Unattenuated light transmission through the interface between two materials with different indices of refraction using magnetic metamaterials

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We report on the application of a magnetic composite metamaterial. The magnetic response of the metamaterial in the light region of the spectrum can realize the Brewster condition for *s*-polarized light. By introducing a uniaxial magnetic metamaterial consisting of stacked two-dimensional arrays of split-ring resonators, the Brewster effect can be produced for both *p* and *s* polarizations simultaneously. We also propose optical components that can interconnect two materials of different refractive index and that can transmit the light across the material boundary without any reflection at the interface.

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The refractive index mismatch at the interface of different materials acts as a potential barrier for light. Therefore, when incident light reaches the interface, the interface inhibits light transmission and part of the light is reflected backward. Fresnel examined this phenomenon and derived the well-known Fresnel formulas, which describe light reflection from material interfaces theoretically.

The Fresnel formulas also predict a peculiar phenomenon whereby, when *p*-polarized light is introduced to a material boundary at a certain incident angle, the light can pass through the boundary without any reflection. This phenomenon, first noted by David Brewster in 1815, is termed the Brewster effect and the incident angle at which it occurs is called Brewster's angle.[1]

The Brewster effect is widely utilized in various optical applications and optical components. For example, one of the most widely used applications is in laser cavities. In order to eliminate the reflection loss of the cavity and to stabilize the lasing condition, glass tubes of gas lasers or crystals of solid state lasers should not exhibit any reflection at the glass window at the end of the tube or at the crystal's surface, which arises from the index mismatch between the glass or crystal and the air outside. Inclining the glass window or the crystal surface at Brewster's angle can prevent this unwanted reflection. However, since the Brewster effect is normally observed only for *p*-polarized light, a laser cavity using a Brewster window or a crystal cut at Brewster's angle inevitably produces linearly polarized laser light.

Since the late 1990s, theoretical and experimental studies on the artificial control of the refractive index have been reported. Some of the most successful studies were reported by Pendry and Smith and their co-workers. Pendry *et al.* theoretically showed that an array of split-ring resonators (SRRs) behaves as a material with an artificially negative permeability (μ) in a particular frequency region.[2] After this work, Smith and colleagues made an artificial composite material containing an array of rods and an array of SRRs.[3–5] Both the rods and the SRRs were made of copper on fiberglass boards, and the boards were assembled three dimensionally. When this composite material was illuminated with microwave radiation, the material behaved macroscopically as a negative-index material, and the transmitted light was refracted with a negative refraction angle. Such novel composite materials are called metamaterials. The most important and interesting point of this work lies in the result that the nonmagnetic material reacted to an external magnetic field and, as a result, it exhibited a negative refractive index; this implies that both the permittivity and the permeability were negative. After their work, many other reports in this field were published.[6–14]

In this paper, we propose an application of such a metamaterial. We discovered that suitably controlling the permeability of the material enables us to produce the Brewster condition even for *s*-polarized light. In addition, we also found that a magnetic metamaterial can exhibit the Brewster effect for both *p*- and *s*-polarized light simultaneously, which allows reflectionless light transmission across the boundary of materials with different refractive indices, regardless of the polarization of the light. The significance of this finding is that the metamaterial can interconnect materials with two different indices while eliminating the reflection arising from the index mismatch, and it can solve the problem of polarization dependence seen in conventional optical components based on the Brewster effect.

First we describe the existence of the Brewster effect for *s*-polarized light. To simplify the discussion, we considered two isotropic and homogeneous materials, material 1 (M_1) and material 2 (M_2), with different optical constants ε_1 and μ_1 and ε_2 and μ_2, as shown in Fig. 1(a).The constants ε and μ represent the relative electric permittivity and the relative magnetic permeability, respectively. When light reaches the boundary, the reflectance for the p polarization, R^p, and that for the s polarization, R^s, are written as

$$R^p = \left(\frac{-\mu_2 \sin \theta'_{12} + \mu_1 \sin \theta_{12} \cos \theta_{12}}{\mu_2 \sin \theta'_{12} + \mu_1 \sin \theta_{12} \cos \theta_{12}} \right)^2 \tag{1}$$

and

$$R^s = \left(\frac{\mu_2 \tan \theta'_{12} - \mu_1 \tan \theta_{12}}{\mu_2 \tan \theta'_{12} + \mu_1 \tan \theta_{12}} \right)^2, \tag{2}$$

respectively, where θ_{12} is the angle of incidence and θ'_{12} is the angle of refraction. These angles are related by Snell's law, as follows:

