

Persistent current and Drude weight in mesoscopic rings

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We study the persistent current and the Drude weight of a system of spinless fermions, with repulsive interactions and a hopping impurity, on a mesoscopic ring pierced by a magnetic flux, using a density matrix renormalization group (DMRG) algorithm for complex fields. Both the Luttinger liquid (LL) and the charge density wave (CDW) phases of the system are considered. Under a Jordan-Wigner transformation, the system is equivalent to a spin-1/2 XXZ chain with a weakened exchange coupling. We find that the persistent current changes from an algebraic to an exponential decay with the system size, as the system crosses from the LL to the CDW phase with increasing interaction U . We also find that in the interacting system, the persistent current is invariant under the impurity transformation $\rho \rightarrow 1/\rho$, for large system sizes, where ρ is the defect strength. The persistent current exhibits a decay that is in agreement with the behavior obtained for the Drude weight. We find that in the LL phase the Drude weight decreases algebraically with the number of lattice sites N , due to the interplay of the electron interaction with the impurity, while in the CDW phase, it decreases exponentially, defining a localization length which decreases with increasing interaction and impurity strength. Our results show that the impurity and the interactions always decrease the persistent current, and imply that the Drude weight vanishes in the limit $N \rightarrow \infty$ in both phases.

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I. INTRODUCTION

The experimental discovery of persistent currents in mesoscopic rings pierced by a magnetic flux,¹⁻³ earlier proposed theoretically,⁴ has revealed interesting new effects. The currents measured in metallic and semiconducting rings, either in a single ring or an array of many rings, generally exhibit an unexpectedly large amplitude, i.e., larger by at least one order of magnitude, than predicted by theoretical studies of electron models with either disorder or electron-electron interaction treated perturbatively.^{5,6} It has been suggested that the interactions and their interplay with disorder are possibly responsible for the large currents observed, expecting that the effect of the interactions could counteract the disorder effect. However, no consensus has yet been reached on the role of the interactions. In order to gain theoretical insight, it is desirable to perform numerical calculations which allow us to consider both interactions and disorder directly in systems with sizes varying from small to large. Analytical calculations usually involve approximations which mainly provide the leading behavior of the properties for large system sizes. Persistent currents in mesoscopic rings strongly depend on the system size, since they emerge from the coherence of the electrons across the entire system. Hence, it is most important to study the size dependence of the current beyond leading order in microscopic models, for a complete understanding of the experimental results. Exact diagonalization was used to calculate persistent currents in systems with very few lattice sites.^{7,8} In this paper, we use the density matrix renormalization group (DMRG) algorithm⁹⁻¹¹ to study a simplified model incorporating interactions and a single impurity, accounting for disorder, in larger system sizes. We consider a system of interacting spinless electrons

on a one-dimensional ring, with a single impurity, and penetrated by a magnetic field. We study an intermediate range of system sizes, where analytical results obtained by bosonization techniques for large system sizes do not yet fully apply. Without impurity, and at half-filling, the system undergoes a metal-insulator transition from a Luttinger liquid¹² (LL) to a charge density wave¹³ (CDW) ground state. The persistent current of the interacting system with an impurity was studied before with the DMRG, in the LL phase.¹⁴ Here we study the persistent current, and also the Drude weight characterizing the conducting properties of the system, in both the LL and the CDW phase, investigating the interplay between the impurity and the interactions in the two phases. In mesoscopic systems, the separation between metallic and insulating behavior is not always obvious, since the localization length can be of the order of or significantly larger than the system size. Hence, a finite Drude weight and a current can be observed in the CDW phase of a mesoscopic system. It is therefore of great interest to characterize the persistent current and the Drude weight in both the LL and the CDW phases of mesoscopic systems. Although the simple model that we consider is not the most appropriate to describe the experimental situation, we hope to obtain useful information for the understanding of the more realistic systems. Under a Jordan-Wigner transformation,¹⁵ the system considered is equivalent to a spin-1/2 XXZ chain with a weakened exchange coupling. Hence, our results also provide insight into the spin transport in this type of system.

