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Hall coefficient and magnetoresistance of two-dimensional spin-polarized electron systems

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Recent measurements of the two-dimensional (2D) Hall resistance show that the Hall coefficient is independent of the applied in-plane magnetic field, i.e., the spin polarization of the system. We calculate the weak-field Hall coefficient and the magnetoresistance of a spin-polarized 2D system using the semiclassical transport approach based on the screening theory. We solve the coupled kinetic equations of the two carrier system including electron-electron interaction. We find that the in-plane magnetic field dependence of the Hall coefficient is suppressed by the weakening of screening and the electron-electron interaction. However, the in-plane magnetoresistance is mostly determined by the change of the screening of the system, and can therefore be strongly field dependent.

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The phenomena of apparent two-dimensional (2D) metallic behavior and the associated 2D metal-insulator transition (MIT) continue to attract a great deal of attention.[1,2] The low-temperature resistivity $\rho(T)$, in a zero applied field, shows remarkably strong "metalliclike" (i.e., $d\rho/dT>0$ for $n>n_c$) temperature dependence for 2D carrier densities above the so-called critical carrier density (n_c) for the 2D MIT whereas, for $n<n_c$, the system exhibits insulating behavior ($d\rho/dT<0$). The application of an in-plane magnetic-field B has interesting effects on the 2D metallic phase, i.e., at a fixed low T the system develops a large positive magnetoresistance with $\rho(B)$ increasing very strongly (by as much as a factor of 4) with B up to a maximum field B_s, and for $B>B_s$, $\rho(B)$ either saturates (or increases slowly with B for $B>B_s$) showing a distinct kink at $B=B_s$.[3] The observed temperature, density, and parallel magnetic dependence of the 2D metallic resistivity[2,4] can be explained by the screening theory in which the strongly temperature-dependent effective screened charged impurity disorder is the qualitative reason underlying the striking metallic behavior of dilute 2D carrier systems.

Remarkably, recent measurements of the 2D Hall resistance[5] in a parallel magnetic field have shown unexpected physical behavior, which is in sharp contrast with the strong in-plane field dependence of the 2D magnetoresistivity. The measured Hall coefficient seems to contradict qualitatively the results based on the screening theory[6] even though the longitudinal magnetoresistance can be explained by the change of the screening as the spin polarization of the system varies. The measured Hall coefficient is found not to vary with the parallel magnetic field (or spin polarization) for fields ranging from 0 to well above B_s, where B_s is the complete spin-polarization field. However, the screening theory shows very strong magnetic-field (spin-polarization) dependence. Because the screening theory, at least in its most elementary formulation,[6] cannot explain these unexpected Hall coefficient data, Vitkalov *et al.*[5] proclaim that the electron-electron intersubband scattering in the spin-polarized system is the main reason for the experimental behavior. The qualitative disagreement between the experimental Hall data of Ref. 5 and the screening theory[6] is therefore a problem in our fundamental understanding of 2D transport because the screening theory[7] *can explain the temperature-dependent Hall coefficient* in *p*-GaAs (Ref. 8) and Si-MOSFET (metal-oxide-semiconductor field-effect transistor) (Ref. 9).

Motivated by this puzzling experimental observation,[5] we investigate in this Rapid Communication, based on the screening model, the spin-polarization dependence of the weak-field Hall resistance and magnetoresistance. For a complete comparison with the experimental Hall coefficient, we include in our calculation the electron-electron scattering between two different spin subbands. In Ref. 5 Vitkalov *et al.* compare the Hall coefficient data with the zero-temperature results of the screening theory[6] in the strong screening limits $q_{TF}/2k_F \gg 1$ (where q_{TF} is the Thomas-Fermi screening wave vector and k_F is the Fermi wave vector). Our calculation, which includes finite temperature and fully wave-vector-dependent screening, is in qualitative agreement with the experimental results[5] on the spin-polarization dependence of the weak-field Hall coefficient although electron-electron scattering might play a role at finite temperature as we show. We therefore resolve the experimental problem posed in Ref. 5.

