Experimental evidence of the ideal de Haas–van Alphen effect in a two-dimensional system

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We have measured the magnetization oscillations—i.e., the de Haas–van Alphen (dHvA) effect—of high-mobility two-dimensional electron systems (2DES's) in modulation-doped AlGaAs/GaAs heterostructures. In the sample exhibiting the highest oscillation amplitude of $2\mu_B^*$ per electron we observe discontinuous jumps in the magnetization when an even integer number of Landau levels (LLs) is filled. This specific feature of the 2DES magnetization was predicted by Peierls more than 70 years ago but never observed in the experiments. Numerical simulations assuming no states between LLs remodel the jumps quantitatively. Measurements on samples with lower mobility reveal a finite background density of states between LLs that depends systematically on the filling factor.

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I. INTRODUCTION

The physics of two-dimensional electron systems (2DES's) in the quantum Hall effect (QHE) regime reveals a rich variety of remarkable phenomena. Central to the understanding of such systems is a detailed knowledge of the density of states (DOS). To explain quantum Hall plateaus found in 2DES's at a strong magnetic field B and low temperature T localized states have been introduced between Landau levels (LLs). A complete DOS was obtained by measuring a thermodynamic quantity of the system, like heat capacity[1] and magnetization M.[2] In these early studies indeed a considerable number of states between LLs and strongly broadened overlapping levels were reported to model the tiny signals. More pronounced oscillations in M—i.e., a pronounced de Haas–van Alphen (dHvA) effect—were only recently observed by Wiegers *et al.*[3] Subsequently Meinel *et al.*[4] and Harris *et al.*[5] reported a similar magnetic behavior. Again the dHvA effect could in all papers only be interpreted with a finite DOS between LLs amounting to at least a few percent of the zero-field DOS.[6–8] The experimental results so far are in contradiction to the theoretical result of Peierls[9] obtained in 1933 where for the ideal 2DES he predicted discontinuous jumps in M. Solid-state physics textbooks have discussed this ideal case for decades as this abrupt jump in M is fundamental to a 2DES.[10] It cannot be observed in three-dimensional electron systems.[9] The absence of such a discontinuity in all experiments has stimulated extensive theoretical and experimental work on the nature of the apparent background DOS. Disorder-induced effects or electron transfer into a reservoir of electronic states were argued to explain the DOS between LLs. In this work, we report the observation of *discontinuous* jumps in the magnetization reflecting a *vanishing* DOS between LLs in a 2DES. At the same time the peak-to-peak amplitude ΔM reaches two effective Bohr magnetons $\mu_B^*=e\hbar/2m^*$ per electron, in further agreement with Peierls' prediction. The experimental results shed new light onto the present discussion and should stimulate further theoretical efforts. For 2DES's where ΔM is smaller, we observe a finite slope in M attributed to a background DOS and to an increased level broadening Γ most likely due to disorder. A model DOS based on Gaussian-broadened Landau levels and an energy-independent background quantitatively explains all our data.

II. EXPERIMENT

The investigated samples were different high-mobility bulk modulation-doped AlGaAs/GaAs heterostructures. We intentionally varied the spacer width and carrier density n_s (Table I). The 2DES mesa structures of area 1.2 mm^2 had no electrical contacts. The magnetization measurements were performed using micromechanical GaAs cantilever magnetometers. Details of the technique, in particular the absolute calibration, have been described elsewhere.[6,11] The cantilevers detect quasistatically the torque $\boldsymbol{\tau}=\mathbf{M}\times\mathbf{B}$ acting on the anisotropic magnetic moment $\mathbf{M}$ of the 2DES in an external

TABLE I. (a) Parameters from DOS model. (b) Zero-field mobility μ at 0.3 K, spacer width, and electron sheet density.

