Mechanism of $d_{x^2-y^2}$-wave superconductivity based on hole-doping-induced spin texture in high T_c cuprates

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A mechanism of $d_{x^2-y^2}$-wave superconductivity is proposed for the high T_c cuprates based on a spin texture with nonzero topological charge density induced by hole doping through Zhang-Rice singlet formation. The pairing interaction arises from a magnetic Lorentz force like the interaction between the holes and the spin textures. The stability of the pairing state against the vortex-vortex interaction and the Coulomb repulsion is examined. The mechanism suggests appearance of a p-wave pairing component by introducing anisotropy in the CuO_2 plane.

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I. INTRODUCTION

High-temperature superconductivity occurs in a moderately hole-doped Mott insulator.[1] The symmetry of the Cooper pairs has been established to be $d_{x^2-y^2}$ wave.[2] To explain the mechanism of $d_{x^2-y^2}$-wave superconductivity, several mechanisms have been proposed. Among others, the antiferromagnetic spin fluctuation theory suggests a $d_{x^2-y^2}$-wave pairing mechanism[3] between d-orbital electrons at copper sites. Another $d_{x^2-y^2}$-wave pairing mechanism is proposed in the resonating valence bond (RVB) theory.[1] In the RVB theory, electrons are assumed to be spin-charge separated. In the slave-boson approach,[4] spinons, which carry the spin degrees of freedom, form a $d_{x^2-y^2}$-wave pairing state. This pairing state is associated with the short-range antiferromagnetic correlations.[5] Combined with Bose-Einstein condensation of holons, which carry the electric charge, the pairing state of the spinons leads to a $d_{x^2-y^2}$-wave superconducting state.

Both of these theories are based on a pairing state between copper site electrons which produces antiferromagnetic correlations. However, from the sign of the Hall coefficient[6] the charge carriers seem to be doped holes. Optical conductivity measurements also support that the charge carriers are doped holes because the Drude weight in the optical conductivity[7] is proportional to the doped hole concentration. If we assume that the charge carriers are doped holes, then it is natural to expect that high-temperature superconductivity is based on a $d_{x^2-y^2}$-wave pairing mechanism between doped holes. In this paper, we propose such a mechanism based on a picture for the doped holes. The picture is that each hole induces a spin texture that is characterized by a nonvanishing topological charge density in the spin system through the suppression of the antiferromagnetic correlations.

In Ref. 8, a half-skyrmion spin texture[9,10] formation is discussed in the single-hole-doped cuprate within the nonlinear σ model (NLSM) description[11] of the Heisenberg antiferromagnet. The formation of the half-skyrmion spin texture can be understood by using an analogy to a superfluid system: Let us consider a two-dimensional boson system which exhibits superfluidity in the ground state. The Bose-Einstein condensate is described by a Gross-Pitaevskii equation.[12] If we suppress the condensate at some point P, then we see that a vortex is formed around P from the analysis of the Gross-Pitaevskii equation. Note that the field described by the Gross-Pitaevskii equation is a classical field. However, the noncondensed component near the vortex core is described by quantum bosons. In case of a charged boson system, the condensate is locally suppressed by a magnetic flux, or a vortex.

We can apply this result to the two-dimensional quantum Heisenberg antiferromagnet. In the Schwinger boson mean-field theory,[13] the Néel ordering is described by Bose-Einstein condensation of the Schwinger bosons.[14] If we assume that the doped hole forms a spin singlet with a localized spin at a copper site, then the condensate is suppressed at that site. Forming a Zhang-Rice singlet in the antiferromagnet plays a similar role to introducing a magnetic flux in a charged boson condensate. As in the case of a superfluid system, a vortex appears around the site. In terms of the spins, the vortex is a spin texture with the nonvanishing topological charge density[15] that is given by

$$q(\mathbf{r}) = \frac{1}{4\pi}\mathbf{n}(\mathbf{r}) \cdot [\partial_x \mathbf{n}(\mathbf{r}) \times \partial_y \mathbf{n}(\mathbf{r})], \tag{1}$$

where the unit vector $\mathbf{n}(\mathbf{r})$ describes the staggered component of the spins. Note that the spin at the core of the spin texture is a quantum spin analogous to the normal component near a vortex core in a superfluid, and so the spin can form a singlet state with the doped hole spin, which is the Zhang-Rice singlet.[16] Based on the Schwinger boson mean-field theory, we find that the topological charge, which is obtained by integrating $q(\mathbf{r})$ over the whole region, is quantized to $\pm 1/2$.[8,17] Because of this quantized value, the spin texture is called a half skyrmion.

