Spin and charge Josephson effects between nonuniform superconductors with coexisting helimagnetic order

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We consider the spin and charge Josephson current between two nonuniform Fulde-Ferrel-Larkin-Ovchinnikov superconductors with helimagnetic order. We demonstrate that the presence of the helimagnetic phase generates a spin Josephson effect and leads to additional contributions to both single-particle and Josephson charge current. It is shown that for such systems the ac effect differs more radically from the dc effect than in the case of a Bardeen–Cooper–Schrieffer superconductor with helimagnetic order considered earlier in the literature [M. L. Kulić and I. M. Kulić, Phys. Rev. B **63**, 104503 (2001)] where a spin Josephson current has also been found. In our system the most interesting effect occurs in the presence of an external magnetic field and in absence of voltage, where we show that the charge Josephson current can be tuned to zero while the spin Josephson current is nonvanishing. This provides a well controlled mechanism to generate a spin supercurrent in absence of charge currents.

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I. INTRODUCTION

Collective spin and charge transport phenomena in ordered many-particle systems are of great importance in modern condensed matter physics. Among them is the dissipationless, Cooper-pair driven, transport in superconductors and in the superfluid ^{3}He. One remarkable consequence of the supercurrent phenomenon is the Josephson effect,[1] which predicts that a superflow exists between two superfluid systems (charged or not) separated by a weak link, and that its value is proportional to the sine of the difference between the phases of the complex order parameter across the link.

Recently, due to the growing interest in spintronics devices, there were a number of works exploring the possibility for dissipationless spin current.[2–6] In singlet superconductors it cannot occur because the total spin of the Cooper pair is zero. However, in unconventional triplet superconductors this may not be the case. Moreover, the B phase of superfluid ^{3}He exhibits both mass and spin-1 supercurrents. The latter was probed in an experiment where two ^{3}He-B superfluids were in contact through a weak link.[7] This led to the observation of a spin Josephson effect, thus establishing the existence of spin supercurrents in the B phase of superfluid ^{3}He. More recently it was pointed out that a spin Josephson effect between two triplet ferromagnetic superconductors may occur.[8]

In the abovementioned scenarios of phase coherence in systems with fermionic pairing the order parameter is uniform. Superconductivity with nonuniform order parameter occurs, for example, in the presence of an exchange field. This class of superconductors is well described by the so-called Fulde–Ferrel–Larkin–Ovchinnikov (FFLO) state.[9,10] Experimentally, it should occur in extremely high-field superconductors, which are obviously of high practical use. Recently, strong evidence has been found that a FFLO state might be realized in the quasi-two-dimensional heavy-fermion superconductor $CeCoIn_5$[11,12] for a magnetic field applied along the *ab* plane. In this respect, the coexistence of a helimagnetic phase induced by the in-plane magnetic field, or as an intrinsic order parameter, may result in various interesting transport phenomena similar to some of the ^{3}He features, though the system considered is in a singlet state.

In this paper we analyze the spin and charge tunneling processes between two FFLO-like helimagnetic superconductors and find that the spin-flip processes associated to the helimagnetic phase result in spin and charge tunneling of the Josephson type, i.e., the phase differences of the superconducting and helimagnetic orders are involved in the tunnel process. Previously a similar analysis was undertaken for helimagnetic superconductors with a uniform superconducting order parameter.[13] In the case of vanishing voltage our results reduce essentially to the ones obtained in Ref. 13. However, at nonzero voltages the corresponding ac Josephson effect in nonuniform helimagnetic superconductors changes the physics more drastically than in the case of Bardeen–Cooper–Schrieffer superconductors. We will also study the effect of a nonzero magnetic field in the Josephson currents at zero voltage. It will be shown that the magnetic field can be used to tune the charge Josephson current to zero, while still having a nonzero spin Josephson current. In this way we provide a mechanism by which a spin supercurrent exists in the absence of charge currents.

The plan of the paper is as follows. In Sec. II we briefly discuss the Josephson effect between two FFLO superconductors from the Ginzburg–Landau theory for the FFLO state. Section III introduces our model for a nonuniform helimagnetic superconductor. There we derive the Green functions of the theory using mean-field approach. In Sec. IV the single-particle and Josephson charge and spin currents are derived using linear response. The effect of an external magnetic field is considered in Sec. V. Our conclusions are presented in Sec. VI.

