# Coherent manipulation of a strongly driven semiconductor quantum well

Emmanuel Paspalakis,[1] Margarita Tsaousidou,[1] and Andreas F. Terzis[2]
[1]*Materials Science Department, School of Natural Sciences, University of Patras, Patras 265 04, Greece*
[2]*Physics Department, School of Natural Sciences, University of Patras, Patras 265 04, Greece*
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We study the interaction of a pulsed electric field with a semiconductor quantum well by using the effective nonlinear Bloch equations. We present analytical solutions for the Bloch equations for specific system parameters, under the rotating wave approximation. We also present conditions that could lead to complete inversion in the system for a wide range of parameters. Our findings are assessed by numerical calculations for a double quantum well based on GaAs.

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## I. INTRODUCTION

The interaction of oscillating electric fields with intersubband transitions in semiconductor quantum wells has led to the prediction of several interesting and potentially useful effects, such as, for example, enhanced nonlinear optics,[1,2] gain without inversion,[3] and electromagnetically induced transparency.[4] Some interesting experiments also exist in this field.[5–13] Also, several useful devices such as lasers,[14] photodetectors,[15] modulators,[16] optical,[17] and quantum[18] switches are based on intersubband transitions in semiconductor quantum wells. In most of these studies atomic-like multi-level theoretical approaches have been used for the description of the dynamics of the intersubband transitions. Many-body effects arising from the macroscopic carrier density have also been included in a significant number of theoretical studies.[19–32] These studies have shown that the optical response and the electron dynamics of the quantum wells can be significantly influenced by changing the carrier density.

In a particular study in this area, Olaya-Castro *et al.*[29] derived effective Bloch equations describing the effect of electron-electron interactions on intersubband transitions in a modulation-doped quantum well with two electronic subbands coupled by an intense ac field. These equations are nonlinear differential equations and the nonlinearities arise due to the macroscopic number of carriers in the semiconductor quantum well. Later, Haljan *et al.*[30] used the generalized nonlinear Bloch equations for a doped double symmetric quantum well and showed the occurrence of high order harmonic generation in the terahertz regime by application of a strong gigahertz electric field. They also showed that the increase of electron sheet density above a critical value can lead to level bifurcation and eventual disappearance of high harmonics.

In this paper we study the potential for coherent control of electron dynamics in a symmetric double quantum well, in the two-subband approximation, that is coupled by a strong pulsed electric field. For the system dynamics we use the effective nonlinear Bloch equations of Olaya-Castro *et al.*[29] We first simplify the nonlinear Bloch equations by using the rotating wave approximation (RWA)[33] and present analytical solutions in cases where the two-subband system interacts with specific pulsed electric fields. Conditions that lead to complete inversion of the electronic population in the two-subband system are also presented. We finally compare our findings with numerical solutions of the effective nonlinear Bloch equations for a realistic semiconductor quantum well structure. Recently, Batista and Citrin[32] studied Rabi oscillations in a two-subband $n$-type modulation-doped quantum well that interacts with a pulsed electric field with time-dependent frequency. The results presented here are complementary to those of the work of Batista and Citrin.[32]

## II. EFFECTIVE NONLINEAR BLOCH EQUATIONS

The system under study is a symmetric double semiconductor quantum well. We assume that only the two lower energy subbands, $n=0$ for the lowest subband and $n=1$ for the excited subband, contribute to the system dynamics. The Fermi level is below the $n=1$ subband minimum, so the excited subband is initially empty. This is succeeded by a proper choice of the electron sheet density. The two subbands are coupled by a time-dependent electric field $E(t)=\mathcal{E}_0 f(t)\sin[\omega t+\phi(t)]$, where $\mathcal{E}_0$ is the electric field amplitude, $f(t)$ is the dimensionless pulse envelope, $\omega$ is the angular frequency and $\phi(t)$ is the time-dependent phase of the electric field. In Ref. 29 Olaya-Castro *et al.* showed that the system dynamics is described by the following effective nonlinear Bloch equations:

