Oscillations of induced magnetization in superconductor-ferromagnet heterostructures

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We study a change in the spin magnetization of a superconductor-ferromagnet (SF) heterostructure, when temperature is lowered below the superconducting transition temperature. It is assumed that the SF interface is smooth on the atomic scale and the mean free path is not too short. Solving the Eilenberger equation we show that the spin magnetic moment induced in the superconductor is an *oscillating sign-changing function* of the product hd of the exchange field h and the thickness d of the ferromagnet. Therefore the total spin magnetic moment of the system in the superconducting state can be not only smaller (screening) but also greater (antiscreening) than that in the normal state, in contrast with the case of highly disordered (diffusive) systems, where only screening is possible. This surprising effect is due to peculiar periodic properties of localized Andreev states in the system. It is most pronounced in systems with ideal ballistic transport (no bulk disorder in the samples, smooth ideally transparent interface), however these ideal conditions are not crucial for the very existence of the effect. We show that oscillations exist (although suppressed) even for arbitrary low interface transparency and in the presence of bulk disorder, provided that $h\tau \gg 1$ (τ is the mean free path). At low interface transparency we solve the problem for arbitrary strength of disorder and obtain oscillating magnetization in ballistic regime ($h\tau \gg 1$) and nonoscillating magnetization in diffusive one ($h\tau \ll 1$) as limiting cases of one formula.

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I. INTRODUCTION

Spin structures of the microscopic states of s-wave superconductors and ferromagnets are opposite to each other.[1,2] Superconducting pairing interaction leads to formation of electron Cooper pairs with opposite projections of spins, whereas the exchange field tends to align electron spins in the same direction. This counteraction on the microscopic level results in a competition between macroscopic superconducting and magnetic states. The suppression of the superconducting order parameter and the transition temperature by the exchange field[2,3] and the reduction of the magnetic spin susceptibility in the superconductors[4,5] are well-known examples of this competition.

Among suitable experimental systems for studying the interplay of the superconductivity and spin magnetism are superconductor-ferromagnet (SF) heterostructures (for reviews see Refs. 6–8). Above the superconducting transition temperature T_c, the superconductor is in its normal state, and the total magnetic moment M_{tot} of such system is given by the intrinsic magnetic moment of the ferromagnet M_{F0}. Below T_c a magnetic moment $M(h)$ induced by the presence of superconductivity appears, and the total spin magnetic moment of the SF system in the superconducting state is $M_{\text{tot}}=M_{F0}+M(h)$. The induced magnetization may be caused by both the Meissner currents (orbital effect) and the spin polarization (spin effect). If the sizes of the ferromagnet and the superconductor are small compared to the London penetration length, then the orbital effect is small compared to the spin effect.[9] In this work we assume this situation, since we want to study the effect related to the spin polarization. Therefore $M(h)$ is the induced *spin* magnetic moment throughout the paper.

What direction of $M(h)$ relative to M_{F0} one would expect? The abovementioned competing behavior of superconducting and magnetic phenomena suggests that $M(h)$ is opposite $[M(h)<0]$ to M_{F0} and thus reduces M_{tot}. In other words, the induced magnetization screens the intrinsic magnetization of the ferromagnet. The idea of spin screening of the ferromagnet's magnetization by the superconductor in SF systems was first brought forward in Ref. 9. In these publications the cases of a ferromagnetic planar film and spherical grain were considered and it was shown that indeed $M(h)<0$.

Experiments carried out on various SF structures confirm indirectly the idea of the screening proposed in Ref. 9. In Ref. 10 a V-$Pd_{1-x}Fe_x$ SF bilayered structure was studied by a magnetic resonance technique and a 50% decrease in M_{tot} was discovered as the temperature was lowered from $T=T_c \approx 4$ K to $T\approx 1.5$ K. In Ref. 11 the neutron reflectometry was performed on multilayered SF structures consisting of ferromagnetic $La_{2/3}Ca_{1/3}MnO_3$ and superconducting $YBa_2Cu_3O_7$ layers. The obtained reflectometry spectra were discussed in context of the screening effect predicted in Ref. 9. In a recent work (Ref. 12), the measurements of magnetization were performed on an SF system consisting of magnetic layers $La_xCa_{1-x}MnO_3$ and the superconductor Nb. The experimental data were analyzed in terms of magnetic properties acquired by the superconductor due to proximity effect.

In Ref. 9 the screening effect was studied under the following assumptions: (i) "diffusive" limit (the mean free path of electrons l is much smaller than both the size of the ferromagnet d and the superconducting coherence length ξ_S) and (ii) the exchange field of the ferromagnet h is small compared to the Thouless energy $E_{\text{Th}}=D/d^2$ (D is the diffusion coefficient in the ferromagnet, throughout the paper we employ units, in which the Planck constant $\hbar=1$):

$$h \ll E_{\mathrm{Th}}, \tag{1}$$

and the ferromagnetic film is thin ($d \ll \xi_S$). Condition (i) allowed us to use Usadel equation and condition (ii) to treat the effect of ferromanget's exchange field h as a pertubation in this equation. For the induced spin magnetic moment $M(h)$ the following result was obtained:

$$M(h) = -[\chi_N - \chi_S(T)]hd, \tag{2}$$

where $\chi_S(T)$ is the magnetic susceptibility of a bulk superconductor and χ_N is the magnetic susceptibility of the superconductor in the normal state $[\chi_S(T_c)=\chi_N]$. [For exact expression for $\chi_S(T)$, see Eq. (15)]. The result (2) is quite universal,[28] since it is independent of the strength of potential disorder and interface transparency.

The following questions arise. First, how robust is this perturbative result (2) to the type of orbital electron dynamics, in particular, what result would be obtained in the opposite case of a clean ballistic system. Second and more interestingly, how does induced magnetization $M(h)$ behave for sufficiently large exchange field, when its effect cannot be considered as a perturbation anymore and how the type of electron dynamics affects this behavior. The theory presented below shows that the behavior of induced magnetization $M(h)$ in ballistic SF systems in nonperturbative regime can be very remarkable.

First, let us define more precisely what we mean by "perturbative" and "nonperturbative" regimes for an SF system in general case. For a generic SF system with arbitrary bulk disorder in S and F regions and arbitrary interface transparency an important energy scale is $\epsilon^*=1/\tau^*$, where τ^* is the characteristic time spent by electron in the ferromagnet. (Ferromagnet is assumed to be of finite size d at least in one dimension.) In a ballistic system without or with relatively weak bulk disorder (the mean free path $l \gtrsim d$) and with not too small interface transparency ($t \sim 1$) this energy scale is the Andreev energy $\epsilon^*=\epsilon_A=v_F/d$ (v_F is the Fermi velocity). In the case of low interface transparency $t \ll 1$ the time τ^* is enhanced due to the fact that electron has to hit the interface $\sim 1/t$ times before it escapes from the ferromagnet, therefore it stays in the ferromagnet $\sim 1/t$ times longer compared to the case of good transparency. Thus, for the case of low interface transparency ($t \ll 1$) one gets $\epsilon^*=\epsilon_A t$ for ballistic system. In the diffusive system ($l \ll d$) with not too small interface transparency ($t \gg l/d$) $\epsilon^*=E_{\mathrm{Th}}=D/d^2$ is the Thouless energy.

