Magnon excitation by spin-polarized direct currents in magnetic nanostructures

S. M. Rezende, F. M. de Aguiar, and A. Azevedo
Departamento de Física, Universidade Federal de Pernambuco, Recife, PE 50670-901, Brazil
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The dynamics of the magnetization in a thin ferromagnetic film traversed by a spin-polarized direct current is studied. In such a system, spin waves (magnons) may be critically driven out of equilibrium by an effective spin-injection field that is proportional to the current density. A direct comparison between the predicted critical current and previous experimental results sheds light on the nature of the excited mode. Beyond the threshold, it is assumed that the spin waves are coupled through nonlinear interactions arising from dipolar and surface anisotropy energies. It is shown that the magnon-magnon interactions play two major roles in the dynamics: (i) They govern and put a limit to the growth in the population of the unstable mode from the thermal level, and (ii) directly contribute to the renormalization of the magnon energy, which manifests itself through a shift in the precession frequency of the magnetic moments with varying current intensity. Numerical results are presented in remarkable quantitative agreement with recent experiments in nanometric magnetic multilayers, where microwave oscillations generated by direct currents have been observed in the postthreshold regime.

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I. INTRODUCTION

The concept of spin waves as elementary excitations in magnetically ordered materials was presented by Bloch[1] in the early 1930s. Bloch proposed that the low-lying excitations of the magnetization consisted of propagating quantized spin deviations instead of localized spin deviations in the molecular field created by the neighboring spins, as was believed until then. With this model he predicted that at low temperatures T (compared to the critical temperature T_c) the magnetization would decrease with temperature with a dependence of the form $T^{3/2}$. It would take more than two decades for this prediction to be firmly confirmed by experiments,[2] but in the following years there was considerable theoretical interest on spin waves, or magnons, as their quanta were to be called later. In 1934, Heller and Kramers[3] developed the semiclassical picture of a spin wave consisting of spins precessing around the equilibrium direction with a phase angle varying along the propagation direction. Soon after, Landau and Lifshitz published their famous paper introducing the torque equation of motion for the magnetization and predicting the magnetic resonance phenomenon.[4] In 1940, Holstein and Primakoff[5] presented a boson formulation that became a widely used quantum-mechanical formalism for studying spin waves mainly because it allows a simple treatment of the interactions between them. By the 1950s, spin waves were thought to be interesting objects but not quite exciting because they seemed to be only a mathematical entity with a small role in the thermodynamic properties of magnetic systems.

With the development of microwave spectroscopy and ferromagnetic resonance (FMR) techniques in the early 1950s, ferrite materials started to attract much attention due to their different properties and potential for technological application in microwave devices.[6,7] Two remarkable phenomena experimentally discovered by Bloembergen and Damon[8] and Bloembergen and Wang,[9] namely, the premature saturation of the FMR and the appearance of a subsidiary resonance at high microwave power levels, were extensively investigated during that decade; but for years, their explanation proved elusive. In the late 1950s, Suhl[10] showed that these instabilities were due to the nonlinear dynamical excitation of spin waves by the uniform mode driven at certain critical values of the microwave field. The evidence of spin waves as real physical entities stimulated a variety of microwave experiments in the early 1960s and different schemes for the nonlinear driving of spin waves were devised, such as the parallel pumping technique.[11–13] In low-loss ferromagnetic materials, such as yttrium iron garnet (YIG), it was found that spin-wave packets could be generated by pulsed microwave fields and propagate along a sample to carry and process information.[13–16] In the 1960s and 1970s, several techniques were then employed to probe and characterize the energy-momentum dispersion of spin waves in a wide range of magnetically ordered materials, such as inelastic neutron scattering,[2,17–19] Raman and Brillouin light scattering,[20–22] and magneto-optical spectroscopy.[19] On the theoretical side, the semiclassical picture of spin waves was thoroughly explored[2,7,13,23–26] and the quantum picture was studied by means of several approaches, such as Green's functions and various many-body techniques.[26–28] The nonlinear spin interactions were extensively studied and used to calculate from first principles properties of magnons, such as relaxation rates and energy renormalization with increasing temperature,[13,19,26,29,30] and to explain the behavior of spin waves above the microwave power threshold in the Suhl (perpendicular) and parallel pumping processes.[31] Contrary to widespread belief, interacting spin-wave theory actually accounts for the properties of magnetic systems at quite high levels of excitation. For instance, it can be used to explain thermodynamic properties at temperatures approaching the critical temperature.[32]

In the 1980s and early 1990s, there was a spur in spin wave research along two different lines. On one hand, the occurrence of universal dynamical scenarios in high-power FMR in YIG turned spin waves into one of the most interesting physical systems with which to study chaotic phenomena.[33–39] On the other hand, the scattering of light by spin waves in thin metallic magnetic films and multilayers turned out to be an important tool with which to investigate these structures.[40–46]

An important achievement has revived the interest in spin-wave excitations in recent years, namely, the proposal made both by Berger[47] and Slonczewski[48] that a spin-polarized direct current would drive the magnetization in a ferromagnetic thin metallic film. As shown by Slonczewski,[48] electrons in such a current traversing a magnetic multilayer with alternating ferromagnetic (FM) and paramagnetic metallic layers, become spin polarized by the passage through a FM layer and produce a spin transfer to a subsequent FM layer. This spin transfer results in a change of the magnetization, which can be seen as originating from an effective torque (which is proportional to the current density). Thus, as the magnetic structures shrink to nanoscale dimensions, this spin-transfer-induced (STI) torque is expected to dominate over the torque produced by the classical Oersted-Ampère field created by charges in motion. Slonczewski then predicted that the STI torque created by a high-density direct current could excite spin waves and switch the magnetization. This proposal stimulated many investigations in recent years, not only because of the novel physical mechanisms and phenomena involved, but also due to the drive for new technological applications in the field of spintronics. On the theoretical side the investigations might fall in two broad categories. In one line of research, aimed at understanding the physical mechanisms of the spin transfer torque, there has been some controversy regarding the detailed origin of and the correct mathematical expression for the driving torque,[47–52] an issue that has not yet been settled. Another set of investigations focuses on studying the dynamics of the magnetization excited by spin-polarized direct currents. The most widely used approach is to solve the Landau-Lifshitz-Gilbert (LLG) equation, generalized to include the STI torque.[53–58] The theoretical work and the possibility of observing different phenomena and properties have stimulated a wealth of experiments to verify the predictions, to test the proposed models, and to develop applications.[59–72]

Early evidence of the excitation of spin waves by a direct current was reported by Tsoi *et al.*[59] in magnetotransport measurements made with point contacts onto magnetic multilayers. By varying the current intensity at a fixed value of the external magnetic field, they observed sudden changes in the resistance that were attributed to the onset of spin-wave growth at certain critical values of the current. The fact that the critical current showed a strong dependence on the direction and intensity of the applied dc field was clear evidence of the spin-wave excitation. However, this evidence was very indirect and no information on the spatial nature of the excited modes was provided in the magnetoresistance experiments.[59–63,65]

Clear-cut signatures of the presence of high-frequency spin waves have been reported by Tsoi,[64] Kiselev *et al.*,[66,69] Rippard *et al.*[67,68] and Krivorotov *et al.*[70] In Ref. 64, the magnetic multilayer is placed in a microwave cavity and subjected, simultaneously, to a direct current from a point contact and a microwave radiation field. The observed mixing of the two frequencies demonstrates that high-frequency spin waves are indeed driven by the current. The most recent papers[66–70] report the direct observation of voltage oscillations with frequencies in the microwave range and represent a definitive demonstration that a precession of the magnetization is induced by direct currents traversing a multilayer structure. These experiments allow precise measurements of the precession frequency, amplitude, and relaxation rate, revealing a rich dynamics and intriguing features not predicted by the simple models proposed thus far.

The purpose of this paper is to present a model that explains recent experimental observations of the dynamics of the magnetization in a FM film traversed by a spin-polarized direct current in the framework of spin-wave theory incorporating nonlinear interactions. Actually, the LLG equation for the magnetization has been explored by several authors in this regard.[53–58] However, although the LLG equation contains all the ingredients to explain the experimental results, its full micromagnetic problem can only be solved, numerically, in a way that tends to hide the underlying physics. In the model presented here, the excitations of the magnetization are considered to be standing spin-wave modes interacting through four-magnon processes. The driving STI torque has the form proposed by Slonczewski.[48] The spin-wave approach thus renders a clear picture of the roles of the nonlinear interactions. Moreover, in some configurations it provides analytical expressions that can be used to calculate quantities measured experimentally. As demonstrated in Secs. II–V, the model accounts for most of the recent experimental observations in thin magnetic films. In particular, it explains the downward frequency shifts (redshifts) with increasing current observed when the external field is applied on the film plane, and upward frequency shifts (blueshifts) with the field perpendicular to the film. A brief account of this work has been published elsewhere.[71]

The paper is organized as follows: In Sec. II, we discuss the nature of the spin-wave modes in very thin films, using the results of earlier work by several authors in order to establish the background for the remainder of the paper. Section III is devoted to analyzing the spin-wave excitations driven by a spin-polarized direct current. The critical current is calculated in the framework of noninteracting spin waves. In Sec. IV, we derive the equations of motion for the spin-wave amplitude, incorporating the effect of nonlinear interactions, which are essential to explain the experimental observations. It is shown that in the configuration of field applied perpendicularly to the film, the equations of motion can be solved analytically. In Sec. V, we present numerical solutions of the equations of motion and compare the results to recent experiments. Section VI summarizes the main results.

