

Vibration-mode-induced Shapiro steps and back action in Josephson junctions

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A model of a superconducting tunnel junction coupled to a mechanical oscillator is studied at zero temperature in the case of linear coupling between the oscillator and tunneling electrons. It is found that the Josephson current flowing between two superconductors is modulated by the motion of the oscillator. Coupling to harmonic oscillator produces additional Shapiro steps in I - V characteristic of Josephson junction whose position is given by the frequency of the vibration mode. We also find a new velocity-dependent term originating from the back action of the ac Josephson current. This term is periodic in time and vanishes at zero bias voltage.

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I. INTRODUCTION

The coupling of the charge carriers to vibrational modes and localized spins in electronic devices has been a subject of intense investigation recently. Vibrational modes and spins possess dynamic internal degrees of freedom, unlike static impurities or defects. As a consequence, they affect the electronic dynamics in these devices. Interesting I - V characteristics (i.e., peaks in differential tunneling conductance) in molecular electronics¹⁻⁶ may indicate a strong influence from the electronic-vibrational coupling. A step structure (rather than peak structure) in the differential tunneling conductance has also been observed in the STM-based inelastic tunneling spectroscopy around a local vibrational mode on surfaces.⁷ The vibrational effects on the conductance of molecular quantum dots and single electron transistors were also examined.⁸⁻¹⁶ These studies might have implications in charge-based quantum computation.

Previous work has focused on the tunneling between two normal metals, where only the single-particle process is involved in transport. In these systems, the damping due to the coupling of the oscillator with the low-energy quasiparticles at the Fermi surface is substantial. In Refs. 17 and 18, the dynamics of a single spin embedded in the tunneling barrier between two superconductors has been addressed. It was found that the superconducting correlation can lead to non-trivial modification of spin dynamics. Recently, microfabrication techniques have progressed to the point where it is now possible to make extremely compliant vacuum tunneling electrodes that may not remain mechanically stationary during the tunneling process.¹⁹ In this paper, we consider, for the first time, the Josephson effect in a superconducting tunneling junction coupled to a mechanical oscillator in the tunnel barrier. Both effects of the oscillator motion on the tunnel current and the tunneling electrons on the oscillator dynamics are studied. To our knowledge, none of these effects have been addressed before. We find the following: (i) In the tun-

nel junction, the Josephson current is modulated by the motion of the oscillator; the Fourier spectrum of the Josephson current exhibits peaks at frequency $\omega_J \pm \omega_0$, $2\omega_J$, and $\omega_J \pm 2\omega_0$, where $\omega_J = 2$ eV and ω_0 are the Josephson frequency and the vibrational mode of the oscillator, respectively. These additional peaks are the result of coupling of ac Josephson current at ω_J and oscillation at ω_0 . (ii) Electron tunneling through the junction leads to a novel time-dependent change in oscillator energy that, in principle, could be measured. If no voltage bias is applied across the junction (i.e., dc Josephson effect in equilibrium), the oscillator energy is time independent. When a nonzero voltage bias is applied, the oscillator will display time-dependent energy variations.

The outline of this paper is as follows: In Sec. II, we introduce the model system and present the theoretical derivations of the Josephson current in the presence of the mechanical oscillator, which is used to construct an effective Hamiltonian for the oscillator dynamics. In Sec. III, we discuss the Josephson back action on the oscillator dynamics and present the numerical results on the supercurrent modulation. Concluding remarks are given in Sec. IV.

II. OUTLINE OF THE CALCULATIONS

A. Model system

Our physical system is illustrated in Fig. 1.²⁰ It consists of a mechanically compliant cantilevered superconducting tip of mass m_c , placed some distance from a stationary bulk superconducting counterelectrode. The movable tip assembly, which we shall refer to as the “mechanical oscillator” or “cantilever,” is modeled as a spring with Hook’s law force constant k_c . A voltage bias is applied across these electrodes. The Hamiltonian describing this system

