref. 14, and is absent (apart from a residual correction to the velocities) when one keeps the electronic density along the string, rather than the number of electrons, fixed. We will therefore not discuss it further. Higher-order terms in the small parameter  $\lambda/a$ , which are generated in the averaging process, are typically nonlocal and can be shown to be irrelevant in the renormalization group sense.

Since in the present model  $\langle [u(x, \tau) - u(x, \tau')]^2 \rangle = 2\lambda^2 \ln(1 + v_u |\tau - \tau'|/a)$ , the function

$$F(\tau) = \frac{1}{2\sqrt{\pi\lambda}} \ln^{-1/2} \left( 1 + \frac{v_u |\tau|}{a} \right), \quad (33)$$

which appears in the averaged disorder part of the action, Eqs. (10) and (11), lacks a cutoff-independent length scale; nor is it a power law. Consequently, an attempt to carry out a renormalization procedure for  $\delta$ -correlated Gaussian disorder, as was done for the rigid string, runs into the problem of not yielding a simple multiplicative renormalization. Stated differently, we are unable to assign a single scaling dimension to the disorder operator. In order to overcome this difficulty we consider instead the more general case of correlated impurity potentials

$$\begin{aligned} \overline{\langle \eta(x, y) \eta(x', y') \rangle} &= \delta(x - x') D_f(y - y') \\ &= \delta(x - x') \int dk D_f(k) e^{ik(y - y')}, \\ \overline{\langle \xi^*(x, y) \xi(x', y') \rangle} &= \delta(x - x') D_b(y - y') \\ &= \delta(x - x') \int dk D_b(k) e^{ik(y - y')}, \end{aligned} \quad (34)$$

and employ functional renormalization to study the flow of  $D_{b,f}(y - y')$ . As a result of the different disorder correlations one should replace

$$\begin{aligned} D_{b,f} F(\tau) &\rightarrow \int dk D_{b,f}(k) e^{-(k^2/2) \langle [u(0, \tau) - u(0, 0)]^2 \rangle} \\ &= \int dk D_{b,f}(k) \left( \frac{v_u |\tau|}{a} \right)^{-\lambda^2 k^2}, \end{aligned} \quad (35)$$

in Eqs. (11) and (12). Note that the last equality holds approximately for  $v_u |\tau|/a > 1$  and is therefore appropriate for use in the effective action, Eq. (11). For a consistent treatment we have to use the same form in the relations, Eqs. (12), where in order to assure convergence of the  $\tau$  integral we need to assume  $D_{b,f}(k) = 0$  for  $|k| > \lambda^{-1}$ . Since at the bare level our stiff string satisfies  $\lambda \ll a$  and, as we shall demonstrate,  $\lambda$  tends to decrease in the course of the renormalization, this requirement is not stringent. The fact that the temporal correlations introduced by each disorder Fourier component are algebraic makes it possible to follow the same renormalization scheme we have used in the case of the rigid string. Finally, we comment that because both the electronic and disorder parts of the action contain the string coordinates, averaging over the latter introduces cross-terms between the two parts. These terms, however, are higher derivative terms compared to those originating from the averaged disorder part and are therefore less relevant. We will therefore neglect them in the following.

Introducing the dimensionless disorder components

$$\mathcal{D}_b(k) = \frac{2D_b(k)a}{\pi v_s^2 \lambda} \left(\frac{v_s}{v_c}\right)^{K_c} \left(\frac{v_u}{v_s}\right)^{-\lambda^2 k^2},$$

$$\mathcal{D}_f(k) = \frac{2D_f(k)a}{\pi v_c^2 \lambda} \left(\frac{v_u}{v_s}\right)^{-\lambda^2 k^2}, \quad (36)$$

we obtain the following renormalization equations

$$\begin{aligned} \frac{dK_c}{d\ell} &= -\frac{1}{2} \frac{v_c}{v_s} \int dk \lambda \left[ K_c^2 - \frac{1}{2} \left(\frac{v_c}{v_s}\right)^{K_c-2} \right. \\ &\quad \times \left. \left( \frac{3-K_c-K_s-y-\lambda^2 k^2}{1-\lambda^2 k^2} \right) (1+K_c^2) \right] \mathcal{D}_b(k), \\ \frac{dv_c}{d\ell} &= -\frac{1}{2} \frac{v_c^2}{v_s} \int dk \lambda \left[ K_c - \frac{1}{2} \left(\frac{v_c}{v_s}\right)^{K_c-2} \right. \\ &\quad \times \left. \left( \frac{3-K_c-K_s-y-\lambda^2 k^2}{1-\lambda^2 k^2} \right) \left( \frac{1}{K_c} - K_c \right) \right] \mathcal{D}_b(k), \\ \frac{dK_s}{d\ell} &= -\frac{1}{2} y^2 K_s^2 - \frac{1}{2} K_s^2 \int dk \lambda \mathcal{D}_b(k) + \frac{1}{4} \left(\frac{v_c}{v_s}\right)^{K_c} \\ &\quad \times \int dk \lambda \frac{(3-K_c-K_s-y-\lambda^2 k^2)(1+K_s^2) + 4K_s^2 y}{1-\lambda^2 k^2} \mathcal{D}_b(k), \\ \frac{dv_s}{d\ell} &= -\frac{v_s}{2} \int dk \lambda \left[ K_s - \frac{1}{2} \left(\frac{v_c}{v_s}\right)^{K_c} \left( \frac{3-K_c-K_s-y-\lambda^2 k^2}{1-\lambda^2 k^2} \right) \right. \\ &\quad \times \left. \left( \frac{1}{K_s} - K_s \right) \right] \mathcal{D}_b(k), \\ \frac{dy}{d\ell} &= (2-2K_s)y - \left\{ 1 - \left(\frac{v_c}{v_s}\right)^{K_c} \left[ 1 - \frac{y}{4} \left( \frac{1}{K_s} - K_s \right) \right] \right\} \\ &\quad \times \int dk \lambda \mathcal{D}_b(k) + \frac{1}{4} \left(\frac{v_c}{v_s}\right)^{K_c} \\ &\quad \times \left\{ 4(K_s - K_c) - y \left[ 1 - K_s^2 + (2-K_c) \right. \right. \\ &\quad \times \left. \left. \left( \frac{1}{K_s} - K_s \right) \right] \right\} \int dk \lambda \frac{\mathcal{D}_b(k)}{1-\lambda^2 k^2}, \\ \frac{d\mathcal{D}_f(k)}{d\ell} &= (1-\lambda^2 k^2) \mathcal{D}_f(k), \\ \frac{d\mathcal{D}_b(k)}{d\ell} &= (3-K_c-K_s-y-\lambda^2 k^2) \mathcal{D}_b(k). \quad (37) \end{aligned}$$