II. SPIN-WAVE MODES IN THIN FILMS

In this section, we review the theory of spin waves in a thin ferromagnetic film. This is done for completeness, aiming at the derivation of the results in Sec. IV. Many authors have studied spin waves in thin films over the years.[16,40–46,72–74,84] The effect of the external driving and relaxation will be considered in Sec. III. The Hamiltonian for the magnetic system in the film is written as

$$\mathcal{H} = \mathcal{H}_Z + \mathcal{H}_{an} + \mathcal{H}_{\text{exc}} + \mathcal{H}_E + \mathcal{H}_S + \mathcal{H}_{\text{dip}}, \tag{1}$$

where the terms on the right-hand side represent, respectively, the Zeeman, volume anisotropy of crystalline or shape origin, volume exchange, interlayer exchange, surface anisotropy, and dipolar contributions. They can be written in terms of the spin $\mathbf{S}_i$ at the lattice site i as[38,44,46]

$$\mathcal{H}_Z = -\sum_i g\mu_B \mathbf{H}_0 \cdot \mathbf{S}_i, \tag{2}$$

$$\mathcal{H}_{\text{exc}} = -\sum_{ij} J_{ij}\mathbf{S}_i \cdot \mathbf{S}_j, \tag{3}$$

$$\mathcal{H}_S = \frac{K_S A}{N}\sum_i \left(\frac{\hat{\mathbf{n}} \cdot \mathbf{S}_i}{S}\right)^2, \tag{4}$$

$$\mathcal{H}_{\text{dip}} = \frac{1}{2}(g\mu_B)^2 \sum_{ij}\left[\frac{\mathbf{S}_i \cdot \mathbf{S}_j}{r_{ij}^3} - \frac{3(\mathbf{S}_i \cdot \boldsymbol{r}_{ij})(\mathbf{S}_j \cdot \boldsymbol{r}_{ij})}{r_{ij}^5}\right], \tag{5}$$

where $\mathbf{H}_0$ is the external static magnetic field applied to the film, g is the Landé g-factor, μ_B is the Bohr magneton, J_{ij} is the exchange interaction constant between the spins at sites i and j, K_s is the surface anisotropy constant (energy per unit area), A is the film area, N is the number of spins in the volume of the film, $\hat{\mathbf{n}}$ is the unit vector perpendicular to the film plane, and $\mathbf{r}_{ij}$ is the vector connecting sites i and j. For simplicity, we consider the field applied along a symmetry direction of the film so that the contribution of the anisotropy is represented by an effective field H_{an} to be added to the external field. The interlayer exchange is also considered to contribute with an additive term to the external field. We treat the excitations of the magnetic system with the approach of Holstein and Primakoff,[5] which consists of three transformations that allow the spin operators to be expressed in terms of boson operators that create or destroy magnons. In the first transformation, the components of the local spin operator are related to the creation and annihilation operators of spin deviation at site i, respectively a_i^+ and a_i, which satisfy the boson commutation rules $[a_i, a_j^+] = \delta_{ij}$ and $[a_i, a_j] = 0$. Using a coordinate system with $\hat{z}$ along the equilibrium direction of the spins, defining $S_i^+ = S_i^x + iS_i^y$ and $S_i^- = S_i^x - iS_i^y$, where i is the imaginary unit, it can be shown that the relations that satisfy the commutation rules for the spin components and the boson operators are[5]

$$S_i^+ = (2S)^{1/2}\left(1 - \frac{a_i^+ a_i}{2S}\right)^{1/2} a_i, \tag{6a}$$

$$S_i^- = (2S)^{1/2} a_i^+ \left(1 - \frac{a_i^+ a_i}{2S}\right)^{1/2}, \tag{6b}$$

$$S_i^z = S - a_i^+ a_i, \tag{6c}$$

where S is the spin and $n_i = a_i^+ a_i$ is the operator for the number of spin deviations at site i. One of the main advantages of this approach is that the nonlinear interactions are treated analytically by expanding the square root in (6a) and (6b) in Taylor series. We use only the first two terms of the expansion, so that

$$S_i^+ \simeq (2S)^{1/2}\left(a_i - \frac{a_i^+ a_i a_i}{4S}\right) \tag{7a}$$

and

$$S_i^- \simeq (2S)^{1/2}\left(a_i^+ - \frac{a_i^+ a_i^+ a_i}{4S}\right). \tag{7b}$$

In order to find the normal modes of the system, we use the linear approximation whereby only the first terms in (6c) and (7) are kept, i.e., $S_i^+ \simeq (2S)^{1/2} a_i$, $S_i^- \simeq (2S)^{1/2} a_i^+$, and $S_i^z \simeq S$. With these transformations, one can express the Hamiltonian (1) in a quadratic form, containing only lattice sums of products of two boson operators. The second step is to introduce a transformation from the localized field operators to collective boson operators $a_{\boldsymbol{k}}^+ \, a_{\boldsymbol{k}}$. In general, this transformation is

$$a_i = \sum_{\boldsymbol{k}} \phi_{\boldsymbol{k}}^i a_{\boldsymbol{k}}, \tag{8a}$$

$$a_i^+ = \sum_{\boldsymbol{k}} \phi_{\boldsymbol{k}}^{i*} a_{\boldsymbol{k}}^+. \tag{8b}$$

The condition that the new collective operators satisfy the boson commutation rules, $[a_{\boldsymbol{k}}, a_{\boldsymbol{q}}^+] = \delta_{\boldsymbol{kq}}$ and $[a_{\boldsymbol{k}}, a_{\boldsymbol{q}}] = 0$, requires that the transformation coefficients satisfy the orthonormality relations

$$\sum_{\boldsymbol{k}} \phi_{\boldsymbol{k}}^i \phi_{\boldsymbol{k}}^{j*} = \delta_{ij}, \tag{9a}$$

$$\sum_i \phi_{\boldsymbol{k}}^i \phi_{\boldsymbol{q}}^{i*} = \delta_{\boldsymbol{kq}}, \tag{9b}$$

where $\boldsymbol{k}$ and $\boldsymbol{q}$ denote spatial wave vectors and the right-hand sides are Kroenecker δ's. In a laterally unbounded film having a completely uniform internal field, the collective modes are traveling waves; thus, the appropriate transformation coefficients are expressed in terms of plane-wave functions

$$\phi_{\boldsymbol{k}}^i = \frac{1}{N^{1/2}} e^{i\boldsymbol{k}\cdot\boldsymbol{r}_i}, \tag{10}$$

where $\boldsymbol{k}$ is a vector spanning the whole wave-vector space. Since driving with spin-polarized electrons should equally excite modes with wave vectors $\boldsymbol{k}$ and $-\boldsymbol{k}$ and, moreover, the films used in experiments are bounded, we consider that a direct current excites standing wave modes. Hence, the transformation coefficients used here are

$$\phi_{\boldsymbol{k}}^j = \frac{1}{(2N)^{1/2}}(e^{i\boldsymbol{k}\cdot\boldsymbol{r}_j} + e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_j}) = \left(\frac{2}{N}\right)^{1/2} \cos \boldsymbol{k}\cdot\boldsymbol{r}_j, \tag{11}$$

where $\boldsymbol{k}$ denotes a set of discrete wave vectors that satisfy the boundary conditions. With transformations (8) and (11) and using the orthonormality relations, one can show that the quadratic Hamiltonian becomes

$$\mathcal{H}^{(2)} = \hbar \sum_{k} \left(A_k a_k^+ a_k + \tfrac{1}{2} B_k a_k a_k + \tfrac{1}{2} B_k^* a_k^+ a_k^+\right), \quad (12)$$

where A_k and B_k are coefficients related to the parameters of the Hamiltonian (1)–(5). In order to diagonalize the quadratic Hamiltonian, it is necessary to introduce new collective boson operators c_k^+ and c_k, satisfying the commutation rules $[c_k, c_q^+] = \delta_{kq}$ and $[c_k, c_q] = 0$. One can show[5] that the new operators are related to a_k^+ and a_k through the Bogoliubov transformation

$$a_k = u_k c_k + v_k c_k^+, \quad (13a)$$

$$a_k^+ = u_k c_k^+ + v_k^* c_k, \quad (13b)$$

where $u_k^2 - |v_k|^2 = 1$, as appropriate for a unitary transformation. The coefficients of this transformation must be such that the quadratic Hamiltonian acquires the diagonal form

$$\mathcal{H}^{(2)} = \hbar \sum_{k} \omega_k c_k^+ c_k, \quad (14)$$

because this leads to the Heisenberg equation of motion

$$\left(\frac{dc_k}{dt}\right)_{\text{kinetic}} = \frac{1}{i\hbar}[c_k, \mathcal{H}^{(2)}] = -i\omega_k c_k. \quad (15)$$

This equation has stationary solution of the form $e^{i\omega_k t}$, which assures that c_k is the operator for the normal-mode excitations of the magnetic system. Hence, c_k^+ and c_k are the creation and annihilation operators for magnons. It can be shown[2,5,16,19,26,27,29,38] that the frequency ω_k of the eigenmodes and the coefficients of the transformations (13) are given by

$$\omega_k = (A_k^2 - |B_k|^2)^{1/2}, \quad (16)$$

$$u_k = \left(\frac{A_k + \omega_k}{2\omega_k}\right)^{1/2}, \quad (17)$$

and

$$v_k = \pm (u_k^2 - 1)^{1/2} = \pm \left(\frac{A_k - \omega_k}{2\omega_k}\right)^{1/2}, \quad (18)$$

where the sign of v_k in (18) is the opposite one of the parameter B_k.