Sample	*hh*#2	*hh*#1	*bo*#1
		(a)	
Γ [meV]	$0.26\sqrt{B_\perp[\mathrm{T}]}$	$0.08\sqrt{B_\perp[\mathrm{T}]}$	$0.01\sqrt{B_\perp[\mathrm{T}]}$
ξ	0.026ν	0.023ν	0[a]
$n_{gap}[1/\mathrm{cm}^2]$	$\sim10^{10}$	$\sim10^{10}$	
		(b)	
$\mu[\mathrm{cm}^2/\mathrm{Vs}]$	0.7×10^6	1.4×10^6	9×10^6
Spacer [nm]	20	30	40
n_s [1/cm^2]	5.3×10^{11}	4.75×10^{11}	3.18×10^{11}

[a]The best fit is for $\xi=0$ over a broad parameter range (see text). Assuming an error bar for $B_\perp$ of ±1 m T leads to an upper boundary for ξ of $5\times10^{-4}\nu$. This yields an upper boundary for n_{gap} for *bo*#1 that is two orders of magnitude lower than for the other samples.

magnetic field **B**. The normal to the 2DES was tilted at 15° with respect to **B**. The cantilevers were mounted on the cold finger of a vacuum loading ^{3}He system. Data were taken after brief illumination with a red-light-emitting diode. A smooth background signal arising from the magnetization of the cantilever structure itself is removed from the experimental data by subtracting a polynomial in $1/B$.[2,3,5–7,11]

In an ideal 2DES in a perpendicular magnetic field $B_\perp$ the DOS condenses into discrete LLs with separation $\hbar\omega_c$ and degeneracy $N_L=2N_\nu=2eB_\perp/h$. At $T=0$ K the magnetization $M=-\partial F/\partial B$ is expected to oscillate in a sawtoothlike manner with discontinuities at even integer filling factor $\nu=N_\nu/n_s$ (Ref. 9): whenever a level with energy $E_j=(j+1/2)\hbar\omega_c$ is emptied and the Fermi energy jumps to the next lower-lying level, the magnetization should experience a discontinuity. Here, $\omega_c=eB_\perp/m^*$ denotes the cyclotron frequency with effective mass m^*, j is the LL index, and F is the free energy. Since the jump in the magnetization, ΔM, is directly related to the jump in the chemical potential, $\Delta\chi$, one expects a peak-to-peak amplitude $\Delta M=\Delta\chi/B_\perp=\hbar\omega_c/B_\perp=2\mu_B^*$ per electron for the ideal 2DES at even integer ν.[3]

III. RESULTS AND DISCUSSION

The magnetization of sample *bo*#1 is displayed in Fig. 1. The elevated temperature $T=2$ K was chosen because nonequilibrium eddy currents were not induced by the sweeping magnetic field[12,13]; i.e. they were completely absent up to the maximum field of 16 T. The exact temperature in the experiment is important since at low T eddy currents are known to obscure the equilibrium dHvA effect in the vicinity of integer ν on which we focus in this paper. For $\nu=4$ at $B=3.2$ T eddy currents were already absent at $T=1.1$ K [Fig. 1(c)]. In Fig. 1 the dHvA oscillations exhibit discontinuous jumps on the high-field side of each sawtooth and the peak-to-peak dHvA amplitude is close to $2\mu_B^*$ at $B_\perp=6.5$ T [Fig. 1(b)]. At lower T, ΔM becomes $2\mu_B^*$. The observed discontinuity is striking in particular if one considers the relatively high temperature compared to Peierl's theoretical prediction. The oscillation at $\nu=1$ is due to spin splitting of LLs and about $0.4\mu_B^*$ at $T=2$ K. The residual rounding of the oscillations at the local maxima and minima is due to the experimental temperature. The discontinuities in the magnetization curve directly reflect a DOS with very sharp peaks at E_j and a vanishing number of states between these LL peaks as will be discussed in detail below. Figures 1(b) and 1(c) show high-resolution measurements of M around $\nu=2$ and $\nu=4$ as open circles. The data acquisition was adjusted to record each data point in a field interval of ~0.001 T, and all data points are shown. The maximum experimental resolution is set by the amplitude of the current ripple in the power supply which corresponds to 1.6 mT. In between the maxima and minima, discontinuous jumps are reproducibly observed. In particular, we observe no difference between upsweeps and downsweeps of the magnetic field at the temperatures and magnetic fields shown in Fig. 1 and can thus rule out that any residual signals from eddy currents are present in the data and might influence the discrete jumps in M. Due to the finite experimental resolution, we give an upper boundary for the width of $\Delta B_\perp \leqslant 2$ mT. Such an abrupt behavior has not been reported before. The solid circles in (b) represent a model calculation which assumes no states between LLs. This model describes the discrete jumps quantitatively over a broad temperature and field range and will be discussed in the following.

