Spin-orbit-induced correlations of the local density of states in a two-dimensional electron gas

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We study the local density of states (LDOS) of two-dimensional noninteracting electrons in the presence of spin-orbit (SO) coupling. Although SO coupling has no effect on the average density of states, it manifests itself in the correlations of the LDOS. Namely, the correlation function acquires two satellites centered at energy difference equal to the SO splitting, $2\omega_{SO}$, of the electron Fermi surface. For a smooth disorder the satellites are well separated from the main peak. Weak Zeeman splitting $\omega_Z \ll \omega_{SO}$ in a parallel magnetic field causes an anomaly in the shape of the satellites. We consider the effect of SO-induced satellites in the LDOS correlations on the shape of the correlation function of resonant-tunneling conductances at different source-drain biases, which can be measured experimentally. This shape is strongly sensitive to the relation between ω_{SO} and ω_Z.

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I. INTRODUCTION

The density of states, ν_0, in two-dimensional (2D) electron gas is energy independent. It remains energy independent in a parallel magnetic field, which causes the spin splitting, $2\omega_Z$, of the electron spectrum. The density of states, corresponding to each spin branch, is equal to $\nu_0/2$. Spin-orbit (SO) coupling also results in the splitting of the electron spectrum into two branches.[1] In this case the density of states in each branch, $\mu=\pm 1$, depends on energy

$$\nu_\mu(\epsilon) = \frac{\nu_0}{2}\left[1 + \mu\frac{\omega_{SO}}{2\epsilon}\right], \tag{1}$$

where $\pm\omega_{SO}$ is the splitting of a state which had the energy $\epsilon \gg \omega_{SO}$ in the absence of the SO coupling. As can be seen from Eq. (1), the *net* density of states is still identically equal to ν_0. Since the magnitude of the splitting, $2\omega_{SO}$, at the Fermi level $\epsilon=E_F$ is inversely proportional to the length of the spin rotation, this magnitude represents an important characteristic of the 2D structure, containing the electron gas. The importance of the spin-rotation length was appreciated since 1990, when the proposal for device application of the spin-polarized currents was put forward.[2] This proposal has lately attracted a lot of interest.

If the magnitude of the SO splitting is large enough, it can be inferred from the beating pattern of the Shubnikov–de Haas oscillations, as was demonstrated by Dorozhkin and Olshanetskii[3] for Si-based structures, and subsequently[4] by Luo *et al.* for narrow-gap heterostructures. While for large ω_{SO} (several meV), as in Ref. 4, the beats of the Shubnikov–de Haas oscillations yield a rather accurate value of SO splitting, extracting small splittings from the beating pattern is complicated in two regards: (i) very low magnetic fields (with cyclotron energy smaller than ω_{SO}) and, correspondingly, very low temperatures are required to observe the beatings; (ii) presence of even moderate disorder suppresses the Shubnikov–de Haas oscillations by the Dingle factor, which gets large at low fields.

In the present paper we demonstrate that the disorder can, actually, *reveal* the SO splitting even if ω_{SO} is smaller than the single particle scattering rate, τ^{-1}. In a disordered sample the local density of states (LDOS) fluctuates randomly in space and these fluctuations contain information about SO coupling. Our main point is that this information, which is lost in the average LDOS, is preserved in the correlation function of LDOS at two different energies, $P(\epsilon_1,\epsilon_2)$. This function, whose magnitude is inversely proportional to the disorder, exhibits peaks at $(\epsilon_1-\epsilon_2)=\pm 2\omega_{SO}$. Even for $\omega_{SO}\tau \ll 1$ the peaks are well pronounced when the disorder is smooth, so that the transport relaxation time, τ_{tr}, is much longer than τ. This is illustrated in Fig. 1. Smooth disorder implies that the momentum transfer in a single scattering act is small compared to the Fermi momentum, k_F. In this way, the states with energies $\epsilon+\omega_{SO}$ and $\epsilon-\omega_{SO}$ remain correlated after many, $\sim\tau_{tr}/\tau$, scattering acts. Therefore, the SO-induced satellites in $P(\epsilon_1,\epsilon_2)$ are sharp even in the presence of a strong disorder, $\tau^{-1} > \omega_{SO}$, if the condition $\omega_{SO}\tau_{tr} \gg 1$ is

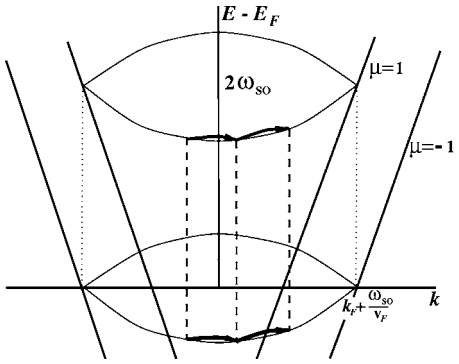

FIG. 1. Schematic illustration of the processes responsible for SO-induced peaks in LDOS correlator at $\epsilon=\pm 2\omega_{SO}$.

met. In terms of underlying physics, the observation that a small SO-splitting can be resolved in a smooth potential is quite analogous to the observation made in Ref. 5 concerning the manifestation of the Landau levels in $P(\epsilon_1,\epsilon_2)$ in the presence of a strong disorder, when the Landau level structure in the average density of states is completely smeared out.[6] Let us emphasize, though, that in our case (unlike Ref. 5) the density of states in the absence of disorder is constant, which does not contain ω_{SO}. Thus, it is only in the presence of disorder that LDOS becomes sensitive to the SO splitting.

Concerning the experimental consequences of SO-induced satellites in $P(\epsilon_1,\epsilon_2)$, we note that the local density of states governs the probability of electron tunneling from the impurity in the barrier into the 2D gas.[7–9] This fact was recently utilized in the resonant-tunneling spectroscopy[10–14] of the LDOS. More specifically, the correlator of the LDOS determines the behavior of correlation function, $C(\delta V)$, of the fluctuations of the tunneling conductance with the change, δV, of the source-drain bias. From the analysis of $C(\delta V)$, measured experimentally, the authors[13] were able to extract the quantitative information about the structure of states and their inelastic lifetime in the disordered emitter. In the present paper we demonstrate that the SO-induced correlations in the LDOS give rise to the new characteristic features in $C(\delta V)$. We also demonstrate that these new features are extremely sensitive to a weak parallel magnetic field.

The paper is organized as follows. In Sec. II we derive the analytic expression for $P(\epsilon_1-\epsilon_2)$ in the case of a smooth disorder. In Sec. III we give the qualitative explanation for the ratio between the amplitudes of the main peak in $P(\epsilon_1-\epsilon_2)$ and of the SO-induced satellites. In Sec. IV we use the shape of $P(\epsilon_1-\epsilon_2)$ to calculate the SO-induced features in the correlator, $C(\delta V)$, of the tunneling conductances. Concluding remarks are presented in Sec. V.

