Intrinsic spin current for an arbitrary Hamiltonian and scattering potential

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We have described electron spin dynamics in the presence of the spin-orbit interaction and disorder using the spin-density matrix method. Exact solution is obtained for an arbitrary two-dimensional spin-orbit Hamiltonian and arbitrary smoothness of the disorder potential. Spin current depends explicitly on the disorder properties, namely the smoothness of the disorder potential, even in the ballistic limit when broadening by scattering is much smaller than the spin-orbit-related splitting of the energy spectrum. In this sense, universal intrinsic spin current does not exist.

I. INTRODUCTION

Spin-orbit coupling brings about a number of interesting effects, one of which is generation of a spin flux in the plane perpendicular to the charge current direction. This phenomenon occurs in the paramagnetic system and has been very well known for quite a long time, see Ref. 1. It results from the fact that in the presense of spin-orbit coupling, the scattering by impurities has an asymmetric character.[2] Extrinsic contribution exists only beyond the Born approximation in the scattering amplitude and leads to an accumulation of the spin density near the sample surface.[1]

It has been recently claimed[3,4] that an analogous phenomenon can exist even without scattering by impurities, i.e., in the ballistic regime, the corresponding contribution being called intrinsic. Later several papers appeared in which the effect of scattering by impurities was taken into account for the Rashba model,[5–10] besides, the extrinsic contribution was described[11,12] to explain the recent experiment done by Awschalom's group,[13] see also Ref. 14.

Here we study the dependence of the intrinsic contribution to the spin current on the form of the Hamiltonian and properties of the disorder potential using the spin-density matrix method.[15] We have considered an arbitrary Hamiltonian, for example the generalized two-dimensional (2D) Rashba Hamiltonian with an arbitrary momentum dependence of the spin-orbit term or the 3D Luttinger Hamiltonian, and the arbitrary smoothness of the disorder potential. In the case of the generalized 2D Rashba model and the arbitrary smoothness of the disorder potential, an exact solution for spin current in the ballistic limit $\Delta\tau \gg 1$ is obtained, where Δ is the spin splitting of the electron spectrum and τ is the transport scattering time. Spin current depends explicitly on the disorder properties, namely the smoothness of the disorder potential, even in the ballistic limit. In this sense, universal intrinsic spin current does not exist. The problem of 2D holes with the p^3 spin-orbit term has been numerically considered recently by several groups.[16,17] Analytically it was studied in Ref. 18 for the δ-scattering case. The case of the arbitrary smoothness of the disorder potential has been analytically considered very recently in Refs. 19–21; some results were also reported in Ref. 22. Below we will compare the results obtained here with those from Ref. 20. I also refer the reader to the introductory part of that paper where a complete list of references on spin current is presented.

II. SPIN CURRENT, KINETIC EQUATION

A. Hamiltonian

We consider the generalized 2D Rashba model, which can be applied, for instance, for the case of 2D holes. The Hamiltonian of the problem is

$$\mathcal{H}(\hat{\mathbf{p}}) = \frac{p^2}{2m} + \frac{\alpha p^N}{2}\vec{\sigma}\cdot\vec{\Omega}(\mathbf{p}), \quad \epsilon_M(p) = \frac{p^2}{2m} + M\alpha p^N, \quad (1)$$

where α is the spin-orbit coupling constant, $\Omega_x = \cos(N\phi)$, $\Omega_y = \sin(N\phi)$, $\Omega_z = 0$, $\epsilon_M(p)$ are the eigenvalues, and $M = \pm 1/2$ are the helicity values. The eigenfunctions are

$$\chi_{M\mathbf{p}} = \sum_{\mu=\pm 1/2} D_{\mu M}^{(1/2)}(\vec{\Omega})u_\mu = \sum_{\mu=\pm 1/2} e^{-i\mu N\phi}d_{\mu M}^{(1/2)}\left(\frac{\pi}{2}\right)u_\mu,$$

where $D_{\mu M}^{(1/2)}(\vec{\Omega})$ is the rotation matrix,[2] ϕ is the angle of $\mathbf{p}$, N is the winding number, and u_μ is the eigenfunction of the $\hat{\sigma}_z$ operator.

