Superfluid density and competing orders in *d*-wave superconductors

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We derive expressions for the superfluid density ρ_s in the low-temperature limit $T \rightarrow 0$ in *d*-wave superconductors, taking into account the presence of competing orders such as spin-density waves, id_{xy} pairing, etc. Recent experimental data for the thermal conductivity and for elastic neutron scattering in $La_{2-x}Sr_xCuO_4$ suggest there are magnetic field induced anomalies that can be interpreted in terms of competing orders. We consider the implications of these results for the superfluid density and show in the case of competing spin-density wave order that the usual Volovik-like $\sqrt{H}$ depletion of $\rho_s(H)$ is replaced by a slower dependence on applied magnetic field. We find that it is crucial to include the competing order parameter in the self-consistent equation for the impurity scattering rate.

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I. INTRODUCTION

The doping and temperature dependence of the superfluid density $\rho_s(x,T)$ and its correlation with the critical temperature T_c in high-temperature superconductors (HTSC) have been intensively studied since their discovery.[1] The last few years are also marked by new observations[2–5] particularly for the underdoped side of the phase diagram.[6–9] This special attention paid to the superfluid density is not surprising, because all properties of $\rho_s(x,T)$ mentioned above show that HTSC differ significantly from previously known superconductors. An important example is the temperature dependence of the in-plane penetration depth[10] $\lambda(T)$ which provided one of the first strong evidence[11] for unconventional pairing symmetry and for the important role of gapless low-energy quasiparticle excitations.

Another interesting dependence of ρ_s is its variation with the external magnetic field H. This dependence contains information on the quasiparticles and on their interaction with the vortices present in the vortex phase of HTSC. Since these are extreme type-II superconductors, a huge vortex phase extends from the lower critical field $H_{c1} \sim 10-100$ G to the upper critical field $H_{c2} \sim 100$ T. The leading term in the field dependence of $\rho_s(H) \sim \sqrt{H}$ is well understood Refs. 12–14 using the local Doppler-shift approximation. This very useful approximation was introduced by Volovik,[15] who predicted, that in contrast to conventional superconductors, in *d*-wave systems the density of states (DOS) is dominated by contributions from the excited quasiparticle states rather than the bound states associated with vortex cores. It was shown that in an applied magnetic field H the extended quasiparticles DOS, $N(\omega=0,H) \sim \sqrt{H}$ rather than $\sim H$ of the conventional case. The validity of the Doppler-shift approximation was discussed in Refs. 16 and 17 using the quasiclassical Eilenberger equations. It was shown, in Ref. 17 that for the superfluid density, this approximation works reasonably well at low temperatures.

The characteristic $\sqrt{H}$ behavior has been observed in the specific heat and thermal conductivity (see Ref. 18 for a review of experiments). The dependence $\rho_s(H)$ can also be directly extracted from mutual-inductance technique

measurements[7,19,20] or, for example, from muon spin rotation (μSR) measurements (see Ref. 21 for a review). A big advantage of the former method is that it provides directly the desired dependence of $\rho_s(H)$. Moreover, the measured inductance is directly related to the superfluid density, so that in contrast to the thermal conductivity discussed below there is no need to subtract a phonon contribution from the raw data.[18] On the other hand, the analysis of μSR experiments involves additional assumptions about the internal magnetic field distribution in the vortex state[21] which are contained in a model dependent function $f(H)$. This important field dependent function enters the relationship $\sigma(H)=f(H)\rho_s(H)$ between the measured muon depolarization rate $\sigma(H)$ and $\rho_s(H)$.

Since 1997 there is ongoing discussion (see Ref. 22 for a historical overview) about the dependence of the thermal conductivity $\kappa(H)$ and its deviations from the expected $\sim \sqrt{H}$ behavior. The crucial fact is that when the magnetic field is applied perpendicular to the CuO$_2$ planes, the thermal conductivity show a transition from a field-dependent regime $\kappa(H) \sim \sqrt{H}$ to a field-independent, plateau-like regime. The latest experimental results in underdoped $La_{2-x}Sr_xCuO_4$ (LSCO) (Refs. 23 and 24) at $H=0$ for $0 \leqslant x \leqslant 0.22$ and (Refs. 25 and 26) at $0 < H < 16$ T for $0.06 \leqslant x \leqslant 0.22$ are interpreted in Ref. 27 in terms of a competing spin-density-wave order in the underdoped regime.

A theoretical background for this interpretation was proposed in Ref. 28, where it was shown that in the presence of a spin-density wave gap m (Ref. 29) for $H=0$ at $T \rightarrow 0$

$$\frac{\kappa}{T} = \frac{k_B^2}{3} \frac{v_F^2 + v_\Delta^2}{v_F v_\Delta} \frac{\Gamma_0^2}{\Gamma_0^2 + m^2}. \tag{1}$$

Here v_F is the Fermi velocity, v_Δ is the gap velocity, and Γ_0 is the impurity scattering rate which in Ref. 28 is assumed to be independent of the gap m (we use units with $\hbar=c=1$, unless stated explicitly otherwise, and from Sec. II also set $k_B=1$).

As one can see from Eq. (1) for $m=0$ and $v_F=v_\Delta$ the minimal value of the thermal conductivity is $\kappa_{min}/T=2k_B^2/3$. The opening of the gap m leads to a suppression of the ther-

mal conductivity and allows for values of κ/T which are less than κ_{min}/T, as is indeed observed in LSCO for $x=0.06$.[24] The presence of a nonzero field-dependent gap $m(H)$ also allows one to explain the behavior of $\kappa[H,m(H)]$ observed in underdoped LSCO.[25,26]

There are two additional experiments that support the idea of the presence of a competing order in LSCO. The first is an angle-resolved photoemission study[30] that indicates the existence of a finite gap over the entire Brillouin zone, including the $d_{x^2-y^2}$ nodal line in LSCO for $x<0.03$. And an even more exciting observation was made for a LSCO sample with $x=0.144$ and $T_c=37$ K by elastic neutron scattering[31] which showed that a static incommensurate spin-density-wave order develops above a critical field $H_0 \approx 3$ T. This picture is supported by the latest measurements[32] made on a sample with the same doping which showed $\kappa(H)/T$ increasing for $0<H\lesssim 0.5$ T and decreasing at higher field. However, a more heavily doped LSCO sample with $x=0.15$ does not show any decrease of $\kappa(H)/T$ up to 17 T. All these observations fuel interest in the investigation of competing orders.

The purpose of the present work is to investigate the influence of competing orders on the superfluid density ρ_s and its field dependence. We demonstrate that it is crucial to include the effect of the opening of the spin-density-wave gap m on the value of the impurity scattering rate $\Gamma(m)$. When this effect is taken into account the the resulting dependence $\rho_s\{H,m(H),\Gamma[m(H)]\}$ resembles the experimental results on very thin films of LSCO.[20]

Although the latest experiments on LSCO (Refs. 31 and 32) are mainly interpreted[27] in terms of a competing spin density wave (SDW) order, the earlier experiments on BSCO (see Refs. in 22 and 33) were also interpreted using a *complex d_{xy}* component generated in a *d*-wave superconductor in the magnetic field.[34,35] The removal of the *d*-wave node in optimally doped YBCO in a magnetic field was observed in the in-plane tunneling conductance.[36] The authors of Ref. 36 interpreted their observation in terms of competing id_{xy} or is order. Here we also consider the influence of competing superconducting id_{xy} and is orders on the superfluid density ρ_s. This problem was considered previously by Modre *et al.*[37] who calculated the London penetration depth for mixed superconducting order in zero magnetic field. We point out that the superfluid density ρ_s may provide a way to distinguish SDW and superconducting orders.

The paper is organized as follows. In Sec. II we introduce the 4×4 Dirac formalism convenient for the description of competing order with the underlying *d*-wave superconductivity. The general representation for the superfluid density is written in Sec. III and the difference between $\rho_s(T=0)$ for competing (with *d*-wave superconductivity) superconducting and SDW orders is considered. The dependence of the impurity scattering rate Γ on the values of competing gaps both in the Born and unitary limits is discussed in Sec. IV. In Sec. V we derive analytical expressions for $\rho_s(T)$ in the presence of competing orders at $H=0$ in the low-temperature limit. The main results of the paper for the field dependence $\rho_s(H)$ at $T=0$ are presented in Sec. VI. In Sec. VII we give a summary of the results obtained.
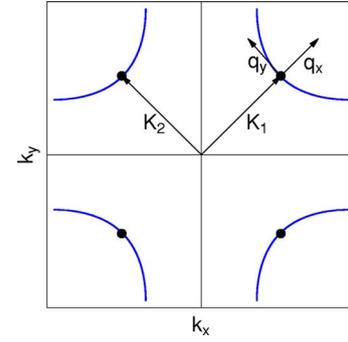
FIG. 1. (Color online) The schematic graph of the Fermi surface with the vectors $\mathbf{K}_i$, $i=1,2$, and $\mathbf{q}$. The SDW ordering vectors are $\mathbf{Q}_i=2\mathbf{K}_i$.