II. CALCULATION OF $P(\epsilon_1-\epsilon_2)$

A. Hamiltonian and eigenfunctions

We choose the conventional form, $\alpha(\hat{\boldsymbol{\sigma}}\times\boldsymbol{k})\boldsymbol{n}$, of the SO-term originating from the confinement potential asymmetry.[1] Here α is the coupling constant, $\hat{\boldsymbol{\sigma}}$ is the spin operator, and $\boldsymbol{n}$ is the unit vector normal to the two-dimensional plane. In a parallel magnetic field that induces the Zeeman splitting $2\omega_Z$, the Hamiltonian of a free electron is

$$\hat{H}=\frac{\hbar^2k^2}{2m}+\alpha(\hat{\boldsymbol{\sigma}}\times\boldsymbol{k})\boldsymbol{n}+\omega_Z\hat{\sigma}_x, \tag{2}$$

where m is the electron mass. The eigenfunctions of the Hamiltonian Eq. (2) are classified according to the chirality, $\mu=\pm1$, and form two branches of the spectrum, as illustrated in Fig. 1. In the vicinity of the Fermi surface the spectrum, $E_\mu(\boldsymbol{k})$, can be simplified

$$E_\mu(k)=E_F+\epsilon_\mu(\boldsymbol{k}), \tag{3}$$

where $\epsilon_\mu(\boldsymbol{k})$ is defined as

$$\epsilon_\mu(\boldsymbol{k})=\hbar v_F(k-k_F)+\mu\Omega(\boldsymbol{k}), \tag{4}$$

and

$$\Omega(\boldsymbol{k})=\sqrt{\omega_{SO}^2+\omega_Z^2+2\omega_{SO}\omega_Z\sin\phi_{\boldsymbol{k}}}. \tag{5}$$

Here $v_F=\hbar k_F/m$ is the Fermi velocity, $\omega_{SO}=\alpha k_F$, and $\phi_{\boldsymbol{k}}$ is the azimuthal angle of $\boldsymbol{k}$. It is convenient to introduce the projection operators $\hat{\Lambda}_\mu(\boldsymbol{k})$, which are defined through the eigenfunctions

$$\chi_\mu(\boldsymbol{k})=\frac{1}{2^{1/2}}\begin{pmatrix}1\\ -i\mu\exp(i\varphi_{\boldsymbol{k}})\end{pmatrix}, \tag{6}$$

of the Hamiltonian Eq. (2) as $\hat{\Lambda}_\mu(\boldsymbol{k})=\chi_\mu(\boldsymbol{k})\chi_\mu^\dagger(\boldsymbol{k})$, so that[15]

$$\hat{\Lambda}_\mu(\boldsymbol{k})=\frac{1}{2}\begin{pmatrix}1 & i\mu\exp(-i\varphi_{\boldsymbol{k}})\\ -i\mu\exp(i\varphi_{\boldsymbol{k}}) & 1\end{pmatrix}, \tag{7}$$

where the angle $\varphi_{\boldsymbol{k}}$ is related to the azimuthal angle $\phi_{\boldsymbol{k}}$ as

$$\tan\varphi_{\boldsymbol{k}}=\tan\phi_{\boldsymbol{k}}+\frac{\omega_Z}{\omega_{SO}\cos\phi_{\boldsymbol{k}}}. \tag{8}$$

In terms of projection operators the Hamiltonian can be presented in a simple form $\hat{H}=\Sigma_\mu E_\mu(\boldsymbol{k})\hat{\Lambda}_\mu(\boldsymbol{k})$.

B. Correlator of the LDOS in the chirality representation: Diffusive regime

In the presence of disorder, the LDOS at energy ϵ at point $\boldsymbol{r}$ is defined as

$$\nu(\boldsymbol{r},\epsilon)=\frac{1}{2\pi i}\mathrm{Tr}[\hat{G}^A(\boldsymbol{r},\boldsymbol{r},\epsilon)-\hat{G}^R(\boldsymbol{r},\boldsymbol{r},\epsilon)], \tag{9}$$

where the advanced and retarded Green's functions are 2×2 matrices in the chirality space, and the trace is taken over different chiralities. The main contribution to the LDOS correlator comes from the cross terms $G^{(R)}G^{(A)}$

$$P(\epsilon-\epsilon')=\frac{1}{\nu_0^2}\langle\delta\nu(\boldsymbol{r},\epsilon)\,\delta\nu(\boldsymbol{r},\epsilon')\rangle$$
$$=\frac{1}{2\pi^2\nu_0^2}\,\mathrm{Re}\langle\mathrm{Tr}\,\hat{G}^R(\boldsymbol{r},\boldsymbol{r},\epsilon)\mathrm{Tr}\,\hat{G}^A(\boldsymbol{r},\boldsymbol{r},\epsilon')\rangle, \tag{10}$$

where $\delta\nu(\boldsymbol{r},\epsilon)$ is the deviation of the LDOS from the disorder-free value, ν_0. It is easy to see from Eq. (4) that $\nu_0=m/\pi\hbar^2$, and does not depend on energy. To evaluate the correlator Eq. (10) it is convenient to use the chirality representation of the Green functions

$$\hat{G}^{R,A}(\epsilon,\boldsymbol{p})=\sum_\mu\frac{\hat{\Lambda}_\mu(\boldsymbol{p})}{\epsilon-\epsilon_\mu(\boldsymbol{p})\pm\dfrac{i}{2\tau}}=\sum_\mu\hat{\Lambda}_\mu(\boldsymbol{p})G_\mu^{R,A}(\epsilon,\boldsymbol{p}), \tag{11}$$

where τ is the scattering time. As in the absence of the SO coupling, the LDOS correlator represents the sum of the diffusion and cooperon contributions.[16] Generalized expressions for the corresponding contributions in the chirality representation read

$$P_D(\epsilon-\epsilon')=\frac{1}{2\pi^2\nu_0^2}\,\mathrm{Re}\int\frac{d\boldsymbol{q}}{(2\pi)^2}\int\frac{d\boldsymbol{p}}{(2\pi)^2}\int\frac{d\boldsymbol{p}'}{(2\pi)^2}\sum_{\{\mu,\eta\}}\Gamma^{\mu_1\eta_1}_{\mu_2\eta_2}(\boldsymbol{p},\boldsymbol{p}',\boldsymbol{q},\epsilon-\epsilon')\Phi^{*}_{\mu_1,\eta_1}(\boldsymbol{p},\boldsymbol{p}')\Phi_{\mu_2,\eta_2}(\boldsymbol{p},\boldsymbol{p}')\delta_{-\mu_1,\eta_1}\delta_{-\mu_2,\eta_2}G^R_{\mu_1}(\epsilon,\boldsymbol{p}+\boldsymbol{q})G^R_{\eta_1}(\epsilon,\boldsymbol{p}'+\boldsymbol{q})G^A_{\mu_2}(\epsilon',\boldsymbol{p})G^A_{\eta_2}(\epsilon',\boldsymbol{p}'),\tag{12}$$

$$P_C(\epsilon-\epsilon')=\frac{1}{2\pi^2\nu_0^2}\,\mathrm{Re}\int\frac{d\boldsymbol{q}}{(2\pi)^2}\int\frac{d\boldsymbol{p}}{(2\pi)^2}\int\frac{d\boldsymbol{p}'}{(2\pi)^2}\sum_{\{\mu,\eta\}}\Gamma^{\mu_1\eta_1}_{\mu_2\eta_2}(\boldsymbol{p},\boldsymbol{p}',\boldsymbol{q},\epsilon-\epsilon')\Phi_{\mu_1,\eta_1}(\boldsymbol{p},\boldsymbol{p}')\Phi^{*}_{\mu_2,\eta_2}(\boldsymbol{p},\boldsymbol{p}')G^R_{\mu_1}(\epsilon,\boldsymbol{q}-\boldsymbol{p})G^R_{\eta_1}(\epsilon,\boldsymbol{q}-\boldsymbol{p}')G^A_{\mu_2}(\epsilon',\boldsymbol{p})G^A_{\eta_2}(\epsilon',\boldsymbol{p}')=P_D(\epsilon'-\epsilon),\tag{13}$$

where

$$\Phi_{\mu,\eta}(\boldsymbol{p},\boldsymbol{p}')=\chi^{\dagger}_{\mu}(\boldsymbol{p})\chi_{\eta}(\boldsymbol{p}')=\tfrac{1}{2}\{1+\mu\eta\exp(i[\varphi_p-\varphi_{p'}])\}.\tag{14}$$