Comparison of h with ϵ^* determines how strongly the exchange field h affects the spectrum of SF system compared to the spectrum of the corresponding superconductor-normal–metal (SN) system with $h=0$. If

$$h/\epsilon^* \ll 1, \tag{3}$$

then the effect of exchange field is small and consequently physical quantities, such as the induced magnetization $M(h)$, can be studied perturbatively. We refer to the regime (3) as perturbative. On the contrary, when

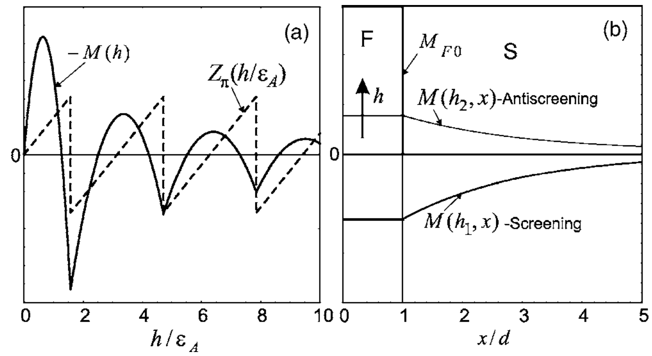

FIG. 1. (a) Oscillations of induced magnetic moment $M(h)$ of a clean SF system as a function of $h/\epsilon_A=2hd/v_F$ [the solid line shows $-M(h)$]. The graph is plotted for the case of zero temperature $T=0$ and thin F-layer $d \ll \xi_S$. At $hd \ll v_F$ we have $M(h)=-\chi_N hd$, indicating complete screening in the case itinerant ferromagnet. Dashed graph shows the function $Z_\pi(h/\epsilon_A)$ [see Eq. (13)]. (b) The geometry of the system and spatial distribution of the density $M(h,x)$ of the induced magnetization ($h=h_1$: screening, $h=h_2$: antiscreening).

$$h \gtrsim \epsilon^*, \tag{4}$$

the spectrum of the SF system is significantly altered by the presence of exchange field h, and we refer to the regime (4) as nonperturbative.

A very interesting property of Andreev spectrum of SF system in nonperturbative regime is its periodicity as a function of parameter h/ϵ^* with the period of the order of unity.[13] [The physical reasons beyond this phenomenon and the origin of the conditions (3) and (4) are given in Sec. II.] This periodicity is known to reveal itself in the oscillations of the Josephson critical current $I_c(h)$ in SFS junctions.[15,16] One could expect to find these periodic features of Andreev spectrum in other macroscopic quantities, such as the induced magnetization $M(h)$. In this paper we extended the analysis of Ref. 9 of induced magnetization in SF structures to nonpertubative case and found that in ballistic systems this is indeed the case.

Namely, we considered an SF bilayered system with the ferromagnet of the thickness d and the superconductor of the size much greater than its coherence length ξ_S [Fig. 1(b)]. Solving the Eilenberger equation we find that in a system without or with relatively weak bulk ($l \gg d$) or surface disorder and with perfect SF interface transparency ($t=1$) the induced spin magnetic moment $M(h)$ is an *oscillating sign-changing function* [Fig. 1(a)] of parameter $h/\epsilon_A=2hd/v_F$ with the quasiperiod approximately equal to π. Therefore, the total magnetic moment

$$M_{\mathrm{tot}} = M_{F0} + M(h) \tag{5}$$

can be either smaller $[M(h)<0]$ or larger $[M(h)>0]$ than that in the normal state $M_{\mathrm{tot}}^N=M_{F0}$ depending on the value of the product hd. The oscillations of $M(h)$ are most pronounced for the system with ideal transport properties: ballistic electron motion and perfect interface transparency. However, these ideal conditions are not crucial for the very existence of oscillations. To verify this we have considered the case of low SF interface transparency ($t \ll 1$) and arbitrary disorder in the ferromagnet, described by the scattering time τ. The limit of low SF interface transparency ($t \ll 1$) is useful from a methodological standpoint, since it allows to solve the Eilenberger equation for the system with arbitrary strength of disorder. This allows one to study not only the limiting diffusive and ballistic cases, but also the crossover between them. It appears that the influence of disorder on the behavior of induced magnetization $M(h)$ is governed by parameter $h\tau$. In the limit $h\tau \gg 1$ *sign-changing oscillations* of $M(h)$ exist (the quasiperiod is still $h^* \approx \pi\epsilon_A$), although their magnitude is suppressed in t as $\sim t^2$ and exponentially in d/l. On the contrary, in the opposite case $h\tau \ll 1$ we get that $M(h)$ *does not exhibit oscillations*, being negative $[M(h) < 0]$ for all $h \ll 1/\tau$. The condition $h\tau \ll 1$ (together with $l \ll d$) corresponds to the "diffusive" limit of the Usadel equation and the results obtained in this case from the Eilenberger equation can be recovered from the Usadel equation.

We mention that nonoscillatory result for $M(h)$ in the diffusive limit is in contrast with the behavior of the Josephson critical current $I_c(h)$ in SFS junctions. Oscillations of $I_c(h)$ are not destroyed by disorder and persist (although exponentially suppressed in h/E_{Th}) even in the "diffusive" limit $h\tau \ll 1$. The period of these oscillations is $h^* \sim \epsilon^* = E_{\mathrm{Th}}$. These oscillations were observed experimentally in Refs. 17–21, for further references see review articles.[6,7] Thus, oscillations of the induced magnetization $M(h)$ turn out to be more sensitive to disorder than those of the Josephson critical current $I_c(h)$.

Our analysis shows that for moderate bulk disorder ($l \gtrsim d$, l may also qualitatively include surface disorder of the interface) and not too small interface transparency ($t \sim 1$) the magnitude of oscillations is still quite noticeable, thus giving hope for experimental check of our predictions. Since for $h\tau \gg 1$ oscillations of $M(h)$ are sustained, oscillatory behavior of $M(h)$ should be attainable even in the presence of disorder in the case of sufficiently strong ferromagnets. Since the exchange field h is hardly variable in the experiment, one may hope to observe the oscillations of $M(h)$ performing measurements on samples with different thickness d. We also note that the case of thin ferromagnetic films $d \ll \xi_S$ is the most interesting for experiment: experimentally relevant exchange fields are $h \gg T_c$, one needs $h \sim \epsilon_A$ to observe oscillations, thus $d/\xi_S \sim T_c/\epsilon_A \sim T_c/h \ll 1$.