II. THE MODEL

The Hamiltonian describing a system of spinless fermions on a ring pierced by a magnetic flux, with repulsive interac-

tions and a single hopping impurity, or defect, is given by,

$$H = H_t + H_U, \quad (1)$$

where

$$H_t = -t \sum_{j=1}^N (e^{i\phi/N} c_j^\dagger c_{j+1} + e^{-i\phi/N} c_{j+1}^\dagger c_j) \\ + (1-\rho)t(e^{i\phi/N} c_N^\dagger c_1 + e^{-i\phi/N} c_1^\dagger c_N) \quad (2)$$

is the hopping term, $\phi=2\pi\Phi/\Phi_0$ contains the magnetic flux Φ in units of the flux quantum $\Phi_0=hc/e$, ρ measures the strength of the defect with values between 0 and 1 ($\rho=1$ corresponding to the defectless case), and

$$H_U = U \sum_{j=1}^N n_j n_{j+1} \quad (3)$$

is the interaction term, with $U>0$ representing the nearest neighbor Coulomb repulsion, and $n_j=c_j^\dagger c_j$, where c_j^\dagger and c_j are the spinless fermion operators acting on the site j of the ring. We consider a system of N sites, with N even, and at half-filling, when $M=N/2$ particles are present. The lattice constant is set to one and periodic boundary conditions, $c_{N+1}=c_1$, are used.

Via the gauge transformation $c_j \rightarrow e^{-i\phi j/N} c_j$, the flux can be removed from the Hamiltonian, but in the impurity term where the flux is trapped, and the quantum phase ϕ is encoded in a twisted boundary condition $c_{N+1}=e^{-i\phi} c_1$. It is then clear that the energy is periodic in ϕ with period 2π , i.e., it is periodic in the flux Φ threading the ring with period Φ_0 .¹⁶

After a Jordan-Wigner transformation, Eqs. (2) and (3) can be rewritten, respectively, as

$$H_J = -\frac{J}{2} \sum_{j=1}^N (S_j^+ S_{j+1}^- + S_j^- S_{j+1}^+) + (1-\rho) \frac{J}{2} (e^{i\phi} S_1^+ S_N^- + e^{-i\phi} S_1^- S_N^+) \quad (4)$$

and

$$H_\Delta = \Delta \sum_{j=1}^N S_j^z S_{j+1}^z \quad (5)$$

with $t=J/2$ and $U=\Delta$, and the boundary conditions $S_{N+1}^+ = (-1)^{M+1} e^{i\phi} S_1^+$ and $S_{N+1}^- = S_1^-$. Hence, the model (1) of spinless fermions is equivalent to a spin-1/2 XXZ chain with a weakened exchange coupling, and twisted boundary conditions in the transverse direction. The half-filled case corresponds to total spin projection $S^z=0$.

The persistent current generated on a ring pierced by a magnetic flux, at temperature $T=0$, can be obtained from the ground state energy $E_0(\phi)$, by taking the derivative with respect to ϕ ,

$$I(\phi) = -\frac{\partial E_0(\phi)}{\partial \phi}. \quad (6)$$

For the spinless fermion system, Eqs. (2) and (3), $I(\phi)$ corresponds to the ground state value of the charge current operator $\hat{I}_c = it \sum_{j=1}^N (c_j^\dagger c_{j+1} - c_{j+1}^\dagger c_j)$, while for the XXZ chain,

Eqs. (4) and (5), it corresponds to the ground state value of the spin current operator $\hat{I}_s = iJ/2 \sum_{j=1}^N (S_j^+ S_{j+1}^- - S_{j+1}^+ S_j^-)$. As a consequence of the periodicity of the energy, the current is also periodic in ϕ , with period 2π . Hence, it can be expressed as a Fourier series,

$$I(\phi) = \sum_{k=1}^{\infty} I_k \sin(k\phi), \quad (7)$$

and the behavior of the current can be analyzed in terms of the coefficients I_k .¹⁴

In the noninteracting case ($U=0$), it has been found that for large system sizes, the current is invariant under the defect transformation $\rho \rightarrow 1/\rho$,^{17,18} i.e.,

$$I(\rho) = I(1/\rho). \quad (8)$$

We shall investigate the existence of this kind of invariance in the interacting case ($U>0$), both in the LL and the CDW phases.