The flow equations for the string parameters may be derived along the lines indicated in Appendix B with the result

$$\begin{aligned} \frac{d\lambda}{d\ell} &= -\frac{1}{4} \frac{v_u}{v_s} \lambda \left\{ [1-f(v,K)] \int dk \lambda^3 k^2 \mathcal{D}_b(k) - f(v,K)(2-K_c \right. \\ &\quad \left. - K_s - y) \int dk \lambda \frac{k^2 \lambda^2}{1-k^2 \lambda^2} [1 - e^{-(1-k^2 \lambda^2) \ell}] \mathcal{D}_b(k) \right\}, \\ \frac{dv_u}{d\ell} &= -\frac{1}{2} \frac{v_u^2}{v_s} \left\{ [1-f(v,K)] \int dk \lambda^3 k^2 \mathcal{D}_b(k) - f(v,K)(2-K_c \right. \\ &\quad \left. - K_s - y) \int dk \lambda \frac{k^2 \lambda^2}{1-k^2 \lambda^2} [1 - e^{-(1-k^2 \lambda^2) \ell}] \mathcal{D}_b(k) \right\}, \quad (38) \end{aligned}$$

where  $f(v,K)$  is given by Eq. (21).

It is evident from the flow equation for the disorder that high-wave-vector components in the correlation function of the scattering potential are less relevant. Consequently, the boundary separating the regions of relevant and irrelevant disorder is determined by the lowest-wave-vector components that are present in the correlation function. For a very weak bare disorder which is  $\delta$  correlated in real space, this would mean that while it acquires, in the course of the renormalization, correlations over longer length scales, the critical point is still given by the condition  $3-K_c-K_s^*=0$ , as in the cases of the static and rigidly oscillating strings. The  $k$  dependence of the disorder scaling dimension also implies that the flow of the electronic parameters in Eq. (37) is dominated by the  $k \sim 0$  components, for which it coincides with that of a static string. Therefore, the effects of string fluctuations enter predominantly via the dependence of the dimensionless disorder strength, Eq. (36), on  $\lambda^{-1}$ . As fluctuations increase, and with them  $\lambda$ , the initial conditions of the flow, for a given scattering potential, shift towards lower values of  $\mathcal{D}_b$ . As a result the extent of the region in which the disorder is irrelevant increases as well. Contrary to the case of the rigid oscillating string there is no available solution which describes the present system in the region of relevant disorder. We are therefore unable to positively identify the nature of the strongly disordered phase. It seems likely, however, that it is localized with magnetic properties which depend on the flow of  $y$  in the manner discussed at the end of the previous section.

Considering the impurity forward-scattering contribution to the suppression of the CDW and SDW correlations we obtain, after taking  $D_f(k) = D_f/2\pi$ , the following results for the function  $W(x, \tau)$  defined previously:

$$W(x,0) = \begin{cases} \frac{3}{\sqrt{2\pi}} \frac{D_f K_c^2 a}{v_c v_u \lambda} \sqrt{\frac{v_u x}{v_c a}} + \mathcal{O}(x^2), & |x| \ll \frac{v_c}{v_u} a, \\ \frac{1}{\sqrt{\pi}} \frac{D_f K_c^2 |x|}{v_c^2 \lambda} \\ \times \left[ \ln^{-1/2} \left| \frac{v_u x}{v_c a} \right| + \mathcal{O}(\ln^{-3/2} |x|) \right], & |x| \gg \frac{v_c}{v_u} a, \end{cases} \quad (39)$$

and



$$W(0, \tau) = \begin{cases} \frac{1}{\sqrt{\pi}} \frac{D_f K_c^2 a}{v_c v_u \lambda} \sqrt{\frac{v_u |\tau|}{a}} + \mathcal{O}(\tau^2), & v_u |\tau| \ll a, \\ \frac{4.95}{4\pi^{3/2}} \frac{D_f K_c^2}{v_c \lambda} |\tau| \ln^{-3/2} \left| \frac{v_u \tau}{a} \right| \\ + \mathcal{O} \left( \ln^{-1/2} \left| \frac{v_u \tau}{a} \right| \right), & v_u |\tau| \gg a. \end{cases} \quad (40)$$

Comparing these results with the corresponding behavior in the case of a rigid string, Eqs. (16) and (17) and Fig. 1, one finds that the more pronounced fluctuations of the elastic string reduce the effects of the forward-scattering part of the disorder, thus leading to a slower decay of the CDW and SDW correlations.

#### IV. FLOPPY STRING

##### A. The model

An obvious way to increase the degree of string fluctuations in the model is to replace the elastic energy term in the string Lagrangian, Eq. (23), by a new term involving higher spatial derivatives of the displacement function  $u$ . Such a simple modification, however, is insufficient, since, as we have demonstrated in the discussion preceding Eq. (30), the dependence of the electronic kinetic energy on the string length will induce such an elastic term even if it is absent from the bare string dynamics.<sup>15</sup> In order to avoid this effect we need to consider a string of fixed length  $L$  and average electronic density  $n$ . We assume that the string is held fixed at one of its end points (which we take as the origin) and that its motion is constrained to be periodic in the  $y$  direction—i.e., that both ends are always at  $y=0$ . Since one end is free to slide along the  $x$  direction, we are unable to characterize the configuration of the string by a single displacement function  $u(x, \tau)$ . Instead we use the arclength  $s \in [0, L]$  to parametrize the string position  $\vec{R}(s, \tau) = [X(s, \tau), Y(s, \tau)]$  in the plane. Taking the bending energy of the string to depend on its curvature  $\sqrt{(\partial^2 X / \partial s^2)^2 + (\partial^2 Y / \partial s^2)^2}$  one obtains

$$L_s = \int_0^L ds \left[ \frac{\rho}{2} \left( \frac{\partial \vec{R}}{\partial \tau} \right)^2 + \frac{\gamma}{2} \left( \frac{\partial^2 \vec{R}}{\partial s^2} \right)^2 \right], \quad (41)$$

together with the constraint

$$(\partial_s X)^2 + (\partial_s Y)^2 = 1, \quad (42)$$

reflecting our choice of parametrization.