As a limiting case of our analysis we obtain that in the clean case for small exchange fields ($h \ll t\epsilon_A$) and a thin ferromagnetic film ($d \ll \xi_S$) the induced magnetization $M(h)$ is given by the universal result Eq. (2). This complements the analysis of Ref. 9 and suggests that this result holds in perturbative regime (3) in SF systems with arbitrary strength of potential disorder.

The paper is organized as follows. In Sec. II we provide qualitative quasiclassical description of Andreev spectrum in SF systems and show how the conditions (3) and (4) and the periodicity of the spectrum arise. In Sec. III we consider the limit of clean sample and perfect interface transparency, when the oscillatory behavior of induced magnetization is most pronounced. We present the system and formalism of the Eilenberger equation used to derive the expression for the induced magnetization and analyze this expression in detail. The connection between the predicted effect and the properties of Andreev spectrum is discussed. In Sec. IV we show that our assumptions about ideal transport properties (perfect interface transparency, ballistic electron motion) of the system are not crucial for the existence of oscillations. We present the results for the case of low SF interface transparency and disordered ferromagnet and show that oscillations exist in such limit provided some conditions on parameters are met. Finally, we conclude with Sec. V. In the Appendix the general formulas for the case of arbitrary interface transparency and clean samples are given.

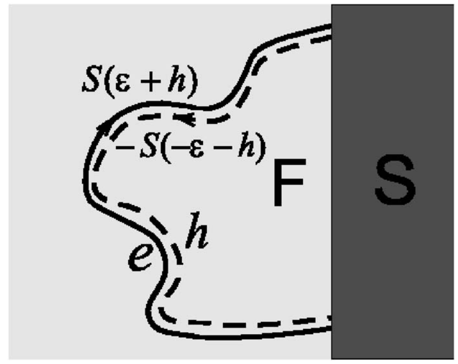

FIG. 2. Semiclassical description of Andreev spectrum of an SF system. Each classical trajectory that starts and terminates at SF interface corresponds to a set of discrete Andreev energy levels, which are obtained from the Bohr-Sommerfeld rule (7).

II. QUALITATIVE PHYSICS OF ANDREEV SPECTRUM IN SF SYSTEMS

The spectrum of a superconductor-ferromagnet (SF) (or superconductor-normal–metal (SN)) system is given by Andreev states, which are the states of electron-hole pairs localized in the $F(N)$ region due to Andreev reflection at $SF(SN)$ interface. A qualitative understanding of the properties of Andreev spectrum in SF systems can be obtained from the following semiclassical picture (Fig. 2). We assume the case of ideal SF interface transparency here for simplicity. (Qualitative analysis of the present section generalizes the discussion of Andreev spectrum of SN systems done in Sec. II of Ref. 22 to SF systems.)

First, consider an SN system without exchange field. Suppose an electron with an energy ϵ relative to the Fermi level ϵ_F inside the superconducting energy gap Δ ($|\epsilon| < \Delta$) is travelling along some classical path in the N region. If the electron hits the SN interface, it is reflected as a hole with the energy $-\epsilon$. A peculiar property of Andreev reflection is that the momentum of reflected hole is opposite (apart from a small angular mismatch $\sim\epsilon/\epsilon_F$) to that of the incident electron. Therefore the reflected hole will travel along the same path as the incident electron but in the opposite direction. If this path hits the interface again, the hole is reflected back as the electron. Within the quasiclassical Bohr-Sommerfeld (BS) approach if the total action of such process is an integer multiple of $2\pi\hbar$, then such electron-hole pairs forms a bound state.

Thus, each classical path that starts and terminates at *SN* interface corresponds to a set of discrete Andreev levels, energies of which can be obtained from the BS rule. For each such path γ of the length L_γ the action of the electron traversing this path is $S(\epsilon)=p(\epsilon)L_\gamma$, where $p(\epsilon)=\sqrt{2m(\epsilon_F+\epsilon)} \approx p_F+\epsilon/v_F$ is the absolute value of electron's momentum. The action takes the form

$$S(\epsilon) = p_F L_\gamma + \epsilon\tau_\gamma,$$

where $\tau_\gamma=L_\gamma/v_F$ is the time of traversal of path γ. The action of the hole (a missing electron) is $-S(-\epsilon)$ and the contributions from Fermi length scales cancel each other. The BS rule gives

$$S(\epsilon) - S(-\epsilon) - 2\phi(\epsilon) = 2\epsilon\tau_\gamma - 2\phi(\epsilon) = 2\pi n,$$

where $\phi(\epsilon)=\arccos(\epsilon/\Delta)$ is the phase of Andreev reflection and n is integer, and thus discrete Andreev levels $\epsilon=\epsilon_\gamma(n)$ corresponding to path γ are determined from the equation

$$\epsilon\tau_\gamma = \pi n + \phi(\epsilon). \tag{6}$$

Generalization to *SF* systems is straightforward (Fig. 2). In *SF* system one should distinguish between Andreev states with electron spins directed along ($\uparrow$) and opposite to ($\downarrow$) the exchange field h. "Up" states $\epsilon_\uparrow$ acquire a shift $-h$ and "down" states $\epsilon_\downarrow$ acquire a shift $+h$ in the F region. The BS rule reads ($+$ and $-$ correspond to $\uparrow$ and $\downarrow$, respectively):

$$S(\epsilon_{\uparrow,\downarrow} \pm h) - S[-(\epsilon_{\uparrow,\downarrow} \pm h)] - 2\phi(\epsilon_{\uparrow,\downarrow}) = 2\pi n.$$

Note that the argument of $\phi(\epsilon_{\uparrow,\downarrow})$ does not acquire the shift $\pm h$ due to exchange field, since exchange field is absent in the superconductor and therefore does not affect the process of Andreev reflection. Thus, the equation for Andreev levels in *SF* system reads

$$(\epsilon_{\uparrow,\downarrow} \pm h)\tau_\gamma = \pi n + \phi(\epsilon_{\uparrow,\downarrow}). \tag{7}$$

The time τ^* defined in Sec. I as a time spent by electron in the F region is the characteristic time for τ_γ. If $h\tau^* \ll 1$, then Andreev levels of the *SF* system obtained from Eq. (7) are only slightly different from those of the corresponding *SN* system with $h=0$ obtained from Eq. (6). This condition corresponds to perturbative regime (3). If $h\tau^* \gtrsim 1$, then Eq. (7) is significantly different from Eq. (6) and this condition corresponds to nonperturbative regime (4). Further, one sees that Eq. (7) is invariant under the periodic translation ($h\tau_\gamma \to h\tau_\gamma + \pi$). Thus the solutions of Eq. (7) are periodic

$$\epsilon_{\uparrow,\downarrow}(h) = \epsilon_{\uparrow,\downarrow}(h + \pi/\tau_\gamma).$$

This periodicity of Andreev spectrum should reveal itself in the behavior of macroscopic quantities of *SF* system as a function of parameter $h\tau^*$. This is indeed the case for Josephson critical current in *SFS* junctions and, as we show in this paper, for the induced magnetization.