The two-particle vertex functions $\Gamma^{\mu_1\eta_1}_{\mu_2\eta_2}(\boldsymbol{p},\boldsymbol{p}',\boldsymbol{q},\epsilon)$ in Eqs. (12) and (13) satisfy the matrix Dyson-type equation

$$\Gamma^{\mu_1\eta_1}_{\mu_2\eta_2}(\boldsymbol{p},\boldsymbol{p}',\boldsymbol{q},\epsilon)=S(\boldsymbol{p}-\boldsymbol{p}')\Phi_{\mu_1,\eta_1}(\boldsymbol{p},\boldsymbol{p}')\Phi_{\mu_2,\eta_2}(\boldsymbol{p},\boldsymbol{p}')+\int\frac{d\boldsymbol{p}_1}{(2\pi)^2}\sum_{\xi_1,\xi_2}K^{\mu_1\xi_1}_{\mu_2\xi_2}(\boldsymbol{p},\boldsymbol{p}_1,\boldsymbol{q},\epsilon)\Gamma^{\xi_1\eta_1}_{\xi_2\eta_2}(\boldsymbol{p}_1,\boldsymbol{p}',\boldsymbol{q},\epsilon)\tag{15}$$

with a kernel

$$K^{\mu_1\xi_1}_{\mu_2\xi_2}(\boldsymbol{p},\boldsymbol{p}_1,\boldsymbol{q},\epsilon)=\Phi_{\mu_1,\xi_1}(\boldsymbol{p},\boldsymbol{p}')\Phi_{\mu_2,\xi_2}(\boldsymbol{p},\boldsymbol{p}')G^R_{\xi_1}(\omega+\epsilon,\boldsymbol{p}_1+\boldsymbol{q})G^A_{\xi_2}(\omega,\boldsymbol{p}_1)S(\boldsymbol{p}-\boldsymbol{p}'),\tag{16}$$

where the function $S(\boldsymbol{p}-\boldsymbol{p}')$ is the Fourier transform of the correlator of the random potential. Upon integration over $|\boldsymbol{p}_1|$ Eq. (15) takes the form

$$\Gamma^{\mu_1\eta_1}_{\mu_2\eta_2}(\boldsymbol{p},\boldsymbol{p}',\boldsymbol{q},\epsilon)=S(\boldsymbol{p}-\boldsymbol{p}')\Phi_{\mu_1,\eta_1}(\boldsymbol{p},\boldsymbol{p}')\Phi_{\mu_2,\eta_2}(\boldsymbol{p},\boldsymbol{p}')+\int\frac{d\phi_{p_1}}{(2\pi)^2}\sum_{\xi_1,\xi_2}\tilde{K}^{\mu_1\xi_1}_{\mu_2\xi_2}(\boldsymbol{p},\boldsymbol{p}_1,\boldsymbol{q},\epsilon)\Gamma^{\xi_1\eta_1}_{\xi_2\eta_2}(\boldsymbol{p}_1,\boldsymbol{p}',\boldsymbol{q},\epsilon),\tag{17}$$

where we have introduced the modified kernel $\tilde{K}^{\mu_1\xi_1}_{\mu_2\xi_2}$, defined as

$$\tilde{K}^{\mu_1\xi_1}_{\mu_2\xi_2}(\boldsymbol{p},\boldsymbol{p}_1,\boldsymbol{q},\epsilon)=\left(\frac{m\tau}{2\pi}\right)\frac{\Phi_{\mu_1,\xi_1}(\boldsymbol{p},\boldsymbol{p}')\Phi_{\mu_2,\xi_2}(\boldsymbol{p},\boldsymbol{p}')S(\boldsymbol{p}-\boldsymbol{p}')}{1-i[\omega-\xi_1\Omega(\boldsymbol{p}_1)+\xi_2\Omega(\boldsymbol{p}_1+\boldsymbol{q})]\tau+i\hbar qv_{\mathrm{F}}\tau\cos(\phi_p-\phi_q)}.\tag{18}$$

Here τ is the elastic scattering time. When the random potential is smooth, the function S restricts the difference $\boldsymbol{p}-\boldsymbol{p}'$ within a narrow domain $|\boldsymbol{p}-\boldsymbol{p}'|\ll k_{\mathrm{F}}$. As we will see below this leads to a drastic simplification of the system Eqs. (17) and (18).

C. Smooth potential; $|\epsilon_1-\epsilon_2|\approx 2\omega_{\mathrm{SO}}$

In the case of a smooth potential the factors $\Phi_{\mu,\eta}(\boldsymbol{p},\boldsymbol{p}')$ for coinciding chiralities, $\mu=\eta$, differ strongly from those with opposite chiralities, $\mu=-\eta$. Indeed, when $|\boldsymbol{p}-\boldsymbol{p}'|$ is small compared to k_{F}, we have $\phi_p\approx\phi_{p'}$ and thus $\varphi_p\approx\varphi_{p'}$, so that $\Phi_{\mu,-\mu}\ll 1$, whereas $\Phi_{\mu,\mu}$ is close to unity and can be presented as

$$\Phi_{\mu,\mu}(\boldsymbol{p},\boldsymbol{p}')=1-\frac{(\varphi_p-\varphi_{p'})^2}{8}.\tag{19}$$

Using Eq. (8), the factor $\Phi_{\mu,\mu}$ can be expressed through the angle, $\phi_p-\phi_{p'}$, between the vectors $\boldsymbol{p}$ and $\boldsymbol{p}'$ as follows:

$$\Phi_{\mu,\mu}(\boldsymbol{p},\boldsymbol{p}')=1-\frac{\omega_{\mathrm{SO}}^2(\omega_{\mathrm{SO}}+\omega_Z\sin\phi_p)^2}{4\Omega^4(\boldsymbol{p})}(\phi_{p'}-\phi_p)^2.\tag{20}$$

The latter simplification allows us to set $\Phi_{\mu,-\mu}=0$ and $\Phi_{\mu,\mu}=1$ everywhere except for the kernel of the Dyson equation. As a result, the system Eqs. (17) gets decoupled into *closed* equations for the elements, $\Gamma^{\mu,\mu}_{\eta,\eta}$ with $\mu\neq\eta$, of the matrix $\Gamma^{\mu_1\eta_1}_{\mu_2\eta_2}$. The underlying reason for this decoupling is that these elements describe the two-particle motion, in course of which one particle moves within the branch μ whereas another particle moves within the branch η. For a smooth potential coupling of these elements to the other branch-nonconserving elements is small in parameter $(1-\Phi_{\mu,\mu})\ll 1$. With the above simplification Eq. (17) takes the form[17]

$$\Gamma^{\mu,\mu}_{\eta,\eta}(\boldsymbol{p},\boldsymbol{p}',\boldsymbol{q},\omega) = S(k_F|\phi_{\boldsymbol{p}} - \phi_{\boldsymbol{p}'}|) + \int \frac{d\phi_{\boldsymbol{p}_1}}{(2\pi)^2}\tilde{K}^{\mu,\mu}_{\eta,\eta}(\boldsymbol{p},\boldsymbol{p}_1,\boldsymbol{q},\omega)\Gamma^{\mu,\mu}_{\eta,\eta}(\boldsymbol{p}_1,\boldsymbol{p}',\boldsymbol{q},\omega), \tag{21}$$

while the kernel Eq. (18) simplifies to

$$\tilde{K}^{\mu,\mu}_{\eta,\eta}(\boldsymbol{p},\boldsymbol{p}_1,\boldsymbol{q},\omega) = \left(\frac{m\tau}{2\pi}\right)\frac{|\Phi_{\mu,\mu}(\boldsymbol{p},\boldsymbol{p}_1)|^2 S(k_F|\phi_{\boldsymbol{p}} - \phi_{\boldsymbol{p}_1}|)}{1 - i\tau[\omega - (\mu - \eta)\Omega(\boldsymbol{p}) - \hbar q v_F \cos(\phi_{\boldsymbol{p}} - \phi_{\boldsymbol{q}})]}. \tag{22}$$