As argued in Ref. 8, the dispersion of the half skyrmion is in good agreement with the results of angle-resolved photoemission spectroscopy (ARPES) in the undoped compounds.[18] The dispersion of the half-skyrmion excitation is the same as that of the quasiparticles in the π-flux phase with a dynamically induced mass.[19,20] In fact, the effective theory of the half skyrmion is similar to that of the π-flux phase: the half skyrmions are described by Dirac fermions with a U(1) gauge field interaction. Anomalously broad peaks observed in ARPES are associated with the self-trapping of the half skyrmions due to the coupling to the longitudinal gauge field.[21]

In Ref. 8, we assume that there is Néel ordering. The presense of the Néel ordering is used for obtaining the quantized topological charge. In the absence of Néel ordering, the topological charge is no longer quantized because the antiferromagnetic correlations decay exponentially with a length scale given by the antiferromagnetic correlation length ξ_{AF}. However, the system preserves the local relationship between the hole density and the topological charge density. We consider spin textures with such topological charge density and hereafter we call such a spin texture a staggered spin vortex.

The rest of the paper is organized as follows. In Sec. II, we describe the model for staggered-spin-vortex formation. The pairing interaction is discussed in Sec. III. We solve the gap equation in Sec. IV and show that the stable pairing state has $d_{x^2-y^2}$-wave symmetry. Section V is devoted to the conclusion.

II. MODEL

We assume that the holes and the localized spins are independent degrees of freedom. Such a model is, for instance, the spin-fermion model, which is derived from the d-p model by applying a canonical transformation. In such a model, the spin system is described by the antiferromagnetic Heisenberg model,

$$H_s = J\sum_{\langle i,j\rangle} \mathbf{S}_i \cdot \mathbf{S}_j. \tag{2}$$

Here the summation is taken over the nearest-neighbor sites and $\mathbf{S}_i$ denote the spin moment at copper sites. As an effective theory of the spin system, we consider the CP^1 model,[22] which is derived from the NLSM by representing $\mathbf{n}(x)$ in terms of the Schwinger bosons: $\mathbf{n}=\Sigma_{\sigma,\sigma'}\bar{z}_\sigma \boldsymbol{\sigma}_{\sigma,\sigma'} z_{\sigma}$:

$$S_{CP^1} = \frac{2}{g}\int d^3x \sum_{\sigma=\uparrow,\downarrow}\left(|(\partial_\mu - i\alpha_\mu)z_\sigma(x)|^2 + \frac{\Delta_{\rm sw}^2}{c_{\rm sw}^2}|z_\sigma(x)|^2\right), \tag{3}$$

where $\Delta_{\rm sw}$ is the antiferromagnetic spin-wave excitation gap and $c_{\rm sw}$ is the spin-wave velocity. The coupling g is given by $g=c_{\rm sw}/\rho_s$ with $\rho_s=Z_\rho J/4$ the spin stiffness constant. Z_ρ is the renormalization factor due to quantum phase fluctuations. In the spin-disordered phase, $\Delta_{\rm sw}\neq 0$, and there is no antiferromagnetic long-range order. The antiferromagnetic correlations are characterized by the correlation length $\xi_{\rm AF}=c_{\rm sw}/\Delta_{\rm sw}$. For Schwinger bosons, the presence of the antiferromagnetic correlations is associated with phase coherence among the Schwinger bosons.

For the kinetic energy of the holes, we assume

$$H_h = \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma}, \tag{4}$$

where $\epsilon_k=k^2/2m$ with m the effective mass. This is an approximate form in the continuum.