The Drude weight was proposed by Kohn as a relevant quantity to distinguish between a metal and an insulator.¹⁹ It is defined as

$$D = N \left. \frac{\partial^2 E_0}{\partial \phi^2} \right|_{\phi=\phi_m}, \quad (9)$$

where ϕ_m is the location of the minimum of $E_0(\phi)$, which depends on the parity of the number of electrons, i.e., $\phi_m=0$ or π for, respectively, an odd or an even number of electrons. For the spinless fermion system D represents the charge stiffness and measures the inverse of the effective mass of the charge carriers.²⁰ In a metallic conductor, D tends to a finite value, whereas in an insulator, D vanishes with the system size N , when $N \rightarrow \infty$. In the insulating state, the Drude weight decays as $D \sim \exp(-N/\xi)$, where ξ measures the localization length. For the XXZ chain, the Drude weight represents the spin stiffness.

In a model of free fermions ($U=0$) with no impurity ($\rho=1$), it is straightforward to see that the leading behavior of the persistent current $I(\phi)$ in the system size N , has a sawtoothlike shape with slope $-v_F/(\pi N)$, where v_F is the Fermi velocity. Thus, the amplitude of the current scales with $1/N$, vanishing in the limit $N \rightarrow \infty$. The discontinuity in $I(\phi)$, that results from a degeneracy of energy levels associated to the translation symmetry,⁸ appears at $\phi=0$ or π for, respectively, an even or an odd number of electrons. In the presence of an impurity ($\rho \neq 1$), bosonization²¹ and conformal field theory¹⁷ calculations predict that the shape of the current $I(\phi)$ is rounded off, and its amplitude decreases with increasing strength of the scatterer potential, still vanishing with the system size as $1/N$. The disorder lifts the degeneracy of the energy levels and the current then varies continuously.⁸

The model, with interactions ($U \neq 0$) and without defect ($\rho=1$), is solvable by the Bethe ansatz for periodic boundary conditions ($\phi=0$)^{22,23} and also for twisted boundary conditions ($\phi \neq 0$).²⁴⁻²⁶ At half-filling, the system exhibits a metal-insulator transition, which occurs at $U/t=2$. For $U/t < 2$, the system is in a gapless LL phase, while for $U/t > 2$ it is in a gapped CDW state.²⁷ The LL phase is characterized by a

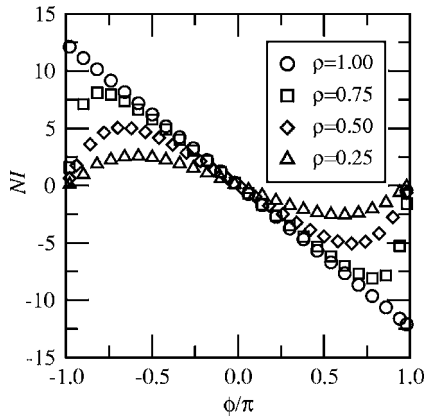


FIG. 1. Persistent current $NI(\phi)$ vs ϕ at fixed $N=26$ for $U=0.80$ and different ρ .

power-law decay of the correlations. Bosonization predicts that in an homogeneous LL, the leading behavior of the persistent current in the system size N , has a sawtoothlike shape with slope $-v_j/(\pi N)$, where v_j is the velocity of current excitations.²⁸ Since translation invariance is preserved in the presence of interactions, the discontinuity in the current still exists for finite U .⁸ A Bethe ansatz calculation shows that the Drude weight of an homogeneous LL in the thermodynamic limit, has a finite value, which decreases with increasing strength of the interaction U/t .²⁰ The LL state is strongly affected by the presence of an impurity,²⁹⁻³² and bosonization yields that the current then vanishes as $I \sim N^{-1-\alpha_B}$, with $\alpha_B > 0$.²¹ The study of the LL phase with $\rho \neq 1$, performed with the DMRG,¹⁴ has in fact found this kind of behavior. The CDW phase is characterized by a localization length ξ , which is associated to the energy gap. From the work of Baxter,³³ the Drude weight in the gapped phase, is expected to behave as $D \sim \exp(-N/\xi)$, vanishing for an infinite system size. This behavior implies that although the system is insulating in the infinite system size limit, for a finite system, provided N/ξ is not too large, D is still finite and a current can be observed. The localization length can then be extracted from the size dependence of the Drude weight.