II. DIRAC FORMALISM FOR DESCRIPTION OF COMPETING ORDERS IN *d*-WAVE SUPERCONDUCTORS

We begin with the action for a *d*-wave superconductor written in imaginary time-momentum representation

$$S = -\int d\tau \int d\mathbf{k}\, \psi^\dagger(\tau,\mathbf{k})[\hat{I}\partial_\tau + \tau_3\xi(\mathbf{k}) - \tau_1\Delta(\mathbf{k})]\psi(\tau,\mathbf{k}),$$

(2)

where

$$\Psi^\dagger(\tau,\mathbf{k}) = [c_\uparrow^\dagger(\tau,\mathbf{k}), c_\downarrow(\tau,-\mathbf{k})]$$

(3)

is the Nambu spinor and $c_\sigma^\dagger(\tau,\mathbf{k})$ and $c_\sigma(\tau,\mathbf{k})$ with $\sigma=\uparrow,\downarrow$ are, respectively, creation and annihilation operators. Most of the time we will rely on the nodal approximation, so that the precise form of the dispersion law of the quasiparticles $\xi(\mathbf{k})$ and the *d*-wave superconducting gap $\Delta(\mathbf{k})$ is not essential.

It is impossible to consider other than *is* competing order while remaining within a 2×2 formalism. Depending on the physical assumptions made about the nature of the competing order, there are different possibilities for constructing a four-component field from Nambu spinors and switching to a 4×4 formalism (see, e.g., Refs. 38–41). Since we are mostly interested in competing spin density wave which forms on top the superconducting state, we choose our spinors as was done in Refs. 28 and 41

$$\Psi_i^\dagger(t,\mathbf{k}) = [c_\uparrow^\dagger(t,\mathbf{k})\ c_\downarrow(t,-\mathbf{k})\ c_\uparrow^\dagger(t,\mathbf{k}-\mathbf{Q}_i)\ c_\downarrow(t,-\mathbf{k}+\mathbf{Q}_i)],$$

(4)

where $\mathbf{Q}_i=2\mathbf{K}_i$ is the wave vector that connects the nodes within the diagonal pair $i=1,2$. Further since we are interested in the low-temperature ($T \ll T_c$) properties of the system, we consider only the vicinity of the nodes $\mathbf{k}=\mathbf{K}_i+\mathbf{q}$ with $|\mathbf{q}| \ll |\mathbf{K}_i|$ as shown in Fig. 1. Using that $\xi(\mathbf{k})=-\xi(\mathbf{k}-\mathbf{Q}_i)$ and $\Delta(\mathbf{k})=-\Delta(\mathbf{k}-\mathbf{Q}_i)$ for $\mathbf{k}\approx\mathbf{K}_i$, and then linearizing the spectrum as $\xi(\mathbf{k})=v_F q_x+O(q^2)$ and $\Delta(\mathbf{k})=v_\Delta q_y+O(q^2)$, one arrives at the low-energy action[41]

$$S = -\int d\tau \int d\mathbf{k} \Psi_1^\dagger(\tau,\mathbf{k})[\hat{I}_4 \partial_\tau + M_1 v_F q_x + M_2 v_\Delta q_y]\Psi_1(\tau,\mathbf{k})$$

$$+ (1 \to 2, x \leftrightarrow y), \tag{5}$$

where $M_1 = \sigma_3 \otimes \tau_3$ and $M_2 = -\sigma_3 \otimes \tau_1$, respectively. It is useful to reformulate the model (5) in the form of QED$_{2+1}$, because this allows us to rely on the algebraic properties of 4×4 reducible representation of γ matrices which satisfy Clifford (Dirac) algebra

$$\{\gamma_\mu, \gamma_\nu\} = 2\hat{I}_4 g_{\mu\nu}, \quad g_{\mu\nu} = \mathrm{diag}(1,-1,-1), \quad \mu,\nu = 0,1,2. \tag{6}$$

Another advantage of the QED$_{2+1}$ formulation is that it also allows us to classify different competing orders in terms of different types of Dirac masses.[28,38–41] We introduce the Dirac conjugated spinor $\bar{\Psi}_i = \Psi_i^\dagger \gamma_0$, where γ_0 is the 4×4 matrix that anticommutes with M_1 and M_2 and such that $\gamma_0^2 = \hat{I}_4$. This choice is not unique, but we will follow the same conventions as in Refs. 28 and 41 choosing $\gamma_0 = \sigma_1 \otimes \tau_0$. Accordingly we define $\gamma_{1,2}$ via $M_1 = \gamma_0 \gamma_1$ and $M_2 = \gamma_0 \gamma_2$, so that $\gamma_1 = -i\sigma_2 \otimes \tau_3$ and $\gamma_2 = i\sigma_2 \otimes \tau_1$ satisfy Eq. (6). Finally we arrive at the action

$$S = -\int d\tau \int d\mathbf{k} \bar{\Psi}_1(\tau,\mathbf{k})[\gamma_0 \partial_\tau + \gamma_1 v_F q_x + \gamma_2 v_\Delta q_y]\Psi_1(\tau,\mathbf{k})$$

$$+ (1 \to 2, x \leftrightarrow y). \tag{7}$$

One may consider quasiparticle gaps m_i of different nature encoding it in the matrix structure $O_i = (\hat{I}_4, i\gamma_5, \gamma_3, \gamma_3\gamma_5)$. Here the matrices γ_3 and γ_5, anticommuting with the matrices γ_ν, and are

$$\gamma_3 = i\sigma_2 \otimes \tau_2, \quad \gamma_5 = \sigma_3 \otimes \hat{I}_2. \tag{8}$$

Then, different gaps m_i correspond to different types of Dirac masses added to the action (7). In particular, the mass m_1, with $O_1 = \hat{I}_4$, describes the (incommensurate) cos spin-density wave (SDW), and the mass m_2 with $O_2 = i\gamma_5$, describes sin SDW. The masses m_3 and m_4 with $O_3 = \gamma_3$ and $O_4 = \gamma_3\gamma_5$, correspond to the id_{xy} and is pairing, respectively.[28,41] In the present paper we concentrate mainly on the SDW gap, $m_1 \equiv m$ with the corresponding bare Matsubara Green's function

$$G_0(i\omega_n,\mathbf{k}) = -\frac{i\omega_n \gamma_0 - v_F k_1 \gamma_1 - v_\Delta k_2 \gamma_2 + m\hat{I}}{\omega_n^2 + v_F^2 k_1^2 + v_\Delta^2 k_2^2 + m^2} \tag{9}$$

and on id_{xy} gap, $m_3 = \Delta_{d_{xy}}$ with the corresponding Green's function

$$G_0(i\omega_n,\mathbf{k}) = -\frac{i\omega_n \gamma_0 - v_F k_1 \gamma_1 - v_\Delta k_2 \gamma_2 - \Delta_{d_{xy}} \gamma_3}{\omega_n^2 + v_F^2 k_1^2 + v_\Delta^2 k_2^2 + \Delta_{d_{xy}}^2}. \tag{10}$$

Finally we should define the electric current operator in 4×4 formalism. In Nambu formalism it reads

$$\mathbf{j}(\tau,\mathbf{q}=0) = e\int d\mathbf{k} \cdot \mathbf{v}_F(\mathbf{k})\psi^\dagger(\tau,\mathbf{k})\hat{I}_2\psi(\tau,\mathbf{k}),$$

$$\mathbf{v}_F(\mathbf{k}) \equiv \frac{\partial \xi(\mathbf{k})}{\partial \mathbf{k}}, \quad d\mathbf{k} \equiv \frac{d^2 k}{(2\pi)^2}, \tag{11}$$

and $\xi(\mathbf{k}) = -\xi(\mathbf{k} - \mathbf{Q}_i)$, we arrive at the expression

$$\mathbf{j}(\tau,\mathbf{q}=0) = e\int_{\mathrm{HBZ}} d\mathbf{k} \mathbf{v}_F(\mathbf{k})\Psi^\dagger(\tau,\mathbf{k})\sigma_3 \otimes \hat{I}_2 \Psi(\tau,\mathbf{k})$$

$$= e\int_{\mathrm{HBZ}} d\mathbf{k} \mathbf{v}_F(\mathbf{k})\bar{\Psi}(\tau,\mathbf{k})\gamma_0 \gamma_5 \Psi(\tau,\mathbf{k}), \tag{12}$$

where the integration is over the halved Brillouin zone (HBZ), i.e., over the domain with $k_y > 0$ in Fig. 1. This implies that after the nodal approximation is used, one should include in the integration only two neighboring nodes with $i = 1,2$ as reflected in Eq. (7), because the opposite nodes are already included in the 4×4 formalism.