To make progress we need to assume that the string is massive and possesses a large bending modulus  $\gamma$ , such that its fluctuations about the  $x$  axis take place over large length scales. Specifically we require that

$$\bar{\lambda} \equiv \frac{\lambda}{a} = \frac{1}{a\sqrt{\rho\gamma}} \ll 1. \quad (43)$$

As we now demonstrate, under this condition the constraint, Eq. (42), may be solved approximately  $\partial_s X \approx 1 - \frac{1}{2}(\partial_s Y)^2$ , to imply, given the specified boundary conditions, that

$$X(s, \tau) = s + \Delta X(s, \tau) = s - \frac{1}{2} \int_0^s ds' [\partial_{s'} Y(s', \tau)]^2. \quad (44)$$

Using this result in the string Lagrangian, Eq. (41), one indeed finds,<sup>16</sup> to lowest order in  $\bar{\lambda}$ ,

$$\langle (\partial_s Y)^2 \rangle = \sqrt{\frac{\bar{\lambda}}{2\pi^2}}, \quad (45)$$

in agreement with the assumption which led to the expansion, Eq. (44).

As long as the system maintains, in addition to condition (43), that its typical string wave velocity (given in this model by  $\gamma$ ) is such that  $\gamma/a < v_{FKF}$ , we may neglect backward scattering by string fluctuations and describe the low-energy electronic dynamics by the Lagrangian

$$L_e = \int_0^L ds \left\{ \sum_{\alpha=c,s} \left[ -i \partial_\tau \phi_\alpha \partial_s \theta_\alpha + \frac{v_\alpha K_\alpha}{2} (\partial_s \theta_\alpha)^2 + \frac{v_\alpha}{2K_\alpha} (\partial_s \phi_\alpha)^2 \right] + \frac{g_{1\perp}}{2(\pi a)^2} \cos(\sqrt{8\pi} \phi_s) - e A_0 \left( n + \sqrt{\frac{2}{\pi}} \partial_s \phi_c \right) - v_c K_c \sqrt{\frac{2}{\pi}} \frac{e}{c} A_1 \partial_s \theta_c + v_c K_c \frac{e^2}{\pi c^2} A_1^2 \right\}, \quad (46)$$

where, provided the electrons interact through contact interactions  $V(s-s') \propto \delta(s-s')$ , the velocities and Luttinger parameters are given by Eq. (3). While in the arclength parametrization the metric remains trivial, the electrons are still coupled to gauge potentials, which to second order in derivatives of  $Y$  read

$$A_0(s, \tau) = -\frac{m}{2e} (\partial_\tau \vec{R})^2 \approx \frac{m}{2e} (\partial_\tau Y)^2,$$

$$A_1(s, \tau) = -\frac{imc}{e} \partial_s \vec{R} \cdot \partial_\tau \vec{R} \approx -\frac{imc}{e} (\partial_s Y \partial_\tau Y + \partial_\tau \Delta X). \quad (47)$$

Note that the term proportional to  $n$  in Eq. (46) leads to the renormalization of the string mass density according to  $\rho \rightarrow \rho + mn$ . However, as expected, the coupling between the string and electrons does not modify the form of the string elastic energy term.

For a  $\delta$ -correlated random potential the averaged disorder part of the action contains the factor

$$D_{f,b} \delta[X(s, \tau) - X(s', \tau')] \delta[Y(s, \tau) - Y(s', \tau')] \approx D_{f,b} \delta(s-s') \delta[Y(s, \tau) - Y(s', \tau')] \times \left\{ 1 + \frac{1}{2} [\partial_s Y(s, \tau)]^2 \right\}, \quad (48)$$

in which, to lowest order in  $D_{f,b} \bar{\lambda}$ , we may replace the curly brackets by 1.

### B. Renormalization and phase diagram

Integrating out the string degrees of freedom from the electronic Lagrangian, Eq. (46), yields at the level of the first cumulant (quadratic in derivatives of  $Y$ ), a simple Luttinger liquid action, Eq. (2). The contribution from the second cumulant (fourth order in derivatives of  $Y$ ) modifies the values of  $v_c$ ,  $K_c$ , and the coefficient of the symplectic term in the action by a small amount of order  $(mv_c\lambda)^2\bar{\lambda}^{1/2}(\gamma/v_c)K_c$ . In addition, however, it also introduces the terms

$$\begin{aligned} & \frac{\beta L (mv_c\lambda K_c)^2}{8\pi} \frac{1}{\sqrt{\lambda\gamma}} \left\{ \frac{1}{2} \sum_{k,k' \neq 0, \omega} |\omega|^{3/2} \theta_c(k, \omega) \theta_c(k', -\omega) \right. \\ & + \sum_{k \neq 0, \omega} \frac{\omega^2}{[\lambda^2 \gamma^2 k^4 + \omega^2]^{1/4}} \cos \left[ \frac{1}{2} \arctan \left( \frac{2|\omega|}{\lambda \gamma k^2} \right) \right] \\ & \left. \times \theta_c(k, \omega) \theta_c(-k, -\omega) \right\}, \end{aligned} \quad (49)$$

of which the first originates from the breaking of translation invariance along the  $x$  direction due to the boundary conditions on the string. These terms are relevant in the long-wavelength limit and dominate over the quadratic terms in the Luttinger action for  $k \rightarrow 0$  and  $\omega < (K_c/4\pi)^2 (mv_c\lambda)^4 (v_c^2/\lambda\gamma) \equiv v_s/\bar{a}$ , with the effect of suppressing slow fluctuations of  $\theta_c$  (effectively increasing the value of  $K_c$  at long time scales). Based on the flow equations we have derived before we expect that this fact would lead to a reduction in the scaling dimension of the backward-scattering disorder at scales larger than  $\bar{a}$ . In the following we will ignore the contribution, Eq. (49) (and possibly other relevant terms which emerge at higher orders in the cumulant expansion), thereby limiting our renormalization treatment to the range  $a < \bar{a}$ . In that sense our results should be viewed as an upper limit on the domain of relevant disorder.