III. BALLISTIC CASE, IDEAL *SF* INTERFACE TRANSPARENCY

A. System and method

We start our analysis with the case of ideal ballistic electron transport: absence of bulk disorder and perfect interface transparency $t=1$. In this case the oscillations of induced magnetization are most pronounced.

We consider an *SF* bilayered system pictured in Fig. 1(b): the superconductor occupies the half space $x>d$ and the ferromagnetic layer of the thickness d is located in the region $0<x<d$. We assume that there is no bulk disorder neither in the F layer nor in the superconductor and that the interfaces are ideal, namely, the superconductor-ferromagnet interface ($x=d$) is perfectly transparent and the ferromagnet-vacuum (*FV*) interface ($x=0$) is specular.

We study the problem solving the Eilenberger equation[23] for the quasiclassical Green's function $\check{g}(\omega,\boldsymbol{n},\boldsymbol{r})$. Here $\omega=\pi T(2m+1)$ is the fermionic Matsubara frequency (T is the temperature and m is an integer), $\boldsymbol{n}$ is the unit vector representing the direction of the electron momentum on the Fermi surface, and $\boldsymbol{r}$ is the radius vector. Due to the geometry considered $\check{g}(\omega,\boldsymbol{n},\boldsymbol{r})=\check{g}(\omega,n_x,x)$, where n_x is the projection of $\boldsymbol{n}$ on the x direction, $-1\leq n_x\leq 1$. The Green's function $\check{g}(\omega,n_x,x)$ is a matrix in the tensor product of Gor'kov-Nambu and spin spaces:

$$\check{g} = \hat{g}_1 \circ \tau_1 + \hat{g}_2 \circ \tau_2 + \hat{g}_3 \circ \tau_3,$$

$$\hat{g}_i = g_i^0 \hat{1} + g_i^z \hat{\sigma}_z. \tag{8}$$

Here τ_i, $i=1,2,3$ are the Pauli matrices in the Gorkov-Nambu space and $\hat{1}$, $\hat{\sigma}_z$ are the unity and Pauli matrices in the spin space (z denotes the direction of the exchange field in the spin space). Diagonal representation (8) in the spin space is possible in the case of homogeneous magnetization, which is assumed here.

The quasiclassical approach implies that the following conditions are satisfied: $h\ll\epsilon_F$, $p_F d\gg 1$, where ϵ_F, p_F are the Fermi energy and momentum. However, we stress that these conditions are required only for the applicability of the method used, but not for the very existence of the oscillations. The results are still valid qualitatively in the case of strong ferromagnet of atomic thickness ($h\lesssim\epsilon_F$, $p_F d\gtrsim 1$), although the effect is reduced. The same concerns possible mismatch of electronic properties (such as density of states ν_F and Fermi velocity v_F) in the ferromagnet and superconductor. It is assumed here that they are the same.

The density of the induced spin magnetization can be expressed in terms of the quasiclassical Green's function in the following way:

$$M(h,x) = \mu_B(\langle\psi_\uparrow^+(x)\psi_\uparrow(x)\rangle - \langle\psi_\downarrow^+(x)\psi_\downarrow(x)\rangle) = \frac{2\pi}{i}\mu_B\nu_F T\sum_\omega \int_0^1 dn_x g_3^z(\omega,n_x,x),$$

where μ_B is the Bohr magneton and ν_F is the electron density of states at Fermi level per single spin projection. The total induced magnetic moment of the system (per unit square in the plane parallel to SF interface) is obtained by the integration of $M(h,x)$ with respect to x:

$$M(h) = \int_0^{+\infty} dx M(h,x). \tag{9}$$

The relation between the intrinsic magnetic moment of the ferromagnet M_{F0} and the exchange field h acting on the electrons depends on the model of the ferromagnet.[14] Usually $M_{F0}=M_{F0el}+M_{F0loc}$ is combined of the contribution $M_{F0el}=\chi_N hd$ produced by free electrons and the contribution $M_{F0loc}=\alpha\chi_N hd$ produced by localized magnetic moments (α is some phenomenological constant and χ_N is the normal metal spin susceptibility). In the case of itinerant ferromagnet the localized magnetic moments are absent ($\alpha=0$) and $M_{F0}=M_{F0el}=\chi_N hd$.

We emphasize that magnetic moment $M(h)$ Eq. (9) expressed in terms of the quasiclassical Green's function is the magnetic moment *induced by the presence of superconductivity* in the system. It is determined by the properties of the energy spectrum on the scale $\sim T_c$ near the Fermi level. It does not include the part $M_{F0el}=\chi_N hd$ of the intrinsic magnetic moment M_{F0} of the ferromagnet produced by free electrons that originates from the energy shift of the entire electron band. The latter cannot be taken into account within the quasiclassical approach and should be added separately. The total magnetic moment of the system is given by Eq. (5).

The Eilenberger equation for the system without disorder reads

$$v_F n_x \partial_x \check{g} + [(\omega\hat{1} - ih(x)\hat{\sigma}_z)\circ\tau_3 + \Delta(x)\hat{1}\circ\tau_2, \check{g}] = 0, \tag{10}$$

where $h(x)$ is the exchange field in energy units, $\Delta(x)$ is the superconducting order parameter, and square brackets stand for the commutator. The exchange field is contained in the F layer only and assumed to be constant within the layer: $h(x)=h$, if $0<x<d$, and $h(x)=0$, if $x>d$. We assume there is no BCS interaction between electrons in the F layer and therefore we always have $\Delta(x)=0$ for $0<x<d$.

In principle, the exact order parameter $\Delta(x)$ has to be found self-consistently from the solution of the Eilenberger equation (10) supplemented by the self-consistency equation for the superconducting order parameter. The order parameter $\Delta(x)$ approaches a bulk BCS value Δ at large distances from the F-layer $x \gg \xi_S$, but is partially suppressed near the F layer. Computation of $\Delta(x)$ self-consistently is a hard analytical problem. Fortunately, our main result about the oscillatory sign-changing behavior of $M(h)$ is not sensitive to the exact shape of $\Delta(x)$. Therefore, we assume that $\Delta(x)=\Delta$ for all $x>d$ and perform calculations under this assumption.