III. GENERAL REPRESENTATION FOR THE SUPERFLUID DENSITY

The superfluid stiffness (or density divided by the carrier mass m^*) is given by[42]

$$\Lambda_s^{ij}(T,H) \equiv \frac{\rho_s^{ij}}{m^*} = \tau_{ij} - \Lambda_n^{ij}(T,H), \tag{13}$$

where τ_{ij} is the diamagnetic (or stress) tensor and Λ_s is related to the London penetration depth λ_L by the standard expression $\Lambda_s = c^2/(4\pi e^2 \lambda_L^2)$. In Eq. (13) Λ_n is the normal fluid density divided by the carrier mass, calculated within the "bubble approximation" with dressed fermion propagators (i.e., with self-energy Σ due to the scattering on impurities included) but neglecting vertex and Fermi liquid corrections

$$\Lambda_n^{ij} = -T\sum_{n=-\infty}^{\infty} \int_{\mathrm{HBZ}} \frac{d^2 k}{(2\pi)^2} v_{Fi}(\mathbf{k})v_{Fj}(\mathbf{k})$$

$$\times \mathrm{tr}[G(i\omega_n,\mathbf{k})\gamma_0\gamma_5 G(i\omega_n,\mathbf{k})\gamma_0\gamma_5]. \tag{14}$$

As shown in Ref. 42 the vertex corrections can be neglected if the impurity scattering potential is isotropic in $\mathbf{k}$ space. Likewise the Fermi liquid corrections can be taken into account along the lines of Ref. 42. When $\Sigma = 0$ it is more convenient to begin with the sum over Matsubara frequencies in Eq. (14) as done after Eq. (16). In a more generic case $\Sigma \neq 0$, the momentum integration has to be done first. Using the spectral representation for the Green's function $G(i\omega_n,\mathbf{k})$ with the spectral function given by the discontinuity of the fermion Green's function

$$A(\omega,\mathbf{k}) = -\frac{1}{2\pi i}[G_R(\omega+i0,\mathbf{k}) - G_A(\omega-i0,\mathbf{k})], \tag{15}$$

one can easily sum over Matsubara frequencies in Eq. (14) and represent Λ_n in the form[13]

$$\Lambda_n^{ij} = \int_{\text{HBZ}} \frac{d^2k}{(2\pi)^2} \int_{-\infty}^{\infty} d\omega \tanh \frac{\omega}{2T} \frac{v_{Fi} v_{Fj}}{4\pi i}$$
$$\times \text{tr}[G_A(\omega,\mathbf{k})\gamma_0\gamma_5 G_A(\omega,\mathbf{k})\gamma_0\gamma_5$$
$$- G_R(\omega,\mathbf{k})\gamma_0\gamma_5 G_R(\omega,\mathbf{k})\gamma_0\gamma_5] \qquad (16)$$

which is more convenient for the integration over $\mathbf{k}$. In Eqs. (15) and (16) $G_{R,A}(\omega,\mathbf{k}) = G(i\omega_n \rightarrow \omega \pm i0, \mathbf{k})$ are retarded and advanced Green's functions.

Properties of Λ_s for different competing orders. To simplify the formal consideration, in this section we will assume that there is perfect nesting, $\xi(\mathbf{k}) = -\xi(\mathbf{k} - \mathbf{Q}_i)$ and $\Delta(\mathbf{k}) = -\Delta(\mathbf{k} - \mathbf{Q}_i)$ for all values of the momentum $\mathbf{k}$ and not only in the vicinity of nodes. Then for the diamagnetic tensor we obtain

$$\tau_{ij} = T \sum_{n=-\infty}^{\infty} \int_{\text{HBZ}} \frac{d^2k}{(2\pi)^2} \text{tr}[\sigma_3 \otimes \tau_3 G(i\omega_n,\mathbf{k})\gamma_0] \frac{\partial^2 \xi(\mathbf{k})}{\partial k_i \partial k_j}$$
$$= -\int_{\text{HBZ}} \frac{d^2k}{(2\pi)^2} \frac{\partial^2 \xi(\mathbf{k})}{\partial k_i \partial k_j} \frac{2\xi(\mathbf{k})}{E(\mathbf{k})} \tanh \frac{E(\mathbf{k})}{2T}, \qquad (17)$$

where depending on the kind of competing order $E(\mathbf{k}) = \sqrt{\xi^2(\mathbf{k}) + \Delta^2(\mathbf{k}) + m^2 + \{\Delta_{d_{xy}}^2\}}$. Here and in what follows the notation $m^2 + \{\Delta_{d_{xy}}^2\}$ implies that either the competing SDW order with the gap m or the competing id_{xy} order with the gap $\Delta_{d_{xy}}$ is considered.

For the second term of Eq. (13) evaluating the trace in Eq. (14) we obtain

$$\Lambda_n^{ij} = -T \sum_{n=-\infty}^{\infty} \int_{\text{HBZ}} \frac{d^2k}{(2\pi)^2} v_{Fi}(\mathbf{k}) v_{Fj}(\mathbf{k})$$
$$\times \frac{4[-\omega_n^2 - m^2 + \{\Delta_{d_{xy}}^2\} + \xi^2(\mathbf{k}) + \Delta^2(\mathbf{k})]}{[\omega_n^2 + \xi^2(\mathbf{k}) + \Delta^2(\mathbf{k}) + m^2 + \{\Delta_{d_{xy}}^2\}]^2}. \qquad (18)$$

One may notice that there is an important difference between SDW and id_{xy} cases in the sign before m^2 and $\Delta_{d_{xy}}^2$ in the numerator of the Matsubara sum. Evaluating the sum first for $\Delta_{d_{xy}}^2$ we arrive at

$$\Lambda_n^{ij} = \int_{\text{HBZ}} \frac{d^2k}{(2\pi)^2} v_{Fi}(\mathbf{k}) v_{Fj}(\mathbf{k}) \frac{1}{T} \frac{1}{\cosh^2 \frac{E(\mathbf{k})}{2T}},$$

$$E(\mathbf{k}) = \sqrt{\xi^2(\mathbf{k}) + \Delta^2(\mathbf{k}) + \Delta_{d_{xy}}^2}. \qquad (19)$$

Note that the same result holds for *is* order. Making an integration by parts one may check that the superfluid density ρ_s remains finite when $\Delta(\mathbf{k}) = 0$, because the second gap is also superconducting and $\rho_s = 0$ only when both gaps are zero, $\Delta(\mathbf{k}) = \Delta_{d_{xy}} = 0$.

For the competing SDW order

$$\Lambda_n^{ij} = \int_{\text{HBZ}} \frac{d^2k}{(2\pi)^2} v_{Fi}(\mathbf{k}) v_{Fj}(\mathbf{k})$$
$$\times \left[\frac{2m^2}{E^3(\mathbf{k})} \tanh \frac{E(\mathbf{k})}{2T} + \frac{\xi^2(\mathbf{k}) + \Delta^2(\mathbf{k})}{TE^2(\mathbf{k})} \frac{1}{\cosh^2 \frac{E(\mathbf{k})}{2T}} \right].$$
$$(20)$$

It is easy to check that when $\Delta(\mathbf{k}) = 0$, while m remains finite, the superfluid density ρ_s becomes zero. In contrast to the superfluid density, the thermal conductivity is blind with respect to quantum numbers distinguishing the gaps m and $\Delta_{d_{xy}}$ (Δ_s), so that Eq. (1) is valid for all competing SDW, *is* and id_{xy} orders. Using the nodal approximation[42] one can estimate that the depletion of the condensate caused by developing SDW order at $T = 0$

$$\Lambda_n = \frac{v_F}{\pi v_\Delta} |m| \approx N(0) v_F^2 \frac{|m|}{\Delta}, \qquad (21)$$

where $N(0)$ is the density of states (DOS) per spin in the normal state and Δ is the amplitude of the *d*-wave gap.