In order to complete the characterization of the normal-mode excitations of the magnetic system in the film, it is necessary to relate the coefficients A_k and B_k to the parameters of the Hamiltonian (1)–(5). It is straightforward to show[2,5,19,26,27,29] that the Zeeman (2) and exchange (3) energies contribute only to A_k with terms that do not depend on the direction of the field. They are given by $\gamma(H_0 + Dk^2)$, where $\gamma = g\mu_B/\hbar$ is the gyromagnetic ratio and $D = 2J_\delta S a^2 / g\mu_B$ is the exchange stiffness, J_δ being the nearest-neighbor exchange constant, and a the lattice parameter of the film. The general expressions for A_k and B_k for an arbitrary direction of the external field are quite lengthy due to the role of the dipolar energy.[16,38,73] For the sake of comparison to recent experiments, we focus hereafter on three distinct configurations, namely, (i) the applied field and the wave vector are both perpendicular to the plane of the film, (ii) the field is perpendicular to the film plane and the wave vector is in the plane, and (iii) the field is in the film plane and wave vector is in the plane, at an arbitrary angle with the field. Case (i) enjoys a simplifying vanishing contribution of the volume dipolar energy. In this case, it can be shown[19,29] that $B_k = 0$. Thus, from (16)–(18), we see that $\omega_k = A_k$, $u_k = 1$, $v_k = 0$, and the frequency of the spin-wave mode is

$$\omega_k = \gamma(H_0 + H_{an} + H_E + Dk^2 - 4\pi M_{\text{eff}}), \quad (19)$$

where H_{an} is the anisotropy field, H_E is the interlayer exchange field, and M_{eff} is the effective magnetization defined by $4\pi M_{\text{eff}} = 4\pi M_s + H_s$, where $M_s = g\mu_B NS/V$ is the saturation magnetization, $V = Ad$ being the volume of the sample, d is the film thickness, and $H_s = 2K_s/M_s d$ is the surface anisotropy field. Because the translational invariance is broken in the vicinity of the film surfaces, the wave number is quantized as $k = p\pi/d$, where $p = 1, 2, 3, \ldots$. This is valid for boundary conditions of unpinned spins on the film surfaces, which is well justified for materials with small anisotropies, such as permalloy.[16,74]

Cases (ii) and (iii), in which the wave vector is on the film plane, are somewhat more complicated due to the role of the dipolar energy. The standard procedure is to separate its contributions into two parts, one arising from the surfaces and one from the volume. Here we follow the approach of Arias and Mills 44 to calculate both contributions for a very thin film ($kd \ll 1$). Considering a coordinate system with the z direction perpendicular to the plane, choosing the x axis along the wave vector, and assuming that the dynamic components of the magnetization do not vary across the thickness of the film, the contributions of the volume and surface dipolar energies as well as the surface anisotropy energy can be calculated as in Ref. 44 and expressed in terms of the operators a_k^+ and a_k. One can then show that the quadratic part of the Hamiltonian (1)–(5) can be written in the form (12) with

$$A_k = \gamma(H_0 + H_{an} + H_E + Dk^2 - 4\pi M_{\text{eff}} + \pi M_s kd) \quad (20a)$$

and

$$B_k = \gamma \pi M_s kd, \quad (20b)$$

where the terms with the factor $\pi M_s kd$ arise from the volume dipolar interaction, whereas the term $4\pi M_{\text{eff}}$ arises from the surface dipolar and surface anisotropy energies. Using (16), one can write the frequency for the case in which the field is perpendicular to the plane and the wave vector is on the film as

$$\omega_k = \gamma(H_0 + H_{an} + H_E + Dk^2 - 4\pi M_{\text{eff}} + 2\pi M_S kd)^{1/2}(H_0 + H_{an} + H_E + Dk^2 - 4\pi M_{\text{eff}})^{1/2}. \quad (21)$$

Note that for the uniform ($\boldsymbol{k} = 0$) mode, Eqs. (19) and (21) coincide, as they should, and become

$$\omega_k = \gamma(H_0 + H_{an} + H_E - 4\pi M_{\text{eff}}). \quad (22)$$

Finally, consider that the field and the wave vector lie in the film plane. This is the configuration most frequently used

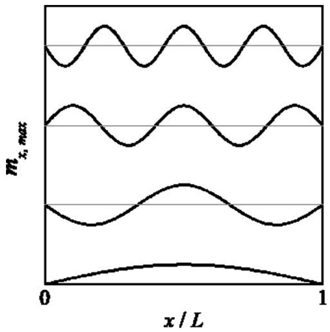

FIG. 1. Schematic representation of the lateral variation of the amplitude of the dynamic components of the magnetization for low-order standing-wave modes in a thin bounded film.

in experiments and, thus, of particular interest. We take the field and magnetization in the plane of the film along the z direction of a Cartesian coordinate system. The x axis is on the plane, and the y axis is normal to the plane. The wave vector of the standing-wave mode is assumed to be in the plane, at an angle $\theta_{\boldsymbol{k}}$ with the field. Again the quadratic part of the full Hamiltonian (1)–(5) can be written in the form (12), with

$$A_{\boldsymbol{k}} = \gamma(H_0 + H_{an} + H_E + Dk^2 + 2\pi M_{\text{eff}} - \pi M_S kd \cos^2\theta_{\boldsymbol{k}}) \tag{23a}$$

and

$$B_{\boldsymbol{k}} = -\gamma[2\pi M_{\text{eff}} - \pi M_S kd(1 + \sin^2\theta_{\boldsymbol{k}})]. \tag{23b}$$

Using (16), one can write the frequency for the case in which the field and the wave vector lie in the plane as

$$\omega_{\boldsymbol{k}} = \gamma(H_0 + H_{an} + H_E + Dk^2 + 2\pi M_S kd \sin^2\theta_{\boldsymbol{k}})^{1/2} \times (H_0 + H_{an} + H_E + Dk^2 + 4\pi M_{\text{eff}} - 2\pi M_S kd)^{1/2}. \tag{24}$$

Note that for $k=0$ this equation yields the well-known expression for the frequency of the ferromagnetic resonance (FMR) mode for the field in the plane,

$$\omega_0 = \gamma(H_0 + H_{an})^{1/2}(H_0 + H_{an} + 4\pi M_{\text{eff}})^{1/2}. \tag{25}$$

In laterally confined films the wave number in Eqs. (21) and (24) is quantized.[16] We assume that the dynamic components of the magnetization are pinned at the border of the film, due to the abrupt change in the surface anisotropy. In this case, the wave number is $k=p\pi/L$, where L is the relevant lateral dimension of the film and p is an odd integer. Figure 1 depicts the lateral variation of the amplitude of the magnetization along the direction of the wave vector in the film for standing wave modes with $p=1$, 3, 5, and 7.

As pointed out in Ref. 44, a unique feature of the spin-wave mode in very thin films with a wave vector in the plane is that in some range of angle $\theta_{\boldsymbol{k}}$ the initial slope of the frequency vs wave-number curve is negative, due to the role of the dipolar energy. As the wave number increases, the exchange energy that varies with k^2 eventually dominates so that the $\omega_{\boldsymbol{k}}$ vs k curve exhibits a minimum at some value of k. This is clearly seen in Fig. 2, showing a plot of Eq. (24)

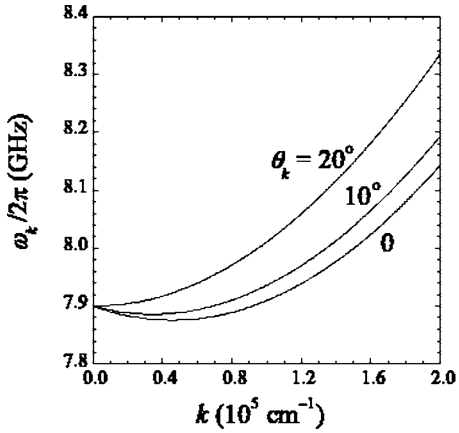

FIG. 2. Dispersion relation for spin waves in a very thin film with the field and wave vector in the plane of the film. The curves are plots of Eq. (24) with the parameters appropriate for the sample used in Ref. 67 for three values of the angle between the wave vector and the field.

for $\theta_{\boldsymbol{k}}=0$, $10°$, and $20°$, with the following parameters appropriate for the permalloy film used in the experiments reported in Ref. 67: $4\pi M_s=10.0$ kG, $4\pi M_{\text{eff}}=8.0$ kG, $g=1.78$, $H_0=1.0$ kOe, $H_E+H_{an}=0.104$ kOe, $D=3\times10^{-9}$ Oe cm^2. An important consequence of the shape of the dispersion relation in Fig. 2 is that there is a manifold of modes with wave-vector direction in a certain range $\theta_{\boldsymbol{k}}<\theta_c$ that are degenerate with the $k=0$ magnon, so that they can scatter energy in a two-magnon process. This has been shown to result in an extrinsic relaxation mechanism that tends to dominate the damping of the FMR mode in ultrathin films since it varies with $1/d^2$.[44] The implications of the shape of the curves in Fig. 2 to the interpretation of the experimental observations of Refs. 66–70 will be discussed later.