Substituting Eq. (22) into Eq. (21) yields an integral equation for the vertex $\Gamma^{\mu,\mu}_{\eta,\eta}$. Note, that this equation differs from the standard equation for the vertex in the absence of the SO coupling only by the factor $|\Phi_{\mu,\mu}(\boldsymbol{p},\boldsymbol{p}_1)|^2$ in the integrand, where $\Phi_{\mu,\mu}(\boldsymbol{p},\boldsymbol{p}_1)$ is defined by Eq. (19). Thus, if the small difference of this factor from unity is neglected, the solution for $\Gamma^{\mu,\mu}_{\eta,\eta}$ would contain a conventional diffusive pole at $\omega = (\mu - \eta)\Omega(\boldsymbol{p}) - iDq^2$, where $D = v_F^2\tau_{tr}/2$ is the diffusion coefficient, and

$$\tau_{tr}^{-1} = \frac{1}{2}\int \frac{d\boldsymbol{p}}{(2\pi)^2}(\phi_{\boldsymbol{k}} - \phi_{\boldsymbol{p}})^2 S(|\boldsymbol{k} - \boldsymbol{p}|)\,\delta[E_\mu(\boldsymbol{k}) - E_\mu(\boldsymbol{p})] \tag{23}$$

is the transport relaxation time. It is much longer than the scattering time, τ, which is given by Eq. (23) in which the factor $(\phi_{\boldsymbol{k}} - \phi_{\boldsymbol{p}})^2$ is replaced by 2. Taking into account the small correction originating from the difference $1 - |\Phi_{\mu,\mu}(\boldsymbol{p},\boldsymbol{p}_1)|^2$ amounts to the imaginary shift of the pole position by $i\tau_{int}^{-1}$, where τ_{int} is defined as

$$\tau_{int}^{-1}(\boldsymbol{k}) = \frac{1}{4}\int \frac{d\boldsymbol{p}}{(2\pi)^2}(\varphi_{\boldsymbol{k}} - \varphi_{\boldsymbol{p}})^2 S(|\boldsymbol{k} - \boldsymbol{p}|)\,\delta[E_\mu(\boldsymbol{k}) - E_\mu(\boldsymbol{p})]. \tag{24}$$

The meaning of τ_{int} is the interbranch scattering time. Indeed, a general expression for the scattering time between the branches with different chiralities can be written as

$$\tau^{-1}_{\mu,-\mu}(\boldsymbol{k}) = \int \frac{d\boldsymbol{p}}{(2\pi)^2}\mathrm{Tr}[\hat{\Lambda}_\mu(\boldsymbol{k})\hat{\Lambda}_{-\mu}(\boldsymbol{p})]S(|\boldsymbol{k} - \boldsymbol{p}|)\,\delta[E_\mu(\boldsymbol{k}) - E_{-\mu}(\boldsymbol{p})] = \int \frac{d\boldsymbol{p}}{(2\pi)^2}[1 - |\Phi_{\mu,\mu}(\boldsymbol{k},\boldsymbol{p})|^2]S(|\boldsymbol{k} - \boldsymbol{p}|)\,\delta[E_\mu(\boldsymbol{k}) - E_{-\mu}(\boldsymbol{p})]. \tag{25}$$

Comparing Eqs. (24) and (25), we see that they differ only by the arguments of the δ functions. This difference is negligible when $\omega_{SO} \ll E_F$, and thus we have $\tau_{\mu,-\mu}(\boldsymbol{k}) \approx \tau_{int}(\boldsymbol{k})$. We also see that the integral Eq. (24) in the expression for τ_{int}^{-1} differs from the integral in Eq. (23) for τ_{tr}^{-1} only by the replacement $\varphi_{\boldsymbol{k}} \to \phi_{\boldsymbol{k}}$. Using the relation Eq. (20) between the two angles, we can express τ_{int} through the transport relaxation time

$$\tau_{int}(\phi) = 2\tau_{tr}\frac{(\omega_Z^2 + \omega_{SO}^2 + 2\omega_Z\omega_{SO}\sin\phi)^2}{\omega_{SO}^2(\omega_{SO} + \omega_Z\sin\phi)^2}. \tag{26}$$

With shifted diffusion pole, the final expression for the vertex $\Gamma^{\mu\mu}_{\eta\eta}$ takes the form

$$\Gamma^{\mu\mu}_{\eta\eta}(\boldsymbol{p},\boldsymbol{p}',\boldsymbol{q},\omega) = \frac{iS(k_F|\phi_{\boldsymbol{p}} - \phi_{\boldsymbol{p}'}|)}{\tau[\omega - (\mu - \eta)\Omega(\boldsymbol{p}) + iDq^2 + i\tau_{int}^{-1}(\boldsymbol{p})]}. \tag{27}$$

Substituting this form into Eq. (12) yields the following expression for the diffusion contribution to the correlator of LDOS

$$P_D(\epsilon) = \frac{\tau}{\pi}\sum_{\mu\neq\eta}\mathrm{Re}\int_0^{1/v_F\tau_{tr}} dqq\int_0^{2\pi}\frac{d\phi_{\boldsymbol{p}}}{2\pi} \times \int_0^{2\pi}\frac{d\phi_{\boldsymbol{p}'}}{2\pi}\frac{S(k_F|\phi_{\boldsymbol{p}} - \phi_{\boldsymbol{p}'}|)}{-i[\epsilon - (\mu - \eta)\Omega(\boldsymbol{p})] + Dq^2 + \tau_{int}^{-1}(\boldsymbol{p})}. \tag{28}$$

Two integrations (over q and over $\phi_{\boldsymbol{p}'}$) in Eq. (28) can be readily performed resulting in the following expression for SO-induced satellites $P_D(\epsilon)$

$$P_D(\epsilon) = -\frac{1}{4\pi E_F\tau_{tr}}\sum_{\mu\neq\eta}\mathcal{L}_{\mu,\eta}(\epsilon), \tag{29}$$

where the functions $\mathcal{L}_{\mu,\eta}(\epsilon)$ are the following azimuthal averages

$$\mathcal{L}_{\mu,\eta}(\epsilon) = \int_0^{2\pi}\frac{d\phi}{2\pi}\ln\left[[\epsilon - (\mu - \eta)\Omega(\phi)]^2\tau_{tr}^2 + \frac{\tau_{tr}^2}{\tau_{int}^2(\phi)}\right]. \tag{30}$$

The ratio $\tau_{tr}/\tau_{int}(\phi)$ in Eq. (30) is defined by Eq. (26). This ratio is equal to 1/2 for $\omega_Z = 0$ and is ~ 1 for $\omega_Z \sim \omega_{SO}$. Since $\mathcal{L}_{\mu,\eta}(\epsilon) = \mathcal{L}_{\eta,\mu}(-\epsilon)$ the cooperon contribution, $P_C(\epsilon)$, to the LDOS correlator coincides with $P_D(\epsilon)$, as it should be expected on general grounds. Thus, Eqs. (29) and (30) constitute our final result. The functions $\mathcal{L}_{\mu,\eta}(\epsilon)$, which determine the energy dependence of the correlator, exhibit a singular dependence on a weak magnetic field, $\omega_Z \ll \omega_{SO}$ for $\mu = -\eta$, as demonstrated in the next section. Here we note that the analytical expression Eq. (27) was obtained using a standard diffusive approximation, which amounts to expansion of denominator in Eq. (22). Validity of this expansion sets the upper limit $q \sim 1/v_F\tau_{tr}$ in the integral (28). Contribution to LDOS correlator from larger momenta is discussed below in Sec. II F.