Finally, averaging the disorder part of the action over the string dynamics one obtains an expression similar to that of Eq. (11) with

$$F(\tau) = \frac{1}{\lambda} \left( \frac{\lambda}{4\pi\gamma|\tau|} \right)^{1/4}, \quad (50)$$

which can be treated according to the lines indicated in the Appendixes to yield the following renormalization equations for the electronic and string parameters:

$$\begin{aligned} \frac{dK_c}{d\ell} &= -\frac{1}{2} \frac{v_c}{v_s} \left[ K_c^2 - \frac{2}{3} \left( \frac{v_c}{v_s} \right)^{K_c-2} \right. \\ & \quad \left. \times \left( \frac{11}{4} - K_c - K_s - y \right) (1 + K_c^2) \right] \mathcal{D}_b, \\ \frac{dv_c}{d\ell} &= -\frac{1}{2} \frac{v_c^2}{v_s} \left[ K_c - \frac{2}{3} \left( \frac{v_c}{v_s} \right)^{K_c-2} \right. \\ & \quad \left. \times \left( \frac{11}{4} - K_c - K_s - y \right) \left( \frac{1}{K_c} - K_c \right) \right] \mathcal{D}_b, \end{aligned}$$

$$\begin{aligned} \frac{dK_s}{d\ell} &= -\frac{1}{2} y^2 K_s^2 + \frac{1}{3} \left( \frac{v_c}{v_s} \right)^{K_c} \left[ \left( \frac{11}{4} - K_c - K_s - y \right) (1 + K_s^2) \right. \\ & \quad \left. - \frac{3}{2} \left( \frac{v_s}{v_c} \right)^{K_c} K_s^2 + 4K_s^2 y \right] \mathcal{D}_b, \end{aligned}$$

$$\begin{aligned} \frac{dv_s}{d\ell} &= -\frac{1}{2} \left[ K_s - \frac{2}{3} \left( \frac{v_c}{v_s} \right)^{K_c} \right. \\ & \quad \left. \times \left( \frac{11}{4} - K_c - K_s - y \right) \left( \frac{1}{K_s} - K_s \right) \right] v_s \mathcal{D}_b, \end{aligned}$$

$$\begin{aligned} \frac{dy}{d\ell} &= (2 - 2K_s)y + \frac{1}{3} \left( \frac{v_c}{v_s} \right)^{K_c} \left\{ 3 \left[ 1 - \left( \frac{v_s}{v_c} \right)^2 \right] + 4(K_s - K_c) \right. \\ & \quad \left. - y \left[ 1 - K_s^2 + \left( \frac{11}{4} - K_c \right) \left( \frac{1}{K_s} - K_s \right) \right] \right\} \mathcal{D}_b, \end{aligned}$$

$$\frac{d\mathcal{D}_f}{d\ell} = \frac{3}{4} \mathcal{D}_f, \quad \frac{d\mathcal{D}_b}{d\ell} = \left( \frac{11}{4} - K_c - K_s - y \right) \mathcal{D}_b,$$

$$\begin{aligned} \frac{d\bar{\lambda}}{d\ell} &= -\bar{\lambda} - \left( \frac{\gamma}{\pi v_s} \right)^{1/2} \left\{ \frac{1}{4} - f(v, K) \left[ \frac{1}{4} + (2 - K_c - K_s - y) \right. \right. \\ & \quad \left. \left. \times (1 - e^{-\ell/4}) \right] \right\} \bar{\lambda}^{3/2} \mathcal{D}_b, \end{aligned}$$

$$\frac{d\gamma}{d\ell} = -\frac{1}{2} \left[ K_s - \frac{2}{3} \left( \frac{v_c}{v_s} \right)^{K_c} \left( \frac{11}{4} - K_c - K_s - y \right) \left( \frac{1}{K_s} - K_s \right) \right] \gamma \mathcal{D}_b, \quad (51)$$

where  $f(v, K)$  is given in Eq. (21) and where the dimensionless disorder amplitudes for this model are defined according to

$$\begin{aligned} \mathcal{D}_b &= \frac{\sqrt{2} D_b}{\pi^{5/4} v_s^2} \left( \frac{a}{\lambda} \right)^{3/4} \left( \frac{v_s}{\gamma} \right)^{1/4} \left( \frac{v_s}{v_c} \right)^{K_c}, \\ \mathcal{D}_f &= \frac{\sqrt{2} D_f}{\pi^{5/4} v_c^2} \left( \frac{a}{\lambda} \right)^{3/4} \left( \frac{v_s}{\gamma} \right)^{1/4}. \end{aligned} \quad (52)$$

The enhanced fluctuations of the floppy string do result in a shift of the critical point which demarcates the border between regions of relevant and irrelevant flow for weak initial disorder potential. The position of this point is given by the condition  $11/4 - K_c - K_s^* = 0$ , which for a spin rotationally invariant system and in the case where  $y$  flows to zero implies that the system is delocalized for  $K_c > 7/4$  [we note once again that due to our inability to treat terms such as Eq. (49), this should be viewed as a lower limit on the extent of the delocalized region]. This is to be contrasted with the previously studied models for which delocalization occurs under similar conditions for  $K_c > 2$ . For a generalized model of a stiff and massive fixed-length string with elastic energy that is proportional to  $(\partial^j \vec{R} / \partial s^n)^2$  one finds (ignoring potentially relevant terms which may arise in the course of the renormalization) a delocalized phase for  $K_c > (3n+1)/2n$ . In close

analogy to the situation in the other models we have considered in this study the fact that  $\mathcal{D}_b \propto \lambda^{-3/4} \gamma^{-1/4} = \rho^{3/8} \gamma^{1/8}$  implies that enhancing the string fluctuations by making  $\rho$  and  $\gamma$  smaller has the effect of shifting the initial conditions of the flow towards smaller values of  $\mathcal{D}_b$ , thereby increasing the extent of the region in parameter space in which the disorder is irrelevant.

Owing to the enhanced string fluctuations one also finds that the forward-scattering-induced decay of the CDW and SDW correlations is diminished (see Fig. 1) and is given by

$$W(x, 0) = \frac{7\pi}{6 \cos(\pi/8)} K_c^2 \mathcal{D}_f \left( \frac{v_c}{v_s} \right)^{1/4} \left( \frac{|x|}{a} \right)^{3/4},$$

$$W(0, \tau) = \frac{\sqrt{2}\pi}{6 \cos^2(\pi/8)} K_c^2 \mathcal{D}_f \left( \frac{v_c}{v_s} \right)^{1/4} \left( \frac{v_c |\tau|}{a} \right)^{3/4}, \quad (53)$$

at least for  $|x|, v_s |\tau| < \bar{a}$  such that we may neglect the higher-order corrections to the electronic effective action, Eq. (49).