$$\dot{S}_1(t)=[\omega_{10}-\gamma S_3(t)]S_2(t)-\frac{S_1(t)}{T_2}, \tag{1}$$

$$\dot{S}_2(t)=-[\omega_{10}-\gamma S_3(t)]S_1(t)+2\left[\frac{\mu E(t)}{\hbar}-\beta S_1(t)\right]S_3(t)-\frac{S_2(t)}{T_2}, \tag{2}$$

$$\dot{S}_3(t)=-2\left[\frac{\mu E(t)}{\hbar}-\beta S_1(t)\right]S_2(t)-\frac{S_3(t)+1}{T_1}. \tag{3}$$

Here, $S_1(t)$ and $S_2(t)$ are, respectively, the mean real and imaginary parts of polarization and $S_3(t)$ is the mean population inversion per electron (difference of the occupation probabilities in the upper and lower subbands). Also, $\mu$ is the electric dipole matrix element between the two subbands and the parameters $\omega_{10},\beta,\gamma$ are given by

$$\omega_{10}=\frac{E_1-E_0}{\hbar}+\frac{\pi e^2}{\hbar\varepsilon}N\frac{L_{1111}-L_{0000}}{2}, \tag{4}$$

$$\gamma=\frac{\pi e^2}{\hbar\varepsilon}N\left(L_{1001}-\frac{L_{1111}+L_{0000}}{2}\right), \tag{5}$$

$$\beta=\frac{\pi e^2}{\hbar\varepsilon}NL_{1100}. \tag{6}$$

Here, $N$ is the electron sheet density, $\varepsilon$ is the relative dielectric constant, $e$ is the electron charge, $E_0$, $E_1$ are the eigenvalues of energy for the ground and excited states in the well, respectively, and $L_{ijkl}=\int\int dzdz'\,\xi_i(z)\xi_j(z')|z-z'|\xi_k(z')\xi_l(z)$, with $i,j,k,l=0,1$. Also, $\xi_i(z)$ is the wave function for the $i$th subband along the growth direction ($z$ axis). Finally, in Eqs. (1)–(3) the terms containing the population decay time $T_1$ and the dephasing time $T_2$ describe relaxation processes in the quantum well and have been added phenomenologically in the effective nonlinear Bloch equations. If there is no relaxation in the system $T_1,T_2\to\infty$, then $S_1^2(t)+S_2^2(t)+S_3^2(t)=1$.

In comparison with the atomic optical Bloch equations[33] we note that in the effective nonlinear Bloch equations the transition frequency should be renormalized by a time-independent term [see Eq. (4)] and by a time-dependent term depending on the parameter $\gamma$ and $S_3(t)$. The parameter $\gamma$ consists of two compensating terms: the self-energy term and the vertex term. In addition, the applied field contribution is screened by the induced polarization term $S_1(t)$ with coefficient $\beta$.

We will proceed with the RWA.[33] We introduce the variables $U(t)$, $V(t)$, $Z(t)$ as

$$S_1(t)=U(t)\sin[\omega t+\phi(t)]-V(t)\cos[\omega t+\phi(t)],$$

$$S_2(t)=V(t)\sin[\omega t+\phi(t)]+U(t)\cos[\omega t+\phi(t)],$$

$$S_3(t)=Z(t). \tag{7}$$

Then, Eqs. (1)–(3) are written as

$$\begin{aligned}\dot{U}(t)&=[\delta-\dot{\phi}(t)-(\gamma-\beta)Z(t)]V(t)\\&\quad-[\Omega_0(t)+\beta U(t)]Z(t)\sin[2\omega t+2\phi(t)]\\&\quad+\beta V(t)Z(t)\cos[2\omega t+2\phi(t)]-\frac{U(t)}{T_2},\end{aligned} \tag{8}$$