D. Shape of the satellites

As it follows from Eq. (26), in a weak magnetic field we have $\tau_{int}(\phi) \approx 2\tau_{tr} = \mathrm{const}(\phi)$. Then for the satellites, centered at $\epsilon = \pm 2\omega_{SO}$ we can identify the ϕ dependence of the integrand in Eq. (30). This dependence comes from $\Omega(\phi)$, defined by Eq. (5). For $\omega_Z \ll \omega_{SO}$, we have $\Omega(\phi) \approx \omega_{SO} + \omega_Z\sin\phi$. Upon substituting this form into Eq. (30), the

integration can be performed analytically with the use of the following identity

$$\frac{1}{2\pi}\int_0^{2\pi} d\phi \ln(a+b\sin\phi)^2 = \begin{cases} \ln\left[\frac{1}{2}a+\frac{1}{2}\sqrt{a^2-b^2}\right]^2 & \text{for } |a|>|b| \\ 2\ln\frac{1}{2}|b| & \text{for } |a|<|b|, \end{cases} \tag{31}$$

yielding

$$P(\epsilon) = -\left(\frac{1}{\pi E_F \tau_{tr}}\right)\begin{cases} \ln|\epsilon-2\omega_{SO}|\tau_{tr} & \text{for } |\epsilon-2\omega_{SO}|>\omega_Z \\ \ln\omega_Z\tau_{tr} & \text{for } |\epsilon-2\omega_{SO}|\leqslant\omega_Z. \end{cases} \tag{32}$$

Equation (32) suggests that in the limit $\omega_Z \ll \omega_{SO}$ the SO satellites of the LDOS correlator exhibit plateaus of width $2\omega_Z$ centered at $\epsilon=\pm 2\omega_{SO}$. We will discuss the numerical results for the shape of the satellites later after calculating the shape of the central peak.

E. Smooth potential; $|\epsilon_1-\epsilon_2|\ll\omega_{SO}$

In this section we solve the system of Eqs. (17) with kernels defined by Eq. (18) for small energy difference $|\epsilon_1-\epsilon_2|\ll\omega_{SO}$. First, we point out that the vertex functions $\Gamma^{\mu,\mu}_{\mu,\mu}$ and $\Gamma^{-\mu,\mu}_{-\mu,\mu}$ are relevant, since in Eq. (18) ξ_1 must be equal to ξ_2. At the first glance, in Eq. (17) for $\Gamma^{\mu,\mu}_{\mu,\mu}$, only the term in the r.h.s. containing the same $\Gamma^{\mu,\mu}_{\mu,\mu}$ should be kept. Indeed, the coupling of this term to $\Gamma^{-\mu,\mu}_{-\mu,\mu}$ is determined by the kernel $\tilde{K}^{-\mu,\mu}_{-\mu,\mu}$, which, as it is seen from Eq. (18), contains a square of the small parameter $\Phi_{-\mu,\mu}$. However, unlike the case of the satellites, the "feedback" becomes important for small $|\epsilon_1-\epsilon_2|$. Namely, Eq. (17) for $\Gamma^{-\mu,\mu}_{-\mu,\mu}$ contains in the r.h.s. the coupling to $\Gamma^{\mu,\mu}_{\mu,\mu}$ with the same small coefficient $\propto\Phi^2_{-\mu,\mu}$. Specifics of the small energy difference, as compared to satellites, is that *both* $\Gamma^{\mu,\mu}_{\mu,\mu}$ and $\Gamma^{-\mu,\mu}_{-\mu,\mu}$ are resonant in this case. Thus Eq. (17) reduces to the following system of coupled equations:

$$\Gamma^{\mu,\mu}_{\mu,\mu}(\boldsymbol{p},\boldsymbol{p}',\boldsymbol{q},\omega) = S(k_F|\phi_{\boldsymbol{p}}-\phi_{\boldsymbol{p}'}|) + \int\frac{d\phi_{\boldsymbol{p}_1}}{(2\pi)^2}\tilde{K}^{\mu,\mu}_{\mu,\mu}(\boldsymbol{p},\boldsymbol{p}_1,\boldsymbol{q},\omega)\Gamma^{\mu,\mu}_{\mu,\mu}(\boldsymbol{p}_1,\boldsymbol{p}',\boldsymbol{q},\omega) + \int\frac{d\phi_{\boldsymbol{p}_1}}{(2\pi)^2}\tilde{K}^{\mu,-\mu}_{\mu,-\mu}(\boldsymbol{p},\boldsymbol{p}_1,\boldsymbol{q},\omega)\Gamma^{-\mu,\mu}_{-\mu,\mu}(\boldsymbol{p}_1,\boldsymbol{p}',\boldsymbol{q},\omega), \tag{33}$$

$$\Gamma^{-\mu,\mu}_{-\mu,\mu}(\boldsymbol{p},\boldsymbol{p}',\boldsymbol{q},\omega) = S(k_F|\phi_{\boldsymbol{p}}-\phi_{\boldsymbol{p}'}|)|\Phi_{\mu,-\mu}(\boldsymbol{p},\boldsymbol{p}_1)|^2 + \int\frac{d\phi_{\boldsymbol{p}_1}}{(2\pi)^2}\tilde{K}^{-\mu,-\mu}_{-\mu,-\mu}(\boldsymbol{p},\boldsymbol{p}_1,\boldsymbol{q},\omega)\Gamma^{-\mu,\mu}_{-\mu,\mu}(\boldsymbol{p}_1,\boldsymbol{p}',\boldsymbol{q},\omega) + \int\frac{d\phi_{\boldsymbol{p}_1}}{(2\pi)^2}\tilde{K}^{-\mu,\mu}_{-\mu,\mu}(\boldsymbol{p},\boldsymbol{p}_1,\boldsymbol{q},\omega)\Gamma^{\mu,\mu}_{\mu,\mu}(\boldsymbol{p}_1,\boldsymbol{p}',\boldsymbol{q},\omega). \tag{34}$$

Solution of the system (33) and (34), yields[22]

$$\Gamma^{\mu\mu}_{\mu\mu}(\boldsymbol{p},\boldsymbol{p}',\boldsymbol{q},\omega) = \frac{i}{2\tau}S(k_F|\phi_{\boldsymbol{p}}-\phi_{\boldsymbol{p}'}|)\left\{\frac{1}{\omega+iDq^2+i\tau_f^{-1}(\boldsymbol{p})}+\frac{1}{\omega+iDq^2+2i\tau_{int}^{-1}(\boldsymbol{p})}\right\}, \tag{35}$$

$$\Gamma^{-\mu\mu}_{-\mu\mu}(\boldsymbol{p},\boldsymbol{p}',\boldsymbol{q},\omega) = \frac{i}{2\tau}S(k_F|\phi_{\boldsymbol{p}}-\phi_{\boldsymbol{p}'}|)\left\{\frac{1}{\omega+iDq^2+i\tau_f^{-1}(\boldsymbol{p})}-\frac{1}{\omega+iDq^2+2i\tau_{int}^{-1}(\boldsymbol{p})}\right\}, \tag{36}$$

where the diffusion pole in the first term is cut by the inelastic time, τ_f. Substituting $\Gamma^{\mu\mu}_{\mu\mu}$ and $\Gamma^{-\mu\mu}_{-\mu\mu}$ into the general expression Eq. (12) and keeping only the terms with coefficients $\Phi_{\mu\mu}\approx 1$, we arrive at the final result

$$P(\epsilon) = -\left(\frac{1}{2\pi E_F\tau_{tr}}\right)\left[\ln\left(\epsilon^2\tau_{tr}^2+\frac{\tau_{tr}^2}{\tau_f^2}\right)+\ln(\epsilon^2\tau_{tr}^2+1)\right]. \tag{37}$$

Note that each term in the sum (37) is comprised of two (equal) contributions from $\Gamma^{1,1}_{1,1}$ and $\Gamma^{-1,-1}_{-1,-1}$.