B. Spin current

We will calculate the q_{yz} component of the spin current. This quantity is defined as

$$
\begin{aligned}
q_{yz} &= \sigma_s E_x \\
&= \text{Tr}\int \frac{d^2p}{(2\pi)^2}\hat{f}(\mathbf{p})\frac{1}{2}(\hat{S}_z\hat{V}_y + \hat{V}_y\hat{S}_z) \\
&= -\frac{1}{2}\int \frac{d^2p}{(2\pi)^2}\frac{p_y}{m}[f_{+-}(\mathbf{p}) + f_{-+}(\mathbf{p})]. \quad (2)
\end{aligned}
$$

Here $\hat{f}(\mathbf{p})$ is the Wigner spin-density matrix, $\hat{V}_y$ is the y component of the velocity operator, and $\hat{S}_z = (1/2)\hat{\sigma}_z$ is the spin operator. The last expression in Eq. (2) is given in the helicity basis. The different components of the spin-density matrix have the following relations to the average spin components:

$$\langle S_z \rangle \propto (f_{+-} + f_{-+}),$$

$$\langle \vec{S} \cdot [\vec{n} \cdot \vec{\Omega}] \rangle \propto (f_{+-} - f_{-+}),$$

$$\langle \vec{S} \cdot \vec{\Omega} \rangle \propto (f_{++} - f_{--}), \tag{3}$$

where **n** is the unit vector normal to the 2D plane (z axis).

C. Kinetic equation

The general expression for the quantum kinetic equation in the case of spin-orbit interaction, when the Hamiltonian and the Wigner distribution function are matrices over the spin indexes, was derived, for example, in Refs. 15 and 19. In our case, this equation is simple and reads

$$\frac{\partial \hat{f}(\mathbf{p})}{\partial t} + e\mathbf{E}\frac{\partial \hat{f}(\mathbf{p})}{\partial \mathbf{p}} + \frac{i}{\hbar}[\mathcal{H}(\hat{\mathbf{p}}), \hat{f}] = \text{St}\{\hat{f}(\mathbf{p})\}. \tag{4}$$

Here e is the charge of the carriers. The last term on the left-hand side is the commutator and the expression for the collision term is given below. Now we write Eq. (4) in the helicity basis where the Hamiltonian is diagonal. When doing that, we should take into account the fact that eigenfunctions $\chi_{M\mathbf{p}}$ depend on the direction of the momentum **p**, thus the matrix elements of the derivative $\partial \hat{f}/\partial \mathbf{p}$ in this basis do not coincide with the quantities $\partial f_{MM'}/\partial \mathbf{p}$,

$$\left(\frac{\partial \hat{f}}{\partial \mathbf{p}}\right)_{MM'} = \frac{\partial f_{MM'}}{\partial \mathbf{p}} - \frac{i}{\hbar}[\hat{\mathbf{a}}, \hat{f}]_{MM'}, \quad \mathbf{a}_{MM'} = i\hbar\chi_{M\mathbf{p}}^\star \frac{\partial \chi_{M'\mathbf{p}}}{\partial \mathbf{p}}.$$

We see that there appears the commutator of the vector matrix $\hat{\mathbf{a}}$ with $\hat{f}$. Thus for Eq. (4) in the linear response regime ($\mathbf{E}\|x$), we obtain

$$eE\cos\phi\frac{\partial f_{MM}^{(0)}}{\partial p}\delta_{MM'} - \frac{iN}{2}\frac{\sin\phi}{p}eE[f_{M'M'}^{(0)}(p) - f_{MM}^{(0)}(p)]$$

$$+ \frac{i}{\hbar}[\epsilon_M(p) - \epsilon_{M'}(p)]f_{MM'}(\mathbf{p}) = \text{St}[\hat{f}(\mathbf{p})]_{MM'}. \tag{5}$$