III. NUMERICAL METHOD

We use the DMRG to numerically calculate the ground state energy of the spinless fermion system as a function of

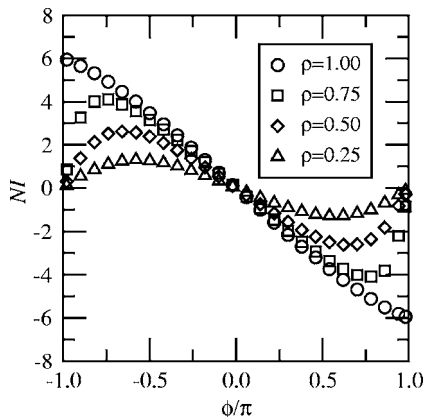


FIG. 2. The same as in Fig. 1, but for $U=3.00$.

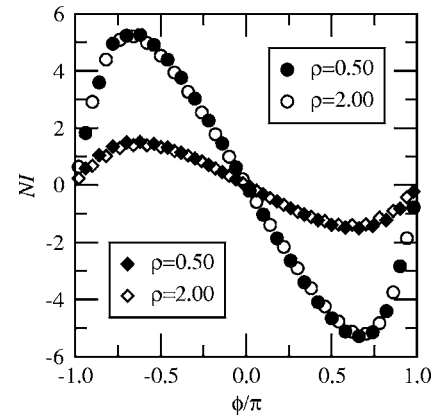


FIG. 3. Symmetry of the current $NI(\phi)$ in ρ (empty symbols) vs $1/\rho$ (filled symbols) for $U=0.80$ (circles) at $N=18$, and $U=3.00$ (diamonds) at $N=58$.

the magnetic flux, $E_0(\phi)$, for fixed interaction U and impurity strength ρ , in rings up to $N=82$ sites, keeping up to 300 density matrix eigenstates per block.³⁴ The DMRG is applied to the Hamiltonian (1) after performing the gauge transformation, which removes the flux into a twisted boundary condition.¹¹ The states of the system are characterized by the quantum numbers associated to the eigenvalues of the local occupation number n_j and the total number of particles $M = \sum_{j=1}^N n_j$ operators, which commute with the Hamiltonian (1).³⁵ For each set of N , ρ , and U , we obtained the ground state energy E_0 for 50 values of ϕ in the periodicity interval $-\pi < \phi \leq \pi$, and using Chebyshev interpolation,³⁶ we determined the corresponding current (6) and Drude weight (9), by numerical differentiation. We developed a DMRG algorithm for complex Hamiltonian matrices, which allowed us to calculate the detailed form of the persistent current I as a function of ϕ ¹⁴ and to obtain the Drude weight. In a previous approach, the DMRG was used to calculate the so-called phase sensitivity ΔE_0 , which is the difference of the ground state energy at flux $\phi=0$ and π , and can be considered a crude measure of the persistent current.³⁷⁻³⁹ Although the calculation of ΔE_0 requires considerably less computational effort than the calculation of $E_0(\phi)$, because then the Hamiltonian matrix is real, the phase sensitivity does not provide information on the shape of the current and the value of the Drude weight.

IV. RESULTS

We now present the results obtained for the persistent current and the Drude weight, where we take $t=1$ and the interaction U is in units of t . Figures 1 and 2, exhibit $NI(\phi)$ plotted versus ϕ , for respectively, $U=0.80$ and $U=3.00$, which correspond, respectively, to the LL and the CDW phase, considering different impurity strengths ρ , on a fixed system size $N=26$. We can see that the effect of the impurities, in both phases, is to reduce the intensity of the current and to round off the shape of $I(\phi)$. The amplitude of the current decreases rapidly with increasing values of $|\rho-1|$, and also with increasing strength of the interaction U . Figure