Equation (10) must be supplied by proper boundary conditions at SF and FV interfaces.[24] The limit of the ideal transparency of SF ($x=d$) interface implies that the Green's function is continuous:

$$\check{g}(\omega,n_x,x=d-0) = \check{g}(\omega,n_x,x=d+0).$$

At the FV ($x=0$) interface the specular reflection condition reads

$$\check{g}(\omega,n_x,x=0) = \check{g}(\omega,-n_x,x=0).$$

At $x \gg \xi_S$ the solution approaches the BCS bulk result

$$g_i^z = g_1^0 = 0, \quad g_2^0 = f_S = \frac{\Delta}{\sqrt{\omega^2+\Delta^2}}, \quad g_3^0 = g_S = \frac{\omega}{\sqrt{\omega^2+\Delta^2}}.$$

B. Analysis

Under the made assumptions the solution of Eq. (10) is straightforward and we obtain

$$g_3^z(\omega,n_x,x) = -\frac{i}{2} f_S^2 \frac{\sin 2H}{[\cosh\Omega + |g_S|\sinh\Omega]^2\cos^2 H + [\sinh\Omega + |g_S|\cosh\Omega]^2 \sin^2 H} \begin{cases} 1, & 0<x<d, \\ \exp\left(-\dfrac{\sqrt{\omega^2+\Delta^2}}{\epsilon_A}\dfrac{x/d-1}{|n_x|}\right), & x>d. \end{cases} \tag{11}$$

Here $\Omega=|\omega|/(\epsilon_A|n_x|)$, $H=h/(\epsilon_A|n_x|)$ and $\epsilon_A=v_F/(2d)$ is the Andreev energy: $\hbar/\epsilon_A$ is the time the electron travels from the SF interface and back within the F layer with the velocity perpendicular to the interface.

Inserting Eq. (11) into Eq. (9) and integrating over x one obtains $M(h)$. The key point of our analysis is that $ig_3^z(\omega,n_x,x)$ is a periodic sign changing [note $\sin 2H$ in the numerator in Eq. (11)] function of $H=h/(\epsilon_A|n_x|)$ and depends on the exchange field solely via this parameter.

General properties of $M(h)$ can be summarized as follows: (1) $M(h)$ depends on the strength of exchange field solely via the combination $h/\epsilon_A=2hd/v_F$, (2) for any temperature $T<T_c$ and any ratio d/ξ_S the induced magnetic moment $M(h)$ is an oscillating sign-changing function of h/ϵ_A with a quasiperiod $h^*/\epsilon_A \approx \pi$, the amplitude of the oscillations decays monotonically as h/ϵ_A increases, (3) hence, $M(h)$ can either have the same $[M(h)>0]$ or opposite $[M(h)<0]$ direction as M_{F0}, depending on h/ϵ_A, (4) at $h/\epsilon_A \ll 1$ we get $M(h)<0$, which indicates the screening of M_{F0}, and (5) the magnitude of oscillations of $M(h)$ is largest at $T=0$ and decreases as T increases; $M(h)=0$ at $T=T_c$.

The spatial dependence of $M(h,x)$ shown in Fig. 1(b) is governed by $g_3^z(\omega,n_x,x)$: $M(h,x)$ is constant within the F layer and decays exponentially over the distance ξ_S into the superconductor. If $d \sim \xi_S$, then parts of $M(h)$ located in the F layer

$$M_F(h) = \int_0^d dx M(h,x)$$

and in the superconductor

$$M_S(h) = \int_d^{+\infty} dx M(h,x)$$

are of the same order. If $d \gg \xi_S$, then the induced magnetic moment is located predominantly in the F layer and $M(h) \approx M_F$, $M_S/M_F \sim \xi_S/d \ll 1$. In the opposite limit $d \ll \xi_S$ the induced magnetization is located mainly in the region of the superconductor of the size ξ_S near the F layer and $M(h) \approx M_S$, $M_F/M_S \sim d/\xi_S \ll 1$.

Below we concentrate on the experimentally more relevant situation of a thin F layer $d \ll \xi_S$ and illustrate the announced properties of $M(h)$ explicitly for this particular case. In this regime, the expression for $M(h) \approx M_S(h)$ can be reduced to the form

$$M(h) = -d\mu_B \nu_F \epsilon_A \pi T \sum_\omega \frac{\Delta^2}{\sqrt{\omega^2+\Delta^2}} \times \int_0^1 dn_x n_x \frac{\sin 2H}{\omega^2 + \Delta^2 \cos^2 H}. \tag{12}$$

First, we consider the zero temperature limit $T=0$. Replacing the sum over ω by the integral $T\Sigma_\omega \cdots = \int_{-\infty}^{\infty} d\omega/(2\pi) \cdots$, we obtain

$$M(h) = -2d\mu_B \nu_F \epsilon_A \int_1^\infty \frac{dt}{t^3} Z_\pi\left(\frac{h}{\epsilon_A} t\right), \tag{13}$$

where $Z_\pi(x)=x$, if $-\pi/2 < x < \pi/2$, and periodically continued to all x (linear "zigzag-type" function with a period π). The integral with respect to n_x can easily be calculated and we obtain the function shown in Fig. 1(a).

Close to superconducting transition point $[(T_c-T)/T_c \ll 1]$ we obtain

$$M(h) = -d\mu_B \nu_F \epsilon_A \pi T \sum_\omega \frac{\Delta^2}{|\omega|^3} \int_1^{+\infty} \frac{dt}{t^3} \sin\left(2\frac{h}{\epsilon_A} t\right).$$

We see that $M(h)$ is again an oscillating sign-changing function of h/ϵ_A with quasiperiod $h^*/\epsilon_A \approx \pi$, although the amplitude of oscillations is parametrically smaller than that at $T=0$ by $[\Delta(T)/T_c]^2 \sim (T_c-T)/T_c \ll 1$.

C. Perturbative regime ($h \ll t\epsilon_A$)

In the limit $h \ll \epsilon_A$ from Eq. (12) we get

$$M(h) = -\chi_N \pi T \sum_\omega \frac{\Delta^2}{(\omega^2+\Delta^2)^{3/2}} hd, \tag{14}$$

where $\chi_N = 2\mu_B \nu_F$ is the bulk spin susceptibility of the normal metal. Since

$$\chi_S(T) = \chi_N \left(1 - \pi T \sum_\omega \frac{\Delta^2}{(\omega^2+\Delta^2)^{3/2}}\right) \tag{15}$$

is the bulk spin susceptibility of the superconductor, Eq. (14) can be rewritten in the form Eq. (2). From the formulas given in Appendix one obtains the same result in the case of clean samples and low SF interface transparency ($t \ll 1$), provided that $h \ll t\epsilon_A$. The total magnetic moment produced by free electrons is $M_{F0el} + M(h) = \chi_S(T) hd$. At zero temperature $\chi_S(T=0)=0$ and $M(h) = -M_{F0el} = \chi_N hd$, i.e., the induced magnetic moment $M(h)$ totally screens the part M_{F0el} of the intrinsic moment M_{F0} produced by free electrons. An interesting feature is that M_{F0el} is located in the ferromagnet, whereas $M(h)$ is spread over the distance ξ_S from the F layer in the superconductor. Since for $h \ll \epsilon_A$ and $d \ll \xi_S$ the exact order parameter $\Delta(x)$ is only slightly suppressed due to the ferromagnetic proximity effect, this result is justified even if the self-consistency condition for $\Delta(x)$ is taken into account.