Here $f_{MM}^{(0)}(p)$ is the equilibrium Fermi function corresponding to the helicity value M. The collision term in the helicity basis has the form[19,23]

$$\text{St}[\hat{f}(\mathbf{p})]_{MM'} = \int \frac{d^2\mathbf{p}_1}{(2\pi\hbar)^2} \sum_{M_1 M_1'} \{[\delta(\epsilon_{M_1}(p_1) - \epsilon_M(p))$$

$$+ \delta(\epsilon_{M_1'}(p_1) - \epsilon_{M'}(p))]K_{M_1 M_1'}^{MM'}(\omega_{\mathbf{p}\mathbf{p}_1})$$

$$\cdot f_{M_1 M_1'}(\mathbf{p}_1) - \delta(\epsilon_{M_1}(p) - \epsilon_{M_1'}(p_1))$$

$$\times [K_{M_1' M_1'}^{MM_1}(\omega_{\mathbf{p}\mathbf{p}_1}) \cdot f_{M_1 M'}(\mathbf{p})$$

$$+ f_{MM_1}(\mathbf{p}) \cdot K_{M_1' M_1'}^{M_1 M'}(\omega_{\mathbf{p}\mathbf{p}_1})]\}, \tag{6}$$

where the kernel in the Born approximation in the scattering amplitude is

$$K_{M_1 M_1'}^{MM'}(\omega_{\mathbf{p}\mathbf{p}_1}) = \frac{\pi}{\hbar}|U(\mathbf{p} - \mathbf{p}_1)|^2 \cdot n_i D_{MM_1}^{(1/2)}(\omega_{\mathbf{p}\mathbf{p}_1})D_{M'M_1'}^{(1/2)\star}(\omega_{\mathbf{p}\mathbf{p}_1}). \tag{7}$$

Here n_i is the 2D impurity density and $U(\mathbf{p} - \mathbf{p}_1)$ is the Fourier component of the impurity potential. The quantities $D_{MM_1}^{(1/2)}(\omega_{\mathbf{p}\mathbf{p}_1})$ depend only on the scattering angle $\theta = \phi - \phi_1$,

$$D_{1/2,1/2}^{(1/2)} = D_{-1/2,-1/2}^{(1/2)} = \cos(N\theta/2),$$

$$D_{1/2,-1/2}^{(1/2)} = D_{-1/2,1/2}^{(1/2)} = -i\sin(N\theta/2). \tag{8}$$

III. SOLUTION

In this paper, we consider only the case of small spin-orbit splitting, $\Delta/E_F \ll 1$, $\Delta = \alpha p_F^N$, and we will be keeping only the terms linear in this small parameter. Note that expanding Eq. (6) with respect to this parameter, for the collision term in the spin basis we obtain the expression that is identical to that given by Eq. (23) of Ref. 19. We will be solving the problem in the helicity basis, which is much more convenient and the solution can be obtained much easier. Moreover, we will consider only the ballistic case, $\Delta \gg \hbar/\tau$, when one expects the intrinsic value for the spin current. In the case when the impurity potential has azimuthal symmetry, it is possible to obtain a solution for the scattering potential with an arbitrary correlation length R. The most simple cases correspond to the following situations: (i) $m\tilde{\alpha}/\hbar \ll 1/R$, (ii) $1/R \ll m\tilde{\alpha}/\hbar$, $\tilde{\alpha} = \alpha p_F^{N-1}$; $\hbar/m\tilde{\alpha}$ is the spin-orbit length. We will see that these two cases correspond to different answers for the spin current. The first case means a relatively short-ranged potential and was considered in Refs. 19 and 20. Note that it includes the limit of small-angle scattering when $m\tilde{\alpha}/\hbar \ll 1/R \ll k_F$.