To conclude this section, we express the components of a local magnetization vector $\mathbf{M}_i=g\mu_B(N/V)\mathbf{S}_i$ in terms of the magnon operators. Using transformations (6) and (13), writing the Cartesian components as $S_i^x=(S_i^+ + S_i^-)/2$, $S_i^y=(S_i^+ - S_i^-)/2i$ and replacing the expectation values of the operators $c_{\boldsymbol{k}}^+$ and $c_{\boldsymbol{k}}$ by classical variables $c_{\boldsymbol{k}}^*$ and $c_{\boldsymbol{k}}$, one obtains

$$M_x(\boldsymbol{r}_i) = \frac{M_S}{(NS)^{1/2}}\left(\frac{1-n_j}{2S}\right)^{1/2}\sum_k \cos \boldsymbol{k}\cdot\boldsymbol{r}_i(u_{\boldsymbol{k}}+v_{\boldsymbol{k}})(c_{\boldsymbol{k}}+c_{\boldsymbol{k}}^*), \tag{26a}$$

$$M_y(\boldsymbol{r}_i) = \frac{M_S}{i(NS)^{1/2}}\left(\frac{1-n_j}{2S}\right)^{1/2}\sum_k \cos \boldsymbol{k}\cdot\boldsymbol{r}_i(u_{\boldsymbol{k}}-v_{\boldsymbol{k}})(c_{\boldsymbol{k}}-c_{\boldsymbol{k}}^*), \tag{26b}$$

and

$$M_z(\boldsymbol{r}_i) = M_S\left(\frac{1-n_i}{S}\right), \tag{26c}$$

where

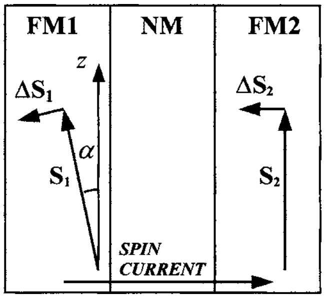

FIG. 3. Illustration of the basic process of spin transfer due to a spin-polarized current traversing a three-layer magnetic structure.

$$n_i = \frac{2}{N}\sum_{\boldsymbol{k},\boldsymbol{q}} \cos \boldsymbol{k}\cdot\boldsymbol{r}_i \cos \boldsymbol{q}\cdot\boldsymbol{r}_i (u_k c_k^* + v_k c_k)(u_q c_q + v_q c_q^*). \tag{26d}$$

Note that if neither interactions nor driving are present, the equations of motion for c_k and c_k^+ are given by (15) and its complex conjugate, which have simple time-harmonic solutions. Thus, for one standing-wave mode, the components of the magnetization described by (26) correspond to the classical picture of the spin wave, in which the magnetization precesses about the equilibrium direction, with an amplitude of precession that varies along the direction of the wave vector. In the case of a traveling wave, characterized by the transformation coefficient (10), it is the phase of the precession that varies along the direction of propagation. One advantage of the spin-wave approach is that any well-behaved spatially varying magnetization can be described by the Fourier expansion in Eqs. (26).

III. EXCITATION OF SPIN-WAVE MODES WITH A DIRECT CURRENT

In this section, we consider the driving of the noninteracting spin-wave system in a thin FM film by the passage of a spin-polarized direct current. The effects of the magnon interactions will be taken into account in Sec. IV. The mechanism of driving is that proposed by Berger[47] and Slonczewski[48] for a direct current flowing perpendicularly to the plane of a magnetic multilayer with intercalated FM and nonmagnetic metallic thin films. Their initial proposal was followed by other interpretations for the driving mechanisms.[49–52] Here we follow the approach of Slonczewski,[48,54] which considers that the current creates a mutual transference of spin angular momentum between the magnetic layers. The basic physical mechanism is illustrated in Fig. 3, showing a three-layer structure consisting of two FM layers intercalated by a nonmagnetic metallic layer NM, with a field applied along the z direction. The macroscopic vectors $\mathbf{S}_1$ and $\mathbf{S}_2$ represent the respective total spins of the layers, $\mathbf{S}_2$ is assumed to be fixed along z while $\mathbf{S}_1$ is instantaneously at an angle α with the z direction. As electrons flow through layer FM1, they become partially polarized with spins along $\mathbf{S}_1$, traverse the NM layer, and impinge on the interface with layer FM2. The probabilities for reflection or transmission at the interface depend on the electron spin; down spins ($-z$ direction) have a higher probability of being reflected. The overall effect is a mutual transfer of spin angular momentum between the two FM layers, resulting in changes $\Delta\mathbf{S}_1$ and $\Delta\mathbf{S}_2$ that are perpendicular to $\mathbf{S}_1$ and $\mathbf{S}_2$ with time rate proportional to $\sin \alpha/2$ and to the electron flow, which is determined by the electric current.The time derivative of the spin angular momentum can be seen as caused by an effective torque, so that the STI driving is associated to an effective magnetic field $\mathbf{H}_{\rm STI}$ acting on the magnetization $\mathbf{M}$ of the layer FM1, given by[48,54]

$$\mathbf{H}_{\rm STI} = \frac{\beta J}{\gamma S}\hat{\mathbf{z}} \times \mathbf{S}, \tag{27a}$$

with

$$\beta = \varepsilon \hbar \gamma / 2 d M_S e, \tag{27b}$$

where J is the electric current density traversing the multilayer in the perpendicular direction and ε is a spin transfer efficiency parameter, which depends on the materials of the multilayer.[48,51,54] Note that we assume J to be uniform in the region of interest. This is a good approximation for an experiment done with nanopillars[60,66,69,70] but not as good for experiments using extended films with point contacts.[59,62,64,65,67,68] The essential feature of the STI field is that it exerts a torque on the magnetization that tends to deviate it away from equilibrium, producing an effect opposite to that of the relaxation and effectively driving its motion. As a result, when the current exceeds a critical value J_c, the damping is overcome leading to a rapid growth of spin-wave modes supported by the film. As shown in Sec. IV, the saturation process and other phenomena at higher currents are governed by nonlinear effects. The contribution to the equation of motion from driving of the spin-wave modes by the STI field produced by the direct current can be calculated with the torque equation

$$\frac{d\mathbf{S}_i}{dt} = -\gamma \mathbf{S}_i \times \mathbf{H}. \tag{28}$$

Equations (27) and (28) yield

$$\frac{dS_i^{\pm}}{dt} = \frac{\beta J}{S} S_i^z S_i^{\pm}. \tag{29}$$

Using (7), (8), and (13) to transform the spin operators into the magnon operators, one obtains, in the linear approximation,

$$\left(\frac{dc_k}{dt}\right)_{\rm drive} = \beta J c_k. \tag{30}$$

Adding (30) to (15) and introducing the relaxation rate η_k in a phenomenological manner, it follows that

$$\frac{dc_k}{dt} = \left(\frac{dc_k}{dt}\right)_{\rm kinetic} + \left(\frac{dc_k}{dt}\right)_{\rm relax} + \left(\frac{dc_k}{dt}\right)_{\rm drive} = -i\omega_k c_k - (\eta_k - \beta J) c_k. \tag{31}$$

Note that the relaxation could have been introduced rigorously, considering the interaction between the magnon system with a heat bath, and this would lead to a result identical to (31).[75] The solution of the linearized equation of motion (31) is straightforward, namely,

$$c_{\mathbf{k}}(t) = c_{\mathbf{k}}(0)e^{-i\omega_{\mathbf{k}}t}e^{-(\eta_{\mathbf{k}}-\beta J)t}. \tag{32}$$

This result means that when the current density exceeds the critical value $J_c = \eta_{\mathbf{k}}/\beta$, the spin-wave mode with lowest relaxation rate grows exponentially. This is what produces a change in the magnetic state and a corresponding step on the magnetoresistance vs current characteristics observed in experiments. Writing the relaxation rate as $\eta_{\mathbf{k}} = \eta_0 + \alpha_G \omega_{\mathbf{k}}$ where $\alpha_G \omega_{\mathbf{k}}$ is the Gilbert contribution and η_0 is a residual value independent of the frequency, one obtains for the critical current

$$I_c = \frac{2A_c e d M_s}{\hbar \varepsilon \gamma}(\eta_0 + \alpha_G \omega_{\mathbf{k}}). \tag{33}$$

where A_c is the area of the current cross section, assumed to be uniform in a certain region of the film and determined by the characteristics of metal contact. Note that the second term in (33) varies with magnetic field and is responsible for the field dependence of the critical current observed in experiments. For films magnetized perpendicularly to the plane, the frequency is given by (19) so that the critical current can be written as

$$I_c = I_{c0} + bH_0 \tag{34a}$$

where

$$I_{c0} = \frac{2A_c e d M_s}{\hbar \varepsilon}[\eta_0/\gamma + \alpha_G(H_{an} + H_E + Dk^2 - 4\pi M_{\rm eff})] \tag{34b}$$

and

$$b = \frac{2A_c e d M_s \alpha_G}{\hbar \varepsilon}. \tag{34c}$$

In the magnetoresistance experiments with point contacts, the critical current is indicative of the onset of the spin-wave growth, but no direct evidence is obtained about the nature of the modes that are excited by the STI field. In the model proposed by Slonczewski,[54] these modes are cylindrical waves propagating radially away from the current beam established by the point contact. The model yields an expression for the critical current identical to (34a), with the same slope b as given by (34c), but with a constant term that is determined by the radiation loss of the radially propagating mode.