As the same result Eq. (2) was obtained in the opposite diffusive limit for $h \ll E_{\mathrm{Th}}$, we make a conjecture that Eq. (2) holds for arbitrary strength of potential disorder, provided the general condition Eq. (3) is met. The universality of result (2) is reminiscent of the properties of the bulk linear spin susceptibility of the superconductor $\chi_S(T)$ [Eq. (15)]. It is also independent of the strength of potential disorder.[5]

D. Andreev states

The oscillations of induced magnetization are closely related to the properties of the energy spectrum of localized Andreev states in the system.[13] The equation for Andreev energy levels $\epsilon_{\uparrow,\downarrow}$ with the electron's spin having the same ($\uparrow$) and opposite ($\downarrow$) direction as the exchange field (corresponding to + and − signs, respectively) reads

$$\frac{\epsilon_{\uparrow,\downarrow} \pm h}{\epsilon_A |n_x|} - \arccos \frac{\epsilon_{\uparrow,\downarrow}}{\Delta} = \pi n, \tag{16}$$

where n is an integer. Note that in the case of ballistic system this equation exactly coincides with Eq. (7) used for qualitative considerations. Equation (16) is invariant under the periodic translation $h/(\epsilon_A|n_x|) \to h/(\epsilon_A|n_x|) + \pi k$ (k is integer) in the same fashion as the Green's function (11) is periodic in H. In the limit $d \ll \xi_S$ there exists only one level for a given n_x and projection of spin[29]

$$\epsilon_{\uparrow,\downarrow}(H) = \pm \Delta \cos H, \quad H \in [0,\pi], \tag{17}$$

and periodically continued to all H. The states with $H \in [0,\pi/2] + \pi n$ ($\epsilon_\uparrow > 0$, $\epsilon_\downarrow < 0$) contribute to the screening of M_{F0} [$\sin 2H > 0$, see Eq. (12)], whereas the states with $H \in [\pi/2,\pi] + \pi n$ ($\epsilon_\uparrow < 0$, $\epsilon_\downarrow > 0$) give rise to the antiscreening of M_{F0} ($\sin 2H < 0$). Due to the property

$$\epsilon_{\uparrow,\downarrow}(\pi/2 + \pi n + \delta H) = \epsilon_{\downarrow,\uparrow}(\pi/2 + \pi n - \delta H),$$

$\delta H \in [0,\pi/2]$, such "up" and "down" states interchange in the energy space but since the spin direction is "attached" to them explicitly, this results in the opposite signs of contributions to $M(h)$.

E. Self-consistency of order parameter $\Delta(x)$

As it has been mentioned, the oscillating behavior of $M(h)$ is insensitive to the exact shape of $\Delta(x)$ and therefore persists if the self-consistency of $\Delta(x)$ is taken into account. This is the case, because the periodic functions of H in Eq. (11) arise from the solution of Eq. (10) in the F layer, where $\Delta(x)=0$ and the general solution can always be found explicitly. Due to this fact ig_z^3 is a periodic sign-changing function of H and, hence, $M(h)$ is a quasiperiodic sign-changing function of h/ϵ_A, independently of the exact shape of $\Delta(x)$. Since for arbitrary T the proof is cumbersome, we illustrate it here in the simplest case, when T is close to T_c. In this limit one can obtain the solution to Eq. (10) as an expansion in $\Delta(x)$. In the lowest (quadratic) order we find for $x>d$:

$$g_3^z(\omega,n_x,x) = -\frac{i}{2\xi^2}\sin\left(2\frac{h}{\epsilon_A}\frac{1}{|n_x|}\right)\frac{1}{\omega^2} \times \int_x^{\infty}\Delta(x')e^{-x'/\xi}dx'\int_d^{\infty}\Delta(y)e^{-y/\xi}dy,$$

where $\xi=v_F|n_x|/2|\omega|$. This yields the form

$$M(h) = -\int_1^{\infty}dt\,\sin\left(2\frac{h}{\epsilon_A}t\right)F(t,h),$$

where $F(t,h)$ is a *positive monotonically decreasing* with respect to t envelope function. The above integral can be both positive and negative depending on the value of h/ϵ_A. This is especially clear for $h/\epsilon_A \gg 1$ when one can integrate by parts to obtain

$$M(h) \approx \frac{\cos(2h/\epsilon_A)}{2h/\epsilon_A}F(1,h).$$

Therefore the induced magnetization is an oscillating function of the parameter $2h/\epsilon_A$ regardless of the exact form of $\Delta(x)$.

IV. LOW *SF* INTERFACE TRANSPARENCY

Our assumptions about ideal transport properties of the system (perfect *SF* interface transparency, ballistic electron motion in the samples) are not crucial for the existence of oscillations of induced magnetization. The oscillations of $M(h)$ exist for arbitrarily low *SF*-interface transparency (see the Appendix). Moreover, they can exist in the presence of bulk disorder in the sample.

To illustrate that we turn to the case of low *SF* interface transparency $t=t(n_x)\ll 1$ ($t(n_x)$ is a transmittance coefficient, see the Appendix). In this limit the proximity effect is weak and one can take the effect of disorder into account by linearizing the Eilenberger equation with collision term.[25,26] We assumed that superconductor is clean and ferromagnet is disordered and described by the mean free path l and scattering time $\tau=l/v_F$.

The Eilenberger equation in the F region $(0<x<d)$ for the "up" component $\check{g}_\uparrow=\check{g}$ reads (we omit the index $\uparrow$ here for brevity):

$$v_F n_x\partial_x\check{g} + \left[(\omega-ih)\tau_3 + \frac{1}{2\tau}\langle\check{g}\rangle,\check{g}\right] = 0, \qquad (18)$$

where $\langle\check{g}\rangle=1/2\int_{-1}^{1}dn_x\check{g}(n_x,x)$ is the angular averaging. In the S region $(x>d)$:

$$v_F n_x\partial_x\check{g} + [\omega\tau_3 + \Delta\tau_2,\check{g}] = 0. \qquad (19)$$

In the zeroth order in interface transparency $[t(n_x)=0]$ the superconductor and ferromagnet are not linked and the solution is:

$$\check{g}_F^{(0)} = \mathrm{sgn}\,\omega\tau_3, \quad 0<x<d,$$

$$\check{g}_S^{(0)} = f_S\tau_2 + g_S\tau_3, \quad x>d.$$

Next, we present the Green's function in the ferromagnet in the form

$$\check{g} = \check{g}_F^{(0)} + \delta\check{g}, \delta\check{g} = \delta g_1\tau_1 + \delta g_2\tau_2 + \delta g_3\tau_3$$

and, leaving only linear in $\delta\check{g}$ terms in Eq. (18), arrive at the following equation for $\delta\check{g}$:

$$v_F n_x\partial_x\delta\check{g} + \left[\left(\omega - ih + \frac{\mathrm{sgn}\,\omega}{2\tau}\right)\tau_3,\delta\check{g}\right] + \frac{\mathrm{sgn}\,\omega}{2\tau}[\langle\delta\check{g}\rangle,\tau_3] = 0. \qquad (20)$$