As we consider the ballistic case here, $\Delta \gg \hbar/\tau$, the problem can be greately simplified, namely, in the collision term, Eq. (6), we can neglect all the nondiagonal components of the spin-density matrix since only diagonal components can be proportional to τ, which follows from Eq. (5). Moreover, since on the left-hand side of Eq. (5) there are only the first harmonics of the angle, the following solution obeys the system of Eqs. (5) and (6):

$$f_{++}(\mathbf{p}) = f_{++}(p)\cos\phi, \quad f_{--}(\mathbf{p}) = f_{--}(p)\cos\phi, \tag{9}$$

$$f_{+-}(\mathbf{p}) = f_{+-}(p)\sin\phi, \quad f_{-+}(\mathbf{p}) = f_{-+}(p)\sin\phi. \tag{10}$$

It can be immediately seen from Eq. (6) with help from the fact that the scattering kernel

$$W(\theta) = \frac{n_i}{2\hbar^3}|U(\theta)|^2, \quad U(\theta) = U(|\vec{p}_1 - \vec{p}|)$$

$$= U(\sqrt{p^2 + p_1^2 - 2pp_1\cos\theta}) \tag{11}$$

is the even function of the scattering angle θ. Indeed, from Eqs. (5) and (6) we obtain

$$eE\frac{\partial f_+^{(0)}}{\partial p} = \frac{a_1 p}{V_+(p)} f_{++}(p) - \frac{p_+}{V_-(p_+)} [a_2 f_{++}(p) - a_3 f_{--}(p_+)]$$

$$- m^2 \tilde{\alpha} \int \frac{d\theta}{2\pi} \frac{dW(\theta)}{dp} \sin^2\left(\frac{N\theta}{2}\right) (1 - \cos\theta)$$

$$\times f_{++}(p)|_{\alpha=0},$$ (12)

$$eE\frac{\partial f_-^{(0)}}{\partial p} = \frac{a_1 p}{V_-(p)} f_{--}(p) + \frac{p_-}{V_+(p_-)} [a_3 f_{++}(p_-) - a_2 f_{--}(p)]$$

$$+ m^2 \tilde{\alpha} \int \frac{d\theta}{2\pi} \frac{dW(\theta)}{dp} \sin^2\left(\frac{N\theta}{2}\right) (1 - \cos\theta)$$

$$\times f_{++}(p)|_{\alpha=0},$$ (13)

$$iN\frac{eE}{p}(f_+^0 - f_-^0) + \frac{i}{\hbar}(\epsilon_+ - \epsilon_-)[f_{+-}(p) + f_{-+}(p)]$$

$$= + ia_4 \left[\frac{p}{V_+(p)} f_{++}(p) - \frac{p}{V_-(p)} f_{--}(p) \right.$$

$$\left. + \frac{p_-}{V_+(p_-)} f_{++}(p_-) - \frac{p_+}{V_-(p_+)} f_{--}(p_+) \right]$$

$$- im^2 \tilde{\alpha} \int \frac{d\theta}{2\pi} \frac{dW(\theta)}{dp} \sin\theta \sin(N\theta) f_{++}(p)|_{\alpha=0}.$$

(14)

In writing these equations, we used Eqs. (8). The coefficients a_i in the above equations are given by the expressions

$$a_1 = \int \frac{d\theta}{2\pi} W(\theta)(\cos\theta - 1)(1 + \cos N\theta),$$

$$a_2 = \int \frac{d\theta}{2\pi} W(\theta)(1 - \cos N\theta),$$

$$a_3 = \int \frac{d\theta}{2\pi} W(\theta)\cos\theta(1 - \cos N\theta),$$

$$a_4 = \int \frac{d\theta}{2\pi} W(\theta)\sin\theta \sin N\theta,$$

$$a_5 = \int \frac{d\theta}{2\pi} W(\theta)(1 - \cos\theta).$$ (15)

Note that here the scattering angle has an arbitrary value and is not assumed to be small. $f_+^0(p), f_-^0(p)$ are the equilibrium Fermi functions, which correspond to the helicity $\pm$; $V_\pm(p) = p/m \pm N\tilde{\alpha}/2$ are the velocity values for a given p for $\pm$ bands. The other quantities entering Eqs. (12)–(14) are defined in the following way:

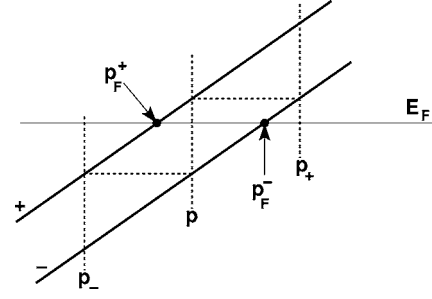$$p_\pm = p \pm m\tilde{\alpha}, \quad V_-(p_+) = \frac{p}{m} - \frac{\tilde{\alpha}}{2}(N - 2),$$