To calculate the Hall coefficient and the magnetoresistance, we solve the coupled kinetic equations for two kinds of carriers. When the parallel magnetic field is applied to the system, the electron densities $n_{\pm}$ for spin up-down are not equal with the total density $n=n_+ + n_-$ fixed. The spin-polarized densities themselves are obtained from the relative shifts in the spin-up and -down bands introduced by the Zeeman splitting associated with the external applied field B. Since there are two groups of electrons (spin up and down), we need to consider inter-spin-band electron-electron scattering, which contributes to resistivity in addition to electron scattering by charged impurities and phonons. In the presence of an applied field, the carrier momentum will relax to equilibrium by electron-electron, electron-impurity, and electron-phonon scattering (which is neglected in this calculation because at low temperatures of interest to us phonon scattering is unimportant). The electron-electron relaxation rate $1/\tau_{ee}$ will only affect the relative momentum (i.e., the electron-electron scattering for the same spin is neglected in this calculation). The electron-electron scattering relaxes the relative velocity or relative momentum between two different populations to zero. In the steady state, the kinetic equations of motion in the presence of an electric field **E** and a magnetic field **B** for spin-up and/or spin-down electrons, taking into account the collisions with spin-down and/or spin-up electrons, have the form ($c=\hbar=1$) (Ref. 11),

$$m_1\frac{\mathbf{v}_1}{\tau_1}+M\frac{n_2}{n}\frac{\mathbf{v}_1-\mathbf{v}_2}{\tau_{ee}}=e\mathbf{E}+e(\mathbf{v}_1\times\mathbf{B}),$$

$$m_2\frac{\mathbf{v}_2}{\tau_2}+M\frac{n_1}{n}\frac{\mathbf{v}_2-\mathbf{v}_1}{\tau_{ee}}=e\mathbf{E}+e(\mathbf{v}_2\times\mathbf{B}), \qquad (1)$$

where m_i is the effective mass ($i=1,2$ denotes up and/or down spin subbands) for each group, $M=nm_1m_2/(m_1n_1+m_2n_2)$, and τ_i is the (energy and temperature dependent) 2D carrier transport scattering time (the so-called momentum relaxation time) determined by the *screened* charged impurity scattering[10] and τ_{ee} is the electron-electron relaxation time for the relative momentum of the spin-polarized system.

By solving the system of equations for $\mathbf{v}_i$ and substituting these velocities into the expression for the current density $\mathbf{j}=n_+e\mathbf{v}_++n_2e\mathbf{v}_-$, we find the resistivities ρ_{xx} and ρ_{xy},

$$\rho_{xx}=\frac{1}{ne}\frac{\langle\tilde{\mu}\rangle\{1+[(n_1\mu_2+n_2\mu_1)/\,n\mu_{ee})]\}+[\mu_1\mu_2(n_1\mu_2+n_2\mu_1)/n]B_z^2}{[\langle\mu\rangle+\mu_1\mu_2/\mu_{ee}]^2+\langle\mu_1\mu_2B_z\rangle^2}, \qquad (2)$$

$$\rho_{xy}=\frac{B_z}{ne}\langle r_H\rangle, \qquad (3)$$

where $\langle\tilde{\mu}\rangle=\langle\mu\rangle+\mu_1\mu_2/\mu_{ee}$ and B_z is the applied magnetic field normal to the 2D layer and r_H, the so-called Hall ratio, is given by

$$\langle r_H\rangle=1+\frac{\langle\mu^2\rangle-\langle\mu\rangle^2}{[\langle\mu\rangle+\mu_1\mu_2/\mu_{ee}]^2+(\mu_1\mu_2B_z)^2}. \qquad (4)$$

$\mu_i=e\tau_i/m_i$ and $\mu_{ee}=e\tau_{ee}/M$ and the average mobility is defined by $\langle\mu\rangle=(n_1\mu_1+n_2\mu_2)/n$ and $\langle\mu^2\rangle=(n_1\mu_1^2+n_2\mu_2^2)/n$. When intersubband electron-electron scattering is weaker than the impurity transport scattering times ($\tau_{ee}\gg\tau_i$), we have $\rho_{xx}=1/ne\langle\mu\rangle$ and $\langle r_H\rangle=\langle\mu^2\rangle/\langle\mu\rangle^2$. In the other limit, $\tau_{ee}\ll\tau_i$, we have $\rho_{xx}=(n_1/n\mu_1+n_2/n\mu_2)/ne$ and $\langle r_H\rangle\to1$. These equations immediately imply that the Hall coefficient will have very weak temperature dependence (i.e., $r_H\sim1$) when the inter-spin-subband electron-electron scattering dominates over the impurity scattering.