IV. INFLUENCE OF THE SECONDARY GAP OPENING ON THE IMPURITY SCATTERING RATE

In this section we consider the influence of nonmagnetic impurities on the residual scattering rate in the presence of a competing order. We begin with the Hamiltonian written in Nambu formalism

$$H_{\text{imp}} = \int d\mathbf{k} \int d\mathbf{k}' V_{\mathbf{k},\mathbf{k}'} \Psi^\dagger(\tau,\mathbf{k}) \tau_3 \Psi(\tau,\mathbf{k}'), \qquad (22)$$

describing the interaction $V(\mathbf{r}) = \Sigma_i V \delta(\mathbf{r} - \mathbf{r}_i)$ with $\mathbf{r}_i$ the positions of a random distribution of impurities. Accordingly in the 4×4 formalism

$$H_{\text{imp}} = \int_{\text{HBZ}} d\mathbf{k} \int_{\text{HBZ}} d\mathbf{k}' V_{\mathbf{k},\mathbf{k}'} \Psi^\dagger(\tau,\mathbf{k}) \hat{I}_2 \otimes \tau_3 \Psi(\tau,\mathbf{k}')$$
$$= \int_{\text{HBZ}} d\mathbf{k} \int_{\text{HBZ}} d\mathbf{k}' V_{\mathbf{k},\mathbf{k}'} \bar{\Psi}(\tau,\mathbf{k})(-\gamma_5\gamma_1)\Psi(\tau,\mathbf{k}').$$
$$(23)$$

Therefore the set of equations for the impurity self-energy reads

$$G_0^{-1}(i\omega,\mathbf{k}) = i\omega\gamma_0 - \xi(\mathbf{k})\gamma_1 - \Delta(\mathbf{k})\gamma_2 - m\hat{I}_4 - \{\Delta_{d_{xy}}\gamma_3\}, \qquad (24a)$$

$$G^{-1}(i\tilde{\omega},\mathbf{k}) = G_0^{-1}(i\tilde{\omega},\mathbf{k}) - \Sigma(i\tilde{\omega}), \qquad (24b)$$

$$\Sigma(i\tilde{\omega}) = \Gamma_{\text{imp}}(-\gamma_5\gamma_1)[c + g(i\tilde{\omega})\gamma_5\gamma_1]^{-1}, \qquad (24c)$$

$$g(i\tilde{\omega}) = \frac{1}{\pi N(0)} \int \frac{d^2k}{(2\pi)^2} G(i\tilde{\omega},\mathbf{k}), \qquad (24d)$$

where $c = 1/[\pi N(0)V]$ is the parameter which controls the strength of impurity scattering and $\Gamma_{\text{imp}} = n_{\text{imp}}/[\pi N(0)]$ with

n_{imp} being the concentration of impurities. Since here we used an unexpanded dispersion $\xi(\mathbf{k})$ and d-wave gap $\Delta(\mathbf{k})$, the integration in Eq. (24d) is done over the whole Brillouin zone to count correctly the contribution from all nodes.

Expanding the self-energy Σ and $g(i\widetilde{\omega})$ in the γ matrices

$$\Sigma(i\widetilde{\omega}) = \Sigma^0(i\widetilde{\omega})\gamma_0 + \Sigma^2(i\widetilde{\omega})\gamma_2 + \Sigma^I(i\widetilde{\omega})\hat{I}_4 + \{\Sigma^3(i\widetilde{\omega})\gamma_3\},$$

$$g(i\widetilde{\omega}) = g^0(i\widetilde{\omega})\gamma_0 + g^2(i\widetilde{\omega})\gamma_2 + g^I(i\widetilde{\omega})\hat{I}_4 + \{g^3(i\widetilde{\omega})\gamma_3\},$$

(25)

one can obtain the system of self-consistent equations for

$$i\widetilde{\omega} = i\omega - \Sigma^0(i\widetilde{\omega}),$$

$$\widetilde{\Delta}(\mathbf{k}, i\widetilde{\omega}) = \Delta(\mathbf{k}) + \Sigma^2(i\widetilde{\omega}),$$

$$\widetilde{m}(i\widetilde{\omega}) = m + \Sigma^I(i\widetilde{\omega}),$$

$$\{\widetilde{\Delta}_{d_{xy}}(\mathbf{k}, i\widetilde{\omega}) = \Delta_{d_{xy}}(\mathbf{k}) + \Sigma^3(i\widetilde{\omega})\}.$$

(26)

Assuming particle-hole symmetry for simplicity, the renormalization of ξ is zero. In particular, when the competing orders are absent, the system of equations reduces to T-matrix equations for d-wave superconductor studied, for example, in Ref. 43. Since in this case the averaging over the Fermi surface gives $g^2(i\widetilde{\omega}) = 0$, the only relevant equation left is for $\widetilde{\omega}$ or Σ^0.

Here our goal is to take into account the influence of a competing order on $\widetilde{\omega}$. When a competing order develops, it affects both the equation mentioned above for $\widetilde{\omega}$ (Σ^0) and the new equation for $\Sigma^I(i\widetilde{\omega})$ $\{\Sigma^3(i\widetilde{\omega})\}$. However, because we do have an explicit gap equation for $m\{\Delta_{d_{xy}}\}$ (see, e.g., Ref. 44), in what follows we will assume that the dependence $\widetilde{m}[\widetilde{\omega}(\omega=0)]$ is given phenomenologically and do not consider an equation for $\Sigma^I(i\widetilde{\omega})$ $\{\Sigma^3(i\widetilde{\omega})\}$. This assumption means that we do not distinguish the values $\widetilde{m}[\widetilde{\omega}(\omega=0)]$ and m, so that in what follows we denote the competing gaps as m, $\Delta_{d_{xy}}$, and Δ_s. These gaps already include the effects of impurities and magnetic field and correspond to their phenomenological values extracted from experiment. Although in what follows we do not consider the equation for $\Sigma^I(i\widetilde{\omega})$, it is useful to stress the analogy between the the effect of nonmagnetic impurities on SDW order, and magnetic impurities in conventional s-wave superconductors[45] that lead to the finite density of states inside the gap. Physically this means that the scattering off random impurities prefers to make the system homogeneous by washing out the nonuniform SDW structure.[46]

Two other important assumptions that we make are the following.

(i) We are interested in the value of the zero energy impurity scattering rate $\Gamma = -\text{Im }\Sigma_R^0[\widetilde{\omega}(\omega\to 0)]$ and do not consider the effects related to the energy dependence of the self-energy $\Sigma^0[\widetilde{\omega}(\omega)]$ studied in Ref. 47. This assumption is justified by the fact that LSCO compound is intrinsically more dirty system than YBCO considered in Ref. 47.

(ii) Although we assume that the values of the competing gaps are field dependent, e.g., $m = m(H)$ we do not include the influence of the Doppler shift on Γ.[47,48]

A. T-matrix equation for competing SDW order

We begin with the equation for Σ^0 for competing SDW order

$$\Sigma_R^0(\widetilde{\omega}) = \frac{\Gamma_{\text{imp}}}{4}\left[\frac{1}{c - g^0(\widetilde{\omega}) - g^I(\widetilde{\omega})} + \frac{1}{-c - g^0(\widetilde{\omega}) + g^I(\widetilde{\omega})} + \frac{1}{c - g^0(\widetilde{\omega}) + g^I(\widetilde{\omega})} - \frac{1}{c + g^0(\widetilde{\omega}) + g^I(\widetilde{\omega})}\right], \quad (27)$$

which is obtained from Eq. (24c) with $g(i\widetilde{\omega} \to \widetilde{\omega})$ given by Eq. (24d) using the decompositions (26).

Since we are interested in the value $\Sigma^0(\widetilde{\omega})$ with $\widetilde{\omega} = \omega - \Sigma^0(\widetilde{\omega})$ at $\omega = 0$, we need only the functions $g^0(\widetilde{\omega})$ and $g^I(\widetilde{\omega})$ calculated for $\widetilde{\omega} = i\Gamma$. Using the nodal approximation we obtain

$$g^0(i\Gamma) = \frac{1}{\pi N(0)}\frac{1}{\pi v_F v_\Delta}\ln\frac{m^2 + \Gamma^2}{p_0^2}i\Gamma, \quad (28a)$$

$$g^I(i\Gamma) = \frac{1}{\pi N(0)}\frac{1}{\pi v_F v_\Delta}\ln\frac{m^2 + \Gamma^2}{p_0^2}m, \quad (28b)$$

where the ultraviolet cutoff p_0 is introduced. Note that for $p_0 \sim 4\Delta$ and $(v_F v_\Delta)^{-1} \sim \pi N(0)/\Delta$, where Δ is the amplitude of the d-wave gap $\Delta(\mathbf{k}) = \Delta/2(\cos k_x a - \cos k_y a)$, the function $g(\omega)$ calculated using the nodal approximation agrees with the expressions given in Refs. 43, 47, and 48. In Eq. (27) the Born limit corresponds to $V \to 0$ ($c \to \infty$), while the unitary limit corresponds to $V \to \infty$, i.e., $c \to 0$.

1. Born limit

In the Born limit Eq. (27) reduces to

$$\Sigma_R^0(\widetilde{\omega}) = \Gamma_{\text{imp}}c^{-2}g^0(\widetilde{\omega}). \quad (29)$$

Due to the fact that only g^0 enters Eq. (29), the Born limit appears to be the same both for SDW and superconducting orders, because g^0 depends either on m^2 or on $\Delta_{d_{xy}}^2$.

Substituting Eq. (28a) in Eq. (29) and solving it with respect to Γ we obtain[49]

$$\Gamma^2 = p_0^2 \exp\left[-\frac{c^2\pi^2 v_F v_\Delta N(0)}{\Gamma_{\text{imp}}}\right] - m^2. \quad (30)$$

For a given impurity concentration this solution is nonzero only if $m < m_{\text{cr}}$, where $m_{\text{cr}} = p_0 \exp[-c^2\pi^2 v_F v_\Delta N(0)/(2\Gamma_{\text{imp}})]$. Because the Born limit is considered, the value of m_{cr} is exponentially small. Nevertheless in the unitary limit there is a possibility to have both finite Γ and rather large values of m.