II. JOSEPHSON EFFECT BETWEEN TWO NONUNIFORM SUPERCONDUCTORS

In order to gain some insight into the physics of the Josephson effect between nonuniform superconductors, let us consider first a FFLO superconductor, where the exchange field is uniform. For this situation a Ginzburg–Landau free energy has been derived in Ref. 14. We can use this result to obtain the charge supercurrent in the absence of magnetic field[14]

$$\mathbf{j} = -i2e[\beta + (\mu - 2\eta)|\psi|^2](\psi^* \nabla \psi - \psi \nabla \psi^*) - 4ie\delta(\nabla\psi^* \nabla^2 \psi - \nabla\psi\nabla^2\psi^*), \tag{1}$$

where β, μ, η, and δ are phenomenological parameters. When two such FFLO superconductors are connected through a tunnel junction, we can consider the charge flow from the left to the right subsystems with appropriate boundary conditions at the tunnel junction. The procedure is similar to the case of ordinary uniform superconductors, except that the above current must be used instead. Let us consider the x component of the current $\mathbf{j}_L$ coming from the left side of the junction, which is given by the expression (1) along the x direction, with ψ replaced by ψ_L. At lowest order we have the boundary conditions $\partial_x \psi_L = \lambda \psi_R$ and $\partial_x^2 \psi_L = \lambda \partial_x \psi_R$ at the tunnel junction, where ψ_R is the order parameter of the right subsystem and λ is a parameter depending on the details of the junction. By assuming an order parameter of the Fulde–Ferrel type, we can approximately write $\psi_L(\mathbf{r}) = \rho_0 e^{i(\theta_L + \mathbf{q}\cdot\mathbf{r})}$ and $\psi_R(\mathbf{r}) = \rho_0 e^{i(\theta_R + \mathbf{q}\cdot\mathbf{r})}$, with ρ_0=const. Note that we are assuming that both sides are made with the same material, so that the amplitude ρ_0 is the same on either side. The current flowing through the junction is then

$$j_{Lx} = 4e\lambda\rho_0^2[2\delta q_x^2 + \beta + (\mu - 2\eta)\rho_0^2]\sin\Delta\theta, \tag{2}$$

where $\Delta\theta \equiv \theta_R - \theta_L$. Note that the amplitude of the Josephson current depends on the FFLO characteristic momentum. The presence of a Josephson effect between two FFLO superconductors is in contrast with the situation of a tunnel junction between a FFLO superconductor and a superconductor having a uniform order parameter. In such a case, it can be shown that the Josephson effect is suppressed, since the uniform state is not able to balance the spatial oscillations from the FFLO state.[15]

III. NONUNIFORM HELIMAGNETIC SUPERCONDUCTORS

A. Helimagnetic superconductors

The study of helimagnetic superconductors has a long story, mainly associated with heavy fermion materials. Particularly interesting is the following model introduced a long time ago,[16] whose free energy is given by

$$\mathcal{F} = |(\nabla - i2e\mathbf{A})\psi|^2 + a|\psi|^2 + \frac{b}{2}|\psi|^4 + \frac{1}{2}(\nabla\mathbf{M})^2 + \frac{r}{2}\mathbf{M}^2 + \frac{u}{8}(\mathbf{M}^2)^2 + \frac{1}{8\pi}(\nabla\times\mathbf{A} - 4\pi\mathbf{M})^2 \tag{3}$$

where $\mathbf{M}$ is the macroscopic magnetization. The above free energy admits a mean-field solution with a helical magnetically ordered state and a uniform superconducting order parameter. A nonuniform order parameter of the Fulde–Ferrel type, $\psi(\mathbf{r}) = \psi_0 e^{i\mathbf{q}\cdot\mathbf{r}}$, does not work in this case, since the wave-vector $\mathbf{q}$ can be gauged away through a gauge transformation $\mathbf{A} \to \mathbf{A} + \mathbf{q}/2e$. Thus, for the above model a possible nonuniformity of the superconducting order parameter does not contain any additional physics with respect to the uniform case, at least not at the macroscopic level.

A mean-field microscopic model having a uniform superconducting order parameter and helimagnetic order would have, in an easy-plane configuration, the Hamiltonian

$$H = \frac{1}{2m}\sum_\sigma \nabla c_\sigma^\dagger(\mathbf{r}) \cdot \nabla c_\sigma(\mathbf{r}) - \mu \sum_\sigma c_\sigma^\dagger(\mathbf{r}) c_\sigma(\mathbf{r}) + \Delta_0 c_\uparrow^\dagger(\mathbf{r}) c_\downarrow^\dagger(\mathbf{r}) + h_\mathbf{q} e^{i\mathbf{q}\cdot\mathbf{r}} c_\uparrow^\dagger(\mathbf{r}) c_\downarrow(\mathbf{r}) + \text{h.c.} \tag{4}$$

The transformation $c_\sigma \to e^{i\sigma\mathbf{q}\cdot\mathbf{r}/2} c_\sigma$ produces a spin current response, since the Hamiltonian becomes

$$H = \frac{1}{2m}\sum_\sigma \nabla c_\sigma^\dagger(\mathbf{r}) \cdot \nabla c_\sigma(\mathbf{r}) - \left(\mu - \frac{q^2}{8m}\right) \times \sum_\sigma c_\sigma^\dagger(\mathbf{r}) c_\sigma(\mathbf{r}) + \Delta_0 c_\uparrow^\dagger(\mathbf{r}) c_\downarrow^\dagger(\mathbf{r}) + h_\mathbf{q} c_\uparrow^\dagger(\mathbf{r}) c_\downarrow(\mathbf{r}) + \text{h.c.} - \mathbf{q}\cdot(\mathbf{j}_\uparrow - \mathbf{j}_\downarrow), \tag{5}$$