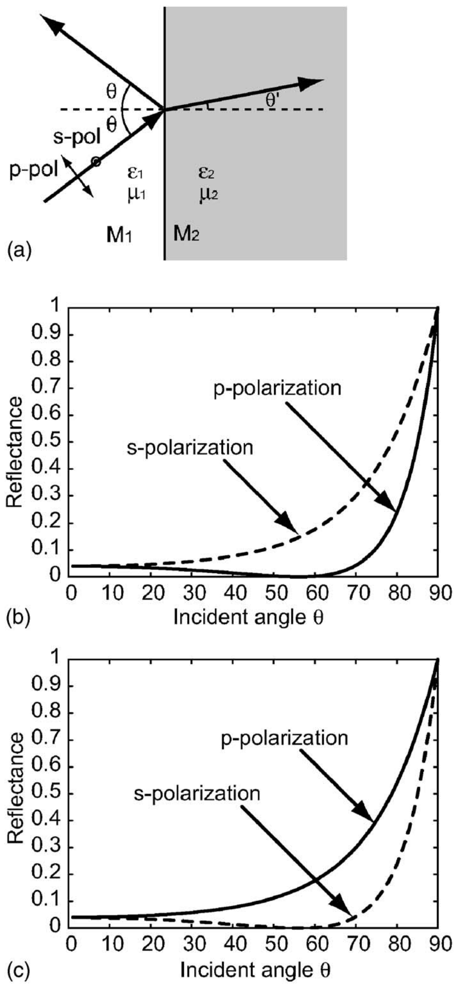

FIG. 1. Illustration depicting the Brewster effect for p- and s-polarized light. (a) Schematic diagram of structure used for theoretical analysis. An incident plane wave reaches a boundary between two homogeneous materials (M_1 and M_2) at an angle of incidence θ. The relative electric permittivity and permeability of M_1 and M_2 are ε_1 and μ_1 and ε_2 and μ_2, respectively. A reflected wave propagates back into the first material. A transmitted wave is refracted and advances into the second material with a refraction angle of θ'. θ and θ' are related by Snell's law. (b) Reflectance as a function of the angle of incidence when M_1 is a vacuum ($\varepsilon_1=1.0$ and $\mu_1=1.0$) and M_2 is glass ($\varepsilon_2=2.25$ and $\mu_2=1.0$). (c) Showing the existence of the Brewster effect also for s-polarized light. This figure shows the reflectance calculated when M_1 is a vacuum and M_2 is a magnetic material with $\varepsilon_2=1.0$ and $\mu_2=2.25$.

$$\sqrt{\varepsilon_1\mu_1}\sin\theta_{12}=\sqrt{\varepsilon_2\mu_2}\sin\theta'_{12}. \tag{3}$$

If both μ_1 and μ_2 are 1.0, Eqs. (1) and (2) are identical to the Fresnel formulas.

Assuming the numerators of Eqs. (1) and (2) to be zero under the condition that the product $\varepsilon_1\mu_1$ is not equal to $\varepsilon_2\mu_2$, the Brewster angles for p- and s-polarized light (θ_B^p and θ_B^s) are

$$\theta_B^p=\tan^{-1}\left(\sqrt{\frac{\varepsilon_2(\varepsilon_1\mu_2-\mu_1\varepsilon_2)}{\varepsilon_1(\varepsilon_1\mu_1-\mu_2\varepsilon_2)}}\right) \tag{4}$$