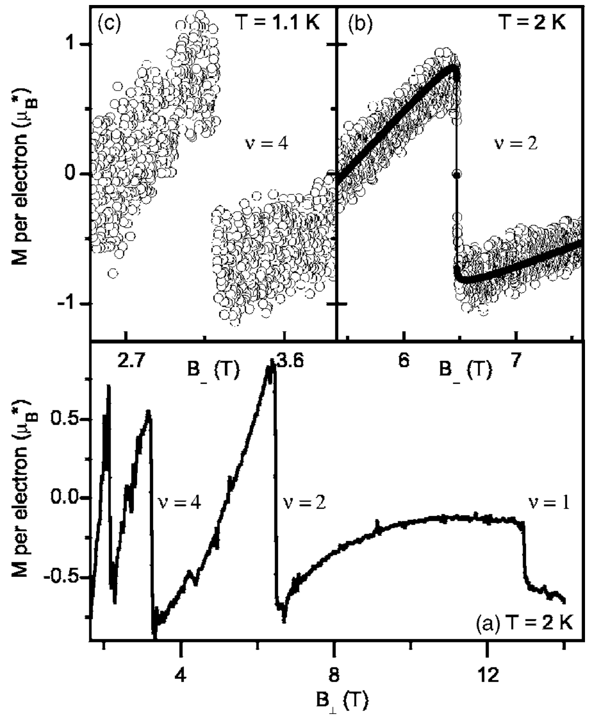

FIG. 1. (a) Experimental magnetization of sample *bo*#1 for $T=2$ K. The thick black line is the coarse behavior, already reflecting very sharp jumps in M. At this elevated temperature eddy currents around integer ν are absent, independent of the used sweep rate. (b), (c) Open circles denote the discrete experimental data points. Data acquisition was adjusted to record one data point in an interval of ~1 mT and the field was swept at a rate of 0.1 T/min. Rounding at the extrema of the oscillations is due to the elevated temperatures of $T=2$ K and $T=1.1$ K in (b) and (c), respectively. The magnetization jumps discontinuously at $\nu=2$ and $\nu=4$, indicating that there are virtually no states in these regions. Solid circles in (b) interconnected by a thin black line denote the result of the model calculation for $T=2$ K with $\Gamma=0.01$ meV$\times\sqrt{B_\perp[\mathrm{T}]}$, $\xi=0$, and a field resolution of 1 mT. The theoretical prediction assumes no states between LLs: i.e., the rounding and slope is only an effect of the finite temperature.

Detailed quantitative information about the DOS is extracted from the experimental curves by means of numerical model calculations. This approach has been successfully applied previously to evaluate the dHvA effect.[2,6–8] For a noninteracting electron system the magnetization $M=-(\partial F/\partial B)|_{n_s,T}$ can be calculated from

$$F=\chi n_s - kT\int D(E)\ln\left[1+\exp\left(\frac{\chi-E}{kT}\right)\right]dE. \qquad (1)$$