The coupling between the holes and the localized spins $\mathbf{S}_j$ has the form of the antiferromagnetic Kondo coupling:

$$H_K = J_K \sum_j (c_j^\dagger \sigma c_j)\cdot \mathbf{S}_j. \tag{5}$$

The parameter J_K is on the order of 1–3 eV. Because of this strong Kondo coupling, the holes suppress phase coherence of the Schwinger bosons locally through Zhang-Rice singlet formation. The suppression of the phase coherence leads to a staggered spin vortex:

$$\nabla \times \boldsymbol{\alpha} = \pi \sum_{s=\pm} s\psi_s^\dagger(\mathbf{r})\psi_s(\mathbf{r}), \tag{6}$$

where $\nabla\times\boldsymbol{\alpha}=\partial_x\alpha_y-\partial_y\alpha_x$ and $\psi_{+(-)}^\dagger$ is the creation operator of the staggered spin vortex (antistaggered spin vortex). The coefficient is determined by considering the limit of $\xi_{\rm AF}\to\infty$. In this limit, the spin texture is the half skyrmion. The gauge flux $\nabla\times\boldsymbol{\alpha}$ is associated with the spin chirality.[23,24] Note that $\langle\nabla\times\boldsymbol{\alpha}\rangle=0$, because in equilibrium there are equal numbers of staggered spin vortices and antistaggered spin vortices, and so $\langle\Sigma_s s\psi_s^\dagger(\mathbf{r})\psi_s(\mathbf{r})\rangle=0$.

By integrating out the Schwinger bosons, we obtain the effective action of the gauge field α_μ. Since $\Delta_{\rm sw}\neq 0$, the action takes the Maxwellian form. That is, the gauge field is massless. However, in the spin-disordered phase, the antiferromagnetic correlation length $\xi_{\rm AF}$ is finite. Therefore, the gauge field propagator decays with a length scale of $\xi_{\rm AF}$. To include this feature, we perform a duality mapping. We write $z_\sigma(x)$ as

$$z_\sigma(x) = \bar{\rho}_\sigma^{1/2}\exp(i\theta_0 + i\theta_v). \tag{7}$$

Here the phase θ_0 is associated with the coherent motion of the Schwinger bosons and the phase θ_v is associated with the staggered spin vortices. Note that this form is applicable for the description outside the vortex cores. After performing a Stratonovich-Hubbard transformation in Eq. (3), we obtain the following Lagrangian density:

$$\mathcal{L} = \frac{g}{8\bar{\rho}}J_\mu^2 + iJ_\mu(\partial_\mu\theta_0 + \partial_\mu\theta_v - \alpha_\mu). \tag{8}$$

We find $\partial_\mu J_\mu=0$ by integrating out θ_0. This constraint is satisfied by introducing a gauge field:

$$J_\mu = \frac{1}{2\pi}\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda. \tag{9}$$

We define the vortex current by

$$j_\mu^v = \frac{1}{2\pi}\epsilon_{\mu\nu\lambda}\partial_\nu\partial_\lambda\theta_v. \tag{10}$$

In terms of the staggered-spin-vortex and antistaggered-spin-vortex fields, the vortex density is given by

$$\rho^v(\mathbf{r}) = \sum_s s\psi_s^\dagger(\mathbf{r})\psi_s(\mathbf{r}). \tag{11}$$

From the equation of continuity, $\partial_t\rho^v+\nabla\cdot\mathbf{j}^v=0$, we find

$$\mathbf{j}^v = \sum_s \frac{s}{2mi}[\psi_s^\dagger \nabla \psi_s - (\nabla \psi_s^\dagger)\psi_s], \tag{12}$$

where we have used the fact that the kinetic energy of the holes is described by Eq. (4). The coupling between the gauge field A_μ and the vortex current j_μ^v is the minimal coupling:

$$\mathcal{L}_{\text{int}} = -ij_\mu^v A_\mu. \tag{13}$$

Since $J_0 = \nabla \times \mathbf{A}/2\pi$ and J_0 describes phase fluctuations of the Schwinger boson density, the nonvanishing gauge flux $\nabla \times \mathbf{A}$ describes the suppression of the phase coherence. Contrary to the gauge flux $\nabla \times \boldsymbol{\alpha}$, the flux quantum for the Meissner phase of $\mathbf{A}$ is 2π because the staggered spin vortices carry a unit gauge charge. Therefore, the relation between the vortex density and the gauge flux is

$$\sum_s \psi_s^\dagger(\mathbf{r})\psi_s(\mathbf{r}) = -\frac{1}{2\pi}\nabla \times \mathbf{A}. \tag{14}$$

The gauge field A_μ is equivalent to the gauge field in the slave-boson mean-field theory.[23]