where

$$\mathbf{j}_\sigma = \frac{i}{4m}[c_\sigma^\dagger \nabla c_\sigma - (\nabla c_\sigma^\dagger) c_\sigma] \tag{6}$$

is the current for the spin σ fermion. Thus, although in the above microscopic model there are no charge currents in the absence of electromagnetic coupling (the momentum of the Cooper pairs is zero), there is a spin current. However, if in the Hamiltonian (4) Δ_0 is replaced by a Fulde–Ferrel mean-field order parameter $\Delta_\mathbf{p} e^{i\mathbf{p}\cdot\mathbf{r}}$ and the transformation $c_\sigma \to e^{i(\mathbf{p}+\sigma\mathbf{q}\cdot\mathbf{r}/2)} c_\sigma$ is done, we obtain a charge current response in addition to the spin current one, i.e.,

$$H = \frac{1}{2m}\sum_\sigma \nabla c_\sigma^\dagger(\mathbf{r}) \cdot \nabla c_\sigma(\mathbf{r}) - \sum_\sigma \left[\mu - \frac{(\mathbf{p}+\sigma\mathbf{q})^2}{8m}\right] \times c_\sigma^\dagger(\mathbf{r}) c_\sigma(\mathbf{r}) + \Delta_\mathbf{p} c_\uparrow^\dagger(\mathbf{r}) c_\downarrow^\dagger(\mathbf{r}) + h_\mathbf{q} c_\uparrow^\dagger(\mathbf{r}) c_\downarrow(\mathbf{r}) + \text{h.c.} - \mathbf{q}\cdot(\mathbf{j}_\uparrow - \mathbf{j}_\downarrow) - \mathbf{p}\cdot(\mathbf{j}_\uparrow + \mathbf{j}_\downarrow). \tag{7}$$

From the above equation we see that there is an excess kinetic energy of amount $(\mathbf{p}+\mathbf{q})^2/(8m)$ and $(\mathbf{p}-\mathbf{q})^2/(8m)$ for the spin up and down electrons, respectively. Setting $\mathbf{p}=\mathbf{q}$ has the effect of producing a (charge) current response only for the up spin electrons while adding no extra kinetic energy to the down spin electrons. The situation in such a state is the one similar to injecting fully polarized electrons in a sample. An important additional property of the $\mathbf{p}=\mathbf{q}$ state is that the magnetic order parameter $\langle c^{\dagger}_{\mathbf{k}+\mathbf{q}/2\uparrow}c_{\mathbf{k}-\mathbf{q}/2\downarrow}\rangle$ can be transformed in the superconducting one $\langle c^{\dagger}_{\mathbf{k}+\mathbf{q}/2\uparrow}c^{\dagger}_{-\mathbf{k}+\mathbf{q}/2\downarrow}\rangle$ through a particle-hole transformation in the down spin channel, i.e., $c_{\mathbf{k}-\mathbf{q}/2\downarrow}\rightarrow c^{\dagger}_{-\mathbf{k}+\mathbf{q}/2\downarrow}$. In other words, the corresponding order parameters can be rotated into one another. The magnetic and superconducting order parameters with a same helical pattern are more coherent: a nonuniform Cooper pair breaking is likely to imply a decay into the helimagnetic state. Due to these interesting properties, we will consider from now on a mean-field microscopic model where both superconducting and magnetic order parameters have the same helical pattern.

Finally, we would like to stress here that there is no contradiction between Eqs. (3) and (7). As a matter of fact, Eq. (3) is not suitable to describe a superconductor with a nonuniform superconducting order parameter. Instead, another form of the free energy has to be used,[14] since in this case higher order derivatives have to be taken into account.

B. Green's functions for nonuniform helimagnetic superconductors

Following the discussion of the previous subsection, let us consider a FFLO-like superconductor with a helimagnetic molecular field. The mean-field Hamiltonian is given in momentum space by

$$H_{\mathrm{MF}}=\sum_{\mathbf{k}\sigma}\varepsilon_{\mathbf{k}}c^{\dagger}_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma}+\sum_{\mathbf{k}}\Delta^{*}_{\mathbf{q}}c_{-\mathbf{k}+\mathbf{q}/2\downarrow}c_{\mathbf{k}+\mathbf{q}/2\uparrow}+\sum_{\mathbf{k}}h_{\mathbf{q}}c^{\dagger}_{\mathbf{k}+\mathbf{q}/2\uparrow}c_{\mathbf{k}-\mathbf{q}/2\downarrow}+\text{h.c.},\tag{8}$$

where $\epsilon_{\mathbf{k}}$ is the quadratic dispersion of the free electrons. We are assuming that the oscillation of the superconducting condensate is characterized by a single wave vector $\mathbf{q}$, i.e., $\psi(\mathbf{r})\propto e^{i\mathbf{q}\cdot\mathbf{r}}$;[9] $h_{\mathbf{q}}$ is a complex mean-field variable describing the helimagnetic phase characterized by the electron-hole singlet pairing $\langle c^{\dagger}_{\mathbf{k}+\mathbf{q}/2\uparrow}c_{\mathbf{k}-\mathbf{q}/2\downarrow}\rangle$. As already discussed, *in our model both the superconducting and helimagnetic order parameters are modulated by the same wave-vector* $\mathbf{q}$. This is important, since for $\mathbf{q}=0$ no coexistence between magnetic and superconducting order will be possible within our model.