and

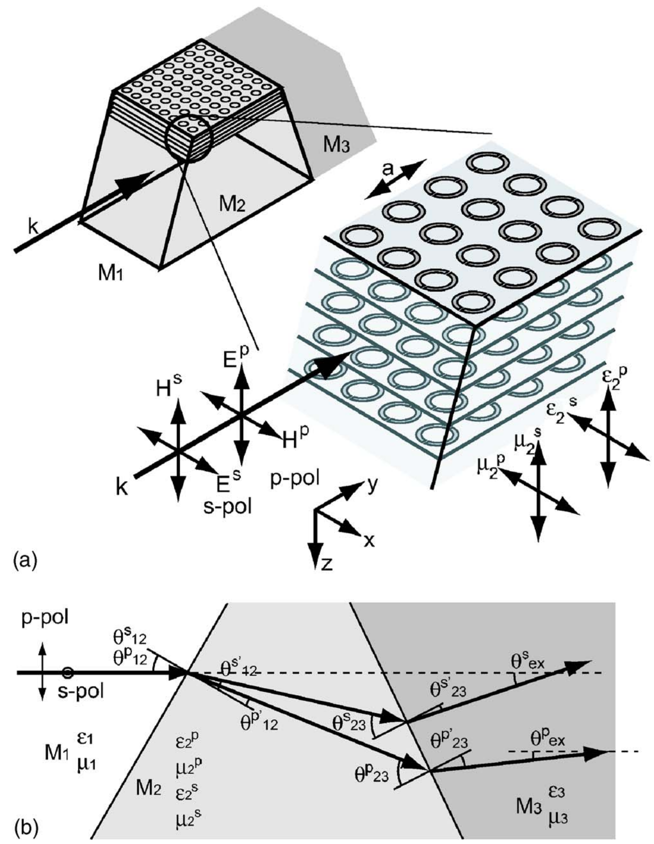

FIG. 2. (Color online) Uniaxial magnetic metamaterial and its calculation model, (a) Incident light whose wave vector is k comes from M_1, enters M_2, and is then transmitted to M_3. The uniaxial magnetic metamaterial consists of two-dimensional arrays of split-ring resonators lying in x-y planes. Because the SRRs react only to the magnetic waves that oscillate along the z direction, they affect only μ_2^s. (b) The calculation model for seeking the electromagnetic material parameters of M_2 that realize reflectionless propagation of light from M_1 to M_3. M_1 and M_3 are isotropic and homogeneous materials whose permittivities and permeabilities are ε_1 and μ_1 and ε_3 and μ_3, respectively. M_2 is a uniaxial material whose material parameters differ according to the polarization of the light; they are defined as $\varepsilon_2^P,\mu_2^P,\varepsilon_2^s$, and μ_2^s. The angles of incidence at both boundaries are represented by θ and the refraction angles are represented by θ'. The difference between the angles of the incident and transmitted light is represented by θ_{ex}.

$$\theta_B^s=\tan^{-1}\left(\sqrt{\frac{\mu_2(\varepsilon_2\mu_1-\mu_2\varepsilon_1)}{\mu_1(\varepsilon_1\mu_1-\mu_2\varepsilon_2)}}\right). \tag{5}$$

Figure 1(b) shows the reflectance curves of R^p and R^s as a function of the angle of incidence θ_{12} calculated under the condition that the material 1 is a vacuum ($\varepsilon_1=1.0,\mu_1=1.0$) and material 2 is glass ($\varepsilon_2=2.25,\mu_2=1.0$). The reflectance curve for p-polarized light drops to zero at $\theta_{12}=56.3°$. This angle is simply Brewster's angle. On the other hand, there is no zero-reflectance point on the curve for s-polarized light, indicating that there is no Brewster's angle for the s polarization. Figure 1(c) shows another result calculated under the condition that ε_1 and μ_1 are the same as in Fig. 1(b), but ε_2 and μ_2 are set to 1.0 and 2.25, respectively. Under this condition, we can see that the reflectance curve for the s polarization falls to zero at the same angle ($\theta_{12}=56.3°$) as in Fig. 1(b). This is the same Brewster effect shown in Fig. 1(b), but for the *s* polarization, not the *p* polarization. These results can be understood from the symmetry between ε and μ in Eq. (4) and Eq. (5). Since the permeability of most materials in nature is approximately unity in the light region of the electromagnetic spectrum, it is believed that the Brewster effect occurs only for *p*-polarized light. However, these results demonstrate that changing the permeability of the materials can realize the Brewster effect also for *s*-polarized light.

From these results, we realized that if we could produce the Brewster effect for both *p*- and *s*-polarized light simultaneously, the light could propagate through the material interface without any reflection at all. This is the fundamental idea in realizing unattenuated transmission of light across the material boundary. However, Eqs. (4) and (5) tell us that the Brewster conditions for each polarization cannot be realized simultaneously, because if the term inside the arctangent of Eq. (4) takes a real value, that of Eq. (5) becomes imaginary, and vice versa. Figures 1(b) and 1(c) also show this problem; in both figures only one polarization exhibits the Brewster effect and the other does not. To overcome this conflict, we introduce the idea of a uniaxial metamaterial whose ε and μ values depend on the direction of the material, analogous to uniaxial crystals. The concept of an anisotropic left-handed metamaterial was first introduced by Grzegorczyk *et al.*, and they reported inversion of the critical angle and Brewster's angle in such a material.[15]