III. PAIRING INTERACTION

The pairing interaction is obtained from Eqs. (14) and (13) by eliminating the gauge field A_μ. We take the Coulomb gauge

$$\nabla \cdot \mathbf{A} = 0. \tag{15}$$

In the momentum space,

$$A_{qx} = \frac{iq_y}{q^2}A_q, \quad A_{qy} = -\frac{iq_x}{q^2}A_q. \tag{16}$$

From Eq. (14) we obtain

$$A_q = -2\pi \sum_{ks} \psi_{ks}^\dagger \psi_{k+q,s}. \tag{17}$$

The staggered-spin-vortex current operator is

$$\mathbf{j}_\mathbf{q}^v = \sum_{\mathbf{k},s} \frac{s}{m}\left(\mathbf{k} + \frac{\mathbf{q}}{2}\right)\psi_{ks}^\dagger \psi_{k+q,s}. \tag{18}$$

Substituting these equations into Eq. (13), we obtain

$$H_{\text{int}} = -\frac{2\pi i}{m\Omega}\sum_{q,s,s'} \frac{\mathbf{q}\times\mathbf{k}'}{q^2} s' \psi_{k+q,s}^\dagger \psi_{k's'}^\dagger \psi_{k'+q,s'}\psi_{ks}, \tag{19}$$

where Ω is the area of the system. Since we are interested in the Cooper pairing, we set $\mathbf{k}+\mathbf{k}'+\mathbf{q}=0$. Thus, we obtain

$$H_{\text{int}} = -\frac{2\pi i}{m\Omega}\sum_{\mathbf{k}\neq\mathbf{k}',s,s'} \frac{\mathbf{k}\times\mathbf{k}'}{|\mathbf{k}-\mathbf{k}'|^2} s' \psi_{k's}^\dagger \psi_{-k',s'}^\dagger \psi_{-k,s'}\psi_{ks}. \tag{20}$$

By making replacement of $s \to s'$, $s' \to s$, $\mathbf{k}' \to -\mathbf{k}'$, and $\mathbf{k} \to -\mathbf{k}$, we obtain

$$H_{\text{int}} = -\frac{2\pi i}{m\Omega}\sum_{\mathbf{k}\neq\mathbf{k}',s,s'} \frac{\mathbf{k}\times\mathbf{k}'}{|\mathbf{k}-\mathbf{k}'|^2} s \psi_{k's}^\dagger \psi_{-k',s'}^\dagger \psi_{-k,s'}\psi_{ks}. \tag{21}$$

Therefore, we may write

$$H_{\text{int}} = -\frac{i\pi}{m\Omega}\sum_{\mathbf{k}\neq\mathbf{k}',s,s'} \frac{\mathbf{k}\times\mathbf{k}'}{|\mathbf{k}-\mathbf{k}'|^2}(s+s') \psi_{k's}^\dagger \psi_{-k',s'}^\dagger \psi_{-k,s'}\psi_{ks}. \tag{22}$$

From this form, it is apparent that the interaction exists for either staggered-spin-vortex pairs or antistaggered-spin-vortex pairs. The Hamiltonian is given by

$$H = \sum_s H^{(s)}, \tag{23}$$

$$H^{(s)} = \sum_\mathbf{k} \epsilon_k \psi_{ks}^\dagger \psi_{ks} - \frac{2\pi i}{m\Omega}\sum_{\mathbf{k}\neq\mathbf{k}'} \frac{\mathbf{k}\times\mathbf{k}'}{|\mathbf{k}-\mathbf{k}'|^2}\psi_{k's}^\dagger \psi_{-k',s}^\dagger \psi_{-k,s}\psi_{ks}. \tag{24}$$

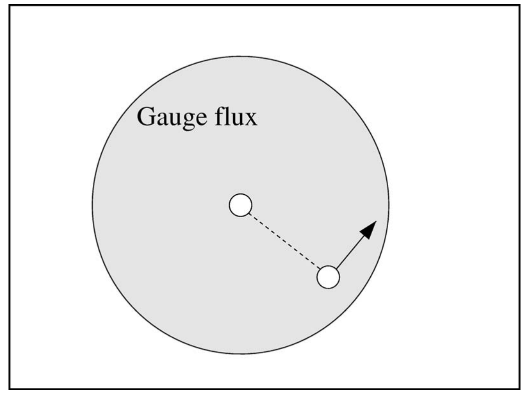

FIG. 1. Pairing interaction between the doped holes. Shaded region represents the gauge flux $\nabla \times \mathbf{A}$ created by the hole at the center. If another hole passes this region with the Fermi velocity as shown in the figure, a magnetic Lorentz-force-like interaction is induced between the two holes.