Here, the chemical potential χ is determined from $n_s=\int f(E,\chi,T)D(E)dE$ with the Fermi-Dirac distribution $f(E,\chi,T)$. We assumed a DOS with the functional form

$$D(E) = \xi D_0 + (1-\xi)N_\nu \sum_{j,s} G(E-E_{j,s}). \qquad (2)$$

The first term is the background DOS, represented as a fraction ξ of the zero-field DOS $D_0 = m^*/\pi\hbar^2$. This background ξD_0 has been found crucial to model the measured thermodynamic quantities in numerous experiments.[1,2,6–8,14] The second term is the DOS of the spin-resolved LLs at energies $E_{j,s} = (j+1/2)\hbar\omega_c + sg\mu_B B$. We start from $|g| = 0.44$ for the noninteracting system in GaAs with $s = \pm 1/2$. In our model calculations we tested Gaussian $[G(E-E_{j,s})]$, Lorentzian, and semielliptical distribution functions for the LL broadening. Semielliptical line shapes were found not to reproduce our experimental data in all cases. Calculations assuming a Lorentzian line shape do not model the discontinuous jumps in the magnetization of sample *bo*#1. This is attributed to the long overlapping tails of the Lorentz curves that effectively constitute a substantial DOS between the LLs and induce a finite slope of the magnetization steps. In Figs. 1(b) and Fig. 2(a), the best agreement is achieved for the Gaussian line shape with a broadening parameter $\Gamma = 0.01\ \mathrm{meV} \times \sqrt{B_\perp[\mathrm{T}]}$ and $\xi = 0$. Note that the single experimental data point within the jump in Fig. 1(b) is consistent with Eq. (1) and $T = 2$ K. In particular, assuming a background DOS even with ξ as low as 1‰ of D_0 results in a finite slope that is in significant disagreement with the experiment. In other words, assuming an error in $B_\perp$ of the order of the field resolution of 1.6 mT results in an upper boundary for ξ of $5 \times 10^{-4}\nu$. This boundary is due to experimental uncertainties and already two orders of magnitude smaller than previously reported values.[3,6] The important new result here is that we find a virtually zero background DOS between LLs which manifests itself in a discontinuity of the magnetization. Before we discuss the implications of this result in detail, we present experimental magnetization data obtained on further heterostructures, in order to establish a systematic picture of the DOS in 2DES's of different mobilities.

The experimental magnetization curves for three different heterostructures at $T = 0.3$ K are displayed as open circles in Fig. 2. To better compare with the model calculations for noninteracting electrons (solid lines)—i.e., to stay within the approach of Peierls—we focus here on the low-field regime, where the influence of the exchange-enhanced spin splitting on the magnetization is still moderate and in addition significant eddy currents are not induced.

In the low-field regime of Fig. 2(a) the sample *bo*#1 still shows a perfect sawtooth with discontinuous jumps. Discrete data points on the high-field side of each oscillation are due to the stronger signal averaging used here. We note that from Fig. 1 to Fig. 2 the temperature has been varied by a factor of 7 and the magnetic field is considerably reduced. Convincingly the model with no gap states describes the data over this broad parameter regime in a quantitative way. Upward spikes occurring at integer ν for $B_\perp \geqslant 1.5$ T in the data set are due to small residual eddy currents. The magnetization of sample *hh*#1 (parameters are in Table I) has still a sawtooth shape in Fig. 2(b), but exhibits a well visible finite and almost constant slope on the high-field side of the oscillations. The amplitudes ΔM are reduced if compared to sample