FIG. 1. Schematics of the $\pm$ energy bands. Momenta $p, p_\pm, p_F^\pm$ are shown; see the text.

$$V_+(p_-) = \frac{p}{m} + \frac{\tilde{\alpha}}{2}(N - 2), \quad \tilde{\alpha} = \alpha p_F^{N-1}.$$ (16)

The quantities $V_-(p_+)$, $V_+(p_-)$ are the velocities in the bands $-$, $+$ for the momenta p_+, p_-; see Fig. 1. These values of the momenta are connected in Eqs. (12)–(14) because of the energy conservation under the elastic scattering. In writing Eqs. (16), we took into account that $m\tilde{\alpha} \ll p_F$, where p_F is the Fermi momentum.

Equations (12)–(14) are written for a given value of p. We see that the elastic scattering by impurities leads to admixture of the spin-density matrix components, which correspond to p_+, p_- values of the momentum. Different combinations of the ratio of the momentum and the velocity values like $p/V_+(p)$ or $p_+/V_-(p_+)$ are just different density of state values corresponding to different points in the momentum space involved in the transitions; see Fig. 1. From the form of the coefficients in Eq. (15), we see that different terms in Eqs. (12)–(14) have a simple physical meaning, namely, they are just "transport" kinetic coefficients multiplied (weighted) by the corresponding overlap factors $\sin^2(N\theta/2)$, $\cos^2(N\theta/2)$, or $\sin(N\theta/2)\cos(N\theta/2)$ depending on the type of transition, i.e., whether it is intraband or interband scattering; see Eq. (8) and Fig. 1. The very last term on the right-hand side in each of the equations (12)–(14) has a somewhat different origin. We can see that the scattering transitions within the band ($+$ or $-$) and between the bands correspond to different values of the p_1 momentum entering the kernel of the collision term, see Eq. (11). In the case of the intraband transitions $p_1 = p$, while for the interband transitions its value is either p_+ or p_-. Thus, in the linear order with respect to $\tilde{\alpha}$ we obtain

$$W(p \to p_\pm) - W(q) = \pm \frac{dW(q)}{dp} \frac{m\tilde{\alpha}}{2},$$

$$W(q) = W\left(2p\left|\sin\frac{\theta}{2}\right|\right).$$ (17)

Note that the quantity $W(q)$ is the value of the scattering kernel for the case of intraband transition when the modulus of the momentum is conserved, $p_1 = p$, and the modulus of the transferred momentum is $q = 2p|\sin\theta/2|$. Hence, we have taken into account the correction to $W(\theta)$ in the corresponding terms of Eqs. (12)–(14), keeping only the correction of

the first order in α, i.e., all the prefectors in the terms that already contain the derivative $dW(q)/dp$ are taken at $\alpha=0$. See, for example, the last term in Eq. (12), where we also used the fact that at $\alpha=0$, $f_{++}(p)=f_{--}(p)$, and is given by Eq. (21); see below.

All the quantities entering Eqs. (12)–(14) can be expressed through the corresponding values at p. Writing Eqs. (12) and (13) for the momenta values $p_{\pm}$, and using the relations

$$\frac{\partial f_+^0(p)}{\partial p}\bigg|_{p=p_-} = \frac{\partial f_-^0(p)}{\partial p}\frac{V_+(p_-)}{V_-(p)}, \quad \frac{\partial f_-^0(p)}{\partial p}\bigg|_{p=p_+}$$

$$= \frac{\partial f_+^0(p)}{\partial p}\frac{V_-(p_+)}{V_+(p)},$$

we obtain

$$f_{--}(p_+) = f_{++}(p)\left[1 - \frac{m\tilde{\alpha}NC}{Ap}\right] - \frac{m\tilde{\alpha}f_{++}(p)|_{\alpha=0}}{2A}\frac{da_1}{dp}$$