A similar pairing interaction was discussed in the composite fermion system at half-filled Landau levels.[25,26] In that system, the composite fermions are spinless fermions and there is no other index associated with internal symmetry. It was shown[26,27] that the pairing interaction leads to a $(p_x \pm ip_y)$-wave pairing state. The sign is determined by the direction of the applied magnetic field perpendicular to the system. This pairing state is consistent with numerical simulations.[28]

Before going into the analysis of the gap equation derived from the Hamiltonian (23) and (24), we give an intuitive view about the origin of the attractive interaction. According to Eq. (14), the gauge flux $\nabla \times \mathbf{A}$ is produced by hole doping. Suppose a hole passes with the Fermi velocity the region of the gauge flux created by another hole. Then, as schematically shown in Fig. 1, the motion of the former hole is equivalent to a charged particle motion under a magnetic field. A magnetic Lorentz-force-like interaction is induced between the two holes. Such an interaction leads to a chiral pairing state. In this pairing mechanism, the gap is scaled by the Fermi energy. Since holes carry either positive or negative gauge charge, we expect there are two chiral pairing states with opposite chiralities. The stable pairing state in the bulk turns out to be a $d_{x^2-y^2}$-wave pairing state as we shall show in the next section.

IV. GAP EQUATION

Now we apply the BCS mean-field theory to Eq. (23). First we consider staggered-spin-vortex pairs described by $H^{(+)}$. In the following, we set $\chi_k = \psi_{k+}$ and $\chi_k^\dagger = \psi_{k+}^\dagger$, to simplify notation. We consider the grand canonical ensemble and define the following mean field:

$$\Delta_k^{(+)} = -\frac{1}{\Omega}\sum_{\mathbf{k}'(\neq \mathbf{k})} V_{\mathbf{k}\mathbf{k}'}\langle \chi_{-\mathbf{k}'}\chi_{\mathbf{k}'}\rangle, \tag{25}$$

where

$$V_{\mathbf{k}\mathbf{k}'} = -\frac{4\pi i}{m}\frac{\mathbf{k}\times\mathbf{k}'}{|\mathbf{k}-\mathbf{k}'|^2}. \tag{26}$$

The mean-field Hamiltonian is

$$H_{\mathrm{MF}}^{(+)} = {\sum_{\mathbf{k}}}' [\xi_k \chi_k^\dagger \chi_k - \xi_k \chi_{-k}\chi_{-k}^\dagger + (\Delta_k^{(+)})^* \chi_{-k}\chi_k + \Delta_k^{(+)}\chi_k^\dagger \chi_{-k}^\dagger], \tag{27}$$

where $\xi_k = \epsilon_k - \mu$ with μ the chemical potential and the summation in the momentum space is taken over half of the Brillouin zone. The gap equation is given by

$$\Delta_{\mathbf{k}}^{(+)} = -\frac{1}{2\Omega}\sum_{\mathbf{k}'(\neq\mathbf{k})} V_{\mathbf{k}\mathbf{k}'}\frac{\Delta_{\mathbf{k}'}^{(+)}}{E_{\mathbf{k}'}}\tanh\frac{\beta E_{\mathbf{k}'}}{2}, \tag{28}$$

with $E_{\mathbf{k}} = \sqrt{\xi_k^2 + |\Delta_{\mathbf{k}}^{(+)}|^2}$.

A similar gap equation is analyzed in Ref. 25 in the context of the composite fermion pairing. We apply the analysis presented in Ref. 25. In order to solve the gap equation, we introduce the following ansatz:[27]

$$\Delta_{\mathbf{k}}^{(+)} = \Delta_k \exp(-i\ell\theta_{\mathbf{k}}), \tag{29}$$

where ℓ is an integer. In Eq. (28) the integral with respect to $\theta_{\mathbf{k}'}$ is reduced to

$$I_\ell(\eta) = \int_0^{2\pi} d\theta \frac{\sin\theta}{\cos\theta - \eta}\exp(i\ell\theta), \tag{30}$$

with $\eta = (k^2 + k'^2)/(2kk')$. The function $I_\ell(\eta)$ is exactly calculated by setting $z = \exp(i\theta)$ and applying a contour integral in the complex plane. From Eq. (28), we see that if the interaction is attractive (repulsive) then the sign of $I_\ell(\eta)$ is positive (negative). For the $\ell > 0$ case, $I_\ell(\eta) > 0$ while for the $\ell < 0$ case, $I_\ell(\eta) < 0$, and $I_{\ell=0}(\eta) = 0$. Therefore, the gap equation has solutions only for $\ell > 0$. Thus, we obtain

$$\Delta_k = \frac{1}{2m}\left[\int_0^k dk' \frac{k'\Delta_{k'}}{E_{k'}}\left(\frac{k'}{k}\right)^\ell + \int_k^\infty dk' \frac{k'\Delta_{k'}}{E_{k'}}\left(\frac{k}{k'}\right)^\ell\right]. \tag{31}$$