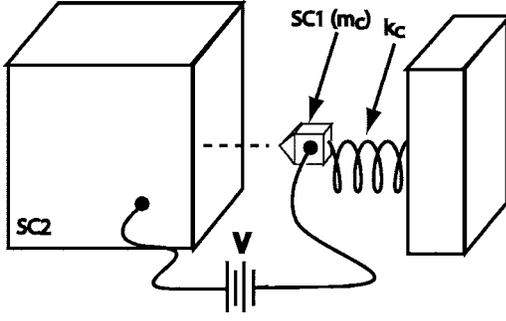


FIG. 1. Schematic view of a mechanically compliant superconducting tunneling junction. The cantilever superconducting electrode (SC1) is modeled as a harmonic oscillator with spring constant k_c and mass m_c , placed some distance from an infinitely massive superconducting counterelectrode (SC2). The device is biased with a voltage V .

$$H = H_L + H_R + H_T. \quad (1)$$

The first two terms are, respectively, the Hamiltonians for electrons in the left and right superconducting leads of the tunnel junction,

$$H_{L(R)} = \sum_{k(p);\sigma} \epsilon_{k(p)} c_{k(p),\sigma}^\dagger c_{k(p),\sigma} + \sum_{k(p)} [\Delta_{L(R)} c_{k(p)\uparrow}^\dagger c_{-k(-p)\downarrow}^\dagger + \text{H.c.}], \quad (2)$$

where we denote the electron creation (annihilation) operators on the left, right (L, R) leads by $c_{k\sigma}^\dagger (c_{k\sigma})$ and $c_{p\sigma}^\dagger (c_{p\sigma})$, respectively. The subscripts $k(p)$ are momenta in the left (right) leads, and σ is the spin index. Finally, in Eq. (2), $\epsilon_{k(p),\sigma}$ and $\Delta_{L(R)}$ are, respectively, the single-particle energies of conduction electrons, and the pair potential (the gap function) in the leads. With no loss of generality, we assume that the superconductors are of a spin-singlet s -wave pairing symmetry, and consider the Josephson tunneling at zero temperature. The last assumption implies that temperature is low compared to relevant frequencies, i.e., $T \ll \omega_0$ and ω_J . For the specific example of a nanomechanical oscillator, the typical frequency can be in the range $\omega_0 \sim 10^8 - 10^9$ Hz, this requires that $T \sim 10 - 100$ mK. The third term in Eq. (1) depicts the tunneling between the superconductors, which may be written as

$$H_T = \sum_{k,p;\sigma} [T_{kp} c_{k\sigma}^\dagger c_{p\sigma} + \text{H.c.}], \quad (3)$$

where the tunneling matrix elements T_{kp} transfer electrons through an insulating barrier. When a local vibrational mode is embedded into the tunneling barrier then, in the linear coupling regime,

$$T_{kp} = T_{kp}^{(0)} [1 + \alpha u], \quad (4)$$

where α describes the coupling between the tunneling electrons and vibrational mode. The quantity u is the displacement operator for the oscillator. The very general equilibrium point ($u=0$) of the mechanical oscillator placed within the junction does not correspond here to a special point of symmetry between the two superconducting leads. Physically,

this linear term captures the modulation of the tunnel barrier triggered by the mechanical oscillator motion. A more detailed analysis of this form is provided, amongst others, in Ref. 21.

As the energy associated with the vibrational mode,

$$\omega_0 = \sqrt{k_c/m_c} \sim 10^{-1} - 10^{-6} \text{ eV} \quad (5)$$

is much smaller than the typical electronic energy on the order of 1 eV, the mechanical oscillation is very slow as compared to the time scale of electronic processes. This allows us to apply the *Born-Oppenheimer approximation* to treat the electronic degrees of freedom as if the local oscillator is static at every instantaneous location. In what follows, we will treat the dynamics of the mechanical oscillator including the back action from the tunneling electrons.