## V. CONCLUSION AND DISCUSSION

In this paper we have set out to explore the way in which geometrical fluctuations of a one-dimensional interacting system affect its renormalization flow in the presence of a random scattering potential. We have found that by inducing temporal variations in the disorder, as seen in the reference frame of the electrons, the geometrical fluctuations increase the region in parameter space where the disorder is an irrelevant perturbation. This is a result of processes in which potential wells diminish in time, thereby releasing electrons that were trapped inside them. Other effects, such as decoherence of interference patterns which lead to localization, also exist but are not captured by our treatment. We expect, however, that the latter are less important in one dimension. Notwithstanding, unless the fluctuations are strong enough, the general features of the phase diagram are unchanged and the localization-delocalization transition for weak bare disorder remains at  $K_c=2$ , in the case of a gapless spin-invariant system. In the hierarchy of models studied by us only in the floppy string model has the critical point shifted towards weaker values of attractive interactions.

Our results thus demonstrate the relative robustness of disorder-induced localization effects in strictly one-dimensional systems. In order to suppress such localization effects it thus seems necessary to introduce some degree of two dimensionality into the system. This can be achieved by allowing the electrons to hop from one one-dimensional chain to the other. A simple realization is provided by ladders. For example, in the maximally gapped phase of a two-leg ladder<sup>17</sup> small disorder is an irrelevant perturbation when the Luttinger parameter  $K_{c+}$ , associated with the gapless total charge mode, is greater than  $3/2$ , thus smaller than for a single chain. In a two-leg bosonic ladder<sup>18</sup> or in the spin-gapped phase of a four-leg ladder<sup>19</sup> disorder is irrelevant for  $K_{c+} > 3/4$ . The results of introducing geometrical fluctuations to such ladder systems are yet to be studied. Understanding such effects would be very interesting, since, de-

spite the fact that the disorder is an irrelevant perturbation, a finite amount of disorder is still quite efficient in destroying the superconducting phase.<sup>17</sup> As we have seen for the case of a single chain, string fluctuations are quite efficient in diminishing the size of the localized region even when they are not able to change the critical point. One could thus expect particularly interesting effects of such fluctuations in the case of ladders.

## ACKNOWLEDGMENTS

This research was supported by the Israel Science Foundation (Grant No. 193/02-1). Part of this work was supported by the Swiss National Science Foundation under Division II.

## APPENDIX A: THE IMPURITY FORWARD-SCATTERING CONTRIBUTION TO THE FLOW EQUATIONS

Here we briefly demonstrate how the forward-scattering component of the disorder influences the renormalization of the electronic parameters of the rigid string effective action, Eq. (11). To this end, we consider the correlation function  $R_c(x_1 - x_2, \tau_1 - \tau_2) = \langle T_\tau e^{i\sqrt{2}\pi\phi_c(x_1, \tau_1)} e^{-i\sqrt{2}\pi\phi_c(x_2, \tau_2)} \rangle$ . To first order in the disorder strength one finds

$$R_c(x_1 - x_2, \tau_1 - \tau_2) = e^{-F_c(\vec{r}_1 - \vec{r}_2)} \left\{ 1 - 2D_f \left( \frac{\bar{K}_c}{2\pi} \right)^2 \sqrt{\frac{M\omega_0}{2\pi}} \int dx \right.$$

$$\times \int_{|\tau - \tau'| > av_s} d\tau d\tau' \frac{1}{\sqrt{1 - e^{-\omega_0|\tau - \tau'|}}} \times \left[ \frac{x - x_1}{(x - x_1)^2 + \bar{v}_c^2(\tau - \tau_1)^2} - \frac{x - x_2}{(x - x_2)^2 + \bar{v}_c^2(\tau - \tau_2)^2} \right]$$

$$\times \left[ \frac{x - x_1}{(x - x_1)^2 + \bar{v}_c^2(\tau' - \tau_1)^2} - \frac{x - x_2}{(x - x_2)^2 + \bar{v}_c^2(\tau' - \tau_2)^2} \right]$$

$$\left. + \text{backward - scattering contribution} \right\}, \quad (A1)$$

with

$$F_c(\vec{r}) = \frac{\bar{K}_c}{2} \ln \left( \frac{x^2 + \bar{v}_c^2 \tau^2}{a^2} \right) + \bar{f}_c \frac{\bar{v}_c^2 \tau^2}{x^2 + \bar{v}_c^2 \tau^2}, \quad (A2)$$

where initially  $\bar{f}_c = 0$ . After carrying out the integration over  $x$  and the center-of-mass coordinate  $(\tau + \tau')/2$  in Eq. (A1), one obtains

$$\begin{aligned}
& R_c(x_1 - x_2, \tau_1 - \tau_2) \\
&= e^{-F_c(\bar{r}_1 - \bar{r}_2)} \left\{ 1 - \frac{\bar{v}_c \bar{K}_c^2 \mathcal{D}_f}{4} \int_{|\eta| > a/v_s} \frac{d\eta}{a} \frac{1}{\sqrt{1 - e^{-(\varpi v_s/a)|\eta|}}} \right. \\
&\quad \times \left[ \ln \left[ \frac{(x_1 - x_2)^2 + \bar{v}_c^2 (\eta + \tau_1 - \tau_2)^2}{\bar{v}_c^2 \eta^2} \right] \right. \\
&\quad \left. \left. + \frac{2(x_1 - x_2)^2}{(x_1 - x_2)^2 + \bar{v}_c^2 (\eta + \tau_1 - \tau_2)^2} \right] \right. \\
&\quad \left. + \text{backward - scattering contribution} \right\}. \quad (\text{A3})
\end{aligned}$$