From the asymptotic forms in $k \to \infty$ and $k \to 0$, we take the following approximate form:[25]

$$\Delta_k = \begin{cases} \Delta\epsilon_F (k/k_F)^\ell & \text{for } k < k_F, \\ \Delta\epsilon_F (k_F/k)^\ell & \text{for } k > k_F, \end{cases} \tag{32}$$

where ϵ_F is the Fermi energy of the holes. The gap Δ is obtained from the following equation:

$$\int_0^1 dx \frac{x^{2\ell+1}}{\sqrt{(x^2-1)^2 + \Delta^2 x^{2\ell}}} + \int_1^\infty dx \frac{x^{1-2\ell}}{\sqrt{(x^2-1)^2 + \Delta^2 x^{-2\ell}}} = 1. \tag{33}$$

The largest gap $\Delta = 0.916$ is obtained for the case of a p wave ($\ell = 1$). The second largest gap $\Delta = 0.406$ is obtained for the case of a d wave ($\ell = 2$). The third largest gap is $\Delta = 0.264$ for $\ell = 3$.

For antistaggered-spin-vortex pairs, we may carry out the same analysis. Since the interaction term has opposite sign compared to the staggered-spin-vortex pair case, the gap function $\Delta_{\mathbf{k}}^{(-)}$ has the following form:

$$\Delta_{\mathbf{k}}^{(-)} = \Delta_k \exp(i\ell\theta_{\mathbf{k}}), \tag{34}$$

with $\ell > 0$.

From the above analyses, the linear combination of the gap functions is

$$\Delta_{\mathbf{k}} = (\Delta_{\mathbf{k}}^{(+)} + \Delta_{\mathbf{k}}^{(-)})/2 = \Delta_k \cos(\ell\theta_{\mathbf{k}}). \tag{35}$$

The relative phase between $\Delta_{\mathbf{k}}^{(+)}$ and $\Delta_{\mathbf{k}}^{(-)}$ is set to be zero. (A nonzero phase can appear if there is a magnetic field.) For the case of $\ell = 1$, the symmetry of the Cooper pair is p_x. Such a state can be stablized only at the boundary of the sample in the square lattice. Meanwhile, for the $\ell = 2$ case, the $d_{x^2-y^2}$-wave pairing state is stable in the bulk. Thus, we may conclude that the above pairing interaction leads to $d_{x^2-y^2}$-wave superconductivity.

The above analysis is carried out for the staggered spin vortices. In terms of those fields, the hole spin states are implicit. For spin-singlet pairing states, the spin states should be

$$\langle \chi_{-k}^\downarrow \chi_k^\uparrow \rangle \tag{36}$$

and

$$\langle \chi_{-k}^\uparrow \chi_k^\downarrow \rangle, \tag{37}$$

where σ in χ_k^σ denotes the hole spin state. The relative phase, which is arbitrary in the above analysis, is π for the spin-singlet pairing state:

$$\langle \chi_{-k}^\downarrow \chi_k^\uparrow \rangle = -\langle \chi_{-k}^\uparrow \chi_k^\downarrow \rangle. \tag{38}$$

Now we consider the effect of the staggered-spin vortex-vortex interaction and the Coulomb interaction. Those repulsive interactions reduce the gap value. In the following we consider those effects separately. In order to evaluate the vortex-vortex interaction effect, we consider a two-vortex solution as follows:

$$\theta_v = \tan^{-1}\frac{y}{x} + \tan^{-1}\frac{y}{x-d}. \quad (39)$$

The interaction energy between the vortices is

$$V_{\mathrm{vv}}(d) = \frac{J}{2}Z_\rho\bar{\rho}\int d^2\mathbf{r}\frac{\hat{z}\times\mathbf{r}}{r^2}\cdot\frac{\hat{z}\times(x-d,y)}{(x-d)^2+y^2}. \quad (40)$$