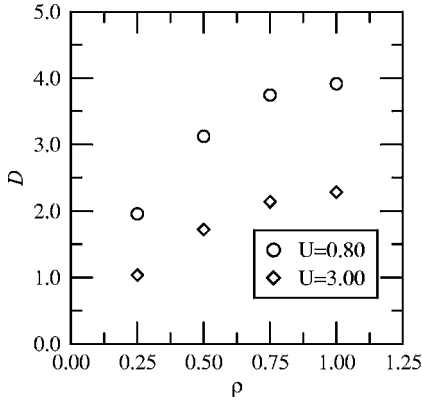


FIG. 4. The Drude weight D vs ρ at fixed $N=26$ for $U=0.80$ and $U=3.00$.

3 shows that the invariance of the current with respect to the defect, described in Eq. (8), found for large system sizes in the noninteracting case, is also observed for the interacting case, both in the LL and in the CDW phases, the system size N required to reach the invariance being larger for larger interaction U . Figure 4 displays the Drude weights associated to the systems with different interactions U and impurities ρ , fixed $N=26$, of the currents presented in Figs. 1 and 2. As one would expect, the Drude weight decreases with increasing $|\rho-1|$ and also increasing U . Figures 5 and 6 present $NI(\phi)$ plotted for several system sizes N , respectively, for $U=0.80$ in the LL phase and $U=3.00$ in the CDW phase with ρ fixed at 0.50. We observe that the current vanishes faster than $1/N$ in both phases, exhibiting a different behavior in each phase, $I(\phi)$ vanishing much faster with N in the CDW than in the LL phase. In order to analyze the behavior of the current in more detail, we have numerically evaluated the coefficients I_k (for $k=1, 2$) of the Fourier expansion (7). The first ($k=1$) and the second ($k=2$) Fourier coefficients of the current, for $U=0.80$ and $U=3.00$, are shown in Fig. 7. One can clearly see that the coefficients I_1 and I_2 behave similar to each other in both phases. However, their behavior in the LL and CDW phase is distinct. In the LL phase, the Fourier coefficients show a power-law decay with N , with the second order coefficient decaying faster, i.e., with a larger exponent,

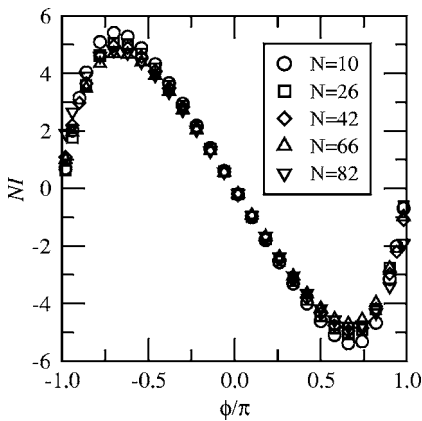


FIG. 5. Persistent current $NI(\phi)$ vs ϕ for $U=0.80$, $\rho=0.50$, and increasing N .

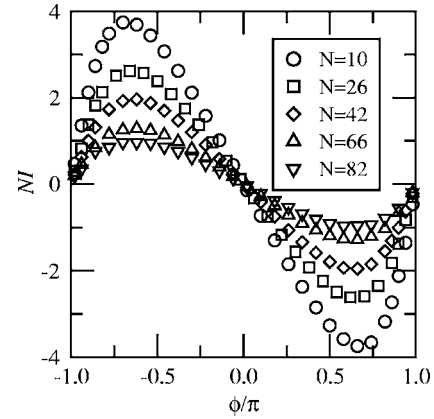


FIG. 6. The same as in Fig. 5, but for $U=3.00$.