First we need to obtain a linear in t solution for δg_2 in the F region. From Eq. (18) we get

$$l^2 n_x^2\partial_x^2\delta g_2 - \alpha_\omega^2\delta g_2 = -\alpha_\omega\langle\delta g_2\rangle, \qquad (21)$$

where $\alpha_\omega=1+2(|\omega|-ih\,\mathrm{sgn}\,\omega)\tau$. The boundary condition with vacuum reads

$$\partial_x\delta g_2(x=0) = 0. \qquad (22)$$

At the *SF* interface one must use Zaitsev boundary conditions (A1) for nonideal interface transparency. In the limit $t\ll 1$ one can expand them in t. First order in t gives

$$\delta g_1(n_x,x=d-0) = i\frac{t}{2}\,\mathrm{sgn}\,\omega\,\mathrm{sgn}\,n_x f_S.$$

Using the τ_2 component of Eq. (20)

$$l n_x\partial_x\delta g_2 + i\alpha_\omega\,\mathrm{sgn}\,\omega\delta g_1 = 0,$$

we arrive at the boundary condition for δg_2 at the *SF* interface

$$\partial_x\delta g_2(x=d-0) = \frac{t(n_x)}{2l|n_x|}f_S\alpha_\omega. \qquad (23)$$

Equation (21) must be solved for $x\in[0,d]$ with boundary conditions (22) and (23). Condition Eq. (22) allows one to symmetrically continue δg_2 to the $[-d,0]$ interval. We perform the Fourier transformation[25,26]

$$\delta g_2(x) = \sum_{n=-\infty}^{+\infty}\delta g_2(n)e^{ik_n x},$$

where $k_n=\pi n/d$ (n is integer) and Fourier coefficients are

$$\delta g_2(n) = \frac{1}{2d}\int_{-d}^{d} \delta g_2(x) e^{-ik_n x} dx.$$

Calculating Fourier coefficient of both sides of Eq. (21) and taking the boundary condition (23) into account, we get

$$\delta g_2(n) = \frac{\alpha_\omega}{L_n}(\langle \delta g_2(n)\rangle + (-1)^n \epsilon_A \tau |n_x| t(n_x) f_S), \qquad (24)$$

where $L_n = l^2 n_x^2 k_n^2 + \alpha_\omega^2$. Angular averaging of Eq. (24) gives

$$\langle \delta g_2(n)\rangle = \frac{(-1)^n \alpha_\omega f_S \epsilon_A \tau \left\langle \dfrac{|n_x| t}{L_n}\right\rangle}{1-\alpha_\omega \left\langle \dfrac{1}{L_n}\right\rangle}$$

and thus

$$\delta g_2(n) = (-1)^n \delta g_2^*(n),$$

$$\delta g_2^*(n) = \frac{\alpha_\omega}{L_n}\epsilon_A \tau f_S \left(\frac{\alpha_\omega \left\langle \dfrac{|n_x| t}{L_n}\right\rangle}{1-\alpha_\omega \left\langle \dfrac{1}{L_n}\right\rangle} + |n_x| t \right). \qquad (25)$$

Next we solve Eq. (19) in the S region and get for $\delta\check{g} = \check{g} - \check{g}_S^{(0)}$

$$\delta\check{g} = c(n_x)(\mathrm{sgn}\, n_x \tau_1 - g_S \tau_2 + f_S \tau_3) e^{-2\sqrt{\omega^2+\Delta^2}(x-d)/v_F |n_x|} \qquad (26)$$

where $c(n_x)$ is a symmetric (yet unknown) function of n_x.

It follows from Eqs. (9) and (26) that the induced magnetic moment is given by [again we assume the case of a thin F film ($d \ll \xi_S$) and therefore the induced magnetic moment $M(h) \approx M_S(h)$ is located in the superconductor]:

$$M(h) = \mu_B \nu_F 2\pi T \sum_\omega \int_0^1 dn_x \frac{v_F n_x}{\sqrt{\Delta^2+\omega^2}} f_S \,\mathrm{Im}\, c(n_x).$$

Expanding Zaitsev's boundary conditions to the second order in t, we obtain

$$\mathrm{Im}\, c(n_x) = -\frac{t(n_x)}{2} g_S \,\mathrm{Im}\, \delta g_2(n_x, x = d-0)$$

and therefore

$$M(h) = -\mu_B \nu_F v_F \pi T \sum_\omega \frac{f_S g_S}{\sqrt{\Delta^2+\omega^2}} \times \int_0^1 dn_x n_x t(n_x) \mathrm{Im}\, \delta g_2(x=d).$$

The needed quantity is

$$\delta g_2(x=d) = \sum_{n=-\infty}^{\infty} (-1)^n \delta g_2(n) = \sum_{n=-\infty}^{\infty} \delta g_2^*(n).$$

Below we analyze two different limiting cases depending on the strength of disorder. It appears that the influence of disorder on behavior of induced magnetization $M(h)$ is governed by parameter $h\tau$ rather than d/l.

A. Quasiballistic case ($h\tau \gg 1$)

If $h\tau \gg 1$, then $\alpha_\omega / L_n \sim 1/\alpha_\omega \sim 1/(h\tau) \ll 1$ and one can neglect the first term in parentheses in Eq. (25) and get

$$\delta g_2^*(n) = \frac{\alpha_\omega}{L_n}\epsilon_A \tau f_S |n_x| t.$$

Summing the series, we get

$$\delta g_2(n_x, n) = \frac{t}{2} f_S \coth\left(\frac{|\omega| - ih\,\mathrm{sgn}\,\omega + \dfrac{1}{2\tau}}{\epsilon_A |n_x|}\right)$$

and

$$M(h) = -\frac{1}{2}\mu_B \nu_F v_F \pi T \sum_\omega \frac{f_S^2 g_S}{\sqrt{\Delta^2+\omega^2}} \times \int_0^1 dn_x n_x t^2(n_x) \mathrm{Im}\coth\left(\frac{|\omega| - ih\,\mathrm{sgn}\,\omega + \dfrac{1}{2\tau}}{\epsilon_A |n_x|}\right).$$

For not too small disorder ($d/l \gtrsim 1$) [and a thin F layer ($d \ll \xi_S$)] we get

$$M(h) = -\frac{1}{2}\mu_B \nu_F v_F \pi T \sum_\omega \frac{\Delta^2 |\omega|}{(\omega^2+\Delta^2)^2} \times \int_0^1 dn_x n_x t^2(n_x) \exp\left(-\frac{2d}{n_x l}\right) \sin 2H.$$

One sees that oscillatory behavior is sustained, although the magnitude of oscillations is suppressed in $t(n_x) \ll 1$ and l/d. Extrapolation of this formula to not too small $t \sim 1$ gives that for moderate disorder $l \gtrsim d$ the magnitude of oscillations is quite comparable to the ideal case Eq. (12). We also mention here, that spin-orbit scattering should also suppress oscillations of $M(h)$. Spin-orbit scattering is neglegible, if the corresponding mean free path $l_{SO} \gg d$. This condition is always satisfied for $l \gtrsim d$, because $l_{SO} \gg l$.