Taking into account the fact that the range of the interaction is limited by the antiferromagnetic correlation length ξ_{AF}, we obtain

$$V_{\mathrm{vv}}(d) \simeq \pi J Z_\rho\bar{\rho}\ln\frac{\xi_{\mathrm{AF}}}{d}. \quad (41)$$

In the momentum space,

$$V_{\mathrm{vv}}(q) = \frac{2\pi\xi_{\mathrm{AF}}^2}{q^2}JZ_\rho\bar{\rho}, \quad (42)$$

with the constraint $q > 2\pi/\xi_{\mathrm{AF}}$.

In the presence of the vortex-vortex interaction $V_{\mathrm{vv}}(q)$, the gap equation is given by

$$\begin{aligned}1 = &\int_0^1 dx\frac{x^{2\ell+1}}{\sqrt{(x^2-1)^2+\Delta^2x^{2\ell}}} + \int_1^\infty dx\frac{x^{1-2\ell}}{\sqrt{(x^2-1)^2+\Delta^2x^{-2\ell}}}\\ &- C_v\left(\int_0^{1-\zeta}dx\frac{x^{2\ell}}{\sqrt{(x^2-1)^2+\Delta^2x^{2\ell}}}\frac{x}{1-x^2}\right.\\ &\left.+\int_{1+\zeta}^\infty dx\frac{x^{-2\ell}}{\sqrt{(x^2-1)^2+\Delta^2/x^{2\ell}}}\frac{x}{x^2-1}\right), \end{aligned}\quad (43)$$

where $\zeta = 2\pi/(k_F\xi_{\mathrm{AF}})$ and the constant C_v is

$$C_v = \frac{\pi J}{8\epsilon_F}Z_\rho\bar{\rho}\xi_{\mathrm{AF}}^2. \quad (44)$$

We take $C_v \simeq 0.5$ as a reasonable value at $x=0.10$ by setting $\bar{\rho}\xi_{\mathrm{AF}}^2 \sim 1$, $J/\epsilon_F \sim 2$. For Z_ρ, the value of $Z_\rho = 0.72$ is used, which is estimated from quantum Monte Carlo simulations.[29] Figure 2 shows the $k_F\xi_{\mathrm{AF}}$ dependence of the gap parameters for each ℓ. The effect of the vortex-vortex interaction is large for strong antiferromagnetic correlations. The gap values are somewhat reduced by the vortex-vortex interaction. While the relevant parameter region would be $\xi_{\mathrm{AF}}k_F/(2\pi) = 1-2$, around $\xi_{\mathrm{AF}}k_F/(2\pi) \sim 3$, the f-wave gap becomes larger than the d-wave gap. Note that as an approximation, C_v is fixed in the above calculation for simplicity. For more precise calculations, we require a detailed analysis of the $\xi_{\mathrm{AF}}k_F$ dependence of $\bar{\rho}$ and Z_ρ.

Another pairing mechanism based on merons, which presumably correspond to the spin texture with nonzero topological charge density, was discussed by Berciu and John.[30,31] From the analysis of an extended Hubbard model, where a nearest-neighbor Coulomb repulsion is included, it was suggested that meron-antimeron pairs lead to d-wave superconductivity. In Refs. 30 and 31, the pairing interaction between merons and antimerons comes from the vortex-

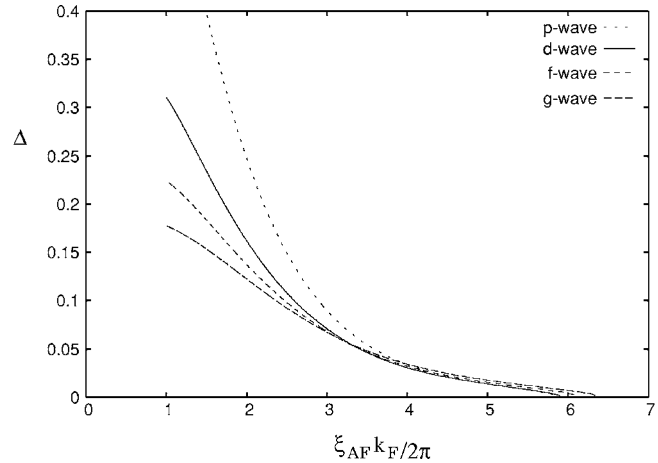

FIG. 2. The gap parameter Δ versus $\xi_{\mathrm{AF}}k_F/(2\pi)$.