Generally, helimagnetism can be induced by the external inhomogeneous magnetic field applied along the x direction or arising from internal (spiral) magnetic order. In the absence of superconductivity, our theory reduces to the one considered in Ref. 4, which corresponds to a magnetic analog of the FFLO state. There the existence of persistent spin currents was demonstrated.

From the mean-field Hamiltonian we see that in the present case not only is the gauge symmetry spontaneously broken, leading to a lack of particle number conservation, but also the spin conservation symmetry is broken due to the helimagnetic phase. Both averages are complex and have therefore an amplitude and a phase, i.e., $\Delta_{\mathbf{q}}=|\Delta_{\mathbf{q}}|e^{-i\theta}$ and $h_{\mathbf{q}}=|h_{\mathbf{q}}|e^{-i\varphi}$. In a bulk system both phases can be gauged away through a global gauge transformation. This is of course not the case when we consider the tunneling processes between two superconductors and, as we will show later, the phase of the helimagnetic order parameter will play an important role.

The mean-field Hamiltonian (8) can be conveniently rewritten in matrix form as $H_{\mathrm{MF}}=(1/2)\Sigma_{\mathbf{k}}\eta^{\dagger}_{\mathbf{k}}M_{\mathbf{k}}\eta_{\mathbf{k}}$, where $\eta^{\dagger}_{\mathbf{k}}=[c^{\dagger}_{\mathbf{k}+\mathbf{q}/2\uparrow}\ c^{\dagger}_{\mathbf{k}-\mathbf{q}/2\downarrow}\ c_{-\mathbf{k}+\mathbf{q}/2\downarrow}\ c_{-\mathbf{k}-\mathbf{q}/2\uparrow}]$ and

$$M_{\mathbf{k}}=\begin{bmatrix}\varepsilon_{\mathbf{k}+\mathbf{q}/2} & |h_{\mathbf{q}}|e^{-i\varphi} & |\Delta_{\mathbf{q}}|e^{-i\theta} & 0\\ |h_{\mathbf{q}}|e^{i\varphi} & \varepsilon_{\mathbf{k}-\mathbf{q}/2} & 0 & -|\Delta_{\mathbf{q}}|e^{-i\theta}\\ |\Delta_{\mathbf{q}}|e^{i\theta} & 0 & -\varepsilon_{-\mathbf{k}+\mathbf{q}/2} & -|h_{\mathbf{q}}|e^{-i\varphi}\\ 0 & -|\Delta_{\mathbf{q}}|e^{i\theta} & -|h_{\mathbf{q}}|e^{-i\varphi} & -\varepsilon_{-\mathbf{k}-\mathbf{q}/2}\end{bmatrix}.\tag{9}$$

The matrix $M_{\mathbf{k}}$ can be easily diagonalized, which leads to the following energy spectrum:

$$E^{\alpha,\beta}_{\mathbf{k}}=\alpha\sqrt{\varepsilon^2_{\mathbf{k}s}+|\Delta_{\mathbf{q}}|^2}+\beta\sqrt{\varepsilon^2_{\mathbf{k}a}+|h_{\mathbf{q}}|^2}\tag{10}$$

and $\alpha,\beta=\pm1$. Here we introduce $\varepsilon_{\mathbf{k}s}=(\varepsilon_{\mathbf{k}+\mathbf{q}/2}+\varepsilon_{\mathbf{k}-\mathbf{q}/2})/2$ and $\varepsilon_{\mathbf{k}a}=(\varepsilon_{\mathbf{k}+\mathbf{q}/2}-\varepsilon_{\mathbf{k}-\mathbf{q}/2})/2$ similarly to Refs. 17 and 18. Since we assume the quadratic dispersion for the free electrons, we have $\varepsilon_{\mathbf{k}s}=\varepsilon_{\mathbf{k}}+q^2/8m$ and $\varepsilon_{\mathbf{k}a}=v_Fq/2\cos x$ and x is the angle between $\mathbf{k}$ and $\mathbf{q}$.