$$\begin{aligned}\dot{V}(t)&=-[\delta-\dot{\phi}(t)-(\gamma-\beta)Z(t)]U(t)\\&\quad+[\Omega_0(t)+\beta U(t)]Z(t)\cos[2\omega t+2\phi(t)]-\Omega_0(t)Z(t)\\&\quad+\beta V(t)Z(t)\sin[2\omega t+2\phi(t)]-\frac{V(t)}{T_2},\end{aligned} \tag{9}$$

$$\begin{aligned}\dot{Z}(t)&=-[\Omega_0(t)+2\beta U(t)]V(t)\cos[2\omega t+2\phi(t)]\\&\quad+[\Omega_0(t)U(t)+\beta[U^2(t)-V^2(t)]]\sin[2\omega t+2\phi(t)]\\&\quad+\Omega_0(t)V(t)-\frac{Z(t)+1}{T_1}.\end{aligned} \tag{10}$$

Here, $\Omega_0(t)=-\mu\mathcal{E}_0 f(t)/\hbar$ is the time-dependent Rabi frequency and $\delta=\omega_{10}-\omega$ is the detuning from resonance. In the case that $T_1,T_2\to\infty$, then $U^2(t)+V^2(t)+Z^2(t)=1$. In the RWA the terms containing $\sin[2\omega t+2\phi(t)]$ and $\cos[2\omega t+2\phi(t)]$ are omitted from Eqs. (8)–(10). Then, we obtain

$$\dot{U}(t)=[\delta-\dot{\phi}(t)-(\gamma-\beta)Z(t)]V(t)-\frac{U(t)}{T_2}, \tag{11}$$

$$\dot{V}(t)=-[\delta-\dot{\phi}(t)-(\gamma-\beta)Z(t)]U(t)-\Omega_0(t)Z(t)-\frac{V(t)}{T_2}, \tag{12}$$

$$\dot{Z}(t)=\Omega_0(t)V(t)-\frac{Z(t)+1}{T_1}. \tag{13}$$

We note that in the RWA the effective nonlinear Bloch equations, Eqs. (11)–(13), reduce to similar equations as those that describe a dense collection of two-level atoms interacting with a laser field.[34,35]

## III. ANALYTICAL SOLUTIONS

In what follows we assume that the system is initially in the lowest subband, so the initial conditions are $S_1(0)=S_2(0)=0$, $S_3(0)=-1$, or $U(0)=V(0)=0$, $Z(0)=-1$. We can present analytical solutions for the nonlinear Bloch equations under the RWA, Eqs. (11)–(13), for two specific cases described below. In both cases the relaxation processes are ignored, so $T_1,T_2\to\infty$. The system we consider is excited at exact resonance, $\omega=\omega_{10}$, and the applied field has a hyperbolic secant form, so $\Omega_0(t)=\bar{\Omega}\,\mathrm{sech}[(t-t_0)/t_p]$, where $\bar{\Omega}=-\mu\mathcal{E}_0/\hbar$ and $f(t)=\mathrm{sech}[(t-t_0)/t_p]$. Here, $t_0$ is the center of the pulse and it is chosen such that the electric pulse is practically zero at $t=0$ (and $t=2t_0$) and $t_p$ is the width of the pulse.

*First case*: If there is no time-dependent phase, $\phi(t)=0$, and $\bar{\Omega}$ is chosen such that $\bar{\Omega}=\sqrt{(\gamma-\beta)^2+1/t_p^2}$, the analytic solution of Eqs. (11)–(13) is given by[35]

$$U(t)=\frac{(\gamma-\beta)t_p}{\sqrt{(\gamma-\beta)^2t_p^2+1}}\,\mathrm{sech}[(t-t_0)/t_p], \tag{14}$$

$$V(t)=\frac{1}{\sqrt{(\gamma-\beta)^2t_p^2+1}}\,\mathrm{sech}[(t-t_0)/t_p], \tag{15}$$

$$Z(t)=\tanh[(t-t_0)/t_p]. \tag{16}$$

Therefore, at $t=2t_0$, $Z(t)\to 1$ and the electrons are transferred in the upper subband. So, there is complete inversion of the system.