2. Unitary limit

LSCO compound is intrinsically a dirtier system than other cuprates, so that the unitary limit is more relevant.
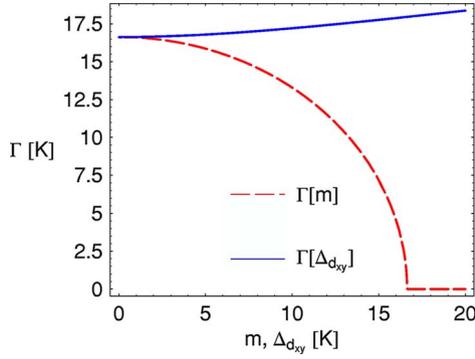
FIG. 2. (Color online) The dependence of the impurity scattering rate Γ on the value of the gaps m and $\Delta_{d_{xy}}$ of the developing SDW and id_{xy} orders, respectively. For $m > m_{ct}$ the value $\Gamma(m) = 0$.

Moreover, thin LSCO films studied in Ref. 20 seems to have particularly large values of $\Gamma \sim 6–50$ K which also indicates the relevance of the unitary limit. When $c \to 0$ Eq. (27) reduces to

$$\Sigma_R^0(\tilde{\omega}) = \Gamma_{\text{imp}} \frac{g^0(\tilde{\omega})}{[g^I(\tilde{\omega})]^2 - [g^0(\tilde{\omega})]^2}. \qquad (31)$$

Now because g^I still enters Eq. (31), the resulting transcendental equation for Γ is

$$\Gamma^2 = \Gamma_{\text{imp}} \pi^2 N(0) v_F v_\Delta \left[\ln \frac{p_0^2}{\Gamma^2 + m^2} \right]^{-1} - m^2, \qquad (32)$$

which differs from that of Ref. 49 by the last term m^2. Due to its presence the dependence $\Gamma(m)$ is rather strong (see Fig. 2) and as in the Born limit there is a critical value m_{cr}^{unit} such that $\Gamma(m) = 0$ for $m > m_{cr}^{\text{unit}}$. As we will argue below the observed deviations of the dependence $\rho_s(H)$ [or $\Lambda_s(H)$] from $\sim \sqrt{H}$ in high fields can be caused by the fact that the field dependence of $m(H)$ affects the behavior of $\rho_s(H)$ via the dependence $\Gamma[m(H)]$. Finally we note that the case of competing is order is also described by Eq. (32).

B. T-matrix equation for competing id_{xy} order

Similarly to Eq. (27) for the competing id_{xy} order we obtain

$$\Sigma_R^0(\tilde{\omega}) = \Gamma_{\text{imp}} \frac{g^0(\tilde{\omega})}{c^2 - [g^0(\tilde{\omega})]^2 + [g^3(\tilde{\omega})]^2}, \qquad (33)$$

where $g^0(i\Gamma)$ is given by Eq. (28a), while for d_{xy} order $g^3 = 0$, analogously to the function g^2 for $d_{x^2-y^2}$ case.

1. Born limit

As was already mentioned in Sec. IV A 1, in the Born limit there is no difference between the consideration of competing SDW and id_{xy} order, so that one may simply replace the gap m by $\Delta_{d_{xy}}$ in the corresponding equations. Moreover, this consideration is also valid for a competing is order. Competing is order with d-wave superconductivity was considered in Ref. 50, where beside addition to the

T-matrix equations for the impurity scattering rate, the optical conductivity order was studied.

2. Unitary limit

Since $g^3 = 0$, instead of Eq. (32) we arrive at the equation[49]

$$\Gamma^2 = \Gamma_{\text{imp}} \pi^2 N(0) v_F v_\Delta \left[\ln \frac{p_0^2}{\Gamma^2 + \Delta_{d_{xy}}^2} \right]^{-1}. \qquad (34)$$

Its solution $\Gamma(\Delta_{d_{xy}})$ is shown in Fig. 2. It demonstrates that a competing id_{xy} does not significantly perturb the value of Γ with respect to the $\Delta_{d_{xy}} = 0$ case, when it reduces to the form[51]

$$\Gamma^2 = \frac{\pi}{2} n_{\text{imp}} v_F v_\Delta \left[\ln \frac{p_0}{\Gamma} \right]^{-1}. \qquad (35)$$

V. SUPERFLUID DENSITY FOR $H = 0$

In Sec. III we discussed in a more formal way how the different competing orders affect the superfluid density. Here instead we concentrate on the simple analytical expressions that demonstrate the dependence of Λ_n on the impurity scattering rate Γ and the competing gaps m and $\Delta_{d_{xy}}$. In the nodal approximation the representation (16) acquires a form convenient for analytical calculations

$$\Lambda_n = -\frac{v_F}{2\pi v_\Delta} J, \qquad (36)$$

where

$$J = -\int_0^\infty d\omega \tanh \frac{\omega}{2T} \tilde{I}(\omega) \qquad (37)$$

with

$$\tilde{I}(\omega) = \frac{1}{2\pi i} \int_0^{p_0} p \, dp [I_A(\omega, p) - I_R(\omega, p)] \qquad (38)$$

and

$$I_{R,A}(\omega, p) \equiv \text{tr}[G_{R,A}(\omega, p) \gamma_0 \gamma_5 G_{R,A}(\omega, p) \gamma_0 \gamma_5]. \qquad (39)$$

Here $p = \sqrt{v_F^2 k_1^2 + v_\Delta^2 k_2^2}$ is the dispersion law of the quasiparticles in the nodal approximation. J in Eq. (36) is twice as big as in Ref. 13 because a 4×4 formalism is used. Now we consider the cases of competing SDW, id_{xy} and is orders.

A. Competing SDW order

Substituting the Green's function (9) with the self-energy $\text{Im} \Sigma_{R,A}(\omega = 0) = \mp \Gamma$ in Eq. (43) one obtains

$$I_{R,A}(\omega, p) = \frac{4[(\omega \pm i\Gamma)^2 + p^2 - m^2]}{[(\omega \pm i\Gamma)^2 - p^2 - m^2]^2}. \qquad (40)$$

Then integration over the energy p we arrive at

$$\tilde{I}(\omega) = -\frac{2}{\pi}\left[\arctan\frac{p_0^2 + \Gamma^2 + m^2 - \omega^2}{2\Gamma\omega} - \arctan\frac{\Gamma^2 + m^2 - \omega^2}{2\Gamma\omega} \right.$$

$$-\frac{p_0^2\Gamma}{\sqrt{p_0^2+m^2}}\frac{1}{(\omega-\sqrt{p_0^2+m^2})^2+\Gamma^2}$$

$$\left. +\frac{p_0^2\Gamma}{\sqrt{p_0^2+m^2}}\frac{1}{(\omega+\sqrt{p_0^2+m^2})^2+\Gamma^2} \right]. \tag{41}$$

Finally integrating over ω for $\Gamma \gg T$ (also for $p_0 \gg m, \Gamma$) we obtain

$$\Lambda_n = \frac{v_F}{\pi^2 v_\Delta}\left[\Gamma\ln\frac{p_0^2}{m^2+\Gamma^2} + m\arctan\frac{2m\Gamma}{\Gamma^2-m^2} \right.$$

$$\left. + \pi|m|\theta(m^2-\Gamma^2) + \frac{\pi^2}{3}\frac{T^2\Gamma}{\Gamma^2+m^2} \right], \tag{42}$$

where θ is the step function. The first term of Eq. (42) can be interpreted as the DOS contribution to the depletion of the condensate even at $T=0$, because the DOS (per spin) in a dirty d-wave superconductor with a finite gap m reads[28]

$$N_m(0) = \frac{2}{\pi^2 v_F v_\Delta}\Gamma\ln\frac{p_0}{\sqrt{\Gamma^2+m^2}}. \tag{43}$$

The second and third terms of Eq. (42) describe the depletion of the condensate because of the development of SDW order and for $\Gamma \to 0$ they reduce to Eq. (21) discussed above. Finally the last term of Eq. (42) $\sim T^2$ shows that the characteristic for d-wave superconductors, i.e., a linear T dependence for Λ_n changes in the presence of impurities and becomes $\sim T^2$. This behavior is indeed observed in thin films[20] over a wide range of the temperatures. Based on Eq. (44) below, in some films it was estimated that Γ can be as big as ≈ 46.6 K.[20] When $m \to 0$, Eq. (42) reduces to the known[13,52,53] expression

$$\Lambda_n(T) = \frac{v_F}{\pi v_\Delta}\left[\frac{2\Gamma}{\pi}\ln\frac{p_0}{\Gamma} + \frac{\pi}{3}\frac{T^2}{\Gamma} - O\left(\frac{T^4}{\Gamma^3}\right) \right]. \tag{44}$$