The field dependence of the critical current has been experimentally studied in detail by Rippard *et al.*[62] in a series of Co/Cu multilayers, and the data were compared to the predictions of Ref. 54. The measured critical currents exhibit a linear dependence on the field with slope b typically on the order of 0.5 mA/T and initial values I_{c0} consistently in the range 2–4 mA. Using mean values for the parameters given in Ref. 62, $4\pi M_s = 16.5$ kG, film thickness $d = 1.2$ nm, $D = 5$ meV nm^2, $\alpha_G = 0.02$, $\varepsilon = 0.2$, and contact diameter 35 nm, the value of b calculated with Eq. (34c) (which is identical to the prediction of Ref. 54), is 0.4 mA/T, which is in quite good agreement with the measured value. However, the measured initial values of I_c are systematically smaller than the value predicted in Ref. 54 by almost one order of magnitude. We attribute this discrepancy to the fact that the radiation loss of the radially propagating mode, assumed in Ref. 54, overwhelms the intrinsic damping of the mode. This is strong evidence that the spin-wave mode excited by the direct current is not the cylindrical wave with wavelength on the order of the contact diameter, $k \approx 10^6$ cm^{-1}, assumed in Ref. 54. Further evidence of this has been recently provided by the experimental results of Refs. 66–70, which indicate that the mode excited by the spin injection current has a frequency close to the ferromagnetic resonance (FMR) value, i.e., it has $k \approx 0$.

The existing experiments on direct-current excitation do not allow a clear-cut determination of the wave number k of the driven spin-wave mode. Some estimates of the value of k are based on a fit of the dispersion relation to the measured frequencies,[67] but the dispersion assumed for the spin wave with wave vector in the plane is not the correct one, as given by Eq. (24). Certainly the mode with lowest critical current is the one with the smallest losses, and it is the one excited first as the current is increased. Regarding losses, it is known that in ultrathin films there are several mechanisms for relaxation, such as the intrinsic or Gilbert damping, spin pumping involving adjacent layers,[72,75–79] and two-magnon scattering.[44,80–84] The latter process relies on the existence of degenerate modes into which the driven mode can scatter energy. Thus, in the experiments of Rippard *et al.*,[67] one might expect that the mode with $k \approx 4 \times 10^4$ cm^{-1} is excited first because, as shown in Fig. 1, it lies at the bottom of the dispersion relation and consequently has no two-magnon relaxation. However, most likely this mode is not the one that produces the observed microwave oscillation because in order to radiate electromagnetic waves, the spin wave should have a k value comparable to that of the microwave, which is $k \approx 1$ cm^{-1}. In other words, in a wave with a high k value the radiated signal averages out to zero. At least for macroscopic samples, it has been demonstrated that the radiation from a precessing magnetization decays rapidly as the wavelength of the spin-wave mode decreases.[85,86] In a magnetic nanostructure, the whole radiation problem has not been tackled and remains a challenging issue. We speculate that as the direct current is increased, the mode at the bottom of the dispersion relation is excited first, causing a step in the magnetoresistance curve. However, the mode that generates the observed oscillation is one with $k \approx 0$, which, having a larger relaxation rate, is excited at a slightly higher critical current. Finally, there is another fact that complicates the question of the mode excited by a direct current, namely, the jumps in the oscillation frequency observed as the current is scanned in experiments with the magnetic field at an angle with the film plane[68] or perpendicular to the plane.[69] The jumps are indicative that the system switches from one mode to another as the current is increased, as will be discussed further in Sec. V.

IV. EFFECTS OF THE MAGNON INTERACTIONS

In order to study the phenomena occurring at currents above the onset of the spin-wave excitation, one needs to take into account the effect of nonlinear processes in the dynamics of the spin evolution. Such processes can be incorporated naturally into the equations of motion by using the full Holstein-Primakoff transformation of the spin operators into boson operators as in Eq. (6). This is done by expanding the square-root terms in (6a) and (6b) and writing the Hamiltonian (1)–(5) as sums of products of more than two magnon operators, which represent the interactions between magnons. In principle, all terms in (1) contribute to the magnon interactions. However, considering that the films used in the experiments of interest are very thin, the contribution of the volume dipolar interaction is small. The terms representing the magnon interactions arising from volume anisotropy, interlayer exchange, and volume exchange are also negligible in the conditions of the experiments in Refs. 66–70. The important contributions arise from the surface dipolar energy (demagnetizing effect) and the surface anisotropy energy. The evaluation of the sums in Eq. (5) can be shown to lead to two terms in the dipolar energy, one arising from the volume and the other from the surfaces. For a film, the latter can be written approximately as[19,26]

$$\mathcal{H}^{s}_{\mathrm{dip}} = \frac{2\pi(g\mu_B)^2 N}{V}\sum_i (\hat{n}\cdot S_i)^2, \tag{35}$$

which turns out to have the same form as the surface anisotropy energy (4). Hence, the sum of (4) and (35) is

$$\mathcal{H}^{s}_{\mathrm{dip}} + \mathcal{H}_s = \hbar\,\gamma\frac{2\pi M_{\mathrm{eff}}}{S}\sum_i (\hat{n}\cdot S_i)^2, \tag{36}$$

where the factor $2\pi M_{\mathrm{eff}}$ accounts for the surface dipolar and anisotropy energy contributions. As in Sec. II, we consider three cases: (i) The applied field and wave vector are perpendicular to the plane of the film, (ii) the field is perpendicular to the film and the wave vector is in the film plane, and (iii) the field and wave vector are in the film plane. Consider, initially, case (i) for which the full equation of motion for the magnon operator can be solved analytically. The z axis is taken normal to the film, so that with transformation (6c), Eq. (36) leads to the following Hamiltonian containing four boson operators

$$\mathcal{H}^{(4)} = \hbar\,\gamma\frac{2\pi M_{\mathrm{eff}}}{S}\sum_i a_i^+ a_i a_i^+ a_i. \tag{37}$$

In order to express this equation in terms of the magnon operators, we use transformations (8) and (13), consider standing-wave modes as in (11), and assume that only one mode $\boldsymbol{k}$ is present. The four-magnon interaction Hamiltonian then becomes

$$\mathcal{H}^{(4)} = \hbar\,\gamma\frac{3\pi M_{\mathrm{eff}}}{NS} c_k^+ c_k^+ c_k c_k. \tag{38}$$

Note that in the derivation of this equation we made use of the following facts: a sum like (9b) with a product of four coefficients of the form (11) leads to a factor $3/2N$ instead of $1/N$ that would hold for propagating waves characterized by the coefficient (10); in the case of field and wave vector perpendicular to the plane, the coefficients in (13) are $u_k=1$ and $v_k=0$; the commutation of c_k^+ with c_k performed to write (38) in the normal ordering, with all creation operators to the left of the annihilation operators, results in a term with two operators like (14) that is negligible because it contains a factor $1/N$ compared to the energy given by (16). The four-magnon interaction (38) contributes to the equation of motion of the operator c_k with a pure imaginary term representing an energy renormalization, which manifests itself as a frequency shift in the microwave oscillation observed in the experiments reported in Refs. 66–70. However, contrary to the role it plays in the Suhl and parallel pumping instability processes,[32] the four-magnon interaction given by (38) does not limit the growth of the spin-wave amplitude. The reason for this is that in those processes the origin of driving with an ac magnetic field is fundamentally different from one with a STI field due to a direct current. In the former, the instability results from the generation of magnon pairs due to the conversion of photons into magnons, whereas, in the process studied here, the spin wave system becomes unstable because the STI torque pulls the magnetization away from the equilibrium direction and balances the effect of the relaxation. It turns out that as the spin-wave amplitude grows, there is a reduction in the z component of the spin, causing a downward deviation of the STI torque from linearity. This is the nonlinear effect that limits the growth of the spin waves generated by the spin-polarized direct current. Including in Eq. (29) the term with two boson operators in S_i^z given by (6c) and adding the contribution of (38), the full equation of motion for c_k becomes

$$\frac{dc_k}{dt} = -i\omega_k c_k - (\eta_k - \beta J)c_k - \frac{3\beta J}{2NS} c_k^* c_k c_k - i S_k c_k^* c_k c_k, \tag{39}$$

where

$$S_k = \gamma 6\pi\frac{M_{\mathrm{eff}}}{NS} \tag{40}$$

is the coefficient arising from the four-magnon interaction. Note that in (39) we have transformed the magnon operators into the classical variables c_k and c_k^*. Note also that (39) and (40) contain a factor 3/2, which is not present in Eq. (11) of our previous paper.[71] The reason for this is that, in Ref. 71, we assumed that the mode driven by the direct current was the $k=0$ uniform mode, whereas here we consider standing-wave modes with $k>0$. In order to interpret the roles of the nonlinear interactions, we rewrite (39) using $n_k=c_k^* c_k$ for the number of magnons,

$$\frac{dc_k}{dt} = -i(\omega_k + S_k n_k)c_k - \left[\eta_k - \beta J\left(\frac{1-3n_k}{2NS}\right)\right]c_k. \tag{41}$$

This equation shows that when a current above the critical value traverses the film, the spin-wave amplitude initially grows exponentially while the number of magnons n_k is negligible. However, as n_k increases and approaches NS the STI driving decreases and its effect is balanced by the relaxation, so that the spin-wave amplitude saturates. On the other hand, the effect of the nonlinear term arising from the surface dipolar and anisotropy interactions is to shift the spin-wave