The matrix Green's function is obtained by inverting the matrix $-i\omega I+M_{\mathbf{k}}$, where I is the identity matrix. The independent elements of the matrix Green's function are

$$G^{\uparrow,\uparrow}_{1}(i\omega_n,\mathbf{k})\equiv\langle c^{\dagger}_{\mathbf{k}+\mathbf{q}/2\uparrow}(i\omega)c_{\mathbf{k}+\mathbf{q}/2\uparrow}(i\omega)\rangle=\frac{(u^{++}_{\mathbf{k}})^2}{i\omega_n-E_{1\mathbf{k}}}+\frac{(u^{--}_{\mathbf{k}})^2}{i\omega_n+E_{1\mathbf{k}}}+\frac{(u^{+-}_{\mathbf{k}})^2}{i\omega_n-E_{2\mathbf{k}}}+\frac{(u^{-+}_{\mathbf{k}})^2}{i\omega_n+E_{2\mathbf{k}}},\tag{11}$$

$$G^{\uparrow,\downarrow}_{2}(i\omega_n,\mathbf{k})\equiv\langle c^{\dagger}_{\mathbf{k}+\mathbf{q}/2\uparrow}(i\omega)c_{\mathbf{k}-\mathbf{q}/2\downarrow}(i\omega)\rangle=-\frac{h_{\mathbf{q}}e^{-i\varphi}(E_{1\mathbf{k}}E_{2\mathbf{k}}+2(i\omega_n)\varepsilon_{\mathbf{k}s}+(i\omega_n)^2)}{(i\omega_n-E_{1\mathbf{k}})(i\omega_n-E_{2\mathbf{k}})(i\omega_n+E_{1\mathbf{k}})(i\omega_n+E_{2\mathbf{k}})},\tag{12}$$

$$F^{\uparrow,\downarrow}_{1}(i\omega_n,\mathbf{k})\equiv\langle c^{\dagger}_{\mathbf{k}+\mathbf{q}/2\uparrow}(i\omega)c^{\dagger}_{-\mathbf{k}+\mathbf{q}/2\downarrow}(-i\omega)\rangle=-\frac{\Delta_{\mathbf{q}}e^{-i\theta}[(i\omega_n)^2-E_{1\mathbf{k}}E_{2\mathbf{k}}-2(i\omega_n)\varepsilon_{\mathbf{k}a}]}{(i\omega_n-E_{1\mathbf{k}})(i\omega_n-E_{2\mathbf{k}})(i\omega_n+E_{1\mathbf{k}})(i\omega_n+E_{2\mathbf{k}})},\tag{13}$$

$$F_2^{\uparrow,\uparrow}(i\omega_n,\mathbf{k}) \equiv \langle c^\dagger_{\mathbf{k+q/2}\uparrow}(i\omega)c^\dagger_{-\mathbf{k-q/2}\uparrow}(-i\omega)\rangle = \frac{2h_{\mathbf{q}}e^{-i\varphi}\Delta_{\mathbf{q}}e^{-i\theta}(i\omega_n)}{(i\omega_n - E_{1\mathbf{k}})(i\omega_n - E_{2\mathbf{k}})(i\omega_n + E_{1\mathbf{k}})(i\omega_n + E_{2\mathbf{k}})}, \tag{14}$$

where we have set $E_{1\mathbf{k}} \equiv E^{+,+}_{\mathbf{k}}$ and $E_{2\mathbf{k}} \equiv E^{+,-}_{\mathbf{k}}$ and the generalized Bogolyubov coefficients are

$$u^{\alpha\beta}_{\mathbf{k}} = \frac{1}{2}\left[1 + (2\delta_{\alpha\beta}-1)\frac{4\varepsilon_{\mathbf{k}a}\varepsilon_{\mathbf{k}s}}{E^2_{1\mathbf{k}} - E^2_{2\mathbf{k}}} + \frac{2\alpha\varepsilon_{\mathbf{k}s}}{E_{1\mathbf{k}} + E_{2\mathbf{k}}} + \frac{2\beta\varepsilon_{\mathbf{k}a}}{E_{1\mathbf{k}} - E_{2\mathbf{k}}}\right]^{1/2}, \tag{15}$$

where $\alpha,\beta = \pm 1$.

IV. JOSEPHSON EFFECT BETWEEN TWO NONUNIFORM HELIMAGNETIC SUPERCONDUCTORS

We will study first the charge single-particle and Cooper-pair Josephson tunneling processes between two FFLO superconductors. We use the standard tunneling Hamiltonian[19,20] in the form

$$H_T = \sum_{\mathbf{k}\mathbf{p}\sigma} T_{\mathbf{k},\mathbf{p}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{p}\sigma} + \text{h.c.}, \tag{16}$$

where $\mathbf{k}$ and $\mathbf{p}$ label single electron momentum eigenstates in the left and right subsystems, respectively. The charge current is given by $I^{\text{charge}} = -e\langle \dot{N}_L(t)\rangle$, where $N_L = \Sigma_{\mathbf{k},\sigma} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma}$ and the spin current is given by $I^{\text{spin}} = -\mu_B\langle \dot{S}_z(t)\rangle$, where $S_z = \Sigma_{\mathbf{k},\sigma}\sigma c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma}$ and μ_B is the Bohr magneton. In the linear response regime the charge and spin currents are given by $I^{\text{charge}}_s = 2e\,\text{Im}[X_{\text{charge}}(\text{eV}+i\delta)]$ and $I^{\text{spin}}_s = 2\mu_B\,\text{Im}[X_{\text{spin}}(\text{eV}+i\delta)]$, where $\delta \to 0^+$. In terms of the Matsubara formalism at lowest order one gets