The flow equations are derived by requiring that the long-range behavior of  $R_c$  remains invariant under a change in the cutoff  $a \rightarrow a + da = a(1 + d\ell)$ . In order to maintain such invariance the missing contribution from the integration over  $(a + da)/v_s > |\eta| \geq a/v_s$  should be compensated by an appropriate change in  $\bar{K}_c$  and  $f_c$  (the latter is related<sup>3</sup> to a change in  $\bar{v}_c$ ). The invariance of the remaining integral over  $|\eta| > (a + da)/v_s$  determines the flow of  $\mathcal{D}_f$  and  $\varpi$ . The equations for  $\bar{K}_s$ ,  $\bar{v}_s$ , and  $\bar{y}$  can be derived in a similar manner by considering the correlation function  $R_s(x_1 - x_2, \tau_1 - \tau_2) = \langle T e^{i\sqrt{2\pi}\phi_s(x_1, \tau_1)} e^{-i\sqrt{2\pi}\phi_s(x_2, \tau_2)} \rangle$ . They, however, do not contain the forward-scattering amplitude. Including the contribution stemming from the backward-scattering piece, as described in Ref. 3, one arrives at

$$\frac{d\bar{K}_c}{d\ell} = -\frac{v_c}{v_s} \frac{1}{\sqrt{1 - e^{-\varpi}}} \left( \frac{\mathcal{D}_b}{2} - \mathcal{D}_f \right) K_c^2,$$

$$\frac{d\bar{v}_c}{d\ell} = -\frac{v_c^2}{v_s} \frac{1}{\sqrt{1 - e^{-\varpi}}} \left( \frac{\mathcal{D}_b}{2} + \mathcal{D}_f \right) K_c,$$

$$\frac{d\bar{K}_s}{d\ell} = -\frac{1}{2} \bar{y}^2 \bar{K}_s^2 - \frac{\mathcal{D}_b K_s^2}{2\sqrt{1 - e^{-\varpi}}},$$

$$\frac{d\bar{v}_s}{d\ell} = -\frac{\mathcal{D}_b K_s v_s}{2\sqrt{1 - e^{-\varpi}}},$$

$$\frac{d\bar{y}}{d\ell} = (2 - 2\bar{K}_s)\bar{y} - \frac{\mathcal{D}_b}{\sqrt{1 - e^{-\varpi}}},$$

$$\frac{d\mathcal{D}_b}{d\ell} = (3 - K_c - K_s - y)\mathcal{D}_b,$$

$$\frac{d\mathcal{D}_f}{d\ell} = \mathcal{D}_f, \quad \frac{d\varpi}{d\ell} = \varpi + \mathcal{O}(\mathcal{D}). \quad (\text{A4})$$

These results are readily extended to the cases of the elastic and floppy strings by replacing the expression for  $F(\tau)$  in Eqs. (A1) and (A3) with its appropriate form for these models, Eqs. (35) and (50).

#### APPENDIX B: THE FLOW EQUATIONS FOR THE STRING PARAMETERS

To obtain the flow equation for the rigid string parameters we consider the correlation function  $R_u(\tau_1 - \tau_2) = \langle T u(\tau_1) u(\tau_2) \rangle$ , where the average is with respect to the effective action, Eq. (18). To first order in the disorder strength one finds

$$\begin{aligned}
R_u(\Delta r) &= \lambda^2 e^{-\varpi|\Delta r|/a} \left\{ 1 - \mathcal{D}_b \mathcal{L} f(v, K) \int_{a_0}^a \frac{d\eta}{a} \frac{1}{(1 - e^{-\varpi\eta/a})^{3/2}} \left( \frac{a}{\eta} \right)^2 \left[ \left( \frac{1}{\varpi} + \frac{|\Delta r|}{a} \right) \left( 1 - \cosh \frac{\varpi\eta}{a} \right) + \frac{\eta}{a} \sinh \frac{\varpi\eta}{a} \right] \right. \\
&\quad - \mathcal{D}_b \mathcal{L} \int_a^{|\Delta r|} \frac{d\eta}{a} \frac{1}{(1 - e^{-\varpi\eta/a})^{3/2}} \left( \frac{a}{\eta} \right)^{K_c + K_s} \left[ \left( \frac{1}{\varpi} + \frac{|\Delta r|}{a} \right) \left( 1 - \cosh \frac{\varpi\eta}{a} \right) + \frac{\eta}{a} \sinh \frac{\varpi\eta}{a} \right] \\
&\quad \left. - \mathcal{D}_f \mathcal{L} K_c \int_{a_0}^{|\Delta r|} \frac{d\eta}{a} \frac{1}{(1 - e^{-\varpi\eta/a})^{3/2}} \left( \frac{a}{\eta} \right)^2 \left[ \left( \frac{1}{\varpi} + \frac{|\Delta r|}{a} \right) \left( 1 - \cosh \frac{\varpi\eta}{a} \right) + \frac{\eta}{a} \sinh \frac{\varpi\eta}{a} \right] \right\} \\
&\quad - \lambda^2 \mathcal{L} \int_{|\Delta r|}^{\infty} \frac{d\eta}{a} \frac{1}{(1 - e^{-\varpi\eta/a})^{3/2}} \left[ \mathcal{D}_b \left( \frac{a}{\eta} \right)^{K_c + K_s} + \mathcal{D}_f K_c \left( \frac{a}{\eta} \right)^2 \right] \\
&\quad \times \left\{ \left( \frac{1}{\varpi} + \frac{|\Delta r|}{a} \right) e^{-\varpi|\Delta r|/a} + e^{-\varpi\eta/a} \left[ \frac{|\Delta r|}{a} \sinh \frac{\varpi|\Delta r|}{a} - \left( \frac{1}{\varpi} + \frac{\eta}{a} \right) \cosh \frac{\varpi|\Delta r|}{a} \right] \right\}, \quad (\text{B1})
\end{aligned}$$

where  $\Delta r \equiv v_s(\tau_1 - \tau_2)$  is assumed larger than  $a$  and  $f(v, k)$  is given in Eq. (21).

The last two integrals in Eq. (B1) are invariant under a change in the cutoff  $a \rightarrow a + da$  provided one uses the previously derived renormalization equations for the electronic and disorder parameters, Eqs. (14), and assumes  $d\varpi/d\ell = \varpi + \mathcal{O}(\mathcal{D})$ . The remaining terms may be brought to the form  $\lambda_{\text{eff}}^2 e^{-\varpi_{\text{eff}}|\Delta r|/a}$  with

$$\begin{aligned}\varpi_{\text{eff}} &= \varpi + \mathcal{D}_b \mathcal{L} \left[ f(v, K) \int_{a_0}^a \frac{d\eta}{a} \frac{1 - \cosh(\varpi \eta/a)}{(1 - e^{-\varpi \eta/a})^{3/2}} \left(\frac{a}{\eta}\right)^2 + \int_a^{|\Delta\tau|} \frac{d\eta}{a} \frac{1 - \cosh(\varpi \eta/a)}{(1 - e^{-\varpi \eta/a})^{3/2}} \left(\frac{a}{\eta}\right)^{K_c + K_s} \right], \\ \lambda_{\text{eff}}^2 &= \lambda^2 - \frac{\mathcal{D}_b \mathcal{L} \lambda^2}{\varpi} \left[ f(v, K) \int_{a_0}^a \frac{d\eta}{a} \frac{1 - \cosh(\varpi \eta/a) + (\varpi \eta/a) \sinh(\varpi \eta/a)}{(1 - e^{-\varpi \eta/a})^{3/2}} \left(\frac{a}{\eta}\right)^2 \right. \\ &\quad \left. + \int_a^{|\Delta\tau|} \frac{d\eta}{a} \frac{1 - \cosh(\varpi \eta/a) + (\varpi \eta/a) \sinh(\varpi \eta/a)}{(1 - e^{-\varpi \eta/a})^{3/2}} \left(\frac{a}{\eta}\right)^{K_c + K_s} \right].\end{aligned}\quad (\text{B2})$$

The renormalization equations for the string parameters, Eq. (20), then follow from the requirement that  $R_u$  remain invariant under a change of the cutoff—i.e.,  $d(\varpi_{\text{eff}}/a)/d\ell = 0$  and  $d\lambda_{\text{eff}}/d\ell = 0$ .