Figure 2 depicts a schematic diagram of a material structure that can completely eliminate light reflection from the material boundary. The model consists of three materials. The incident light comes from material 1 (M_1), passes through material 2 (M_2), and is then transmitted to material 3 (M_3). M_2 is a uniaxial metamaterial that functions as a buffer layer for realizing perfect light transmission from M_1 to M_3. As shown in Fig. 2(a), M_2 consists of an array of SRRs. Because the SRRs lie only in the *x*-*y* planes, they react to a magnetic wave that oscillates along the *z* direction (H_z) and thus change only μ_2^s. We defined the incident angles and the angles of refraction for each polarization at each interface as $\theta_{12}^P, \theta_{12}^{P'}, \theta_{23}^P, \theta_{23}^{P'}, \theta_{12}^s, \theta_{12}^{s'}, \theta_{23}^s$, and $\theta_{23}^{s'}$, as shown in Fig. 2(b).

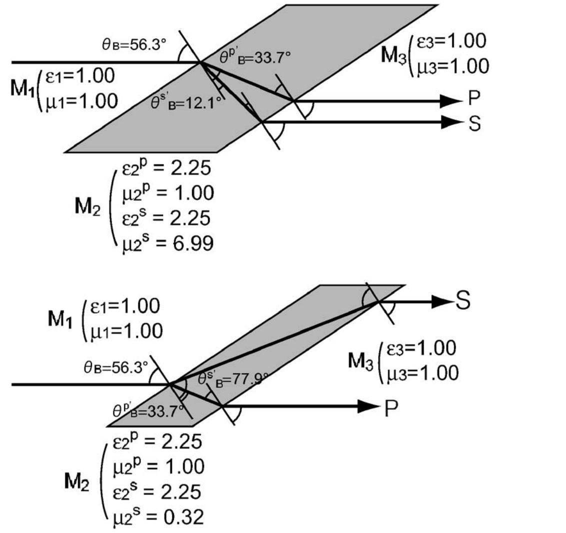

FIG. 3. A Brewster window made of metamaterial. Both sides of the window are air, and the relative permittivity and permeability for *p*-polarized light are 2.25 and 1.0, respectively (i.e., the same as glass). When we fixed the relative permittivity for *s*-polarized light to 2.25, the permeabilities for *s*-polarized light μ_2^s that realize unattenuated light transmission were calculated from Eq. (6) to be 6.99 and 0.32. The upper figure illustrates the case where $\mu_2^s=6.99$ and the lower figure $\mu_2^s=0.32$.

Under the condition of $\varepsilon_1=\varepsilon_3$ and $\mu_1=\mu_3$, the problem becomes simple. When the Shape of M_2 is a parallel plate, the exit angle of transmitted light from M_2 is the same as its incident angle to M_2. Therefore, the constraint for $\theta_{\mathrm{ex}}^P=\theta_{\mathrm{ex}}^S$ is only $\theta_B^P=\theta_B^S$. Using Eqs. (4) and (5), and making $\theta_B^P=\theta_B^S$, the relative permeability of M_2 for the *s* polarization was theoretically derived as a function of $\varepsilon_1, \mu_1, \varepsilon_2^P, \mu_2^P$, and ε_2^S, as follows:

$$\mu_2^s = \frac{(\varepsilon_1^2 - \varepsilon_2^{p^2})\varepsilon_2^s\mu_1^2 \pm \mu_1\sqrt{(\varepsilon_1^2 - \varepsilon_2^{p^2})^2\varepsilon_2^{s^2}\mu_1^2 - 4\varepsilon_1^3\varepsilon_2^p(-\varepsilon_2^p\mu_1 + \varepsilon_1\mu_2^p)(\varepsilon_1\mu_1 - \varepsilon_2^p\mu_2^p)}}{2\varepsilon_1^2(\varepsilon_1\mu_1 - \varepsilon_2^p\mu_2^p)}. \tag{6}$$

The condition for determination of ε_2^P, μ_2^P, and ε_2^S is that the term inside the square root of Eq. (6) should be positive.