Again the first term of Eq. (44) is proportional to the DOS of a dirty d-wave superconductor.[42]

It is also instructive to consider the limit $\Gamma \to 0$ when the T dependence of Λ_n is expected to change from quadratic to linear. To extract this limit one should extract the singular part of $\tilde{I}(\omega)$ in the limit $\Gamma \to 0$, viz.

$$\tilde{I}(\omega) = -\,\text{sgn}\,\omega[\text{sgn}(p_0^2-\omega^2) - \text{sgn}(m^2-\omega^2)]$$

$$+ 2p_0[\delta(\omega-p_0) - \delta(\omega+p_0)], \tag{45}$$

where we again took $p_0 \gg m, \Gamma$. Then

$$J(m) = -4T\ln\left(2\cosh\frac{m}{2T}\right). \tag{46}$$

Obviously for $m=0$ we recover the well-known expression[42]

$$\Lambda_n = \frac{2\ln 2}{\pi}\frac{v_F}{v_\Delta}T. \tag{47}$$

For $|m| \gg T$ we obtain

$$\Lambda_n = \frac{v_F}{\pi v_\Delta}[|m| + 2T\exp(-|m|/T)], \tag{48}$$

where the first term coincides with Eq. (21). For $|m| \ll T$, we arrive at the expression

$$\Lambda_n = \frac{2\ln 2}{\pi}\frac{v_F}{v_\Delta}T\left[1 + \frac{1}{8\ln 2}\frac{m^2}{T^2} \right]. \tag{49}$$

Here we refer for comparison to simple expressions for Λ_n when d-wave superconducting states coexists with the orbital antiferromagnetic (d-density-wave) state.[54]

B. Competing id_{xy} and is orders

The difference between competing SDW and superconducting orders can be traced back to the opposite sign before the m^2 term in the numerator of Eq. (40) and the corresponding sign before $\Delta_{d_{xy}}^2$ in the formula

$$I_{R,A}(\omega,p) = \frac{4[(\omega\pm i\Gamma)^2 + p^2 + \Delta_{d_{xy}}^2]}{[(\omega\pm i\Gamma)^2 - p^2 - \Delta_{d_{xy}}^2]^2}, \tag{50}$$

which is related to the different γ matrix before $\Delta_{d_{xy}}$ in Eq. (10). Note that for competing is order, Eq. (50) turns out to be the same, so that these order differ by the influence of the superconducting gaps $\Delta_{d_{xy}}$ and Δ_s on the impurity scattering rate Γ (see Sec. IV A 2) in the unitary limit. Integrating over p one obtains

$$\tilde{I}(\omega) = -\frac{2}{\pi}\left[\arctan\frac{p_0^2 + \Gamma^2 + \Delta_{d_{xy}}^2 - \omega^2}{2\Gamma\omega} \right.$$

$$-\arctan\frac{\Gamma^2 + \Delta_{d_{xy}}^2 - \omega^2}{2\Gamma\omega} + \frac{\Gamma\Delta_{d_{xy}}}{(\omega-\Delta_{d_{xy}})^2+\Gamma^2}$$

$$-\frac{\Gamma\Delta_{d_{xy}}}{(\omega+\Delta_{d_{xy}})^2+\Gamma^2} - \frac{\Gamma\sqrt{p_0^2+\Delta_{d_{xy}}^2}}{(\omega-\sqrt{p_0^2+\Delta_{d_{xy}}^2})^2+\Gamma^2}$$

$$\left. +\frac{\Gamma\sqrt{p_0^2+\Delta_{d_{xy}}^2}}{(\omega+\sqrt{p_0^2+\Delta_{d_{xy}}^2})^2+\Gamma^2} \right]. \tag{51}$$

Finally integrating over ω for $\Gamma \gg T$ (also for $p_0 \gg \Delta_{d_{xy}}, \Gamma$) we obtain

$$\Lambda_n = \frac{v_F}{\pi^2 v_\Delta}\left[\Gamma\ln\frac{p_0^2}{\Delta_{d_{xy}}^2+\Gamma^2} \right.$$

$$\left. + \frac{\pi^2}{3}\left(\frac{\Gamma}{\Gamma^2+\Delta_{d_{xy}}^2} + \frac{2\Delta_{d_{xy}}^2\Gamma}{(\Gamma^2+\Delta_{d_{xy}}^2)^2} \right)T^2 \right]. \tag{52}$$

Similarly to Eq. (45) in the limit $\Gamma \to 0$ from Eq. (51) we obtain

$$\tilde{I}(\omega) = - \operatorname{sgn} \omega [\operatorname{sgn}(p_0^2 - \omega^2) - \operatorname{sgn}(\Delta_{d_{xy}}^2 - \omega^2)]$$
$$+ 2p_0[\delta(\omega - p_0) - \delta(\omega + p_0)]$$
$$- 2\Delta_{d_{xy}}[\delta(\omega - \Delta_{d_{xy}}) - \delta(\omega + \Delta_{d_{xy}})], \qquad (53)$$

where we again took $p_0 \gg \Delta_{d_{xy}}, \Gamma$. Then

$$J(\Delta_{d_{xy}}) = 2\Delta_{d_{xy}} \tanh \frac{\Delta_{d_{xy}}}{2T} - 4T \ln\left(2 \cosh \frac{\Delta_{d_{xy}}}{2T}\right), \quad (54)$$

which differs from Eq. (46) by the term with tanh. For $\Delta_{d_{xy}} = 0$ we again recover Eq. (47). For $\Delta_{d_{xy}} \gg T$ the dependence $\Lambda_n(T)$ becomes thermally activated due to the secondary gap

$$\Lambda_n = \frac{2}{\pi} \frac{v_F}{v_\Delta} \Delta_{d_{xy}} \exp(-\Delta_{d_{xy}}/T), \qquad (55)$$

while for $\Delta_{d_{xy}} \ll T$ the leading term of $\Lambda_n(T)$ coincides with Eq. (47)

$$\Lambda_n = \frac{2 \ln 2}{\pi} \frac{v_F}{v_\Delta} T \left[1 - \frac{1}{8 \ln 2} \frac{\Delta_{d_{xy}}^2}{T^2}\right]. \qquad (56)$$

Comparing the last equation with Eq. (49) one may notice that they differ by the sign before the T^2 term.

VI. SUPERFLUID DENSITY IN THE VORTEX STATE

The presence of circulating supercurrent around vortices in the mixed state can be taken into account in the semiclassical approach by introducing the Doppler shift in quasiparticle energies, $\omega \to \omega - \mathbf{v}_s(\mathbf{r})\mathbf{k}$.[15] Here $\mathbf{v}_s(\mathbf{r})$ is the superfluid velocity at a position $\mathbf{r}$ which depends on the form of the vortex distribution and $\mathbf{k}$ is the quasiparticle momentum which can be approximated by its value at the node.[51] This distribution is described by the function

$$\mathcal{P}(\epsilon) = \frac{1}{A} \int d^2r \delta[\epsilon - \mathbf{v}_s(\mathbf{r})\mathbf{k}], \qquad (57)$$

where the integration is over the unit vortex cell with the area $A = \pi R^2$. Several choices for $\mathcal{P}(\epsilon)$ were discussed in Ref. 51. Among them are the distribution for the vortex liquid[51]

$$\mathcal{P}(\epsilon) = \frac{E_H^2}{2(\epsilon^2 + E_H^2)^{3/2}} \qquad (58)$$

which is the most convenient for analytic calculations and[51,55]

$$\mathcal{P}(\epsilon) = \frac{1}{\sqrt{\pi}E_H} \exp\left(-\frac{\epsilon^2}{E_H^2}\right) \qquad (59)$$

for the completely disordered vortex state. The characteristic energy scale E_H in Eqs. (58) and (59) is associated with the Doppler shift energy in the vortex state

$$E_H(H) = a \frac{\hbar v_F}{2R} = a \frac{\hbar v_F}{2} \sqrt{\frac{\pi H}{\Phi_0}}, \qquad (60)$$

where a is a geometrical factor of order unity and H is the magnetic field applied perpendicularly to the ab plane. In the

second equality we used the convention of Ref. 51 (see also Refs. 33 and 56 therein) that for $a = 1$ there is one flux quantum $\Phi_0 = hc/2e$ per unit cell of the vortex lattice approximated by a circle of radius $R = (\Phi_0/\pi H)^{1/2}$. The final results depend somewhat on the choice of the distribution function and on the value of a, however, the qualitative results are not sensitive to this choice. In what follows we take the value $v_F = 2.5 \times 10^7$ cm/s[24] which corresponds to $E_H[K] = 38$ K $T^{-1/2}\sqrt{H[T]}$ as used in Ref. 28.