In Figs. 1 and 2 we show our calculated Hall coefficient without intersubband electron-electron scattering. In this case the total conductivity is the sum of the conductivities of each group, $\sigma=\sigma_1+\sigma_2$, and the Hall coefficient is given by $\langle r_H\rangle=\langle\mu^2\rangle/\langle\mu\rangle^2$. Throughout this Rapid Communication, we use the parameters corresponding to Si-MOSFET electron systems following Ref. 5. In Fig. 1 we show our calculated Hall coefficient, $R_H(B)/R_H(0)$, ($R_H=\langle r_H\rangle/ne$) for several carrier densities, $n=1,\ 10,\ 50,\ 100\times10^{10}$ cm^{-2} (which correspond to the screening strength $q_{TF}/2k_F=35,\ 11,\ 5,\ 3.5$, respectively) as a function of in-plane magnetic fields at $T=0$. As the in-plane magnetic field increases the spins are polarized, and at $B=B_s$ the system is completely spin polarized. In the low density limit (strong screening, $q_{TF}/2k_F\gg1$) the normalized Hall coefficient is strongly dependent on the polarization of the system. But for high densities (weak screening) the coefficient is almost independent of the spin polarization. Thus the polarization-dependent Hall coefficient is suppressed as screening effects decrease. We find that the normalized Hall coefficient for a density $n=25\times10^{10}$ cm^{-2} (corresponding to Ref. 5) increases only about 6% at most as the system gets fully spin polarized by the applied field. For $\mu_1\approx\mu_2$, we have $\langle r_H\rangle\sim1$ from Eq. (4). (If both carriers have the same mobility, the system becomes one band model and R_H must be constant.) Thus, the strong variation in the Hall coefficient with spin polarization is related to the very different mobility behavior of the two carriers. Based on our model we find the mobility ratio at $T=0$,

$$\frac{\mu_2}{\mu_1}=\frac{2x^2}{\pi}\sin^{-1}x+4\left(1-\frac{2}{\pi}\sin^{-1}x\right)\left[\frac{1+q_0}{2+q_0(2-\sqrt{1-x^2})}\right]^2, \qquad (5)$$

where $x=k_{F2}/k_{F1}$ and $q_0=q_{TF}/2k_{F1}$ is the screening strength. In the strong screening limits ($q_0\gg1$) μ_2/μ_1 is strongly dependent on the spin-polarization (x) and $\mu_2/\mu_1\approx4$ as $x\to0$. However, in the weak screening limits ($q_0\approx1$) we have $\mu_2/\mu_1\approx1$ for all spin polarizations. (See the inset of Fig. 1.) This is the main reason why the $\langle r_H\rangle$ is almost constant in the weak screening limits.

Since the screening function is suppressed by thermal effects, we expect the Hall coefficient to be suppressed at finite temperatures. In Fig. 2 we show our calculated Hall coefficient, $R_H(B)$, for several temperatures, $T=0,1,2$ K and a fixed density $n=20\times10^{10}$ cm^{-2}. As the temperature increases the normalized Hall coefficient shows suppressed field dependence mostly due to the weakening of the screening. Comparison between our results and the experimental results[5] shows good qualitative agreement. Thus, the observed field independence of the Hall coefficient can be qualitatively explained by the screening theory. However, in Ref. 5 Vikalov *et al.* conclude, by comparing their measured Hall coefficient (which does not vary with the parallel mag-

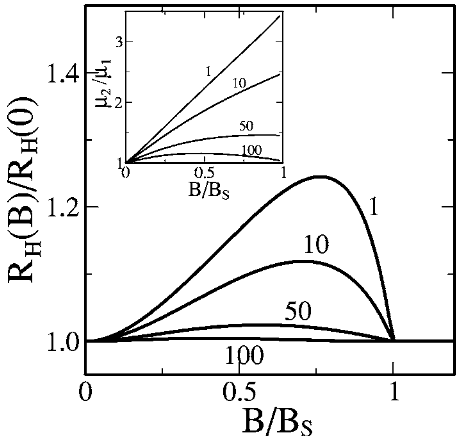

FIG. 1. Normalized Hall coefficient $R_H(B)/R_H(0)$ as a function of the in-plane magnetic field (spin polarization of the system) for different densities, $n=1$, 10, 50, 100×10^{10} cm^{-2} and at $T=0$ K. At $B=B_s$ the 2D system is completely spin polarized. The inset shows the mobility ratio.