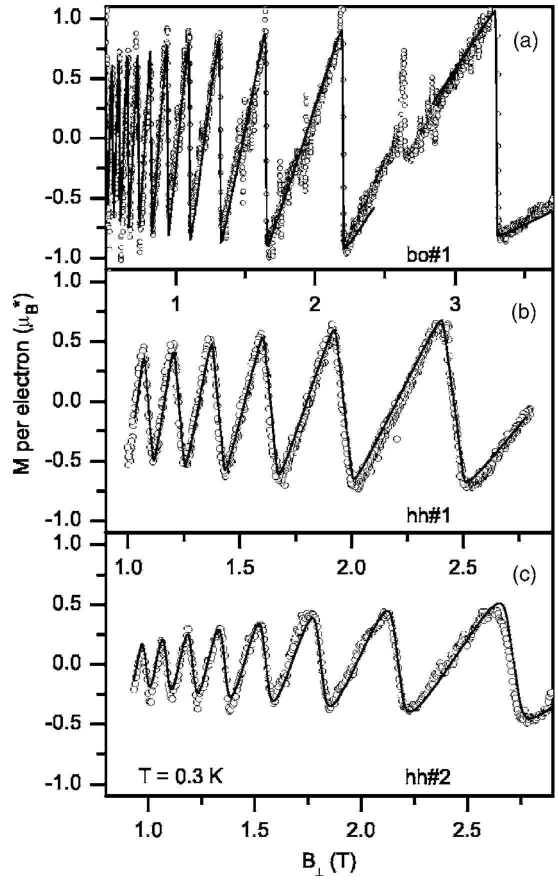

FIG. 2. Comparison of the experimental data at $T = 0.3$ K (open circles) with model calculations. Convincingly, the magnetization of *bo*#1 shows discontinuous jumps at even integer ν also at low $B_\perp$. Here a stronger signal averaging was used to increase the signal-to-noise ratio. The discrete experimental points (open circles) exactly follow the theoretical prediction (thin black line) which assumes no states in the energy gaps. For *hh*#1 and *hh*#2 the magnetization steps have a finite slope. Solid lines denote the results of the model calculations with parameters given in Table I. In (a) an offset has been added to the theoretical curve at $B_\perp > 2.6$ T, since the already significant spin splitting displaces the experimental data vertically. Note that the vertical scale is the same for all three graphs.

bo#1.[15] The dHvA effect of sample *hh*#2 has a further reduced amplitude of only $1\mu_B^*$ at $B_\perp = 2.7$ T and shows stronger smoothing of the oscillations. The negative slopes of the magnetization steps are more gentle. Data points are quasicontinuous.

The only functional form of the DOS that allows us to model all experimental traces is a Gaussian line shape with $\Gamma \propto B_\perp^{1/2}$ and a filling-factor-dependent background $\xi = \kappa\nu$.[16] It consistently and quantitatively explains all our data, equally well at low and at high temperatures. The model parameters are given in Table I(a). One can see that the fraction of background DOS ξ decreases strongly as Γ is reduced. At the same time the mobility μ measured at $B = 0$ in magnetotransport increases by a factor of 10. Our data thus imply a connection between Γ, μ and the background DOS. The parameter Γ is often used to quantify the amount of disorder.[2] In this context, μ and ξ improve as disorder is reduced. From the modeled background DOS one can estimate the number of states in the gap between LLs to be $n_g = \xi D_0/\hbar\omega_c$. In case of sample *hh*#1 and *hh*#2 we get a filling-factor-independent number of states per unit area in the gaps between LLs that evaluates to $n_g \sim 10^{10}$ cm^{-2}. To summarize, our data show (I) that the background DOS can become vanishingly small and (II) that a correlation exists between the amount of disorder (measured by the LL broadening) and the amount of background DOS. (III) In *hh*#1 and *hh*#2 we observe that the number of states per unit area in the gap is roughly independent of ν.

For the following discussion we first reexamine the explanations for the background DOS which was observed in the earlier experiments and which we find in our samples *hh*#1 and *hh*#2. Early single-electron theories predicted an exponential drop in the DOS between LLs.[17] They are inconsistent with the constant background observed in *hh*#1 and *hh*#2 and in previous studies[1,2,6–8,14] leading to a finite slope in M at even ν. For the last decade, the background DOS was believed to be a well-established characteristic of a 2DES.

An elegant explanation for the observed finite slopes on the high-field sides of the dHvA oscillations is the reservoir hypothesis. Here, the finite slope of the magnetization is explained by equilibrium transfer of electrons to a reservoir with field-independent energy spectrum: As was shown by Shoenberg,[18] the abrupt jump in the dHvA oscillations that is on the high-field side of the sawtooth for a canonical ensemble moves towards the low-field side for the case of a grand canonical ensemble. In an intermediate case of oscillating χ *and* n_s slopes as observed on *hh*#1 and *hh*#2 could result. This case was recently discussed by Itskovsky *et al.*[19] for quasi-2D organic metals of the (BEDT-TTF$_2$X) type, where the shape of the Fermi surface gives rise to field independent reservoir states. Zhu *et al.* proposed this hypothesis to explain the background observed in their recent magnetization data on AlGaAs/GaAs systems[7] but stated that the mechanism of transfer and the origin of the reservoir remain unclear. The discontinuities in M we observed in the high-quality sample *bo*#1 cannot be convincingly explained by this hypothesis.