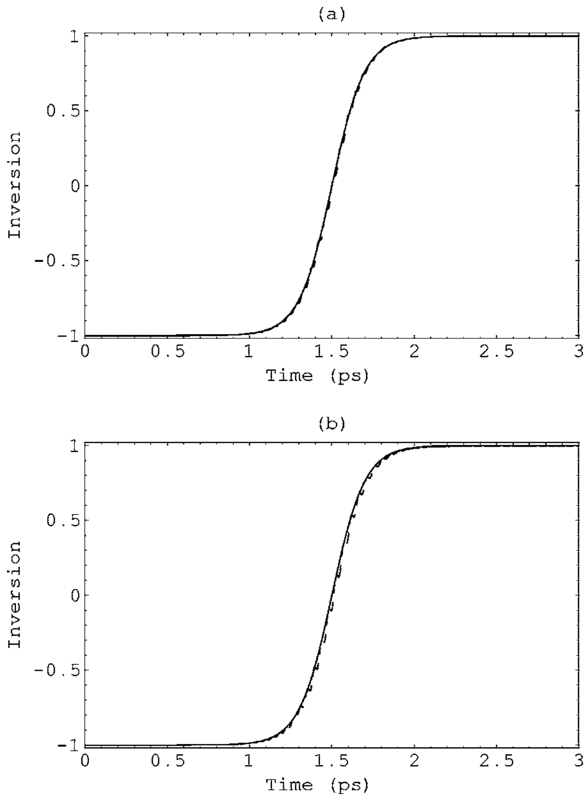


FIG. 1. The time evolution of the inversion, $S_3(t)$, obtained from numerical solution of the nonlinear Bloch Eqs. (1)–(3) (dashed curve) and the analytical result (solid curve). In (a) the first case is presented and in (b) the second case is presented. The parameters of the pulse are $t_0=1.5$ ps and $t_p=0.2$ ps. In Fig. 1(b) $\phi(t)$ is calculated by the integration of $\dot{\phi}(t)$ from 0 to $t$.

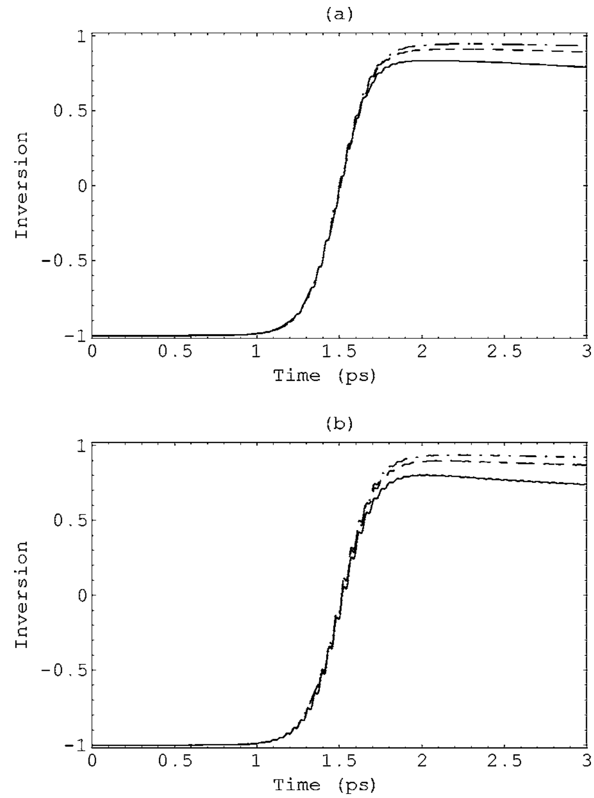


FIG. 2. The time evolution of the inversion, $S_3(t)$, obtained from numerical solution of the nonlinear Bloch Eqs. (1)–(3) for the same parameters as in Fig. 1 but with $\tau=3$ ps (solid curve), $\tau=6$ ps (dashed), and $\tau=10$ ps (dot-dashed curve).