netic field) with the theoretical expectations based on the screening theory[6] calculated in the strong screening limit ($q_{TF}/k_F \gg 1$) and at zero temperature, that the screening theory disagrees qualitatively with the experimental Hall effect results, and therefore the strong electron-electron scattering is a possible explanation for their data. In order to investigate the effects of electron-electron scattering on the Hall coefficient we consider fully Eq. (4), which includes electron-electron scattering as well as the screened charged impurities. We explicitly calculate the electron-electron scattering time τ_{ee} defined as the relaxation time of the relative momentum between spin-up and -down carriers.[12] For an unequal spin population we have τ_{ee},

$$\frac{1}{\tau_{ee}} = \frac{8(k_BT)^2}{3(\pi)^2}\frac{nm^3}{n_1n_2}p\int_0^\pi d\theta \sin\theta \left|\frac{2\pi e^2}{q\varepsilon(q)}\right|^2 [f(p)]^2, \quad (6)$$

where $f(p)=(1+p^2+2p\cos\theta)^{1/2}$ with $p=(n_1/n_2)^{1/2}$, $q=4\pi\sqrt{n_1n_2}\sin(\theta)/f(p)$, and $\varepsilon(q)$ is the total dielectric function of the system. Thus, we have that $\tau_{ee}^{-1} \propto T^2$. In the very low-temperature limit we expect the contribution of the electron-electron scattering to the Hall coefficient to be negligible because of this T^2 dependence of τ_{ee}^{-1}. At low temperatures therefore ($T \ll T_F$) our results (Figs. 1 and 2) neglecting τ_{ee} effects apply.

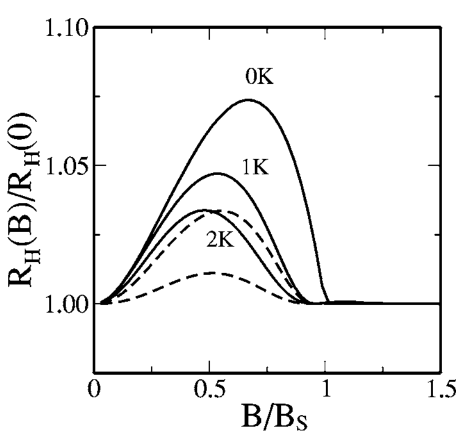

FIG. 2. The calculated Hall coefficient $R_H(B)/R_H(0)$ as a function of the in-plane magnetic field (spin polarization of the system) for different temperatures, $T=0,1,2$ K, and $n=20\times10^{10}$ cm^{-2}. Solid (dashed) lines indicate the results with (without) electron-electron scattering.

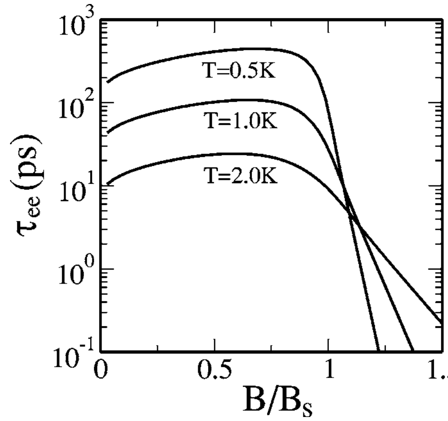

FIG. 3. Calculated inter-spin-subband electron-electron scattering times for different temperatures $T=0.5$, 1.0, 2.0 K as a function of the parallel magnetic field. We use the 2D density $n=20\times10^{10}$ cm^{-2}.

In Fig. 3 we show our calculated inter-spin-subband electron-electron scattering times, τ_{ee}, as a function of the parallel field (or spin polarization). Our results show that τ_{ee} depends strongly on the spin polarization and temperature. (Note in Ref. 5 a constant parameter τ_{ee} is used to fit the Hall coefficient data, which is incorrect.) In general, we find the elastic scattering time due to ionized impurities at the interface to be $\tau_i \approx 7$ ps with an impurity density $n_i=3\times10^{10}$ cm^{-2}, which corresponds to the experimental Si-MOSFET sample of the mobility $\mu \approx 2\times10^4$ V/cm^2 s. Thus, the calculated τ_{ee} is much larger than the elastic scattering time τ_i in the low-temperature limit where the 2D experiments are typically carried out. The Hall coefficient of the spin-polarized system including electron-electron scattering is shown in Fig. 2 (dashed lines) for different temperatures, $T=1,2$ K and $n=20\times10^{10}$ cm^{-2}. Our calculation shows that the electron-electron scattering leads to the further suppression in the field dependence of the Hall coefficient, leading to even better agreement between the experiment[5] and our theory.