A statistical model for inhomogeneities was proposed in Refs. 20. The DOS for disorder having a finite correlation length was calculated in Ref. 21: Both predict a monotonous dependence of the background DOS on disorder and include the case of vanishing background for sufficiently low disorder and low inhomogeneity, respectively. In Ref. 21 the background decreases also strongly with B. These theories could yield good qualitative agreement with the experiment, but no relations to specific sample parameters are made.

We now turn to a discussion of more recent works. Here, it is important that our magnetization experiment maps out the complete DOS $D(E)$ of the system [Eq. (2)], including *all* localized states independent of the microscopic origin. We have found that the samples *hh*#1 and *hh*#2 exhibit a number of states per unit area n_{gap} in the gap that is independent of ν (Table I). Following conventional theories of the QHE these states are expected to be localized around integer filling factors.[22] Indeed, Ilani *et al.*[23] recently succeeded in measuring the localized states in 2DES's directly by using a single-electron transistor electrometer. They found a substantial number of localized states, n_{loc}, centered around integer ν for different samples covering a wide range of mobilities. For samples with mobilities similar to *hh*#1 and *hh*#2 they found $n_{loc} = 2\times 10^{10}$ cm^{-2}. This number was also independent of the filling factor and agrees quantitatively with our observation on *hh*#1 and *hh*#2. From the dHvA effect we now conclude that this considerable amount of states lies within the energy gap—i.e., is located *outside* the Gaussian distribution functions of the LLs. This was not derived in Ref. 23. The states are distributed over a considerable energy interval in the electronic spectrum, leading to the gentle slope in the dHvA trace. In Ref. 23 the authors employed a theory of nonlinear screening developed in Ref. 17 to explain their data. Following this theory we can now predict that the amount of states between LLs should decrease as a function of increasing spacer width and decreasing donor density. Consistent with that, in sample *bo*#1, where the spacer width is largest and the assumed donor density is smallest, we observe a background DOS that is reduced to virtually zero; i.e., there is no substantial number of localized states between LLs. The exact boundary for the crossover between the different magnetic behaviors of *hh*#1 and *bo*#1 is not yet clear. It is interesting, however, that sample *bo*#1 still exhibits well-defined QHE plateaus in transport measurements. This points toward very recent theories that explain the QHE plateaus without assuming disorder-induced localization in the bulk of a 2DES.[24] Our results on *bo*#1 show that if there is still a localized DOS in a high-mobility 2DES, it must be located *inside* LLs and must exhibit very sharp distribution functions of width $\leqslant 0.01$ meV$\times\sqrt{B_\perp[\mathrm{T}]}$ centered around integer ν.

In conclusion, we have reported the measurement of a thermodynamic quantity in a real 2DES that exhibits a discontinuity as was predicted for the ideal disorder-free 2D system.[9] The observation of the ideal dHvA sawtooth with discontinuous jumps in M should stimulate further theoretical efforts on the quantum Hall effects.

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[15] 2DES's as sample *hh*#1 show in M enhanced spin splitting of LLs and fractional quantum Hall states at high magnetic field.

[16] This is different from the results in Ref. 7, where the authors found that their magnetization data could be equally well fitted by field-independent Gaussian and Lorentzian distributions with ξ=const. However, they adjusted the observed signal strength by up to ±10% from sample to sample. Tentatively including such a "scaling factor" in our model, we observe that the difference between a Gaussian broadening having $\Gamma \propto B_{\perp}^{1/2}$ with $\xi = \kappa\nu$ and a Lorentzian one independent of B with ξ=const is indeed small for samples *hh*#1 and *hh*#2. Still, the Lorentzian line shape fails completely to describe the data for sample *bo*#1.

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