$$- \frac{\tilde{\alpha}\tilde{E}}{2Ap}(N-1),$$

$$f_{++}(p_-) = f_{--}(p)\left[1 + \frac{m\tilde{\alpha}NC}{Ap}\right] + \frac{m\tilde{\alpha}f_{++}(p)|_{\alpha=0}}{2A}\frac{da_1}{dp}$$

$$+ \frac{\tilde{\alpha}\tilde{E}}{2Ap}(N-1), \tag{18}$$

where $\tilde{E}$ is given by Eq. (21) and A,C are expressed through coefficients a_i by the formulas $A=(a_1-a_2-a_3)/2$, $C=a_1+a_5$,

$$A = \int \frac{d\theta}{2\pi}W(\theta)(\cos\theta\cos N\theta - 1),$$

$$C = \int \frac{d\theta}{2\pi}W(\theta)(\cos\theta - 1)\cos N\theta. \tag{19}$$

While deriving Eqs. (18), we used the expansions

$$W(p_- \to p_-) = W(q) - m\tilde{\alpha}\frac{dW(q)}{dp},$$

$$W(p_+ \to p_+) = W(q) + m\tilde{\alpha}\frac{dW(q)}{dp}, \tag{20}$$

where the right-hand side of these equations is written for the momentum value p. Note the difference in the coefficients in Eqs. (20) compared to Eq. (17). Using the equation

$$[f_{++}(p) + f_{--}(p)]|_{\alpha=0} = -\frac{\tilde{E}}{ma_5}, \quad \tilde{E} = eE\frac{\partial f_0}{\partial p}, \tag{21}$$

which can be easily found from Eqs. (12) and (13), with f_0 being the equilibrium Fermi function at $\alpha=0$, we find the final expression for the quantity in question, $[f_{+-}(p)+f_{-+}(p)]$, in the case $m\tilde{\alpha}/\hbar \ll 1/R$,

$$N\frac{eE}{p}(f_+^0 - f_-^0)$$

$$+ \frac{m\tilde{\alpha}\tilde{E}}{2a_5}\left[\frac{a_4}{A}\left(\frac{da_1}{dp} + \frac{2N}{p}a_2 - \frac{2(N-1)}{p}a_5\right) - \frac{da_4}{dp}\right]$$

$$= -\frac{(\epsilon_+ - \epsilon_-)}{\hbar}[f_{+-}(p) + f_{-+}(p)], \tag{22}$$

where a_i are defined by Eq. (15).

A. Some limiting cases

Let us check some limiting cases from Eq. (22).

(i) We start with the simplest case of the δ-correlated potential, when scattering is isotropic, i.e., $W(\theta)$ is a constant. Then at $N \neq 1$, the coefficient $a_4=0$, and from Eq. (22) we immediately obtain a simple result,

$$\frac{NeE}{p}(f_+^0 - f_-^0) - \frac{m\alpha\tilde{E}}{p}\delta_{N,1} = -\frac{(\epsilon_+ - \epsilon_-)}{\hbar}[f_{+-}(p) + f_{-+}(p)]. \tag{23}$$

With the help of Eqs. (2) and (10) for the spin conductivity at $N \neq 1$, we obtain

$$\sigma_s = -\frac{eN}{8\pi\hbar}. \tag{24}$$

Note that this result also follows trivially from the diagrammatic calculations since the vertex correction is identically zero. It happens because of a very simple reason, namely, the vector vertex contains the first harmonics but the Green's functions contain only the third one (at $N=3$), and their overlap is zero, as it happens here with the coefficient a_4.

(ii) $N=1$. We know[5–10] that in this case for an *arbitrary* scattering (not necessarily small angle or isotropic), we should obtain a zero value for q_{yz}. Indeed, in this case we have $a_4=-A$, $a_1=-a_4$, $a_2=a_5$, and from Eq. (22) we easily obtain

$$\frac{eE}{p}(f_+^0 - f_-^0) - \frac{m\alpha}{p}eE\frac{\partial f_0}{\partial p} = -\frac{(\epsilon_+ - \epsilon_-)}{\hbar}[f_{+-}(p) + f_{-+}(p)]. \tag{25}$$

Expanding the first function on the left-hand side of this equation with respect to α, we obtain a zero value for the spin current.