than the first one. In the CDW phase, the Fourier coefficients show a dominant exponential decay with N , with the second order coefficient also decaying faster, i.e., with a smaller localization length, than the first one. We observe that for longer rings, stronger interactions and also stronger impurities, the current is increasingly more precisely described by its first Fourier component. In the LL phase, this in fact corresponds to the asymptotic behavior predicted by bosonization in the large N limit, that is $I \sim N^{-1-\alpha_B} \sin \phi$. However, for the system sizes considered here, this asymptotic regime is not reached and the current displays a more complex behavior. The current is composed of a few Fourier components with decreasing weight. Figure 8 presents the first Fourier coefficient of the current for different values of the interaction, $U=0.80, 2.50, 3.00$, fixed $\rho=0.50$, from which we extract the dependence of I_1 on N , in the intermediate range of sizes considered. We observe that for $U=0.80$ in the LL phase, the current varies as $I_1 \sim N^{-1-\alpha_1}$ with $\alpha_1 \approx 0.06$, while for $U=2.50$ and $U=3.00$ in the CDW phase, it varies as $I_1 \sim N^{-1-\delta_1} \exp(-N/\xi_1)$, respectively, with $\xi_1 \approx 259$, $\delta_1=0.11$, and $\xi_1 \approx 68$, $\delta_1=0.10$. The exponent α_1 is given by the slope of the straight line in Fig. 8(a), and the length ξ_1 and the exponent δ_1 were carefully adjusted in order to obtain the best collapse of the data in Fig. 8(b), on a plot of $\ln(N^{1+\delta_1}I_1)$ vs N/ξ_1 . The Drude weights characteriz-

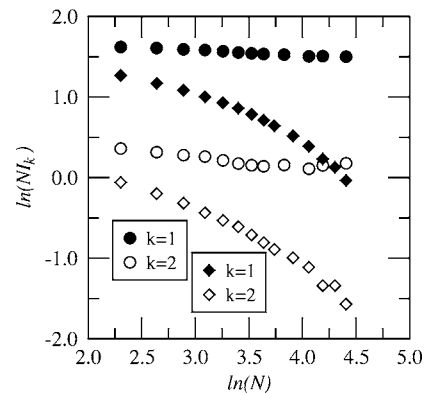


FIG. 7. Fourier coefficients of the current. $\ln(NI_k)$ vs $\ln(N)$ for $U=0.80$ (circles) and $U=3.00$ (diamonds) at $\rho=0.50$ for $k=1$ (filled symbols) and $k=2$ (empty symbols).

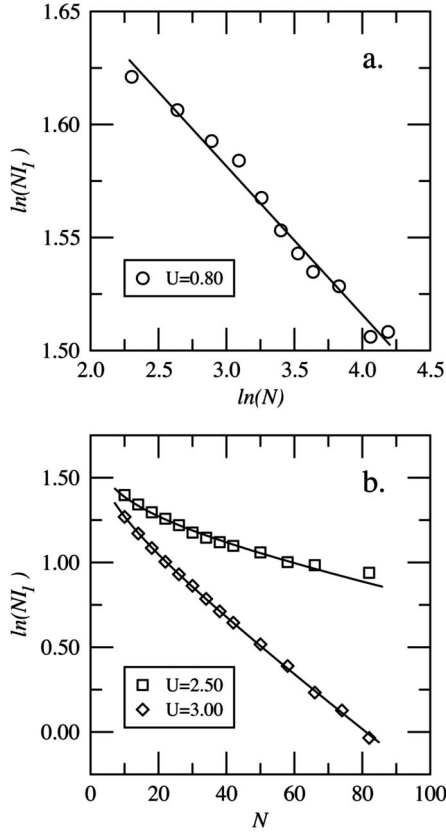


FIG. 8. NI_1 for different U and $\rho=0.50$. (a) $\ln(NI_1)$ vs $\ln(N)$ for $U=0.80$, the line represents $I_1 \sim N^{-1-\alpha_1}$. (b) $\ln(NI_1)$ vs N for $U=2.50$ and 3.00 , the lines represent $I_1 \sim N^{-1-\delta_1} \exp(-N/\xi_1)$, with ξ_1 and δ_1 dependent on U .

ing the systems with different interactions U , fixed $\rho=0.50$, are presented in Fig. 9. Figure 10 clearly shows that the results obtained for the Drude weight confirm the conducting behavior shown by the first coefficient of the currents in Fig. 8. We observe that, for $U=0.80$, the Drude weight varies with the system size as $D \sim N^{-\alpha}$ with $\alpha \approx 0.04$, while for $U=2.50$ and $U=3.00$ it varies as $D \sim N^{-\delta} \exp(-N/\xi)$, respectively, with $\xi \approx 307$, $\delta \approx 0.08$ and $\xi \approx 68$, $\delta \approx 0.06$. The exponent α is given by the slope of the straight line in Fig. 10(a), and the localization length ξ and the exponent δ were care-