F. Ballistic contribution

It might seem from Eq. (28) that the upper limit, $q=1/v_F\tau_{tr}$, narrows dramatically the domain of validity of the results for LDOS correlator. However, this is not the case. Calculation of the average product of two Green functions (in application to the interaction-induced corrections to the DOS) with energy difference exceeding the inverse scattering time was first carried out in Ref. 18. In particular, it was demonstrated in Ref. 18 that, upon using the *exact* vertex function, the logarithmic form of this product persists at $\epsilon\tau\gg 1$. However, consideration in Ref. 18 was restricted to the short-range disorder, while it is essential for us that the dis-

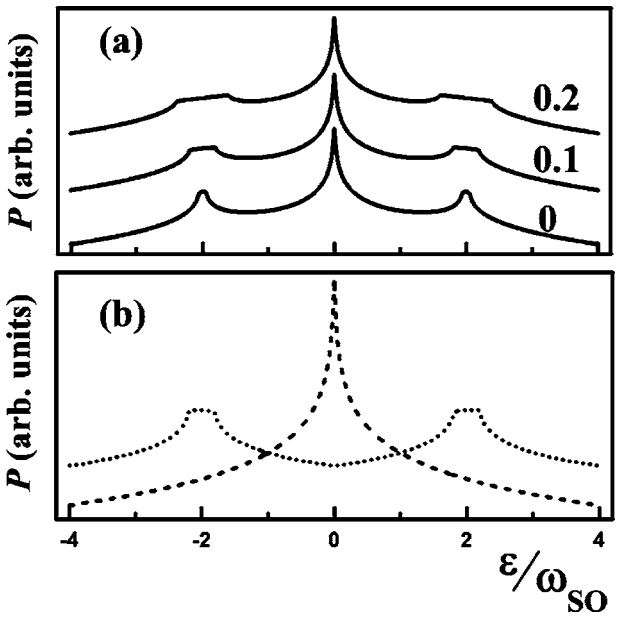

FIG. 2. (a) Correlator of the LDOS, $P(\epsilon_1,\epsilon_2)$, calculated from Eqs. (29) and (37) for $\omega_{SO}\tau_{tr}=50$ and $\tau_f/\tau_{tr}=10$, is plotted versus dimensionless energy ϵ/ω_{SO}. The numbers near the lines show the value of ω_Z/ω_{SO}. For convenience different curves are shifted along the vertical axis. (b) Separate contributions of the main peak (dashed line) and satellites (dotted line) are shown for $\omega_Z/\omega_{SO}=0.1$.

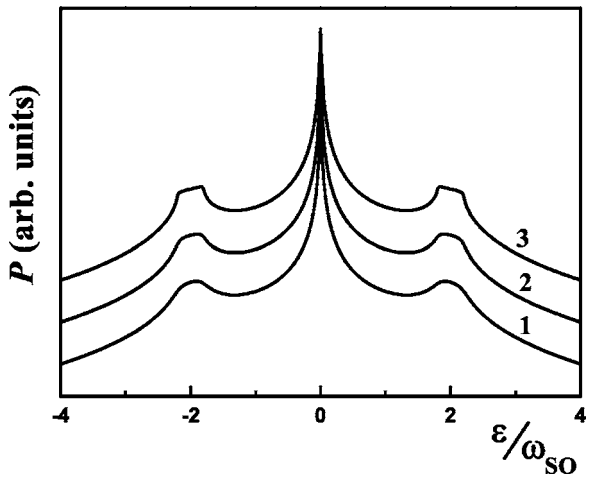

FIG. 3. Correlator of the LDOS, $P(\epsilon_1,\epsilon_2)$, calculated from Eqs. (29) and (37), is plotted versus dimensionless energy ϵ/ω_{SO} for $\omega_Z/\omega_{SO}=0.1$ and different values of $\omega_{SO}\tau_{tr}$ equal to 10 (curve 1); 20 (curve 2); and 50 (curve 3). For convenience different curves are shifted along the vertical axis.

order is smooth. The insight for the situation of a smooth disorder comes from Ref. 19, where it was demonstrated that, as soon as the relevant spatial scale exceeds the correlation radius, d, of the random potential (which is much smaller than the mean free path), the single-particle scattering time in the diffuson must be replaced by the transport time. Thus, the combination of Refs. 18 and 19 suggests that correct result for the LDOS correlator in the ballistic regime and in smooth disorder can be obtained by extending integration in Eq. (28) to high momenta. The same conclusion can be inferred from Ref. 5, where the LDOS correlator in a smooth potential and in a classical magnetic field was studied. In the case of a smooth disorder the adequate physical picture is a semiclassical electron motion along weakly deflected trajectories. In this regard, in a magnetic field the natural cutoff of the momentum integration is the inverse cyclotron radius, R_c, which was assumed in Ref. 5 to be larger than d. This assumption is necessary to ensure that classical trajectories with the length $2\pi R_c$ are longer than d. In the absence of magnetic field, classical trajectories can have an arbitrary length. Level-level correlation comes only from long trajectories (with length exceeding d). Hence the momentum cutoff of the logarithm must be chosen at d^{-1}.

G. Numerical results

Numerical results for the full LDOS correlator are presented in Fig. 2(a). The full correlator is the sum of the main peak Eq. (37) and two satellites given by Eqs. (29) and (30). Separate contributions to the full correlator from the main peak and from the satellites, calculated from Eqs. (37) and (30), respectively, are shown in Fig. 2(b). Integral (30) was evaluated numerically upon substitution of Eq. (26) into the integrand [asymptotical expression Eq. (32) was not used in the numerics]. We emphasize that the plateaus in Fig. 2 are present and remain sharp without neglecting $(\tau_{tr}/\tau_{int})^2$ in the integrand. In Fig. 3 we illustrate how sharp plateaus develop upon gradual increase of the product $\omega_{SO}\tau_{tr}$. The plateaus are also slightly tilted. The reason for this tilt is the effect of the "background" originating from the main peak. In our numerics the ballistic contribution, discussed in the previous section, was neglected. The justification for this is that, under the condition $\omega_{SO}<v_F/d$, the difference between the cutoffs $q=1/v_F\tau_{tr}$ and $q=d^{-1}$ in Eq. (28) amounts to the *energy-independent constant* within the *entire* domain $\epsilon\sim\omega_{SO}$. The condition $\omega_{SO}<v_F/d$ is met in all realistic situations. It ensures that the slopes and behavior between the peaks in Fig. 2 are reliable. On the other hand, the constant coming from ballistic contribution does not affect the correlation function of the tunneling conductance studied in Sec. IV.