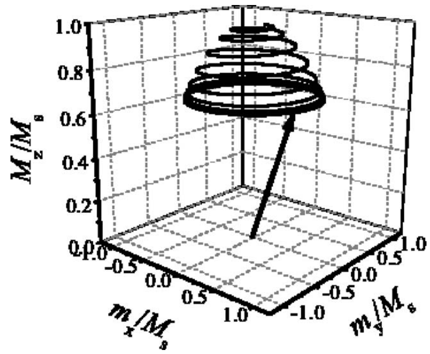

FIG. 4. Trajectory of the magnetization tip in a film with the field and wave vector perpendicular to the plane driven by a spin-polarized current for $I=2I_c$ applied at $t=0$ when the magnetization was near the equilibrium direction z.

frequency upward as the number of magnons increases. Further understanding of this process can be gained by solving (41), which affords a full analytical solution. With (41) and its complex conjugate, one can obtain a simple equation for the magnon number. Using the normalized variables $n'_k = n_k/NS$, $t' = 2\eta_k t$, $r=\beta J/\eta_k = I/I_c$, where r is the driving parameter, the equation takes the form

$$\frac{dn'_k}{dt'} = (r-1)n'_k - \frac{3}{2}rn'^2_k. \tag{42}$$

This is the Bernoulli equation, which has an analytical solution for an excitation r applied at $t=0$ in the form of a step function

$$n'_k(t') = \frac{r-1}{3r/2 - v_0 e^{-(r-1)t'}}, \tag{43}$$

where $v_0 = 3r/2-(r-1)/n'_0$, n'_0 is the (normalized) initial number of magnons, assumed to be the thermal value. Equation (43) shows that with driving $r>1$, or $I>I_c$, the number of magnons increases rapidly and saturates at times $t \gg [(r-1)\eta_k]^{-1}$ with a value

$$n_s = \frac{2NS}{3}\frac{(r-1)}{r} = \frac{2NS}{3}\frac{(I-I_c)}{I}. \tag{44}$$

Equation (42) can be used to calculate the evolution of the components of the magnetization given by (26). Since $u_k = 1$ and $v_k = 0$, the amplitudes of the real and imaginary parts of c_k are equal and given by $n_k^{1/2}$. Figure 4 shows a three-dimensional (3D) plot of the trajectory of the magnetization tip for $I=2I_c$ obtained from the solution (43) and considering the magnetization averaged over the lateral coordinate of the film. The magnetization precesses in a circular manner with increasing cone angle and reaches a stable trajectory after a transient time $t \gg [(r-1)\eta_k]^{-1}$. Equation (44) can be used in Eq. (6c) to calculate the z component and, hence, the cone angle of the magnetization precession. For strong driving, $I \gg I_c$, or $r \gg 1$, the number of magnons saturates at $n_s = 2NS/3$, corresponding to a cone angle of 70.5°. It is likely that the spin-wave approach does provide an accurate description of the phenomenon at such a level of excitation. However, as we shall show, it holds quite well for currents at least as large as $I=2I_c$. For such current, the saturation number is $n_s = NS/3$, corresponding to a cone angle of 48.2°.

Note that the nonlinear terms in Eq. (41) have negligible effect if $I<I_c$ because, in this case, the magnon number has a thermal value that is much smaller than the number of spins N. On the other hand, if $I>I_c$ the magnon number is comparable to NS and both nonlinear terms have important effects on the spin dynamics. Although the term in $\beta J n_k$ limits the growth of the spin-wave amplitude, the term in S_k arising from the surface dipolar and anisotropy energies produces a shift in frequency with increasing driving current given by $\delta\omega_k = S_k n_k$. Thus, with (40) and (44) we obtain the frequency shift for the case of field and wave vector perpendicular to the film plane,

$$\delta\omega_k = \gamma 4\pi M_{\text{eff}} \frac{I-I_c}{I}. \tag{45}$$

Clearly, the frequency shifts upward with increasing current (blueshift) as observed experimentally.[69] Note that Eq. (45) coincides with the result obtained in Ref. 71 because the extra factor of 3/2 appearing in S_k is canceled by the factor 2/3 in the saturation number given by Eq. (44).

Consider now the configuration of case (ii), i.e., the field is normal to the plane of the film and the wave vector is in the plane. Here, the equation of motion for the magnon operator is more complicated because of the transformations (13). Using (13) in (37) one obtains a four-magnon interaction Hamiltonian containing all possible combinations of the operators c_k^+ and c_k, each term with a coefficient involving the parameters u_k and v_k. A similar treatment is carried out for the nonlinear term arising from the STI torque, leading to an equation of motion for c_k in the form

$$\begin{aligned}\frac{dc_k}{dt} = &-i\omega_k c_k - (\eta_k - \beta J)c_k - \frac{3\beta J}{2SN}[(u_k^2+v_k^2)c_k^* c_k c_k \\ &+ u_k v_k(c_k^* c_k^* c_k + c_k c_k c_k)] - iS_k(\beta_1 c_k^* c_k c_k + \beta_2 c_k^* c_k^* c_k \\ &+ \beta_3 c_k^* c_k^* c_k^* + \beta_4 c_k c_k c_k)\end{aligned} \tag{46}$$

where the coefficients beta are given by

$$\beta_1 = u_k^4 + 4u_k^2 v_k^2 + v_k^4, \tag{47a}$$

$$\beta_2 = 3u_k^3 v_k + 3u_k v_k^3, \tag{47b}$$

$$\beta_3 = 2u_k^2 v_k^2, \tag{47c}$$

$$\beta_4 = 2u_k^3 v_k + u_k v_k^3, \tag{47d}$$

where the coefficients u_k and v_k are given by (17), (18), and (20). Note that for $\boldsymbol{k}=0$, $u_k=1$ and $v_k=0$, so that (46) is the same as (39), as it should.

Consider, finally, case (iii) for which the applied field and the wave vector are in the plane of the film. In order to obtain the equation of motion, we first calculate the magnon interactions arising from the surface dipolar and anisotropy energies. Considering a coordinate system with the y axis perpendicular to the plane, the energy (36) contains only the component S_y^2, which can be written in terms of the magnon operators with the expansion in (7b). Using transformations (7b) and (8) in Eq. (36), assuming that only one standing-wave mode is present, and introducing the effect of the modulation of the magnetic charges in the surface dipolar energy, one obtains

$$\mathcal{H}^{(4)} = -\hbar\gamma\frac{3\pi(M_{\text{eff}}-M_s kd/2)}{4NS}(2a_k^+a_k^+a_ka_k - a_k^+a_ka_ka_k - a_k^+a_k^+a_k^+a_k). \quad (48)$$

Calculations similar to the previous case lead to an equation of motion for c_k in the form

$$\frac{dc_k}{dt} = -i\omega_k c_k - (\eta_k - \beta J)c_k - \frac{3\beta J}{2SN}[(u_k^2+v_k^2)c_k^*c_kc_k + u_kv_k(c_k^*c_k^*c_k + c_kc_kc_k)] - iT_k(\alpha_1 c_k^*c_kc_k + \alpha_2 c_k^*c_k^*c_k + \alpha_3 c_k^*c_k^*c_k^* + \alpha_4 c_kc_kc_k), \quad (49)$$

where

$$T_k = \frac{-\gamma 3\pi(M_{\text{eff}}-M_s kd/2)}{4NS}, \quad (50)$$

and the other factors are

$$\alpha_1 = 4u_k^4 - 12u_k^3v_k + 16u_k^2v_k^2 - 12u_kv_k^3 + 4v_k^4, \quad (51a)$$

$$\alpha_2 = -3u_k^4 + 12u_k^3v_k - 18u_k^2v_k^2 + 12u_kv_k^3 - 3v_k^4, \quad (51b)$$

$$\alpha_3 = -4u_k^3v_k + 8u_k^2v_k^2 - 4u_kv_k^3, \quad (51c)$$

$$\alpha_4 = -u_k^4 + 4u_k^3v_k - 6u_k^2v_k^2 + 4u_kv_k^3 - v_k^4, \quad (51d)$$

where u_k and v_k are given by (17), (18), and (23). Note here again that Eqs. (49) and (50) contain factors of 3/2 relative to the equations in Ref. 71 for the same reason explained earlier. Note also that the negative sign of the term appearing in Eq. (16) of Ref. 71 was a misprint; the correct positive sign as in (49) was actually used in the numerical calculations.

Equations (46) and (49) cannot be solved analytically as was done for the case of field and wave vector perpendicular to the film. However, the roles of the nonlinear terms are the same, while the term in βJ limits the growth of the spin-wave amplitude, the terms with S_k in (46) and with T_k in (49) produce a shift in frequency with increasing the driving current. An important point to be noted is that since S_k is positive and T_k is negative, in the case of field perpendicular to the film, the frequency increases with increasing current (blueshift); whereas when the field is in the plane of the film, the frequency decreases with increasing current (redshift), as observed in experiments.