antivortex interaction, and the pairing interaction (22) is not considered. Although the vortex-antivortex interaction is an attractive interaction, the analysis of the gap equation for the vortex-antivortex interaction within our ansatz shows that there is no pairing state for $\xi_{\mathrm{AF}}k_F/(2\pi) < 13$. In this analysis, the coefficient C_v is fixed. More precise analysis requires the ξ_{AF} dependence of C_v. However, C_v is expected to be an increasing function with respect to ξ_{AF}, and the increase of C_v leads to a reduction of the gap values for large ξ_{AF}.

Now we consider the effect of the Coulomb interaction. The Coulomb interaction between the holes is

$$V_q^C = \frac{2\pi e^2}{\epsilon q}, \quad (45)$$

with ϵ the dielectric constant. The gap equation with the Coulomb interaction is

$$\begin{aligned}1 = &\int_0^1 dx\frac{x^{2\ell+1}}{\sqrt{(x^2-1)^2+\Delta^2x^{2\ell}}} + \int_1^\infty dx\frac{x^{1-2\ell}}{\sqrt{(x^2-1)^2+\Delta^2x^{-2\ell}}}\\ &- C_e\left[\int_0^1 dx\frac{x^{2\ell+1/2}}{\sqrt{(x^2-1)^2+\Delta^2x^{2\ell}}}J_\ell\left(\frac{x^2+1}{2x}\right)\right.\\ &\left.+\int_1^\infty dx\frac{x^{1/2-2\ell}}{\sqrt{(x^2-1)^2+\Delta^2/x^{2\ell}}}J_\ell\left(\frac{x^2+1}{2x}\right)\right], \end{aligned}\quad (46)$$

where

$$J_\ell(\eta) = \int_0^{2\pi}d\theta\frac{\cos(\ell\theta)}{\sqrt{\eta-\cos\theta}}, \quad (47)$$

$$C_e = \frac{e^2k_F}{8\sqrt{2}\pi\epsilon}. \quad (48)$$

Figure 3 shows the $1/\epsilon$ dependence of the gap Δ for each ℓ.

V. CONCLUSION

In this paper, we have proposed a mechanism of $d_{x^2-y^2}$-wave superconductivity based on a spin texture with

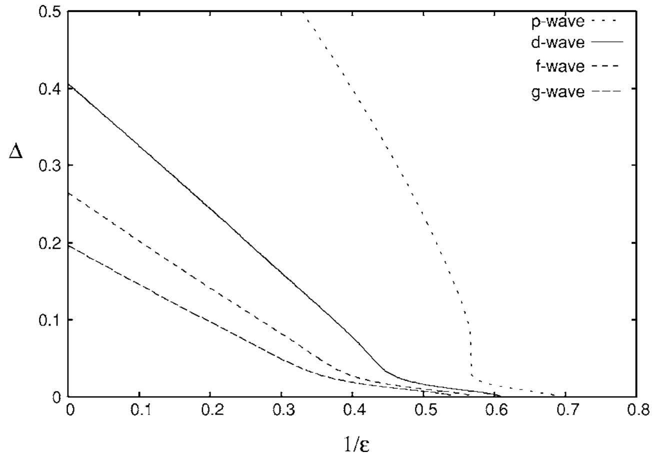

FIG. 3. The gap Δ versus $1/\epsilon$.

nonzero topological charge density. The spin texture formation is based on the Zhang-Rice singlet formation in the background of the antiferromagnetic correlations. In terms of a gauge field that describes antiferromagnetic spin correlations, the spin texture is described by a gauge flux. The interaction between the flux and the hole current induces a magnetic Lorentz-force-like interaction between the holes. Such an interaction leads to chiral pairing states with opposite chiralities. It turns out that the stable pairing state in the bulk is the $d_{x^2-y^2}$-wave pairing state. The stability of the pairing state is examined against the vortex-vortex interaction and the Coulomb interaction.

In our pairing mechanism, the p-wave state is unstable in the bulk. However, this state can be stabilized in the presence of anisotropy. Since the p-wave gap is much larger than the d-wave gap, we expect enhancement of the superconducting transition temperature if we can induce that component. If there is a p-wave component, then the parity is broken locally, while the time-reversal symmetry is not broken unless one chiral pairing state is suppressed.

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