B. The supercurrent in the presence of mechanical oscillations

Whenever a voltage bias is applied across the junction, the Josephson current

$$I_J(t) = e \int_{-\infty}^t dt' [e^{ieV(t+t')} \langle [A(t), A(t')]_- \rangle - e^{-ieV(t+t')} \langle [A^\dagger(t), A^\dagger(t')]_- \rangle], \quad (6)$$

where the operator $A(t) = \sum_{k,p;\sigma} T_{kp} \tilde{c}_{k\sigma}^\dagger(t) \tilde{c}_{p\sigma}(t)$. Here, the transformed electronic annihilation operators

$$\tilde{c}_{k(p)\sigma}(t) = e^{iK_{L(R)}t} c_{k(p)\sigma} e^{-iK_{L(R)}t}, \quad (7)$$

with

$$K_{L(R)} = H_{L(R)} - \mu_{L(R)} N_{L(R)}, \quad (8)$$

and

$$N_{L(R)} = \sum_{k(p),\sigma} c_{k(p)\sigma}^\dagger c_{k(p)\sigma}. \quad (9)$$

The unequal chemical potentials of the two superconductors lead to a voltage bias $\mu_L - \mu_R = eV$. In Eq. (6), we further assumed that the two superconductors are identical and set the constant phase difference between them $\phi_0 = 0$. The Josephson frequency is given by $\omega_J = 2 \text{ eV}$. Hereafter, we set $\hbar = 1$. Inserting Eqs. (3), (4), and (7)–(9) into Eq. (6) and simplifying, we find the supercurrent

$$I_J(t) = J_S^{(0)}(eV) [1 + \alpha u]^2 \sin(\omega_J t) + \Gamma_S(eV) (1 + \alpha u) \alpha \frac{\partial u}{\partial t} \cos(\omega_J t). \quad (10)$$

Here, we applied a local approximation

$$u(t') \simeq u(t) + (t' - t) \frac{\partial u}{\partial t}. \quad (11)$$

The *Born-Oppenheimer approximation* of Eq. (11), albeit making the physics very transparent and the current derivation convenient, is not necessary at this stage. Similar results can be arrived at by employing a more detailed Keldysh contour analysis,²² which parallels that in Refs. 17 and 18.

The physical origin of this effect is the separation in time scales of the fast electronic versus the much slower mechanical degrees of freedom, $E_{k,p} \gg \omega_J, \omega_0$ (with a natural resonant “Larmor frequency” ω_L taking on the role of ω_0 in the context of Ref. 18). Here we wish to provide the reader with a more physically transparent understanding of the reported effects. The quantity J_S^0 in Eq. (10) is the amplitude of the Josephson current in the absence of coupling to the vibrational mode (that is, $\alpha=0$), which is found to be

$$J_S^{(0)}(\text{eV}) = e \sum_{k,p} \frac{|\Delta|^2 |T_{kp}^{(0)}|^2}{E_k E_p} \left(\frac{1}{\text{eV} + E_k + E_p} - \frac{1}{\text{eV} - E_k - E_p} \right), \quad (12)$$

with $|\Delta|$ is the superconducting energy gap and the quasiparticle energies,

$$E_{k(p)} = \sqrt{(\epsilon_{k(p)} - E_F)^2 + |\Delta|^2}. \quad (13)$$

In the second line of Eq. (10), the amplitude

$$\Gamma_S(\text{eV}) = e \sum_{k,p} \frac{|\Delta|^2 |T_{kp}^{(0)}|^2}{E_k E_p} \left(\frac{1}{(E_k + E_p - \text{eV})^2} - \frac{1}{(E_k + E_p + \text{eV})^2} \right). \quad (14)$$

An order of magnitude estimate for the relative ratio between the two terms in Eq. (10) yields

$$\frac{\Gamma_S \omega_J}{J_S^{(0)}} \sim \left(\frac{\text{eV}}{|\Delta|} \right)^2 \ll 1. \quad (15)$$

C. The effective Hamiltonian and oscillator dynamics

In the previous section, we computed the tunneling supercurrent. With its magnitude at hand, we now construct the Hamiltonian and investigate the corresponding dynamics.