V. NUMERICAL RESULTS AND COMPARISON TO EXPERIMENTS

In this section, we present numerical solutions of the equations derived previously in order to compare theory to the experimental results recently reported. Consider, initially, the configuration of the field and wave vector in the plane of the film as in the experiments of Rippard *et al.*[67] and Krivorotov *et al.*[70] The equations for the real and imaginary parts of c_k obtained from (49) have been solved numerically using conditions and material parameters to compare to the reported data. First, it is important to make some considerations about the driven mode. As remarked at the end of Sec. III, the measurements of the oscillations induced by direct currents in multilayers do not allow a clear-cut identification of the excited spin-wave mode. However, the fact that in order to radiate efficiently the mode should have the smallest possible value of k, strongly suggests that the driven mode is the lowest-order standing-wave mode allowed by the structure [i.e., it is a mode with wave function given by (11) with $k=\pi/L$], where L is the relevant lateral dimension. In the case of the experiments of Rippard *et al.*,[67] the lateral dimension of the spin-valve mesa is on the order of 10 μm, so that the wave number is $k \approx 3\times10^3$ cm^{-1}. One can see in Fig. 2 that this value corresponds to a quasi-uniform mode so that the actual k cannot be determined only from the measurement of the oscillation frequency. In the experiments of Krivorotov *et al.*,[70] the sample is a nanopillar with an elliptical shape, with dimensions 130×60 nm^2; hence, we consider L on the order of 100 nm, so that the wave number of the lowest-order mode is $k\approx3\times10^5$ cm^{-1}. In this case, we see from Fig. 2 that the contribution of the exchange energy to the frequency is sizeable so that the value of k could be determined from the frequency as long as the other parameters, saturation and effective magnetizations and anisotropy field, were accurately measured with another technique, such as FMR. Since the information on the driven modes in Refs. 67 and 70 is insufficient, we will assume that they are the lowest-order standing-wave modes with $k\approx3\times10^3$ cm^{-1} and $k\approx3\times10^5$ cm^{-1}, respectively.

Krivorotov *et al.*[70] studied nanopillar-shaped samples made of two 4 nm thick permalloy ($\text{Py}=\text{Ni}_{80}\text{Fe}_{20}$) layers separated by an 8 nm thick Cu spacer layer on top of an antiferromagnetic underlayer for exchange biasing of the bottom Py layer. The current is applied to the multilayer through Cu electrodes on both sides so that it is uniformly distributed over the lateral dimension. The equilibrium direction of the magnetization in the top layer is determined by the balance between Zeeman, shape anisotropy and interlayer coupling, and the angle between the magnetizations in the two layers is estimated to be 30°. When a current I is applied to the multilayer, electrons flow from the top to the bottom layer and a STI torque drives the magnetization of the free layer. The dynamics of the magnetization was observed both with time-resolved measurements and with spectral analysis of the voltage induced by the precessing magnetization in the free layer. We have solved Eq. (49) with the conditions and parameters to compare our model theory to the data presented in Fig. 3 of Ref. 70, namely, $H_0=0$, $\omega_k=2\pi\times4.275$ GHz, $d=4$ nm, $I_c=1.25$ mA, and $4\pi M_{\text{eff}}=0.81$ kG. Since Ref. 70 does do not provide the value of the g-factor, we use the same value measured in Ref. 67, namely, $g=1.78$. We do not understand the reason why the measured g (Ref. 67) is well below the value obtained with FMR for 3d transition metals, which is close to 2.1. We speculate that in the regime of high spin-wave excitation, characteristics of

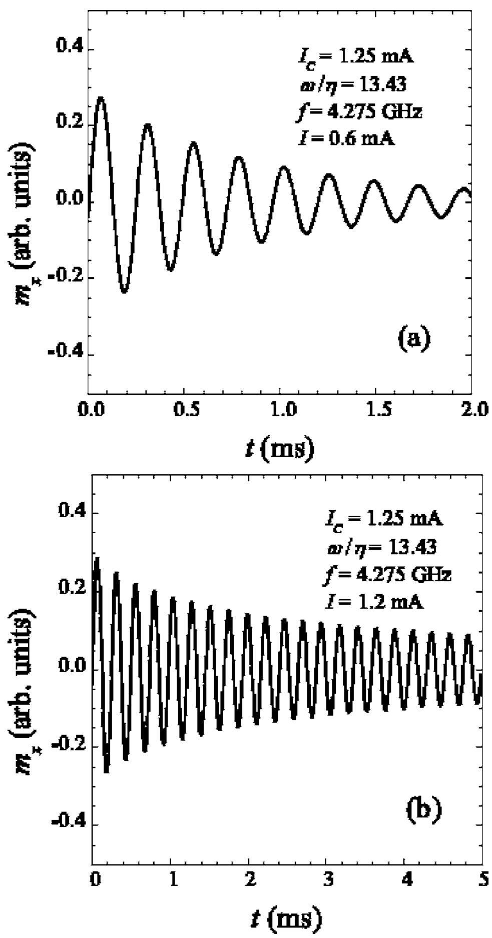

FIG. 5. Time dependence of the magnetization m_x in a film magnetized in the plane, resulting from the application of a spin-polarized direct current $I<I_c$, in the conditions of the experiments reported in Ref. 70: (a) $I=0.6$ mA and (b) $I=1.2$ mA.

the STI driving, the momentum transfer from the magnetic system to the lattice is significantly modified. Also consider $4\pi M_s=10.0$ kG as appropriate for Py. From these we determine the effective field $H_{AEK}=H_{an}+H_E+H_k=0.356$ kOe, where H_k is the contribution of the $\boldsymbol{k}$-dependent terms in (24), to match the observed oscillation frequency. These parameters are used in (16)–(18) and (23) to obtain $u_k=1.32$ and $v_k=0.87$. The relaxation rate obtained by extrapolating to $I=0$ the data of Fig. 3 in Ref. 70 is $\eta_k=2$ ns^{-1}.

Figure 5 shows the time evolution of the m_x component of the magnetization given by (26a), obtained from the integration of (49) for two values of the current $I<I_c$, assuming arbitrary initial values at instant $t=0$ for the real and imaginary parts of c_k. In both cases, the magnetization decays in time, as expected. However, since the effective relaxation rate is $\eta_k-\beta J$, as the current increases the relaxation rate decreases and the decay time increases. For currents above the critical value $I_c=A_c\eta_k/\beta$, instead of decaying, the amplitude of the magnetization grows to saturation and a sustainable precession is established. Figure 6 shows a 3D plot of the trajectory of the magnetization tip for $I=1.7$ mA. Since the amplitude of the magnetization in a standing-wave mode varies across the film, we have calculated the spatially aver-

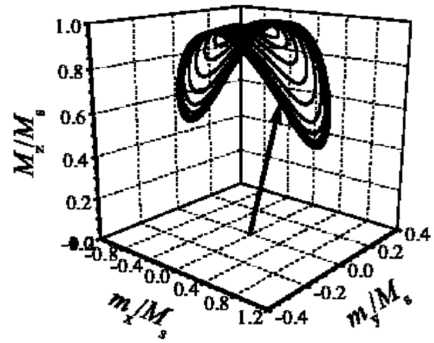

FIG. 6. Trajectory of the magnetization tip in a film with field and wave vector in the plane driven by a spin-polarized current for $I=1.7I_c$ applied at $t=0$ when the magnetization was near the equilibrium direction z. The parameters used in the calculation are appropriate for the sample of Ref. 70.

aged value. Figure 7(a) shows the spectra of m_x for several current values used in experiments to compare to the data of Fig. 3(a) of Ref. 70. The spectra were obtained with a fast Fourier transform of the time series of m_x for several hundred cycles and passed through a frequency filter with the shape of a Lorentzian function to the fourth power and width

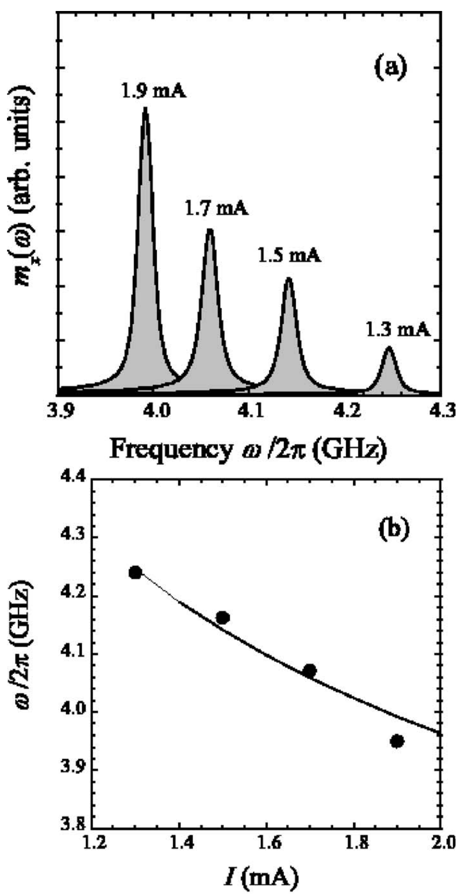

FIG. 7. (a) Spectra of the m_x component of the magnetization obtained by the numerical solution of Eq. (49) for several values of the driving current to compare to the data of Ref. 70. (b) The solid line shows the theoretical frequency vs current while the symbols represent the data of Ref. 70.