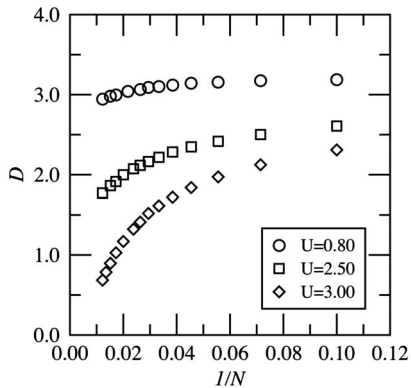


FIG. 9. The Drude weight D as a function of $1/N$, for different U and $\rho=0.50$.

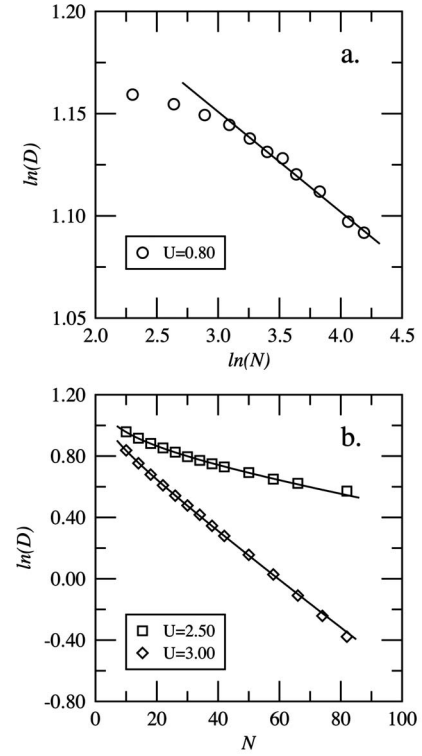


FIG. 10. D for different U and $\rho=0.50$. (a) $\ln(D)$ vs $\ln(N)$ for $U=0.80$, the line represents $D \sim N^{-\alpha}$. (b) $\ln(D)$ vs N for $U=2.50$ and 3.00 , the lines represent $D \sim N^{-\delta} \exp(-N/\xi)$, with ξ and δ dependent on U .

fully adjusted in order to obtain the best collapse of the data in Fig. 10(b), on a plot of $\ln(N^\delta D)$ vs N/ξ . The exponents and localization lengths characterizing the Drude weight are a little different from those characterizing the first Fourier component of the current, as one would expect, since the Drude weight contains the contribution from the various Fourier components. One sees that the localization length ξ and the exponent δ decrease with increasing strength of the interaction U . Also, concerning the impurity influence, Fig. 4 implies that the exponent α in the LL phase increases, and the localization length ξ in the CDW phase decreases, with increasing $|\rho-1|$. As mentioned, in the large N limit, the current is expected to behave as its first Fourier component, which in the LL phase implies that the exponent α_1 should be identified with $\alpha_B=1/K-1$ as calculated from bosonization,²¹ where K is the LL parameter, calculable from the Bethe ansatz.⁴⁰ For $U=0.80$, this leads to $\alpha_B \approx 0.27$, which is much larger than our value of α_1 . We should note that the size dependence found for the first Fourier component of the current, and the Drude weight, characterizes the behavior of an intermediate and limited range of system sizes. If one would consider a larger range of systems, in the LL phase, one would most probably see the data for the larger N bending down, crossing to an asymptotic power-law behavior with the exponent approaching α_B . This was observed in Ref. 14, where the behavior of the first few Fourier components of the current in the LL phase was discussed in detail, with data taken for larger values of N and stronger interaction and impurity strengths. Also, in the CDW phase

we consider systems in an intermediate regime where the localization length is larger or near the system size.⁴¹ For larger systems, the power factors that occur in the first Fourier component of the current and the Drude weight may decline,⁴² possibly leaving a pure exponential behavior in the asymptotic regime.