As an example, we considered a Brewster window with a vacuum on both sides ($\varepsilon_1=\varepsilon_3=1.0$ and $\mu_1=\mu_3=1.0$). If this window is made of glass, the relative permittivity and permeability for *p*-polarized light are $\varepsilon_2^P=2.25$ and $\mu_2^p=1.0$. When ε_2^s was also assumed to be 2.25, the values of μ_2^s that produce the Brewster condition for both *p*- and *s*-polarized light simultaneously were calculated, using Eq. (6), to be 0.322 and 6.991, and the Brewster angle was $\theta_{\mathrm{B}}=56.31°$. Figure 3 illustrates these results.

If the refractive indices of the materials on both sides of the metamaterial are different, the solution cannot be derived analytically. Therefore, a numerical calculation method, such as the simplex method, must be used to seek the optimal relative permittivity and relative permeability of M_2 for both *p* and *s* polarizations (ε_2^p, μ_2^p, ε_2^s, and μ_2^s) so that the exit angles for both *p*- and *s*-polarized light are identical.

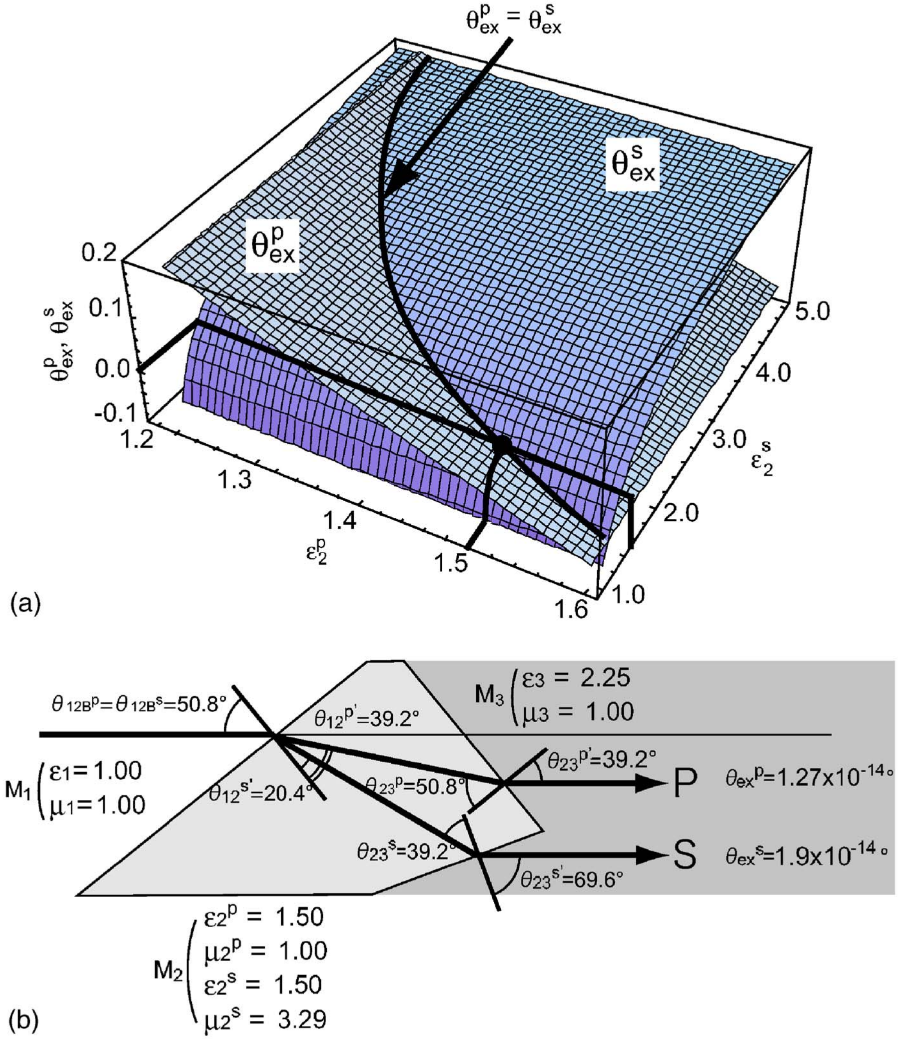

FIG. 4. (Color online) Design of the uniaxial magnetic metamaterial. (a) shows the relations of the exit angles of p- and s-polarized light (θ_{ex}^{p} and θ_{ex}^{s}) according to both ε_2^p and ε_2^s. The combination of ε_2^p and ε_2^s producing the Brewster effect for both p- and s- polarized light is indicated by the line of intersection of two surfaces. With the additional constraint that the directions of the transmitted light beams should be identical to that of the incident light, the solution converged at the point $\varepsilon_2^p=1.5$, $\mu_2^p=1.0$, $\varepsilon_2^s=1.5$, and $\mu_2^s=3.29$. (b) illustrates this for a metamaterial with $\varepsilon_2^p=1.5$, $\mu_2^p=1.0$, $\varepsilon_2^s=1.5$, and $\mu_2^s=3.29$. Incident light having both p- and s- polarized components enters the boundary between M_1 and M_2 with the same Brewster angles $\theta_{12B}^{p}=\theta_{12B}^{s}=50.8°$. After transmission across the boundary, the light splits into two rays corresponding to each polarization, and each ray propagates inside M_2. At the boundary between M_2 and M_3, the rays are refracted again and