*Second case*: If the time-dependent phase is chosen such that $\dot{\phi}(t)=(\beta-\gamma)\tanh[(t-t_0)/t_p]$ and $\bar{\Omega}=1/t_p$, then

$$U(t)=0, \tag{17}$$

$$V(t)=\operatorname{sech}[(t-t_0)/t_p], \tag{18}$$

$$Z(t)=\tanh[(t-t_0)/t_p]. \tag{19}$$

In this case, too, we obtain complete population inversion in the system at $t=2t_0$. We note that the combination hyperbolic secant pulse and hyperbolic tangent derivative for the time-dependent phase have been also used in the Allen-Eberly solution of the optical Bloch equations for a two-level atom interacting with a laser pulse.[33]

We will assess the analytical results by comparing them with numerical solutions of the nonlinear Bloch equations [Eqs. (1)–(3)]. We consider a GaAs/AlGaAs double quantum well. The structure consists of two GaAs symmetric square wells of width 5.5 nm and height 219 meV. The exchange self-energy terms, not present in the effective Bloch equations, can be neglected in this structure, as the dynamics of the system is dominated by the depolarization effects for such values of the quantum well width.[28] The wells are separated by an AlGaAs barrier of width 1.1 nm. Also, the electron sheet density is taken $5\times10^{11}$ cm$^{-2}$. Then, the parameters are calculated to be $E_1-E_0=44.955$ meV, $\pi e^2N(L_{1111}-L_{0000})/2\varepsilon=1.03$ meV, $\hbar\gamma=0.2375$ meV and $\hbar\beta=-3.9$ meV. As we can see from Fig. 1 where the time evolution of the inversion, $S_3(t)$, is presented, apart from a small internal oscillation in the numerical results which arise due to the non-RWA terms, there is a good agreement between the analytical and the numerical results. These internal oscillations disappear for pulses of larger width and, in that case, the analytical result describes more exactly the inversion dynamics.

In order to study the effects of relaxation processes in inversion dynamics, we repeat the numerical calculations including the population decay and dephasing rates in the calculations. As the dephasing is the crucial relaxation process in semiconductor quantum wells we choose $T_1=10\tau$ and $T_2=\tau$. The results of our calculations for three different dephasing times, $\tau=3$, 6, and 10 ps, are shown in Fig. 2. We note that in this case complete inversion is not possible but significant electron transfer still exists. It may be thought that the efficiency of electron transfer can be effectively increased by choosing significantly shorter electric pulses, such that the effect of the relaxation processes is minimized. However, in such case the RWA starts becoming insufficient and the above analytical solutions are not proper for the description of the electron dynamics. The latter should be taken into account during the decrease of pulse width.

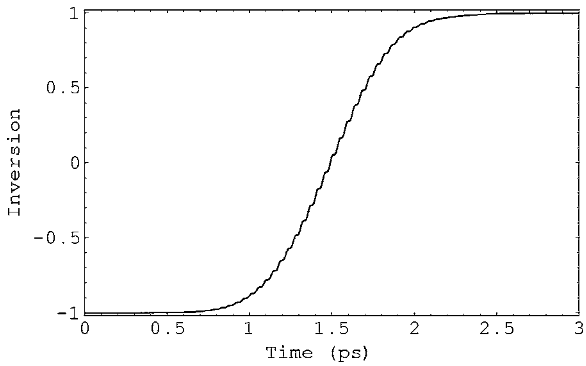


FIG. 3. The time evolution of the inversion, $S_3(t)$, obtained from numerical solution of the nonlinear Bloch equations for a Gaussian pulse with parameters $\bar{\Omega}=(\gamma-\beta)$, $t_0=1.5$ ps, $t_p=0.6$ ps, and $\phi(t)=0$.