III. DISCUSSION

The general case of an arbitrary SO splitting would require a lengthy calculation, involving 4×4 diffusion and cooperon matrices.[20,21] However, in the most interesting case of a strong splitting, when $\omega_{SO}\tau_{int}\gg1$, a considerable simplification occurs: the LDOS correlation function, $P(\epsilon)$, at large enough $\epsilon\gg\tau_{int}^{-1}$ can be expressed in terms of that in the absence of splitting, $P(\epsilon,\omega_{SO}=0)\equiv P_0(\epsilon)$. The splitting energy $2\omega_{SO}$ enters only through the argument of P_0 and the low-energy cutoff $\hbar/\tau_{int}$, which assumes the role of dephasing time.

To establish the relation between P and P_0, we notice[22] that the states with the same energy, E, have two different values of the wave number, $\hbar k_\pm=\sqrt{2mE}\pm\omega_{SO}/v_F$, depending on the branch (see Fig. 1). Similarly, states at energy $E'=E+\epsilon$ have wave numbers $\hbar k'_\pm=\sqrt{2mE+(\epsilon\pm\omega_{SO})}/v_F$. Thus, we

can identify four contributions to $P(\omega)$, coming from correlating each of the $k_\pm$ states with either of $k'_\pm$ states. Since, for $\omega_{SO}\tau_{int} \gg 1$, the degree of correlation between a pair of states depends only on the difference of the wave numbers, all four contributions have the same functional form. It is given by the function $P_0(\epsilon)$ with the corresponding arguments, which can be either $\hbar v_F(k'_+ - k_+) = \hbar v_F(k'_- - k_-) = \epsilon$ or $\hbar v_F(k'_+ - k_-) = \epsilon + 2\omega_{SO}$, or $\hbar v_F(k'_- - k_+) = \epsilon - 2\omega_{SO}$. It must be emphasized that, since separate branches cannot be resolved beyond the time τ_{int}, the correlator P_0 for $|\epsilon| \approx 2\omega_{SO}$ must be computed with a low-energy cutoff $\hbar/\tau_{int}$. Hence the condition $\epsilon\tau_{int} \gg 1$, adopted in this qualitative consideration, ensures that the SO-induced satellites in $P(\epsilon)$ do not overlap with the main peak. Combining the four contributions, we obtain

$$P(\epsilon) = \tfrac{1}{4}[2P_0(\epsilon) + P_0(\epsilon + 2\omega_{SO}) + P_0(\epsilon - 2\omega_{SO})]. \quad (38)$$

Obviously, the above general consideration applies to the both diffusion and cooperon contributions to the LDOS correlator, P. This consideration clarifies why the height of the central peak in $P(\epsilon)$ is twice the height of each of the SO satellites. The factor 2 in the first term of Eq. (38) appears because the argument ϵ occurs twice out of the four possibilities mentioned above. Equation (38) also illustrates why the SO-induced structure in the correlator $P(\epsilon)$ does not emerge when the second term in the Hamiltonian Eq. (2) is treated perturbatively. Namely, although, within perturbation theory, the second and the third terms in Eq. (38) contain corrections that are *linear* in ω_{SO}, these corrections cancel each other out.

IV. MANIFESTATION IN THE TUNNELING SPECTROSCOPY

In the tunneling-spectroscopy experiments[10–14,23–26] the measured quantity is a resonant-tunneling current, I_{sd}, through a localized impurity state as a function of the source-drain bias, V_{sd}. Such an impurity plays the role of a "spectrometer" since its energy position, ϵ_0, changes with V_{sd}. Within a narrow range, δV, of V_{sd} this change is linear, i.e., $\Delta = \epsilon_0(V + \delta V) - \epsilon_0(V) \approx \beta\delta V$, where β is the structure-specific parameter. Experimentally,[10–14,23–26] measurements are performed in the plateau regime when the temperature is much lower than V_{sd}, so that the value of resonant-tunneling current

$$I(V_{sd}) = \frac{e}{h}\int_{-\infty}^{\infty} d\epsilon \frac{\gamma_l(\epsilon)\gamma_r(\epsilon)}{[\epsilon - \epsilon_0(V_{sd})]^2 + [\gamma_l(\epsilon) + \gamma_r(\epsilon)]^2/4} \quad (39)$$

is temperature independent. In Eq. (39), γ_l and γ_r stand for the tunneling widths, associated with the escape from the impurity into the emitter and collector, respectively. These widths are proportional to the LDOS in the electrodes. Typically, the widths γ_l and γ_r differ strongly, $\gamma_r \gg \gamma_l$. In addition, the energy dependence of γ_r is weak, so that it can be considered as a constant. As a result, it is a tunneling coupling to collector that dominates the width of the Lorentzian in Eq. (39). With regard to the weak fluctuating dependence of current, I, on V_{sd} in the plateau regime, it is exclusively due to the energy dependence of γ_l, which, in turn, originates from the fluctuations of the LDOS.

Quantitative analysis of the experimental data is performed (see, e.g., Ref. 13) by plotting the correlator $C(\delta V) = \langle \delta g(V + \delta V)\delta g(V)\rangle$ of the fluctuations of the differential conductance $g = dI_{sd}/dV$ around its average value, $\langle g\rangle$, as a function of δV for different values of $V = V_{sd}$. The expression for $C(\delta V)$ in terms of the LDOS correlator $P(\epsilon - \epsilon')$ is obtained (see Refs. 9 and 27) by, first, calculating the correlator of current fluctuations $\langle \delta I(V_{sd})\delta I(V'_{sd})\rangle$ and then taking derivatives with respect to V_{sd} and V'_{sd}. Using the fact that a convolution of two Lorentzians is also a Lorentzian, the final expression for $C(\delta V)$ can be written in the form

$$C(\delta V) = \langle \delta g(V_{sd} + \delta V)\delta g(V_{sd})\rangle = -\frac{\langle \gamma_l\rangle^2}{4}\frac{\partial^2}{\partial \Delta^2}\left\{\gamma_r \int \frac{d\omega[P(\omega) + P(-\omega)]}{(\omega - \Delta)^2 + \gamma_r^2}\right\}. \quad (40)$$

If, following Ref. 9, we substitute the conventional form $P(\omega) \propto \ln(\omega^2\tau_f^2 + 1)$ into Eq. (40), where τ_f is the "floating up" time of a hole created as a result of the tunneling act, then Eq. (40) yields[9] $C(\delta V) = F_2(\delta V/V_c)$, where the dimensionless function $F_2(x)$ is defined as

$$F_2(x) = \frac{1 - x^2}{(1 + x^2)^2}, \quad (41)$$

and the characteristic value, V_c, is given by $V_c = \beta^{-1}(\gamma_r + \hbar/\tau_f)$. In the presence of the SO coupling the correlator $P(\omega)$ has three peaks, centered at $\omega = 0$ and $\omega = \pm 2\omega_{SO}$. Without Zeeman splitting all three peaks have the same shape, which allows us to express the correlator, Eq. (40), in terms of the function F_2 as follows:

$$C(\delta V) = \frac{1}{4}F_2\left(\frac{\delta V}{V_c}\right) + \frac{1}{4}F_2\left(\frac{\delta V}{V''_c}\right) + \frac{1}{4}F_2\left[\frac{\beta\delta V - 2\omega_{SO}}{\beta V'_c}\right] + \frac{1}{4}F_2\left[\frac{\beta\delta V + 2\omega_{SO}}{\beta V'_c}\right], \quad (42)$$