proceed into M_3 with the same propagation direction. Moreover, the propagation direction is also identical to that of the incident light. The angles of incidence for p- and s-polarized light at the boundary between M_2 and M_3 were 50.8° and 39.2°, respectively, which were also the Brewster angles for the respective polarizations.

To simplify the problem, we fixed the permeability for p-polarized light μ_2^p to 1.0. When ε_2^p was determined, the Brewster angles at the interfaces of M_1-M_2 and M_2-M_3 were uniquely determined by Eq. (4). For the s-polarized light, if ε_2^s is fixed, the value of μ_2^s that realizes $\theta_B^p=\theta_B^s$ can be calculated using Eq. (6). Therefore, the free parameters are ε_2^p and ε_2^s, and we must find the correct combination of ε_2^p and ε_2^s under the restriction that the exit angles for p- and s-polarized light should be identical.

Figure 4 shows an example of the results obtained under the condition that M_1 was a vaccum $(\varepsilon_1=1.0, \mu_1=1.0)$ and M_3 was glass $(\varepsilon_3=2.25, \mu_3=1.0)$. Figure 4(a) shows the relationship between the exit angles of p- and s-polarized light according to ε_2^p and ε_2^s. The solution $\theta_{ex}^{p}=\theta_{ex}^{s}$ is indicated by the line of intersection of two surfaces. When we applied the additional constraint that the directions of the transmitted light beams should be identical to that of the incident light (i.e., the light was transmitted straight through from M_1 to M_3, or in other words $\theta_{ex}^{p}=\theta_{ex}^{s}=0.0$), the solution converged at a point $\varepsilon_2^p=1.5$, $\mu_2^p=1.0$, $\varepsilon_2^s=1.5$, and $\mu_2^s=3.29$. Under this condition, the Brewster angles for p- and s-polarized light at the interface of M_1 and M_2 were identically 50.77°. Figure 4(b) illustrates this result. After refraction at the interface of M_1 and M_2, the incident light splits depending on the polarization, and the split beams travel separately inside M_2 with different propagation directions. Then, at the interface of M_2 and M_3, both light beams are refracted again and exit to M_3 with the same propagation directions. Since the Brewster