$$X_{\text{charge,spin}}(i\omega) = -\frac{1}{\beta}\sum_{\omega_n}\sum_{\mathbf{k},\mathbf{p}}[|T_{\mathbf{k+q/2},\mathbf{p+q/2}}|^2 G_1^{\uparrow,\uparrow}(\mathbf{k},i\omega_n)G_1^{\uparrow,\uparrow}(\mathbf{p},i\omega_n - i\omega_m) \pm |T_{\mathbf{k-q/2},\mathbf{p-q/2}}|^2 G_1^{\downarrow,\downarrow}(\mathbf{k},i\omega_n)G_1^{\downarrow,\downarrow}(\mathbf{p},i\omega_n - i\omega_m) + T_{\mathbf{k+q/2},\mathbf{p+q/2}}T^*_{\mathbf{k-q/2},\mathbf{p-q/2}}G_2^{\uparrow,\downarrow}(\mathbf{k},i\omega_n)G_2^{\downarrow,\uparrow}(\mathbf{p},i\omega_n - i\omega_m) \pm T_{\mathbf{k-q/2},\mathbf{p-q/2}}T^*_{\mathbf{k+q/2},\mathbf{p+q/2}}G_2^{\downarrow,\uparrow}(\mathbf{k},i\omega_n)G_2^{\uparrow,\downarrow}(\mathbf{p},i\omega_n - i\omega_m)], \tag{17}$$

where $+$, $-$ refer to the charge and spin current, respectively. Besides the usual contribution to the single particle charge current involving the product of Green's functions $G_{\uparrow\uparrow}$ and $G_{\downarrow\downarrow}$ from the left and right sides of the junction, there are extra contributions involving the Green's functions $G_{\uparrow\downarrow}$ and $G_{\downarrow\uparrow}$ which give a term proportional to $e^{i(\varphi_L-\varphi_R)}$. In particular, after straightforward calculations we get for the single-particle charge current

$$I^{\text{charge}}_s(\text{eV}) = I_0(\text{eV}) + I_1(\text{eV})\cos\Delta\varphi, \tag{18}$$

where $I_0(\text{eV})$ is the single particle current

$$I_0(\text{eV}) = 2\pi e|T|^2\sum_{\mathbf{k},\mathbf{p},i\neq j}\left(1 + \frac{\varepsilon_{\mathbf{k}a}\varepsilon_{\mathbf{p}a}}{\sqrt{\varepsilon^2_{\mathbf{k}a}+|h_{\mathbf{q}}|^2}\sqrt{\varepsilon_{\mathbf{p}a}+|h_{\mathbf{q}}|^2}}\right) \times[\delta(\text{eV}-E_{i\mathbf{k}}-E_{i\mathbf{p}}) - \delta(\text{eV}+E_{i\mathbf{k}}+E_{i\mathbf{p}})] + \left(1 - \frac{\varepsilon_{\mathbf{k}a}\varepsilon_{\mathbf{p}a}}{\sqrt{\varepsilon^2_{\mathbf{k}a}+|h_{\mathbf{q}}|^2}\sqrt{\varepsilon_{\mathbf{p}a}+|h_{\mathbf{q}}|^2}}\right) \times[\delta(\text{eV}-E_{i\mathbf{k}}-E_{j\mathbf{p}}) - \delta(\text{eV}+E_{i\mathbf{k}}+E_{j\mathbf{p}})] \tag{19}$$

and for $I^{\text{charge}}_1(\text{eV})$ we find

$$I_1(\text{eV}) = 2\pi e|T|^2\sum_{\mathbf{k},\mathbf{p},i\neq j}\left(\frac{|h_{\mathbf{q}}|^2}{\sqrt{\varepsilon^2_{\mathbf{k}a}+|h_{\mathbf{q}}|^2}\sqrt{\varepsilon_{\mathbf{p}a}+|h_{\mathbf{q}}|^2}}\right)[\delta(\text{eV} - E_{i\mathbf{k}} - E_{i\mathbf{p}}) - \delta(\text{eV}+E_{i\mathbf{k}}+E_{i\mathbf{p}}) - \delta(\text{eV}-E_{i\mathbf{k}}-E_{j\mathbf{p}}) - \delta(\text{eV}+E_{i\mathbf{k}}+E_{j\mathbf{p}})] \tag{20}$$

while for the single-particle spin current we obtain

$$I^{\text{spin}}_s(\text{eV}) = \tilde{I}_0(\text{eV})\sin\Delta\varphi \tag{21}$$

with

$$\tilde{I}_0(\text{eV}) = 2\mu_B|T|^2\sum_{\mathbf{k},\mathbf{p},i\neq j}\left(\frac{|h_{\mathbf{q}}|^2}{\sqrt{\varepsilon^2_{\mathbf{k}a}+|h_{\mathbf{q}}|^2}\sqrt{\varepsilon^2_{\mathbf{p}a}+|h_{\mathbf{q}}|^2}}\right) \times\left[\frac{1}{\text{eV}+E_{i\mathbf{k}}+E_{i\mathbf{p}}} - \frac{1}{\text{eV}-E_{i\mathbf{k}}-E_{i\mathbf{p}}} - \left(\frac{1}{\text{eV}+E_{i\mathbf{k}}+E_{j\mathbf{p}}} - \frac{1}{\text{eV}-E_{i\mathbf{k}}-E_{j\mathbf{p}}}\right)\right]. \tag{22}$$