## IV. OTHER CONDITIONS FOR INVERSION

Using the analogy between Eqs. (11)–(13) and those of the dense two-level atoms in a laser field[34,35] we arrive at conditions that lead to complete inversion in the system, if the system is initially in the lower subband. We first assume that there is no relaxation in the system, so $T_1, T_2 \to \infty$. The interaction with the external field is at exact resonance, $\omega=\omega_{10}$, $\phi(t)=0$ and the applied field is taken to have Gaussian shape, with $\Omega_0(t)=\bar{\Omega}\exp[-(t-t_0)^2/t_p^2]$. We note that the shape of the electric pulse is not a very crucial parameter in this case as the same results will be obtained for other pulse shapes, such as, for example, hyperbolic secant pulses.

If the maximum value of the Rabi frequency is chosen such that $\bar{\Omega}=\gamma-\beta$, then, if the quantum well system interacts with a resonant pulsed field with Gaussian form, complete inversion may occur. This can be seen in Fig. 3 obtained from numerical solutions of the nonlinear Bloch equations, Eqs. (1)–(3). Similar results are obtained for hyperbolic secant pulses but we find that a Gaussian pulse creates the inversion in shorter times than a sech pulse (not shown here).

As we can see from Fig. 4, after a critical value, approximately $t_p=0.45$ ps for the system under study, the two-subband system exhibits inversion for a wide range of pulse widths larger than this value. This effect can be explained

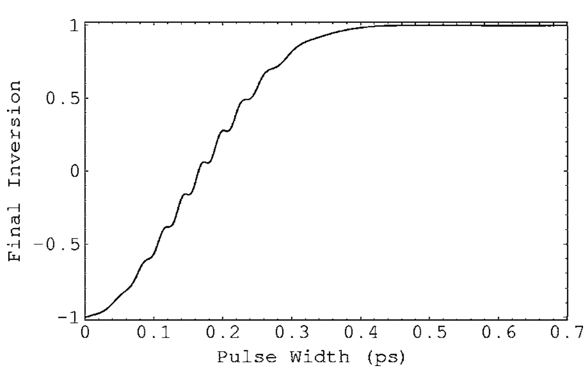


FIG. 4. The inversion $S_3(t)$ at $t=2t_0$ obtained from numerical solution of the nonlinear Bloch equations as a function of the pulse width $t_p$ for a Gaussian pulse with $\bar{\Omega}=(\gamma-\beta)$ and $\phi(t)=0$.

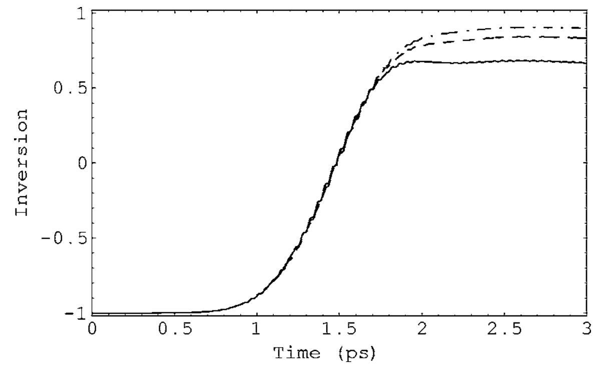


FIG. 5. The same as in Fig. 3 but with $\tau=3$ ps (solid curve), $\tau=6$ ps (dashed curve), and $\tau=10$ ps (dot-dashed curve).

with the quasi-adiabatic following approximation of Ref. 35. Also, by comparing Figs. 2 and 5, we see that the effects of relaxation processes in the present method are slightly more detrimental than previously to the efficiency of electron transfer in the upper subband.