Now the Doppler shift effect can be incorporated in the Green's function formalism that already includes the scattering on impurities by averaging over the distribution[51,57] $\mathcal{P}(\epsilon)$

$$\Lambda_n^{ij}(H) = \int_\infty^\infty d\epsilon \mathcal{P}(\epsilon) \int_{HBZ} \frac{d^2k}{(2\pi)^2} \int_{-\infty}^\infty d\omega \tanh \frac{\omega}{2T} v_{Fi} v_{Fj} \frac{1}{4\pi i}$$
$$\times \operatorname{tr}[G_A(\omega - \epsilon, \mathbf{k}) \gamma_0 \gamma_5 G_A(\omega - \epsilon, \mathbf{k}) \gamma_0 \gamma_5$$
$$- G_R(\omega - \epsilon, \mathbf{k}) \gamma_0 \gamma_5 G_R(\omega - \epsilon, \mathbf{k}) \gamma_0 \gamma_5]. \qquad (61)$$

We note that in 2×2 Nambu formalism the replacement $\omega \to \omega - \epsilon$ for taking into account the Doppler shift in the argument of the Green's function is exact. In 4×4 formalism the corresponding Green's function describes two opposite nodes with the reversed sign of the Doppler shift. Nevertheless one can check that in the latter case the simple prescription $\omega \to \omega - \epsilon$ is approximately valid if we neglect the terms $\sim \epsilon m$. In the nodal approximation similarly to Eq. (36) one may write

$$\Lambda_n(H) = -\frac{v_F}{2\pi v_\Delta} \int_\infty^\infty d\epsilon \mathcal{P}(\epsilon) J[m(H); \epsilon], \qquad (62)$$

where the function $J[\epsilon; m(H)]$ is given below. However, the field dependence of $\Lambda_n(H)$ is not yet completely specified, because one should also provide a field dependence for the gap $m(H) \{\Delta_{d_{xy}}(H)\}$. We will come to this question later in Sec. VI B.

A. Results for the field dependence of the superfluid density for $m, \Delta_{d_{xy}}, \Gamma = $ const

For $T = 0$ and $p_0 \gg m, \Delta_{d_{xy}}, \Gamma$ in SDW case one obtains

$$J(m, T = 0; \epsilon) = -\frac{2}{\pi}\left[\epsilon \arctan \frac{\epsilon^2 - m^2 - \Gamma^2}{2\epsilon\Gamma} + \frac{\pi}{2}|\epsilon|\right.$$
$$+ \Gamma \ln \frac{p_0^2}{\sqrt{[\Gamma^2 + (m-\epsilon)^2][\Gamma^2 + (m+\epsilon)^2]}}$$
$$\left. + m \arctan \frac{2m\Gamma}{\Gamma^2 + \epsilon^2 - m^2} + \pi|m|\theta(m^2 - \Gamma^2 - \epsilon^2)\right]$$
$$\qquad (63)$$

and in the id_{xy} case

$$J(\Delta_{d_{xy}}, T=0; \epsilon) = -\frac{2}{\pi}\left[\epsilon \arctan \frac{\epsilon^2 - \Delta_{d_{xy}}^2 - \Gamma^2}{2\epsilon\Gamma} + \frac{\pi}{2}|\epsilon| \right.$$
$$\left. + \Gamma \ln \frac{p_0^2}{\sqrt{[\Gamma^2 + (\Delta_{d_{xy}} - \epsilon)^2][\Gamma^2 + (\Delta_{d_{xy}} + \epsilon)^2]}} \right].$$
$$(64)$$

Before doing the numerical calculation it is useful to consider analytically the limit $\Gamma \to 0$. One may notice that in this limit the only difference between the SDW and id_{xy} cases is the last two terms $\sim m$ in Eq. (63). Thus we consider firstly the id_{xy} case and then discuss the role of these terms. We obtain from Eq. (64) that

$$J(\Delta_{d_{xy}}, T=0; \epsilon) = -2|\epsilon|\theta(\epsilon^2 - \Delta_{d_{xy}}^2). \quad (65)$$

Substituting Eq. (64) in Eq. (62) and using the vortex liquid distribution (58) we arrive at

$$\Lambda_n(H) = \frac{v_F}{\pi v_\Delta} \frac{E_H^2}{\sqrt{E_H^2 + \Delta_{d_{xy}}^2}}. \quad (66)$$

This means that in spite of the presence of a gap for nodal quasiparticles, the contribution to Λ_n still has the behavior $\sim \sqrt{H}$ if $\Delta_{d_{xy}}(H) \lesssim \sqrt{H}$. The origin of the gapless behavior is the Doppler shift of the quasiparticle energy $E - \mathbf{k} \cdot \mathbf{v}_s(\mathbf{r})$, which is position dependent. There are regions where the shift is larger than the minimal gap $\Delta_{d_{xy}}$ in the spectrum, thus leading to the finite DOS and Λ_n. This point about the DOS was emphasized in Refs. 58 and 59, where it is stressed that regardless of the power with which $\Delta_{d_{xy}} \sim H^\alpha$ opens up in the field, as long as $\alpha \geq 1/2$, the leading term in DOS will always be $\sqrt{H}$ at small fields. Thus in the clean system $\Lambda_n(H)$ would remain $\sim \sqrt{H}$ even in the presence of a nonzero gap $\Delta_{d_{xy}} \propto H^\alpha$ with $\alpha \geq 1/2$. As we already mentioned in the SDW case the last two terms of Eq. (63) contribute in Λ_n. However, if the SDW gap $m(H) \propto \sqrt{H}$, the $\sqrt{H}$ behavior of $\Lambda_n(H)$ would persist, because the contribution of the last two terms of Eq. (63) is $v_F/(\pi v_\Delta)m^2(H)/\sqrt{E_H^2 + m^2(H)}$.

In Figs. 3 and 4 we plot the dependence $\Lambda_s(H)$ for constant field independent gaps m, $\Delta_{d_{xy}}$ for the clean case ($\Gamma = 0$) and for $\Gamma = 16$ K, respectively. This dependence is obtained numerically from Eq. (62) using the distribution Eq. (59). We use $v_F/v_\Delta = 30$ and a rough estimate of the diamagnetic term $\tau = 1500$ K from the Uemura plot. In the clean limit $\Gamma = 0$ at zero temperature, the opening of the gap leaves the condensate unaltered if it is of superconducting character, but depletes it for the spin density wave case. The application of an external magnetic field H oriented perpendicular to the cooper oxide planes creates quasiparticles and decreases the condensate density as compared with its zero field value. Comparing with the pure $d_{x^2-y^2}$ case, the reduction in condensate for a given value of H is less for an additional id_{xy} (is) gap and even less for the SDW case. However, because in this last case the $H=0$ value is already depleted as compared with the pure $d_{x^2-y^2}$ case, the condensate density remains lower for all values of H considered. Looking at Fig. 3
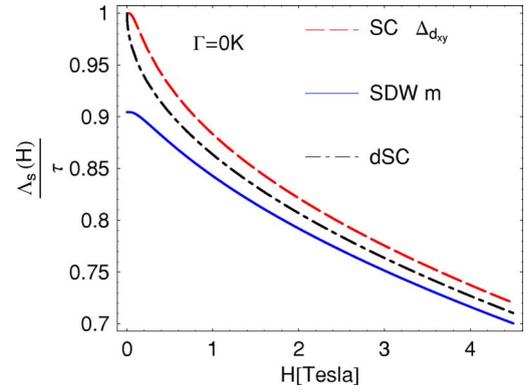
FIG. 3. (Color online) The dependence $\Lambda_s(H)/\tau$ at $T=0$ for d-wave superconductor with additional SDW, id_{xy} gaps $\Delta_{d_{xy}} = m = 15$ K and also without any competing order in the clean, $\Gamma = 0$ K limit.

one may develop the impression that the cases of competing id_{xy} and SDW orders are easily distinguishable experimentally. However, the situation is more complicated because in practice one considers the dependence $\Lambda_s(H) - \Lambda_s(H=0)$ which can hardly distinguish different competing orders. Moreover, in all cases the introduction of impurity (residual) scattering reduces the condensate density and makes the effect of H on it more similar (Fig. 4).

B. Ansatz for gap $m(H)$

In general the competing gap and its doping dependence have to be obtained by solving a self-consistent system of equations for the main d-wave and subdominant gaps (see, e.g., Refs. 8 and 44). However, since the origin of the secondary gap is unknown, here we follow the phenomenological approach of Ref. 28, where it was assumed that

$$m(H,x) = (1 - x/0.16)^{1/2}\theta(0.16 - x)(m_0 + bE_H), \quad (67)$$

where x is the doping, m_0 and b are free parameters.