The first difference in the single-particle charge current occurs due to the FFLO state itself which would be present also in the case of the uniform exchange field. Most interestingly, the presence of the helimagnetic phase and the corresponding breaking of the SU(2) symmetry induces an additional term in the single-particle charge current proportional to $\cos\Delta\varphi$ and generated the corresponding term in the spin current proportional to $\sin\Delta\varphi$. The form of the single-particle spin current resembles the one in the so-called "spin Josephson effect" in ferromagnetic/ferromagnetic junctions[5] which, strictly speaking, is still a single-particle transport. There the charge current vanishes for zero voltage while the spin current remains, leading in this way to the appearance of a persistent spin current across the junction.

For the Cooper-pair tunneling the charge and spin Josephson currents are determined by $I_J^{\text{charge}} = 2e\,\text{Im}[e^{-2\text{eV}t/\hbar}\Phi^{\text{charge}}(\text{eV})]$ and $I_J^{\text{spin}} = 2e\,\text{Im}[e^{-2\text{eV}t/\hbar}\Phi^{\text{spin}}(\text{eV})]$, where

$$\Phi_{\text{charge,spin}}(i\omega) = -\frac{1}{\beta}\sum_{\omega_n}\sum_{\mathbf{k},\mathbf{p}}[T_{\mathbf{k+q/2,p+q/2}}T_{\mathbf{-k+q/2,-p+q/2}}F_1^{\uparrow,\downarrow}(\mathbf{k},i\omega_n)F_1^{\downarrow,\uparrow *}(\mathbf{p},i\omega_n - i\omega_m) \pm T_{\mathbf{k-q/2,p-q/2}}T_{\mathbf{-k-q/2,-p-q/2}}F_1^{\downarrow,\uparrow}(\mathbf{k},i\omega_n)F_1^{\uparrow,\downarrow *}(\mathbf{p},i\omega_n - i\omega_m) + T_{\mathbf{k+q/2,p+q/2}}T_{\mathbf{-k-q/2,-p-q/2}} \times F_2^{\uparrow,\uparrow}(\mathbf{k},i\omega_n)F_2^{\uparrow,\uparrow *}(\mathbf{p},i\omega_n - i\omega_m) \pm T_{\mathbf{k-q/2,p-q/2}}T_{\mathbf{-k+q/2,-p+q/2}}F_2^{\downarrow,\downarrow}(\mathbf{k},i\omega_n)F_2^{\downarrow,\downarrow *}(\mathbf{p},i\omega_n - i\omega_m)]. \tag{23}$$

Once more + or − refer to the charge and spin Josephson current, respectively. Evaluating the sum over Matsubara's frequencies the charge current can be found

$$I_J^{\text{charge}}(\text{eV}) = [J_1(\text{eV}) + J_2(\text{eV})\cos\Delta\varphi]\sin(\Delta\theta + 2\text{eV}t) + [J_3(\text{eV}) + J_4(\text{eV})\cos\Delta\varphi]\cos(\Delta\theta + 2\text{eV}t), \tag{24}$$

where the explicit expressions of the coefficients $J_1(\text{eV})$, and $J_2(\text{eV})$ are given as

$$J_1(\text{eV}) = 2e|T|^2\sum_{\mathbf{k},\mathbf{p},i\neq j}\frac{|\Delta_\mathbf{q}|^2}{\sqrt{\varepsilon_{\mathbf{k}s}^2 + |\Delta_\mathbf{q}|^2}\sqrt{\varepsilon_{\mathbf{p}s}^2 + |\Delta_\mathbf{q}|^2}}\left[\left(1 - \frac{\varepsilon_{\mathbf{k}a}\varepsilon_{\mathbf{p}a}}{\sqrt{\varepsilon_{\mathbf{k}a}^2 + |h_\mathbf{q}|^2}\sqrt{\varepsilon_{\mathbf{p}a}^2 + |h_\mathbf{q}|^2}}\right)\left(\frac{1}{\text{eV} + E_{i\mathbf{k}} + E_{i\mathbf{p}}} - \frac{1}{\text{eV} - E_{i\mathbf{k}} - E_{i\mathbf{p}}}\right) + \left(1 + \frac{\varepsilon_{\mathbf{k}a}\varepsilon_{\mathbf{p}a}}{\sqrt{\varepsilon_{\mathbf{k}a}^2 + |h_\mathbf{q}|^2}\sqrt{\varepsilon_{\mathbf{p}a}^2 + |h_\mathbf{q}|^2}}\right)\left(\frac{1}{\text{eV} + E_{i\mathbf{k}} + E_{j\mathbf{p}}} - \frac{1}{\text{eV} - E_{i\mathbf{k}} - E_{j\mathbf{p}}}\right)\right] \tag{25}$$