From Eq. (10), we construct the modulated part of the Josephson junction energy,

$$H_J = E_J (1 + \alpha u)^2 [1 - \cos(\omega_J t)] + \frac{\Gamma_S}{2e} \alpha (1 + \alpha u) \frac{\partial u}{\partial t} \sin(\omega_J t), \quad (16)$$

where

$$E_J = \frac{J_S^{(0)}}{2e}. \quad (17)$$

Equation (16) is constructed by demanding that the derivative of H_J with respect to the phase yields the supercurrent in Eq. (10). Equation (16) captures the *Josephson back action effect*—it vividly illustrates how *electronic degrees of freedom influence the mechanical oscillator* of Fig. 1 coupled to them.

Consequently, the total mechanical oscillator Hamiltonian is

$$H_{osc} = H_J + \frac{P^2}{2m_c} + \frac{k_c u^2}{2}. \quad (18)$$

This leads to an equation of motion for the oscillator coordinate,

$$m_c \frac{d^2 u}{dt^2} + \gamma_S(t) \frac{\partial u}{\partial t} + k_c u = F(t). \quad (19)$$

Here, the driving force

$$F(t) = -2\alpha E_J (1 + \alpha u) \left[1 - \left(1 + \frac{\Gamma_S \omega_J}{4e E_J} \right) \cos(\omega_J t) \right], \quad (20)$$

with

$$\frac{\Gamma_S \omega_J}{4e E_J} \simeq \left(\frac{\text{eV}}{|\Delta|} \right)^2 \ll 1, \quad (21)$$

and the time-dependent energy nonconserving

$$\gamma_S(t) = -\frac{\alpha^2 \Gamma_S}{e} \sin(\omega_J t). \quad (22)$$

We find that the net effect of Josephson backaction on the oscillator dynamics is two-fold: (a) Coupling to the Josephson current produces modification of the stiffness coefficient—In Eq. (20), the first, linear in the u term, may, alternatively, be lumped into the spring constant k_c and regarded as a *Josephson stiffness*—a shift of the spring constant resulting from electronic correlations; (b) the oscillatory part of the driving force and the time dependence of the Josephson backaction generated $\gamma_S(t)$ [Eq. (22)] lead to coherent back action. Equations (10), (19), (20), and (22) constitute the central result of this work. Below we discuss the experimental consequence of these results.

III. RESULTS AND DISCUSSIONS

A. Josephson back action

As shown by Eq. (22) and Eq. (14), the velocity coefficient $[\gamma_S(t)]$ originates from the coupling of the mechanical oscillator to the tunneling electrons. It is quadratically proportional to the coupling constant α . This term has two novel features.

(i) γ_S depends on the voltage bias. At zero voltage bias (the dc setting), γ_S vanishes since Γ_S is zero. γ_S is finite only when a finite voltage bias is applied across the junction (i.e., ac case). In the low-voltage limit ($\text{eV} \ll |\Delta|$), γ_S is linearly proportional to the voltage bias.

(ii) Once a finite voltage bias is applied, γ_S is also a periodic function of time with the Josephson frequency ω_J . These properties are unique to the coupling of the mechanical oscillator to the superconductors. In the normal metal, there is no quasiparticle energy gap on the Fermi surface.

The physical picture becomes far richer when the oscillator is coupled to the superconductor. On the one hand, in the superconductor, there exists an energy gap on the Fermi surface and the quasiparticles are depleted below this energy. This leads to the quenching of the single-particle tunneling channel; no contribution to the dissipation of the oscillator

due to normal quasiparticles is possible. On the other hand, due to macroscopic quantum coherence in the superconductor, Cooper pairs can tunnel through the barrier between the two superconductors with a probability comparable to that of single-particle tunneling in a normal metal junction. When a static voltage bias is applied, the tunneling of a single Cooper pair requires an energy of 2 eV to overcome the potential barrier. The energy carried by Cooper pair upon tunneling can be transferred to oscillator. For zero bias voltage, the tunneling pairs do not acquire/lose energy (the effective electronic action is time independent, no effective external sources are present, and energy is preserved at all times). In the ac setting, $\sin \omega_J t$ is odd under time reversal and, as a consequence, the term $\sin \omega_J t (\partial u / \partial t)$ is allowed in the effective Hamiltonian, Eq. (16).