(iii) Now consider the small-angle scattering case, $\theta \ll 1$. Then from Eq. (22) it follows that

$$\frac{NeE}{p}(f_+^0 - f_-^0) - \frac{m\tilde{\alpha}\tilde{E}}{p}\frac{2N(N^3 - N + 1)}{(N^2 + 1)}$$

$$- N\frac{(N^2 - 1)}{(N^2 + 1)}m\tilde{\alpha}\tilde{E}\frac{1}{a_5}\frac{da_5}{dp}$$

$$= -\frac{(\epsilon_+ - \epsilon_-)}{\hbar}[f_{+-}(p) + f_{-+}(p)]. \tag{26}$$

Finally, we need to calculate the quantity $(1/a_5)(da_5/dp)$ in

the limit of small-angle scattering. For this quantity, we obtain

$$\frac{1}{a_5}\frac{da_5}{dp} = \frac{1}{p}\frac{\int_0^\infty \frac{dq}{2\pi}\frac{\partial W(q)}{\partial q}qq^2}{\int_0^\infty \frac{dq}{2\pi}W(q)q^2} = -\frac{3}{p}, \qquad (27)$$

where $q=p|\theta|$. With the help of Eq. (27), we finally obtain for the spin conductivity

$$\sigma_s = -\frac{eN}{8\pi\hbar}\left[1 + \frac{(-5+2N-2N^3+3N^2)}{(N^2+1)}\right]$$

$$= \frac{eN}{4\pi\hbar}\frac{(N^2-1)(N-2)}{(N^2+1)}. \qquad (28)$$

The first term (unity) inside the square brackets originated from the first term on the left-hand side of Eq. (26). The result we obtain, i.e., Eq. (28), coincides exactly with the corresponding result of Ref. 20.

(iv) Finally, we consider the case of a very smooth scattering potential when the impurity radius obeys the inequality $1/R \ll m\tilde{\alpha}/\hbar \ll k_F$.[22] Physically, it means that elastic transitions between different bands (with opposite helicity) are forbidden since the quantity $W(\theta)$, see Eq. (11), is exponentially small for these transitions. Then in Eqs. (12)–(14) we should drop all the terms containing $p_\pm$ quantities (and also derivatives dW/dp), and with the use of the fact that in the case considered $\theta \ll 1$, the final result reads

$$\frac{eEN}{p}(f_+^0 - f_-^0) + NeE\left(\frac{\partial f_+^0}{\partial p} - \frac{\partial f_-^0}{\partial p}\right)$$

$$= -\frac{(\epsilon_+ - \epsilon_-)}{\hbar}[f_{+-}(p) + f_{-+}(p)]. \qquad (29)$$

Integrating in Eq. (2) between the points $p_F^\pm = \mp m\tilde{\alpha}/2 + p_F$, see Fig. 1, we obtain for the spin conductivity

$$\sigma_s = -\frac{eN}{8\pi\hbar}(N-1), \qquad (30)$$

which for $N=3$ is twice as large compared to the δ-scattering case.[22]

IV. CONCLUSION

In conclusion, we have studied how intrinsic spin current depends on the form of the Hamiltonian and the scattering potential properties. As an example, we have investigated the generalized 2D Rashba model when the spin-orbit term contains an arbitrary dependence on the electron momentum. We have found an exact analytic solution for the intrinsic spin current in the ballistic limit, when spin-orbit splitting is much larger than the disorder-induced broadening. In contrast to the case of a common, linear in momentum Hamiltonian, the intrinsic current does not necessarily vanish. However, even in the ballistic limit indicated above, its value depends explicitly on the disorder properties. More precisely, the result, being independent of the spin-orbit coupling constant, is different for a different correlation radius of the disorder potential. The situation is similar for the other Hamiltonians. For example, we have also studied this problem for the Hamiltonian of 3D holes.[24] In the diagrammatic language, it means that even in the ballistic limit the vertex correction is always as important as the contribution of the empty loop and physically it is not correct to consider them separately. In this sense, universal intrinsic spin current does not exist.

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