B. Diffusive case ($h\tau \ll 1$)

If $h\tau \ll 1$ and $l \ll d$, then $\alpha_\omega \langle 1/L_n \rangle \to 1$ and the main contribution to $\delta g_2^*(n)$ comes from the first term in parentheses in Eq. (25), which has a (diffusion) pole, and the second term can be neglected:

$$\delta g_2^*(n) = \epsilon_A f_S \langle |n_x| t \rangle \frac{1}{2(|\omega| - ih\,\mathrm{sgn}\,\omega) + D k_n^2},$$

where $D = v_F l/3$ is the diffusion coefficient. Summing the series, we get

$$\delta g_2(x=d) = f_S\langle|n_x|t\rangle\frac{3d}{2l}\frac{\coth\sqrt{\frac{2(|\omega|-ih\,\mathrm{sgn}\,\omega)}{E_{\mathrm{Th}}}}}{\sqrt{\frac{2(|\omega|-ih\,\mathrm{sgn}\,\omega)}{E_{\mathrm{Th}}}}}$$

and

$$M(h) = -\mu_B\nu_F v_F\pi T\sum_\omega\frac{f_S^2 g_S}{\sqrt{\Delta^2+\omega^2}}\times\langle|n_x|t(n_x)\rangle^2\frac{3d}{2l}\,\mathrm{Im}\,\frac{\coth\sqrt{\frac{2(|\omega|-ih\,\mathrm{sgn}\,\omega)}{E_{\mathrm{Th}}}}}{\sqrt{\frac{2(|\omega|-ih\,\mathrm{sgn}\,\omega)}{E_{\mathrm{Th}}}}},$$

where $E_{\mathrm{Th}}=D/d^2$ is the Thouless energy. This result is valid, if $t \ll l/d$ and $h \gg t\epsilon_A$. Since the conditions $h\tau \ll 1$ and $l \ll d$ correspond to "diffusive" regime, this result could be obtained from the Usadel equation.

Interestingly, Im $\delta g_2(x=d)$ *does not oscillate* as a function of h, even though it contains trigonometric functions. Therefore, we obtain that in the diffusive limit $h\tau \ll 1$ the induced magnetization $M(h)$ is not oscillatory and always negative.

V. CONCLUSION

In conclusion, we have shown that in SF systems the total spin magnetic moment in the superconducting state can be both smaller and larger than that in the normal state. The effect is due to peculiar periodic properties of Andreev states in *SF* systems that result in oscillatory sign-changing behavior of the superconductivity-induced magnetization of the system. The predicted effect is expected to be best observable in relatively clean *SF* systems with good quality of interfaces. Practically this means that the mean free path l should be larger than the "exchange length" $l_{\mathrm{exc}}=v_F/h$. This condition can be fulfilled in the case of sufficiently strong ferromagnets. On the other hand l should not be much smaller than the thickness of the ferromagnetic film d. We ignored a change in the magnetic moment $M(h)$ caused by the Meissner currents assuming that the thicknesses of the ferromagnet and superconductor are smaller than the London penetration length. In this case the contribution of these currents to $M(h)$ is small. Spontaneous orbital effects in clean *SF* structures were studied in Ref. 27.

Note added in proof: Recently, a paper by F. S. Bergeret, A. L. Yeyati, and A. Martín-Rodero, Phys. Rev. B **72**, 064524 (2005) was published, in which a similar problem was considered. The main results of this work (the dependence of the induced magnetization on the SF interface transmittance) were obtained numerically.

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APPENDIX: CLEAN SAMPLES, ARBITRARY SF INTERFACE TRANSPARENCY

Boundary conditions for nonideal interface transparency have been derived by Zaitsev.[24] They are expressed in terms of the antisymmetric

$$\check{a}(n_x,x) = [\check{g}(n_x,x) - \check{g}(-n_x,x)]/2$$

and symmetric

$$\check{s}(n_x,x) = [\check{g}(n_x,x) + \check{g}(-n_x,x)]/2$$

parts of Green's function in the following way:

$$\check{a}[(1-t)(\check{s}_+ + \check{s}_-)^2 + (\check{s}_+ - \check{s}_-)^2] = t(\check{s}_+ - \check{s}_-)(\check{s}_+ + \check{s}_-). \tag{A1}$$

Here $\check{a}=\check{a}(n_x,d)$, $\check{s}_+=\check{s}(n_x,d+0)$, $\check{s}_-=\check{s}(n_x,d-0)$. Antisymmetric part $\check{a}(n_x,d)$ is continuous at the boundary $x=d$. The transmittance coefficient $t(n_x)$ can vary from $t(n_x)=0$ for nontransparent interface (e.g., boundary with vacuum) to $t(n_x)=1$ for perfectly transparent interface.

In the case of clean samples and arbitrary transparency $t(n_x)$ one must solve Eilenberger equation (10) with boundary conditions (A1). We obtain $(\omega>0)$

$$g_3^z(\omega,n_x,x) = if_S^2\,\mathrm{Im}\,\frac{\sinh(\Omega-iH)}{\sqrt{[g_S\cosh(\Omega-iH)+(2/t(n_x)-1)\sinh(\Omega-iH)]^2+f_S^2}}\exp\left(-\frac{\sqrt{\omega^2+\Delta^2}}{\epsilon_A}\frac{x/d-1}{|n_x|}\right) \tag{A2}$$

in the superconductor $(x>d)$ and

$$g_3^z(\omega,n_x,x) = 2i\,\mathrm{Im}\,\frac{g_S\cosh(\Omega-iH)+(2/t(n_x)-1)\sinh(\Omega-iH)}{\sqrt{[g_S\cosh(\Omega-iH)+(2/t(n_x)-1)\sinh(\Omega-iH)]^2+f_S^2}} \tag{A3}$$

in the ferromagnet $(0<x<d)$. (For notation, see Secs. III A and III B. One can check that induced magnetization $M(h)$ following from these formulas is an oscillating sign-changing function for arbitrary $t(n_x)$, $0<t(n_x)\leq 1$. The magnitude of oscillations of $M(h)$ is greatest at $t(n_x)=1$ and decreases as $t(n_x)$ decreases; $M(h)=0$ at $t(n_x)=0$.

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[28] It should be noted that it was assumed in Ref. 9 and will be assumed throughout this paper that no spin-orbit or magnetic scattering is present in the system; such processes would, of course, affect this result.
[29] This statement and expression (17) are actually violated when H is close to πn, however, this is not important for the present considerations.