We estimate the magnitude of the velocity coefficient. By taking $|\Delta| = \mathcal{O}(10)$ meV as relevant to superconducting MgB_2 ,²³ $J_S^{(0)} = 0.1$ mA, $eV = 2.5$ meV, and $\alpha = 1 \text{ \AA}^{-1}$, we find

$$\gamma_{S0} = \frac{\alpha^2 \Gamma_S}{e} \sim 10^{-13} \text{ NS/m}. \quad (23)$$

In addition to the back action from the Josephson tunneling, the mechanical oscillator has its own intrinsic damping coefficient γ_0 . For a nanomechanical oscillator with spring constant $k_c \approx 1 \text{ N/m}$, the fundamental vibration frequency $f = \omega_0 / 2\pi \sim 1 \text{ GHz}$, and if we assume the intrinsic quality factor $Q_0 = 10\,000$ (as a lower bound),²⁴ the intrinsic damping coefficient is $\gamma_0 \sim 10^{-14} \text{ NS/m}$, which is an order of magnitude smaller than γ_{S0} .

Here we remark on some of the differences between normal and superconducting junctions when coupled to a mechanical oscillator. These were investigated in Refs. 21, 25, and 26. Not too surprisingly, in normal junctions (much unlike Josephson junctions), a constant time-independent dissipation arises [in the analog of Eq. (19), $\gamma(t) = \gamma > 0$]. Coupling to the tunneling electrons leads to an effective stochastic driving force $F(t)$ acting on the oscillator. Unlike the superconducting junction, $F(t)$ is now a stochastic variable (albeit of a non-white-noise character). These forces further lead (at sufficiently large bias V) to a finite effective temperature,

$$T_{\text{eff}} = \frac{eV}{2}, \quad (24)$$

even when the system itself is at zero temperature. As a consequence, uniform dissipation notwithstanding, the oscillator fluctuations $\langle u^2(t) \rangle$ stabilize (thanks to the driving stochastic forces) about a finite time-independent value in the infinite time limit. This is in contrast to the Josephson junction, where oscillations in $u^2(t)$ persist forever. The tunnel current in the normal junction also differs in fundamental regards from the Shapiro steps that we found in the Josephson junction. For the resultant characteristics, the reader is referred to Ref. 25.

B. Supercurrent and Shapiro steps

To calculate the Josephson current, we need to solve Eq. (19) for the displacement field $u(t)$. In the weak coupling

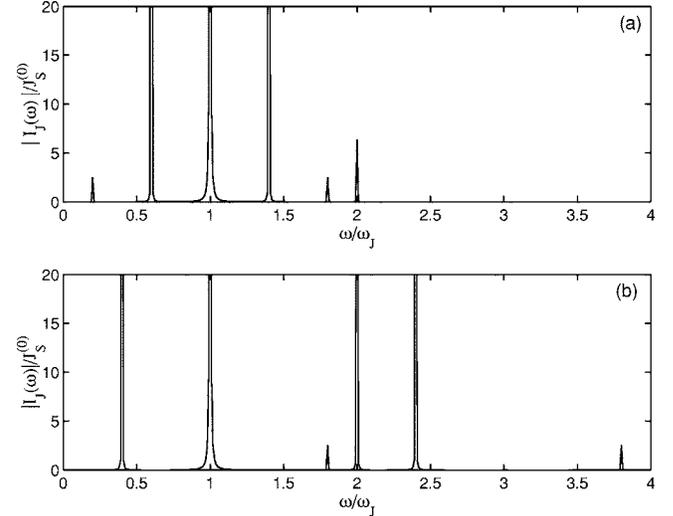


FIG. 2. Absolute magnitude of the Fourier transform $I_J(\omega)$ of the Josephson current as given by Eq. (26) for the vibration mode $\omega_0 = 0.4\omega_J$ (a) and 1.4 (b). The other parameters values are taken to be: $\tilde{\alpha} = 0.1$ and $\tilde{K} = 0.6$.