We note that the above condition is not the only one that could lead to complete inversion in the system. Actually, complete electron transfer to the upper subband occurs for several values of the maximum Rabi frequency, as long as $\bar{\Omega}>\gamma-\beta$, as is shown in Fig. 6. Also, complete return to the initially occupied lower subband can be found for several values of $\bar{\Omega}$ (see Fig. 6). This figure shows a typical switching behavior that could be found in the quantum well system. For the results of Fig. 6 the transfer process takes 3 ps. The effects of relaxation processes in the maximum inversion are shown in Fig. 7. We note that in the presence of relaxation processes complete electron transfer to the upper subband or complete electron return to the lower subband is not possible.

## V. CONCLUSIONS

In this work we have studied the electron dynamics in a symmetric double quantum well, in the two-subband approximation, that is coupled by a strong pulsed electric field.

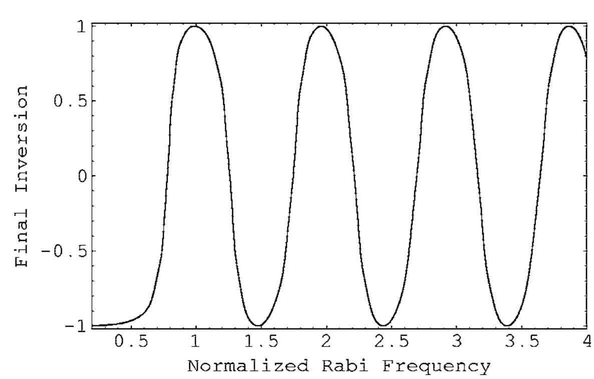


FIG. 6. The inversion $S_3(t)$ at $t=2t_0$ obtained from numerical solution of the nonlinear Bloch equations as a function of the normalized Rabi frequency $\bar{\Omega}/(\gamma-\beta)$ for a Gaussian pulse with parameters $t_0=1.5$ ps, $t_p=0.6$ ps, and $\phi(t)=0$.

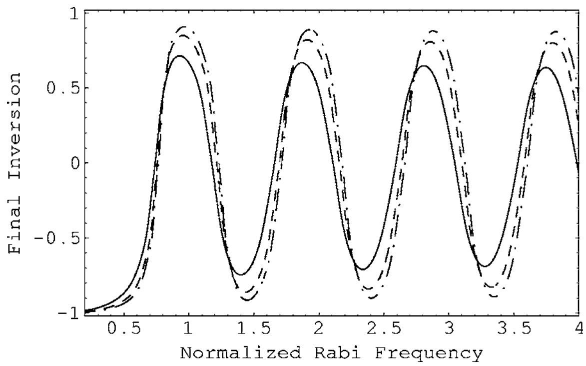


FIG. 7. The same as in Fig. 6 but with $\tau$=3 ps (solid curve), $\tau$=6 ps (dashed curve), and $\tau$=10 ps (dot-dashed curve).

We have used the effective nonlinear Bloch equations[29] for the description of the system dynamics. We have shown that in the RWA the effective nonlinear Bloch equations of the two-subband system are similar to the optical Bloch equations for a dense collection of two-level atoms interacting with pulsed laser fields. Then, we present analytical solutions for the case that the system interacts with hyperbolic secant pulses with or without time-dependent frequency. In addition, conditions that lead to complete inversion of the electronic population in the two-subband system are also presented in the case that the quantum well structure interacts with a Gaussian pulse. Our findings are verified by results obtained from a numerical solution of the nonlinear Bloch equations for a realistic double quantum well based on GaAs. We have found that significant population inversion occurs even in the cases where relaxation processes are taken into account.

We finally note that in the results presented above the pulse area (integral of the time-dependent Rabi frequency) is not a parameter that should get very specific values in order to achieve complete inversion in the two-subband system. This is in contrast to what one may expect from the Rabi solution of the optical Bloch equations for an atomic two-level system interacting with pulsed laser fields.[33] However, the present system of effective Bloch equations is a nonlinear system and its dynamics, as we have shown above, is quite different from the well known optical Bloch equations. Therefore, one may apply the results we present here in order to obtain Rabi oscillations between two subbands in a semiconductor quantum well system.

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