In the case of the Gaussian string we find

$$\begin{aligned}\langle T_{\tau\mu}(x_1, \tau_1) u(x_2, \tau_2) \rangle &= F_u(\vec{r}_1 - \vec{r}_2) \left\{ 1 - \frac{v_u}{2v_s} \int dk \lambda^3 k^2 K_c \mathcal{D}_f(k) \right. \\ &\quad \times \int_{a_0}^{\infty} \frac{d\eta}{a} \left(\frac{\eta}{a}\right)^{-\lambda^2 k^2} \\ &\quad - \frac{v_u}{2v_s} \int dk \lambda^3 k^2 \mathcal{D}_b(k) \left[ f(v, K) \int_{a_0}^a \frac{d\eta}{a} \left(\frac{\eta}{a}\right)^{-\lambda^2 k^2} \right. \\ &\quad \left. \left. + \int_a^{\infty} \frac{d\eta}{a} \left(\frac{\eta}{a}\right)^{(2-K_c-K_s-\lambda^2 k^2)} \right] \right\},\end{aligned}\quad (\text{B3})$$

where  $f(v, K)$  is given by Eq. (21) and

$$\begin{aligned}F_u(\vec{r}_1 - \vec{r}_2) &= -\lambda^2 \ln \left( \frac{2\pi}{L_x} \sqrt{(x_1 - x_2)^2 + v_u^2 (\tau_1 - \tau_2)^2} \right) \\ &\quad - f_u \lambda^2 \frac{v_u^2 (\tau_1 - \tau_2)^2}{(x_1 - x_2)^2 + v_u^2 (\tau_1 - \tau_2)^2},\end{aligned}\quad (\text{B4})$$

with the bare value  $f_u = 0$ . The term containing  $\mathcal{D}_f(k)$  in Eq. (B3) is invariant under  $a \rightarrow a + da$  provided we use the flow equation for  $\mathcal{D}_f(k)$ , as given in Eq. (37), and assume  $d\lambda/d\ell = \mathcal{O}(\mathcal{D})$ ,  $dv_u/d\ell = \mathcal{O}(\mathcal{D})$ , which is consistent with the final result. The renormalization equation for the length  $\lambda$  is derived from the invariance requirement of the remaining logarithmic terms in Eq. (B3) and using the flow equation for  $\mathcal{D}_b(k)$ . In order to obtain the renormalization of the string wave velocity we first note that  $d(f_u \lambda^2)/d\ell = d\lambda^2/d\ell$ . Second, a renormalization  $v_u \rightarrow v_u + dv_u$  in the logarithmic part of  $F_u$  is equivalent to a renormalization of  $f_u \lambda^2$  according to  $d(f_u \lambda^2)/d\ell = (\lambda^2/v_u) dv_u/d\ell$ . Combining the two we find that, to first order in  $\mathcal{D}_b$ ,

$$\frac{dv_u}{d\ell} = \frac{2v_u}{\lambda} \frac{d\lambda}{d\ell},\quad (\text{B5})$$

which results in the renormalization equations (38).

For the floppy string one obtains

$$\begin{aligned}\langle T_{\tau\mu}(s_1, \tau_1) u(s_2, \tau_2) \rangle &= \frac{\bar{\lambda} a}{4\pi} \int dq \frac{1}{q^2} e^{iq\Delta s - \gamma \bar{\lambda} a |\Delta\tau| q^2} \left\{ 1 - \mathcal{D}_b f(v, K) \left(\frac{v_s}{4\pi\gamma\bar{\lambda}}\right)^{1/2} \int_{a_0}^a \frac{d\eta}{a} \left(\frac{a}{\eta}\right)^{11/4} \left[ \left(\frac{v_s}{\gamma\bar{\lambda}q^4 a^4} + \frac{v_s |\Delta\tau|}{q^2 a^3}\right) \right. \right. \\ &\quad \times \left( 1 - \cosh \frac{\gamma\bar{\lambda}q^2 a \eta}{v_s} \right) + \frac{\eta}{q^2 a^3} \sinh \frac{\gamma\bar{\lambda}q^2 a \eta}{v_s} \left. \right] - \mathcal{D}_b \left(\frac{v_s}{4\pi\gamma\bar{\lambda}}\right)^{1/2} \int_a^{v_s |\Delta\tau|} \frac{d\eta}{a} \left(\frac{a}{\eta}\right)^{3/4 + K_c + K_s} \\ &\quad \times \left[ \left(\frac{v_s}{\gamma\bar{\lambda}q^4 a^4} + \frac{v_s |\Delta\tau|}{q^2 a^3}\right) \left( 1 - \cosh \frac{\gamma\bar{\lambda}q^2 a \eta}{v_s} \right) + \frac{\eta}{q^2 a^3} \sinh \frac{\gamma\bar{\lambda}q^2 a \eta}{v_s} \right] - \mathcal{D}_f K_c \left(\frac{v_s}{4\pi\gamma\bar{\lambda}}\right)^{1/2} \int_{a_0}^{v_s |\tau_1 - \tau_2|} \frac{d\eta}{a} \\ &\quad \times \left(\frac{a}{\eta}\right)^{11/4} \left[ \left(\frac{v_s}{\gamma\bar{\lambda}q^4 a^4} + \frac{v_s |\Delta\tau|}{q^2 a^3}\right) \left( 1 - \cosh \frac{\gamma\bar{\lambda}q^2 a \eta}{v_s} \right) + \frac{\eta}{q^2 a^3} \sinh \frac{\gamma\bar{\lambda}q^2 a \eta}{v_s} \right] \left. \right\} - \left(\frac{v_s \bar{\lambda}}{64\pi^3 \gamma}\right)^{1/2} \int_{v_s |\Delta\tau}^{\infty} \frac{d\eta}{a} \\ &\quad \times \left[ \mathcal{D}_b \left(\frac{a}{\eta}\right)^{3/4 + K_c + K_s} + \mathcal{D}_f K_c \left(\frac{a}{\eta}\right)^{11/4} \right] \int dq \frac{1}{q^2} e^{iq\Delta s} \left\{ e^{-\gamma \bar{\lambda} a |\Delta\tau| q^2} \left(\frac{v_s}{\gamma\bar{\lambda}q^4 a^4} + \frac{v_s |\Delta\tau|}{q^2 a^3}\right) \right. \\ &\quad \left. - e^{(\gamma v_s) \bar{\lambda} q^2 a \eta} \left[ \frac{v_s |\Delta\tau|}{q^2 a^3} \sinh \gamma \bar{\lambda} a |\Delta\tau| q^2 - \left(\frac{v_s}{\gamma\bar{\lambda}q^4 a^4} + \frac{\eta}{q^2 a^3}\right) \cosh \gamma \bar{\lambda} a |\Delta\tau| q^2 \right] \right\},\end{aligned}\quad (\text{B6})$$