From the results obtained, we observe that the system with $U=0.80$ and $\rho=0.50$ is characterized by an exponent $\alpha>0$, which is generated by the interplay of the electron interaction with the impurity, and D exhibits then a power-law decay with N , which implies vanishing in the limit $N\rightarrow\infty$. On the other hand, the systems with $U=2.50$ and $U=3.00$ with fixed $\rho=0.50$ are characterized by a localization length ξ , which decreases with increasing interaction and impurity strength, and D exhibits now an exponential decay with N , also vanishing as $N\rightarrow\infty$. Hence, we find that both in the LL and the CDW phases, with an impurity in the system, the effect of the interaction is to decrease the current and the Drude weight. As referred to before, our results also provide insight into the spin transport in a spin-1/2 XXZ chain with a weak link, and a similar behavior to the one above is implied for the spin current and stiffness. So, in the gapless XY phase, the spin stiffness decays with a power law, vanishing in the limit $N\rightarrow\infty$, while in the gapped Ising phase, it decays exponentially, also vanishing as $N\rightarrow\infty$. Comparing our results for the persistent current in the LL phase with those obtained in Ref. 14, we have similarly found that the current vanishes faster than $1/N$. One observes that the model parameters strongly influence when the last asymptotic regime described by bosonization is reached. A calculation of the finite-size corrections to the spin stiffness in a pure spin-1/2 XXZ chain⁴²⁻⁴⁴ has revealed a size dependence in the gapped phase that has a similar form to the one found here. The result that the Drude weight in a ring in the gapless phase with an impurity drops to zero is in agreement with a previous result obtained for a spin chain³⁸ and with renormalization group arguments, which state that the impurity term is relevant leading to a transmission cut.^{31,32} The renormalization group studies find that either a weak barrier or a weak link lead to an insulating state for repulsive interactions, while in the noninteracting case those are marginal perturbations. In turn, our work shows that there is an invariance of the current under the defect transformation $\rho\rightarrow 1/\rho$ in the interacting system, as for the noninteracting system, and that implies that a strong link will also reduce the current and the Drude weight. The observation that with an impurity in the system, the interaction always leads to an additional decrease of the current and the

Drude weight is in agreement with previous results by other authors^{7,8} and can be understood as it is more difficult to move correlated electrons in a scattering potential than independent electrons.

V. SUMMARY

We have studied the behavior of the persistent current and the Drude weight on a mesoscopic ring pierced by a magnetic flux. We considered a model of spinless fermions with repulsive interactions and a hopping impurity, which is also equivalent to a spin-1/2 XXZ chain with a weakened exchange coupling. Using a powerful numerical method, the DMRG with complex fields, we have calculated the detailed form of the current as a function of the magnetic flux, which enabled us to investigate the corrections to the large system-size limit, and also allowed us to obtain the Drude weight. We show that the system at half-filling changes from an algebraic to an exponential behavior as the interaction increases, corresponding to a change from a LL to a CDW phase. We find that the analytical predictions of bosonization for the LL phase, are not yet fully observed in the intermediate range of system sizes considered. In addition, we observe that the invariance of the current under the defect transformation $\rho\rightarrow 1/\rho$, seen in the noninteracting system, is also verified in the interacting system, in both phases. Hence, an isolated strong link is not only useless for increasing the persistent current (as might have been expected), but it rather destroys coherence and reduces the current. The behavior determined for the current is consistent with the behavior determined for the Drude weight, the LL phase being characterized by an exponent $\alpha>0$, which results from the interplay of the interactions with the impurity, while the CDW phase is characterized by a localization length ξ , which decreases with increasing interaction and impurity strength. We find that, both in the LL and the CDW phase, with a defect in the system the interactions always suppress the current, and the Drude weight drops to zero in the limit $N\rightarrow\infty$. Away from half-filling, there is no metal-insulator transition in the pure case, and the system is always metallic. Hence, one does not expect to observe then a change in the current and the Drude weight from an algebraic to an exponential decay. Nevertheless, one still expects to observe that in the system with an impurity, the current and the Drude weight decrease with increasing impurity and interaction strengths.⁸ Therefore, within the model considered, the interactions cannot explain the results observed in the experiments.

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