limit and in view of the fact that Γ_S is much smaller than $J_S^{(0)}$, the main physics can be captured by neglecting the damping terms and the $\alpha\Gamma_S$ and α^2 terms in the driving force. In this limit,

$$u(t) = u_0 \cos(\omega_0 t) + \frac{2\alpha E_J}{M(\omega_0^2 - \omega_J^2)} \cos(\omega_J t) - \frac{2\alpha E_J}{k_c}, \quad (25)$$

and the Josephson current

$$I_J(t) = J_S^{(0)} \left[\left(1 - \frac{4\tilde{\alpha}^2}{\tilde{K}} \right) \sin(\omega_J t) + 2\tilde{\alpha} \cos(\omega_0 t) \sin(\omega_J t) + \frac{2\tilde{\alpha}^2 \sin(2\omega_J t)}{\tilde{K}(1 - \omega_J^2/\omega_0^2)} + \tilde{\alpha}^2 \cos^2(\omega_0 t) \sin(\omega_J t) \right], \quad (26)$$

where

$$\tilde{\alpha} = \alpha u_0,$$

$$\tilde{K} = k_c u_0^2 / E_J. \quad (27)$$

Equation (26) demonstrates clearly that the Josephson current not only oscillates with time with a frequency ω_J , but is also modulated by the vibrational mode of the mechanical oscillator with a frequency ω_0 .

In Fig. 2, we plot the absolute magnitude of the Fourier transform of the Josephson current given by Eq. (26) for various values of the vibration mode frequency ω_0 . The spectrum shows a main peak at the frequency ω_J . In addition, the coupling of the mechanical oscillator and tunneling electrons generates new side peaks at frequencies $\omega_J \pm \omega_0$, $\omega_J \pm 2\omega_0$, and $2\omega_J$. The intensity of these peaks is proportional to the coupling constant. Note that the peak at $2\omega_J$ originates from the second term in $u(t)$ given by Eq. (25), which is a direct manifestation of the feedback effect from the Josephson tunneling. Our calculation implies that a dc component arises if

the voltage bias is equal to one of the Shapiro step values ω_0 and $2\omega_0$. When higher-order effects are taken into account, the equation of motion for the oscillator can only be solved numerically. The main conclusions presented here remain qualitatively unchanged. Apart from the standard rf steps in the I - V characteristics advanced long ago by Shapiro, similar I - V steps were also obtained for a Josephson junction interlaced with piezoelectric layer.²⁷ The origin of the physical coupling that leads to the Shapiro steps there is the inherent dependence of the piezoelectric force (modulating the layer thickness) on the voltage (which governs the Josephson phase), as well as the trivial dependence of the junction capacitance and resistance on the piezoelectric dimensions. The physics considered in Ref. 27 and that investigated by us is radically different.

IV. CONCLUSION

In summary, we studied the Josephson junction coupled to a mechanical oscillator between its two superconducting leads. We found that the Josephson current flowing between two spin-singlet pairing superconductors is modulated by the motion of the oscillator. The coupling of an oscillator of eigenfrequency ω_0 to an ac junction of characteristic frequency ω_J leads to sidebands. We find novel Shapiro steps induced at $\omega_J \pm \omega_0$, $\omega_J \pm 2\omega_0$, and $2\omega_J$. The coupling between

tunneling electron mechanical degrees of freedom leads to a novel nonenergy conserving effect. This time-dependent effect arises from the back action of the supercurrent on the oscillator dynamics.

As far as we know, no measurements of Josephson currents through a vibrational mode between two superconductors have been reported yet. Recent progress in molecular electronics⁴ and nanomechanical resonators^{28,29} holds great promise in attaching single molecules to superconducting leads, or even tune the tunnel barrier of the superconducting junctions by a mechanical cantilever.³⁰ Our predictions are, potentially, within the realm of present technology. Another possible experiment concerns atomically sharp superconducting tip in low-temperature STM in both the quasiparticle tunneling³¹ and Josephson tunneling regimes³² (coined “Josephson STM” or JSTM³³) on conventional superconductors. It is very interesting to extend the JSTM technology by using a superconducting tip to study the Josephson current in the vicinity of a local vibrational mode on the superconducting surface, which may provide a new detection technique.

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