It was demonstrated in Ref. 28 that the experimental data in Refs. 25 and 26 can be qualitatively understood using the
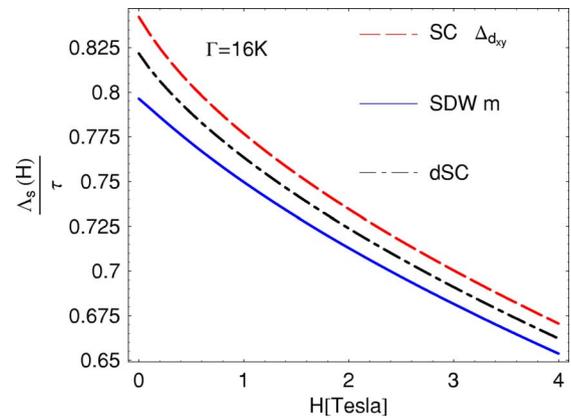
FIG. 4. (Color online) The dependence $\Lambda_s(H)/\tau$ at $T=0$ for d-wave superconductor with additional SDW, id_{xy} gaps $\Delta_{d_{xy}} = m = 15$ K and also without any competing order. The constant $\Gamma = 16$ K is taken.
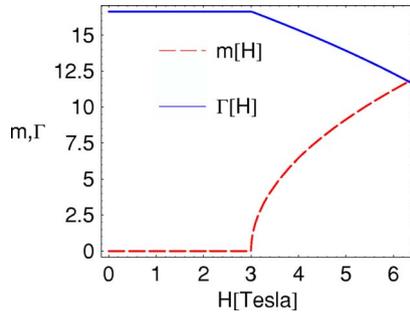
FIG. 5. (Color online) The model dependence of the gap m on the applied field H and the resulting dependence of the impurity scattering rate Γ on m.

gap (67) which is generated below a critical doping x_{cr} =0.16 and increases with magnetic field as $\sqrt{H}$.[34] As mentioned in the Introduction, the present work is partly motivated by the experiments[31,32] made on an x=0.144 LSCO sample, where the SDW gap develops above a critical field $H_0 \approx 3T$. Since here we do not consider the doping dependence of the gap but instead are more interested in the effects related to the critical field, we assume that

$$m(H) = bE_H(H - H_0).\qquad(68)$$

Moreover, while in Ref. 28 a rather large value of b=2.2 was used, in the present paper we consider the case of small values of b=0.17, so that $m(H) \ll E_H$. The dependence $m(H)$ is shown in Fig. 5, where we also plot the dependence $\Gamma(H)$ obtained by solving Eq. (32) for $\Gamma(m)$ with p_0=250 K and $\pi^2 \Gamma_{imp} N(0) v_F v_\Delta$=1500 K². As we already mentioned, there is also a direct influence of the Doppler shift and Andreev scattering on Γ.[47,48] As shown in Ref. 48, in the unitary limit the change of Γ due to the Doppler shift is not significant. Nevertheless these effects and particularly the energy dependence of $\Gamma(H, \omega)$ (Ref. 47) will become important when the value of $m \lesssim m_{cr}^{unit}$ and $\Gamma(\omega=0)$ approaches zero. Finally we note the fact that the explanation of experimental data requires a doping dependent gap, which supports its SDW character. The generation of a id_{xy} gap by a magnetic field can presumably occur at any doping.

C. Results for the field dependence of the superfluid density for field dependent gaps and Γ

Substituting into Eq. (63) $m(H)$ and $\Gamma(H)$ shown in Fig. 5 for the SDW case and into Eq. (65) $\Delta_{d_{xy}}(H)=m(H)$ and Γ =const=$\Gamma[\Delta_{d_{xy}}(H=0)]$ for the id_{xy} case and using Eq. (62) after numerical integration over ϵ with $\mathcal{P}(\epsilon)$ given by Eq. (59) we obtain the results shown in Fig. 6. We use v_F/v_Δ =30 and τ as above. Above H=3T the difference between $\rho_s(H)$ without competing order and $\rho_s(H)$ for the id_{xy} order is hardly noticeable, because we used a small value of b =0.17. Nevertheless, for the SDW order $\rho_s(H)$ in high-fields deviates from a $\sqrt{H}$ behavior quite significantly. This effect is caused by the decrease of $\Gamma(H)$ seen in Fig. 5. We stress that a similar behavior of $\Lambda_s(H) - \Lambda_s(H=0)$ with a crossover from $\sqrt{H}$ behavior in low fields to $\ln H$ dependence in high fields
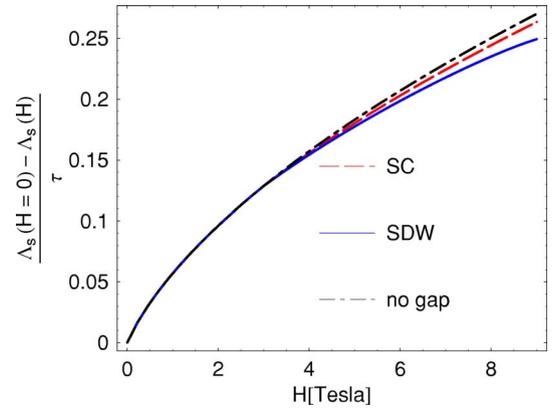
FIG. 6. (Color online) The dependence $[\Lambda_s(0) - \Lambda_s(H)]/\tau$ at T =0 for d-wave superconductor with additional SDW, id_{xy} gaps [see Eq. (68) and Fig. 5] and also without any competing order.

is observed in one of the samples with the lowest T_c studied in Ref. 20.

In contrast to Ref. 28, where the decrease of the thermal conductivity $\kappa(H)$ is directly associated with the opening of the gap and, accordingly, requires rather large values of b, the results presented here for $\rho_s(H)$ are related to the indirect influence of the development of the gap on $\Gamma[m(H)]$ which in turn leads to the deviations of the dependence $\rho_n(H)$ from a simple $\sim \sqrt{H}$ law.

VII. CONCLUSIONS

We have addressed the problem of the superfluid density of a d-wave superconductor with competing order. Both the case of a spin density wave (SDW) and a second (minority) superconducting order with id_{xy} symmetry are treated and compared. The nodal approximation is introduced to treat the main d-wave gap and so the formulation is restricted to low temperatures. The resulting action which corresponds to QED$_{2+1}$ involves a reducible 4×4 representation of Dirac matrices with competing order equivalent to the different types of Dirac masses. Residual impurity scattering is accounted for within a T-matrix formalism which includes as special cases the Born and unitary limit.

SDW and a second superconducting order are, in principle, very different. For example, in the limit when the main d-wave order is set zero, the superfluid density vanishes for the SDW case, but remains finite for the id_{xy} (or is) superconducting case. Also it is found that the SDW order reduces the superfluid density at T=0 [see Eq. (21)] because it competes for the Fermi surface with the d-wave order, but for id_{xy} order there is no such effect. At low temperatures $T \ll m, \Delta_{d_{xy}}$ the superfluid density acquires an exponentially activated form [cf. Eqs. (48) and (55)]. In the opposite limit $T \ll m, \Delta_{d_{xy}} \ll \Delta$, where Δ is the amplitude of the main d-wave gap, the well-known linear in T law (47) is modified [see Eqs. (49) and (56)] and an additional $1/T$ dependence appears with a coefficient proportional to the square of the secondary gap value. The sign of the correction is opposite in the two cases. So far we described results only for the pure

limit. Analytic results are also obtained for the case when impurity scattering is in the limit when the zero frequency value of the impurity self-energy $\Gamma \gg T$. The expressions properly reduce to the known results when the secondary gap is set to zero. In the both cases impurities modify the classic linear in T dependence to a T^2 dependence as is also the case in d-wave superconductor without competing order. The coefficient of T^2 term [see Eqs. (42) and (52)], however, is modified by the presence of the secondary gap and this modification is different for SDW and id_{xy} superconducting order. This is also the case for the change of the zero temperature limit of the superfluid density due to impurity scattering.

We have also considered the influence of the opening of a secondary gap on the magnetic field dependence of the superfluid density for **H** oriented perpendicular to the CuO_2 plane. We have found that the presence of competing orders causes deviations of the field dependence of the superfluid density $\rho_s(H)$ from a simple $\sqrt{H}$ law which is associated with the Doppler shift of the quasiparticle energy. A new and rather important conclusion is that this effect is caused not only by the developing competing order, but also by the influence of this order on the residual impurity scattering rate

at zero frequency. On the experimental side the superfluid density is a directly measurable quantity,[7,19,20] so that the effects discussed here may be well within experimental access. Another advantage of the superfluid density is that it is not blind with regard to the different kinds of competing orders as is thermal transport. However, on a theoretical side, in contrast to the thermal transport, charge transport is renormalized by the vertex and Fermi liquid corrections[42] which were not considered in the present paper. Accordingly the values of v_F, v_Δ, Γ, and m extracted from the measurements of the superfluid density may be in disagreement with the values for the same parameters extracted, for example, from the measurements of thermal conductivity.

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