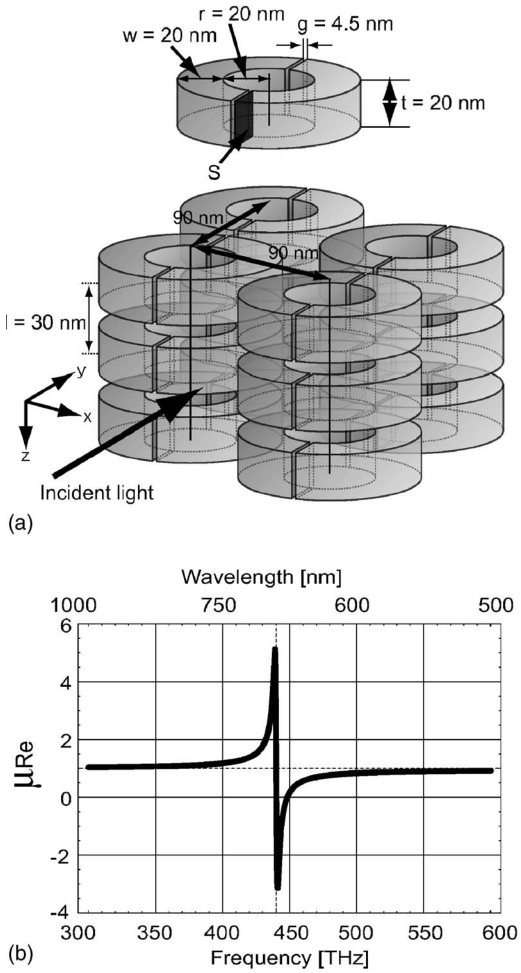

FIG. 5. Design of silver single-ring split-ring resonator. (a) For each SRR, the inner radius (r), the width of the metal lines (w), the gap between two semicircles (g), and the thickness (t) were 20, 20, 4.5, and 20 nm, respectively. The lattice constants were 90×90 nm^2 in the x-y plane and 30 nm along the z axis; these lattice constants correspond to a 15% filling factor (F). (b) The numerically simulated dispersion curve of the real part of the effective permeability of the SRR array. The array of silver single-ring SRRs exhibits $\mu_{\text{eff}}=3.29$ at a wavelength of 680 nm.

angles of p- and s-polarized light are 50.77° and 39.23°, respectively, the angles of interface of M_2 and M_3 for p- and s-polarized light must be prepared individually as shown in Fig. 4(b). The difference of the exit angles between the two polarizations was less than 1×10^{-14} deg. Moreover, the reflectances at both interfaces for both polarizations were less than 2×10^{-5}; this value was simply due to computational errors. These results indicate that the light completely passed through both interfaces without any reflection loss.

Figure 5 shows an example of a structure that can realize this reflectionless light transmission across the material boundary. This structure is composed of an array of single-ring SRRs. When the time-varying external electromagnetic field of incident light is applied to the SRR array, an induced current flows in each resonator, which produces an internal magnetic field that resists the magnetic field of the incident light. Recently, we have derived the following equation that describes the behavior in the entire visible-light region for the single-ring SRRs shown in Fig. 5(a):

$$\mu_{\text{eff}} = \mu_{\text{Re}} + i\mu_{\text{Im}} = 1 - \frac{F\omega^2}{\omega^2 - 1/CL + iZ(\omega)\omega/L}, \qquad (7)$$

where F is the filling factor, C is the geometrical capacitance, L is the geometrical inductance, and $Z(\omega)$ is the impedance of the metal rings.[16,17] In the case where a single-ring SRR is placed in a material whose relative permittivity is ε_r, the parameters F, C, L, and $Z(\omega)$ are written as

$$F = \frac{\pi r^2}{a^2}, \qquad (8)$$

$$C = \epsilon_0 \epsilon_r \frac{S}{2g}, \qquad (9)$$

$$L = \frac{\mu_0 \pi r^2}{l}, \qquad (10)$$

$$Z(\omega) = \frac{2\pi r}{w} \frac{1-i}{\sigma(\omega)\delta(\omega)}, \qquad (11)$$

where r is the inner radius, w is the width of the metal lines, g is the gap between two semicircles, t is the thickness, a is the lattice constant in the x-y plane, l is that along the z axis, and $\sigma(\omega)$ and $\delta(\omega)$ are the conductivity and the penetration depth of the metal, respectively.

By using Eqs. (6)–(10) we examined the effective μ, and we found that an array of single-ring SRRs made of silver and having an inner radius $r=20$ nm, width of the metal lines $w=4.5$ nm, gap between two semicircles $g=4.5$ nm, and thickness $t=20$ nm, as shown in Fig. 5(a), could realize a permeability $\mu=3.29$ at a wavelength of 680 nm. Figure 4(b) shows the effective permeability of this array of silver single-ring SRRs as a function of frequency.

In conclusion, we have investigated a phenomenon whereby the Brewster effect can be produced for both p- and s-polarized light simultaneously. We also proposed a uniaxial metamaterial and its application to an intermediate layer that interconnects two materials of different index to allow perfect transmission of light across the material boundary without any surface reflection. This technology will find applications in laser cavities that can lase randomly polarized or circularly polarized light while still reducing reflection losses. In optical communication systems using optical fibers and optical switching devices, the reflection at the junction of components is a serious problem from the viewpoint of energy loss. Incorporating the above-mentioned metamaterial between such components will allow the reflection loss at the interface to be completely eliminated. As described here, controlling electromagnetic material parameters, such as permittivity ε and permeability μ, will open the door for exotic physical phenomena and their applications.

The results of calculations were performed by using the RIKEN Super Combined Cluster (RSCC).

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