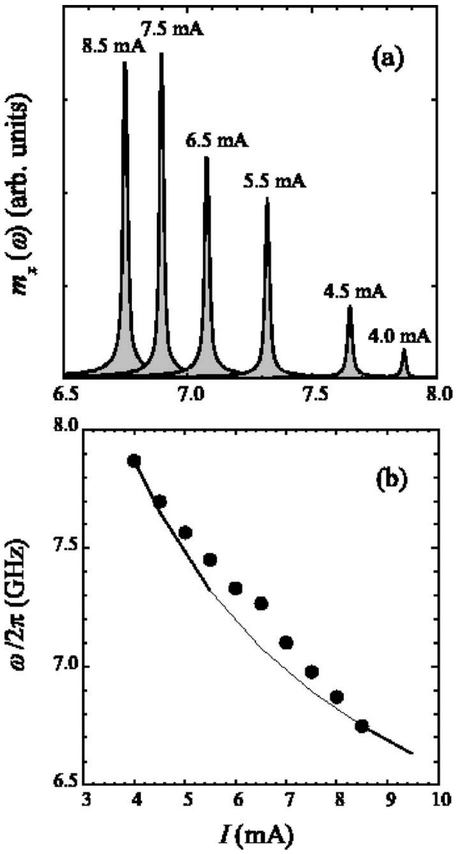

FIG. 8. (a) Spectra of the m_x component of the magnetization obtained by the numerical solution of Eq. (49) for several values of the driving current to compare to the data of Ref. 67. (b) The solid line shows the theoretical frequency vs current while the symbols represent the data of Ref. 67.

similar to the instrumental width of Ref. 70. The solid line in Fig. 7(b) shows the calculated frequency vs current while the symbols represent the data. Note that in order to fit the data, the effective magnetization used in the nonlinear parameter (50) is $4\pi M_{\text{eff}}=7.9$ kG, which is very close to the value given in Ref. 70.

The same in-plane field configuration has been used by Rippard *et al.*[67] to observe voltage oscillations produced by a direct current traversing a Py layer in a spin-valve structure Ta(2.5 nm)/ Cu(50 nm)/ Co90Fe10 (20 nm)/ Cu(5 nm)/ Ni80Fe20 (5 nm)/ Cu(1.5 nm)/ Au(2.5 nm). The current is applied to the multilayer through point contacts with a circular cross section with a nominal diameter of 40 nm. The field dependence of the oscillation frequency is used to measure the g-factor, and the FMR technique is used to obtain $4\pi M_{\text{eff}}$. Numerical solutions of (49) were used to calculate the spectra of the m_x component of the magnetization for several values of the driving current, using the parameters of the experiments: $4\pi M_{\text{eff}}=8$ kG, $4\pi M_s=10.0$ kG, $d=5$ nm, $\omega_k=2\pi\times 7.9$ GHz, $g=1.78$, and $I_c=3.95$ mA, from which we determine the coefficients $u_{\boldsymbol{k}}=1.14$ and $v_{\boldsymbol{k}}=0.55$. In order to fit the calculated frequency shift to the experimental data, a smaller value for the effective magnetization has to be used, namely, $4\pi M_{\text{eff}}=7.5$ kG. The spectra shown in Fig. 8 is remarkably similar to the that in Fig. 1 of Ref. 67. Note

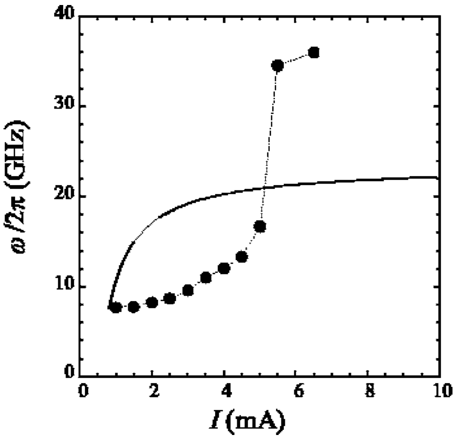

FIG. 9. Frequency vs current for a film magnetized in the perpendicular direction. The curve is a plot of Eq. (45) with parameters appropriate for the sample used in Ref. 69. The symbols represent the data of Ref. 69.

that since, in the parallel configuration the precession of the magnetization is elliptical, the spectra of the M_z component of the magnetization exhibit peaks at the second harmonic, as observed experimentally.[67]

Although the comparison between theory and experiment is very satisfactory for the case of field in the plane, this is not true for the case of field perpendicular to the film. Kiselev *et al.*[69] have reported the observation of voltage oscillations in nanopillar samples, with a 130 nm$\times$70 nm elliptical cross section, structured with composition Cu(80 nm)/ Co(40 nm)/ Cu(10 nm)/ $Ni_{80}Fe_{20}$ (3 nm)/ Cu(2 nm)/ Pt(30 nm), with the field perpendicular to the plane. The data of Ref. 69 exhibit several features which are not explained by the simple theory presented in Sec. IV. The only observation that confirms the prediction of the model is the fact that the oscillation frequency increases with increasing current (blueshift). The data reveal two striking features: the oscillations are observed in a wide range of current, from 0.8 to 10 mA, while in the experiments with the field in the plane the range in only up to $2I_c$; the frequencies are characterized by jumps at certain current values. As discussed in Sec. IV, if we assume that the wave vector is perpendicular to the plane, the spin-wave frequency is given by $\omega_{\boldsymbol{k}}(I=0)+\delta\omega_{\boldsymbol{k}}$, where $\delta\omega_{\boldsymbol{k}}$ is given by (45). The symbols in Fig. 9 represent the data of Ref. 69 while the solid line is the result of the model calculated with the parameters of Ref. 69: $H_0=7.5$ kOe, $\omega_k(I=0)=2\pi\times 7.5$ GHz, $g=2.23$, $4\pi M_{\text{eff}}=5.1$ kG, and $I_c=0.8$ mA. Note that the curve in Fig. 9 bears great resemblance to the theoretical curve in Fig. 2(d) of Ref. 69, obtained by solving the Landau-Lifshitz equation with the STI torque. Evidently both theoretical predictions grossly overestimate the measured frequency shift in the range $I<5$ mA and cannot be correct. In an attempt to remedy this we assume that the mode initially excited by the current is the lowest-order standing-wave mode along the lateral dimension of the film, for which $k\approx 3\times 10^5$ cm^{-1}. However, with the parameters of the sample used in Ref. 69, the coefficients in (46) and (47) are $u_{\boldsymbol{k}}=1.05$ and $v_{\boldsymbol{k}}=0.32$, and the frequency shift obtained from the numerical solution of (46) turns out to be almost identical to the one calculated with (45).

We believe that the failure of the model is an indication that the mode excited in the film with perpendicular magnetization is indeed a standing wave along the lateral dimension. In such a mode the number of magnons varies from nearly NS at the lobe of the mode in the center to zero at the nodes in the edges. Hence, the z component of the magnetization varies along the film by an amount on the same order of magnitude as $4\pi M_{\rm eff}$, resulting in an internal magnetic field that is highly nonuniform. In this case, the plane-wave functions (10) and (11) are not solutions of the equations of motion so that the model completely breaks down. The jumps observed in the oscillation frequency may also be another indication that the excited modes are standing waves along the film. In a nonuniform, symmetrical internal field, the lowest-order mode wave function has a shape that resembles the curve at the bottom of Fig. 1, though not described by a cosine function. The contribution of the exchange energy to the frequency of such a mode is on the order of $\gamma D(\pi/L)^2$, which is 0.92 GHz for the sample used in Ref. 69. As the current increases, the amplitude of the mode increases, so that the magnetoresistance in the center of the film becomes larger at the edges. This results in a redistribution of the current density, which increases near the edges at the expense of the center area. As a consequence, the driving torque decreases for the lowest-order mode and increases for modes with lobes near the edges. With increasing current, one of the higher-order modes is eventually excited when its relaxation rate is overcome by the torque, producing a jump in the oscillation frequency. Note that the excitation of the third-order mode would result in a change in frequency of about $24\gamma D(\pi/L)^2$, which is 18 GHz for the sample of Ref. 69, a value close to the variation observed when the current reaches 5.5 mA, as shown in Fig. 2(c) of Ref. 69.

VI. CONCLUSION

We have presented a spin-wave theory for the driving of the magnetization in a thin film by a direct current traversing a magnetic multilayer. When nonlinear effects due to magnon interactions are incorporated in the formalism, the model accounts for the stabilization of the magnetization precession and the frequency shift occurring with increasing current, predicting downward frequency shifts (redshifts) with increasing current if the external field is applied on the film plane, and upward frequency shifts (blueshifts) with the field perpendicular to the film. Note that the red- and blueshifts in the spin-wave frequencies could also be interpreted as a simple consequence of the decrease in magnetization caused by the magnon excitation, expressed by Eqs. (19), (21), and (24). However, the full nonlinear theory is necessary to explain quantitatively recent experimental observations of microwave oscillations in nanostructures with a magnetic field applied in the film plane. The fact that the theory fails to account for the experimental data when the field is applied perpendicularly to the plane is attributed to the highly nonuniform internal field generated when standing-wave modes are excited.

In closing, we remark that when this paper was in the final preparation stages we learned of a paper by Slavin and Kabos,[87] which presents a nice analytical calculation of the frequency shifts in a film excited by a direct current magnetized by a field in an arbitrary direction. Their results are in general agreement with the ones presented here as well as in our previous paper[71] for the particular cases of field parallel and perpendicular to the film. As such they are also in disagreement with the experimental data[69] for the case of field perpendicular to the film. More recently, Hoefer *et al.*[88] published a calculation of the fully nonlinear equation of motion for the magnetization driven by a current applied to a perpendicularly magnetized ferromagnetic film by a nanocontact. They have considered the spatial variation of the internal static magnetic field thus obtaining a good agreement with the experiments of Ref. 69 for currents up to 5 mA, confirming our conjectures for the reason of the failure of the plane spin-wave theory.

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