Figure 4 shows the calculated magnetoresistance as a function of the in-plane magnetic field for different temperatures and a carrier density $n=2\times10^{11}$ cm^{-2}. This strong positive magnetoresistance can be explained by the systematic suppression of screening as the spin polarization of the system changes.[4,6] The saturation of the magnetoresistance above B_s is attributed to the complete spin polarization of the system. Considering electron-electron scattering in the calculation produces very small quantitative modification in the magnetoresistance even though the electron-electron scattering times are comparable to the elastic impurity scattering times (at $T=2$ K). In Fig. 4 the solid (dashed) lines indicate calculated results without (with) electron-electron interaction. Even though the electron-electron scattering rate is

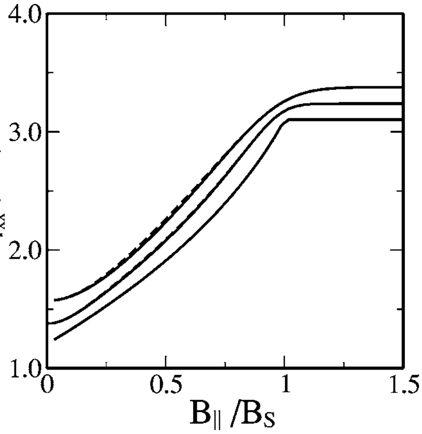

FIG. 4. Calculated magnetoresistance as a function of the in-plane magnetic field for different temperatures, $T=0,1,2$ K (from bottom to top). The results are shown for the carrier density $n=2\times10^{11}$ cm^{-2}. The solid (dashed) lines indicate calculated results without (with) electron-electron inter-spin-subband scattering.

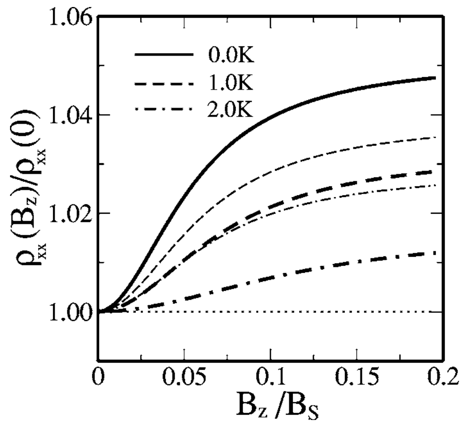

FIG. 5. Calculated magnetoresistance as a function of perpendicular magnetic-field (B_z) to the 2D plane for different temperatures, $T=0,1,2$ K (from bottom). The results are shown for the carrier density $n=2\times10^{11}$ cm^{-2} and a fixed in-plane magnetic field $B_{\parallel}=0.5B_s$. The thick (thin) lines represent results with (without) electron-electron interaction.

stronger than the elastic scattering rate, the calculated magnetoresistance shows the same behavior. Thus, the experimentally measured strong positive magnetoresistance with increasing in-plane magnetic field is induced mostly by the change of the screening properties, not by the inter-spin-subband electron-electron scattering.

In Fig. 5 we show that calculated longitudinal magnetoresistance ρ_{xx} as a function of the perpendicular magnetic field (B_z) for different temperatures, $T=0,1,2$ K. The in-plane parallel magnetic field solely gives rise to the spin polarization of the system. The results are shown for a carrier density $n=2\times10^{11}$ cm^{-2} and a fixed in-plane magnetic field $B_{\parallel}=0.5B_s$. For an unpolarized system (i.e., a single subband with an isotropic scattering rate) classical transport theory predicts no magnetoresistance (horizontal dotted line in Fig. 5), because the Hall field exactly compensates the Lorentz force and the carriers drift in the direction of the applied field. In fact, single subband systems typically exhibit a negative magnetoresistance due to quantum corrections arising from weak localization or electron-electron interactions.[13] For a system with two different Fermi wave vectors (two occupied subbands) the classical transport theory predicts a positive magnetoresistance varying quadratically with magnetic field B_z at low fields and saturating at higher fields. In Fig. 5 we have the positive magnetoresistance as the magnetic field increases for a spin-polarized system. The positive magnetoresistance is again reduced by thermal effects (weakening of screening), and also by the electron-electron scattering (assuming that there are no quantum corrections).

In conclusion, we calculate the weak-field Hall coefficient and the magnetoresistance of a spin-polarized system based on the screening theory. We find that the spin-polarization dependence of the Hall coefficient is strongly suppressed by the weakening of the screening. The electron-electron inter-spin-subband scattering gives rise to an additional suppression of the Hall coefficient, but in the low-temperature experimental regime the inter-spin-subband electron-electron scattering is not quantitatively important. Our theory provides a very good explanation for recent experiments[5] on the magnetic-field dependence of the 2D Hall coefficient.

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