where $V'_c = V_c + \beta^{-1}\hbar/\tau_{int} = \beta^{-1}(\gamma_r + \hbar/\tau_f + \hbar/\tau_{int})$, $V''_c = V_c + 2\beta^{-1}\hbar/\tau_{int} = \beta^{-1}(\gamma_r + \hbar/\tau_f + 2\hbar/\tau_{int})$, and τ_{int} is the intersubband scattering time defined by Eq. (26). It is seen from Eq. (42) that the SO peaks in $P(\omega)$ give rise to the satellites in $C(\delta V)$ at $\delta V = 2\beta^{-1}\omega_{SO}$. Thus, the SO coupling has a noticeable effect on the correlator of the tunnel conductances when $2\beta^{-1}\omega_{SO} \geq V'_c$. In Fig. 4 the correlator C is plotted vs $\delta V/V'_c$ for three different values of the dimensionless SO splitting $\omega'_{SO} = \omega_{SO}/\beta V'_c$. It is seen that at $\omega'_{SO} = 0.5$ the correlator $C(\delta V/V'_c)$ develops a characteristic "shoulder," while at $\omega'_{SO} = 0.7$ it exhibits a well-developed additional maximum. In the experiments[11–14] the characteristic width of the impurity level, γ_r, was $\lesssim 1$ K. This suggests that the SO satellites should be well resolved in tunneling spectroscopy of the narrow-gap semiconductors, where ω_{SO} is of the order of several meV. Concerning the GaAs-based structures, the SO splitting there is much smaller (of the order of 1 K), and depends strongly on the details of the confinement potential. Therefore, the observation of the additional peaks in tunnel-

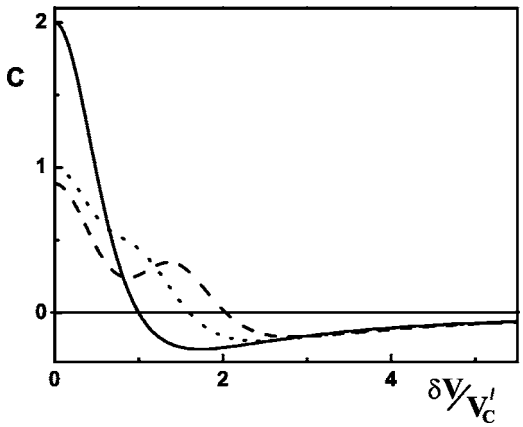

FIG. 4. The correlator $C(\delta V)=\langle \delta_g(V)\delta_g(V+\delta V)\rangle$ of differential tunneling conductances is plotted from Eq. (42) versus dimensionless voltage difference $\delta V/V_c'$ for different values of the SO splitting: $\omega_{SO}=0$ (solid line); $\omega_{SO}=0.5V_c'$ (dotted line); $\omega_{SO}=0.7V_c'$ (dashed line). For simplicity, we have assumed $\gamma_r \gtrsim \hbar/\tau_{int}$, so that $V_c' \approx V_c \approx V_c''$.

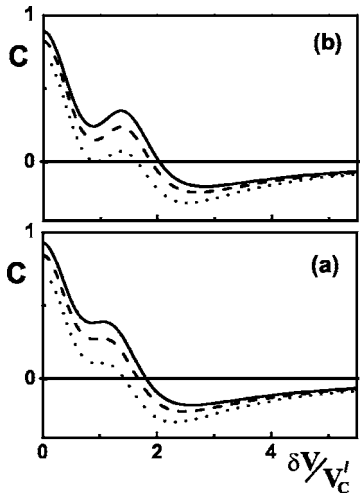

FIG. 5. Evolution of the SO-related features in the correlator $C(\delta V)$ with parallel magnetic field is shown for same $\omega_{SO}=0.5V_c'$ (a) and $\omega_{SO}=0.7V_c'$ (b) as in Fig. 4. Solid lines $\omega_Z=0$; dashed lines $\omega_Z=0.1\omega_{SO}$; dotted lines $\omega_Z=0.2\omega_{SO}$.

ing spectroscopy would require a higher "spectrometer resolution." The characteristic signature of the SO-related features in the correlator $C(\delta V)$ would be a strong sensitivity of the correlator to a weak parallel magnetic field. In Fig. 5 we plot $C(\delta V/V_c')$ for the same values $\omega_{SO}'=0.5$ and $\omega_{SO}'=0.7$ as in Fig. 4 and for different ratios ω_Z/ω_{SO}. The curves were calculated using Eqs. (29) and (37) for $P(\omega)$. We see that both the shoulder at $\omega_{SO}'=0.5$ and additional maximum at $\omega_{SO}'=0.7$ do not disappear with increasing ω_Z, but rather get shifted down. Remarkably, the effect of the parallel field is noticeable already at very small $\omega_Z/\omega_{SO}\sim 0.1$.

V. CONCLUSIONS

In the present paper we have demonstrated that the intrinsic SO splitting of the electron spectrum in the absence of disorder manifests itself in a disorder-induced effect, mesoscopic fluctuations of the local density of states. This observation suggests that the splitting, $2\omega_{SO}$ must show up in the resonant tunneling spectroscopy in the form of additional peaks in the correlator $C(\delta V)$. The fact that the positions of these peaks do not depend on the disorder offers a possibility to measure experimentally the magnitude of the splitting. Note that, unlike the magnetotransport experiments, where the SO-related features are quickly washed out with increasing temperature,[28] the resonant tunneling current in the plateau regime depends on temperature rather weakly.[12] As it was demonstrated in Sec. II, the correlator, P, of the LDOS develops a plateau in a weak parallel magnetic field $\omega_Z \ll \omega_{SO}$. This singular behavior manifests itself in the anomalous sensitivity of the correlator of the tunneling conductances to ω_Z, as illustrated in Fig. 5.

Note in conclusion that the form of the SO term in Hamiltonian Eq. (2) implies that the SO coupling originates from the interface 1. In realistic heterostructures it is possible that the dominating mechanism of the SO coupling is the absence of the inversion symmetry in the bulk.[29] If the corresponding splitting, ω_D, of the 2D electron spectrum[30] resulting from this mechanism is much larger than ω_{SO}, all the above results remain valid upon replacement of ω_{SO} by ω_D. In the absence of Zeeman splitting, both mechanisms 1 and 29 can be easily incorporated into the theory by replacing $\Omega(\boldsymbol{k})$ in the energy spectrum Eq. (4) by $\tilde{\Omega}(\boldsymbol{k})=(\omega_{SO}^2+\omega_D^2+2\omega_{SO}\omega_D \sin 2\phi_{\boldsymbol{k}})^{1/2}$.

As a final remark, we note that electron-electron interactions were completely neglected in our calculation. The reason for this is that, while the correlator $P(\epsilon_1-\epsilon_2)$ depends only on the difference $(\epsilon_1-\epsilon_2)$, the effect of electron-electron interactions on P depends crucially on *absolute positions* of the energies ϵ_1,ϵ_2 with respect to the Fermi level, E_F. Indeed, if ϵ_1,ϵ_2 are close to E_F (more precisely, within ω_{SO} from E_F), then the *average* density of states, ν_0, which we assumed to be constant, acquires a logarithmic interaction-induced correction depending on $|\epsilon-E_F|$ (zero-bias anomaly). Moreover, these corrections also exhibit SO-induced satellites[17] at $|\epsilon-E_F|=\pm 2\omega_{SO}$. Thus, our experimental predictions [Eq. (42) and Figs. 3 and 4] pertain to the domain of the source-drain voltages, V_{SD}, for which the position, $\epsilon_0(V_{SD})$, of the localized state in the barrier is well below E_F. We note that in experimental papers,[12–14] in which the quantitative analysis of the fluctuations of the tunnel conductance has been performed, the domain of V_{SD} used in the analysis was well above the threshold for the resonant tunneling (at the threshold the electrons tunnel from the Fermi level). Within this domain the inelastic scattering time, τ_f, is temperature independent. On the other hand, within this domain, τ_f depends on the energy of the electron (measured from the Fermi level) as a power law.[13]

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