where  $\Delta s \equiv s_1 - s_2$  and  $\Delta \tau \equiv \tau_1 - \tau_2$ . The  $q$  integrals in the above are dominated by the small- $q$  region, therefore allowing us to expand the hyperbolic functions in the integrands. Using the flow equations for the electronic and disorder parameters, Eqs. (51), and assuming  $d\bar{\lambda}/d\ell = -\bar{\lambda} + \mathcal{O}(\mathcal{D})$  and  $d(\gamma/v_s)/d\ell = 0$ , the scale invariance of the last two  $\eta$  integrals in Eq. (B6) is then readily verified. The remaining two terms may be expressed as

$$\frac{\bar{\lambda}_{\text{eff}} a}{4\pi} \int dq \frac{1}{q^2} e^{iq\Delta s - \gamma \bar{\lambda}_{\text{eff}} a \Delta \tau q^2},$$

where

$$\bar{\lambda}_{\text{eff}} = \bar{\lambda} - \frac{\sqrt{\gamma} \bar{\lambda}^{3/2} \mathcal{D}_b}{4\sqrt{\pi v_s}} \left[ f(v, K) \int_{a_0}^a \frac{d\eta}{a} \left( \frac{a}{\eta} \right)^{3/4} + \int_a^{v_s |\Delta \tau|} \frac{d\eta}{a} \left( \frac{a}{\eta} \right)^{K_c + K_s - 5/4} \right]. \quad (\text{B7})$$

The requirement that  $d(\bar{\lambda}_{\text{eff}} a)/d\ell = 0$  results in the renormalization equations for the string parameters in Eqs. (51).

<sup>1</sup>For a review see P. A. Lee and T. V. Ramakrishnan, *Rev. Mod. Phys.* **57**, 287 (1985).

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<sup>3</sup>T. Giamarchi and H. J. Schulz, *Phys. Rev. B* **37**, 325 (1988).

<sup>4</sup>A. M. Finkel'stein, *Z. Phys. B: Condens. Matter* **56**, 189 (1984).

<sup>5</sup>C. Castellani, C. Di Castro, P. A. Lee, M. Ma, S. Sorella, and E. Tabet, *Phys. Rev. B* **30**, R1596 (1984).

<sup>6</sup>A. Punnoose and A. M. Finkel'stein, *Science* **310**, 289 (2005).

<sup>7</sup>D. E. Feldman and Y. Gefen, *Phys. Rev. B* **67**, 115337 (2003).

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<sup>10</sup>For recent reviews of some of the experimental evidence for the existence of mesostructures in the cuprates and their possible relation to the mechanism of high-temperature superconductivity see E. W. Carlson, V. J. Emery, S. A. Kivelson, and D. Orgad, in *The Physics of Conventional and Unconventional Superconductors*, edited by K. H. Bennemann and J. B. Ketterson (Springer-Verlag, Berlin, 2004), Vol. 2; S. A. Kivelson, I. P. Bindloss, E. Fradkin, V. Oganessian, J. M. Tranquada, A. Kapitulnik, and C. Howald, *Rev. Mod. Phys.* **75**, 1201 (2003).

<sup>11</sup>For reviews, see V. J. Emery, in *Highly Conducting One-Dimensional Solids*, edited by J. T. Devreese, R. P. Evrard, and V. E. van Doren (Plenum, New York, 1979), p. 327; T. Giamarchi, *Quantum Physics in One Dimension* (Oxford University

Press, Oxford, 2004).

<sup>12</sup>J. V. Jos e, L. P. Kadanoff, S. Kirkpatrick, and D. R. Nelson, *Phys. Rev. B* **16**, 1217 (1977).

<sup>13</sup>The relation  $dK_s/d\ell = 1/2(dy/d\ell)$  holds up to first order in  $\mathcal{D}_b$  and second order in  $y$ , separately. The terms of order  $y\mathcal{D}_b$  in the flow equations do not obey this relation. We expect terms of order  $y^2\mathcal{D}_b^2$  in the expansion of  $R_s$ , which were not taken into account here, to fix this problem.

<sup>14</sup>D. Orgad, *Phys. Rev. B* **69**, 205104 (2004).

<sup>15</sup>This is true also for the case where the electronic density along the string,  $n$ , is kept fixed rather than the number of particles. Under such conditions the total kinetic energy of the electrons fluctuates as they flow in and out of the string in order to compensate for the changes in the string length. The result is an induced elastic term with  $\sigma = E_F n$ , where  $E_F$  is the (fixed) Fermi energy of the electrons.

<sup>16</sup>The result is sensitive to the regularization scheme used in evaluating the average. Here and in the following we employ a hard cutoff  $1/a$  in the integral over wave vectors and a hard cutoff  $\gamma/a$  in the frequency integrals.

<sup>17</sup>E. Orignac and T. Giamarchi, *Phys. Rev. B* **53**, R10453 (1996); **56**, 7167 (1997).

<sup>18</sup>E. Orignac and T. Giamarchi, *Phys. Rev. B* **57**, 11713 (1998).

<sup>19</sup>E. Arrigoni and S. A. Kivelson, *Phys. Rev. B* **68**, 180503(R) (2003).