and

$$J_2(\text{eV}) = 2e|T|^2\sum_{\mathbf{k},\mathbf{p},i\neq j}\frac{|\Delta_\mathbf{q}|^2}{\sqrt{\varepsilon_{\mathbf{k}s}^2 + |\Delta_\mathbf{q}|^2}\sqrt{\varepsilon_{\mathbf{p}s}^2 + |\Delta_\mathbf{q}|^2}}\left(\frac{|h_\mathbf{q}|^2}{\sqrt{\varepsilon_{\mathbf{k}a}^2 + |h_\mathbf{q}|^2}\sqrt{\varepsilon_{\mathbf{p}a}^2 + |h_\mathbf{q}|^2}}\right)\left[\left(\frac{1}{\text{eV} + E_{i\mathbf{k}} + E_{j\mathbf{p}}} - \frac{1}{\text{eV} - E_{i\mathbf{k}} - E_{j\mathbf{p}}}\right) - \left(\frac{1}{\text{eV} + E_{i\mathbf{k}} + E_{i\mathbf{p}}} - \frac{1}{\text{eV} - E_{i\mathbf{k}} - E_{i\mathbf{p}}}\right)\right]. \tag{26}$$

$J_3(\text{eV})$ and $J_4(\text{eV})$ are found similarly

$$J_3(\text{eV}) = 2\pi e|T|^2\sum_{\mathbf{k},\mathbf{p},i\neq j}\frac{|\Delta_\mathbf{q}|^2}{\sqrt{\varepsilon_{\mathbf{k}s}^2 + |\Delta_\mathbf{q}|^2}\sqrt{\varepsilon_{\mathbf{p}s}^2 + |\Delta_\mathbf{q}|^2}}\left\{\left(1 - \frac{\varepsilon_{\mathbf{k}a}\varepsilon_{\mathbf{p}a}}{\sqrt{\varepsilon_{\mathbf{k}a}^2 + |h_\mathbf{q}|^2}\sqrt{\varepsilon_{\mathbf{p}a}^2 + |h_\mathbf{q}|^2}}\right)[\delta(\text{eV} - E_{i\mathbf{k}} - E_{i\mathbf{p}}) - \delta(\text{eV} + E_{i\mathbf{k}} + E_{i\mathbf{p}})] + \left(1 + \frac{\varepsilon_{\mathbf{k}a}\varepsilon_{\mathbf{p}a}}{\sqrt{\varepsilon_{\mathbf{k}a}^2 + |h_\mathbf{q}|^2}\sqrt{\varepsilon_{\mathbf{p}a}^2 + |h_\mathbf{q}|^2}}\right)[\delta(\text{eV} - E_{i\mathbf{k}} - E_{j\mathbf{p}}) - \delta(\text{eV} + E_{i\mathbf{k}} + E_{j\mathbf{p}})]\right\} \tag{27}$$

and

$$J_4(\text{eV}) = 2\pi e|T|^2\sum_{\mathbf{k},\mathbf{p},i\neq j}\frac{|\Delta_\mathbf{q}|^2}{\sqrt{\varepsilon_{\mathbf{k}s}^2 + |\Delta_\mathbf{q}|^2}\sqrt{\varepsilon_{\mathbf{p}s}^2 + |\Delta_\mathbf{q}|^2}}\left(\frac{|h_\mathbf{q}|^2}{\sqrt{\varepsilon_{\mathbf{k}a}^2 + |h_\mathbf{q}|^2}\sqrt{\varepsilon_{\mathbf{p}a}^2 + |h_\mathbf{q}|^2}}\right)[\delta(\text{eV} - E_{i\mathbf{k}} - E_{j\mathbf{p}}) - \delta(\text{eV} + E_{i\mathbf{k}} + E_{j\mathbf{p}}) - \delta(\text{eV} - E_{i\mathbf{k}} - E_{i\mathbf{p}}) + \delta(\text{eV} + E_{i\mathbf{k}} + E_{i\mathbf{p}})]. \tag{28}$$

Since $J_3(0)=0$ and $J_4(0)=0$, Eq. (24) becomes for $V=0$

$$I_J^{\text{charge}}(0) = [J_1(0) + J_2(0)\cos\Delta\varphi]\sin\Delta\theta. \tag{29}$$

The above result is identical, up to the precise expressions of the current amplitudes, to the zero voltage result of Ref. 13, though there the superconducting order parameter is uniform. Thus, a nonzero voltage in helimagnetic FFLO-like superconductors affects the Josephson current in an essential way.

For the Josephson spin current we find

$$I_J^{\text{spin}}(\text{eV}) = \sin\Delta\varphi[\tilde{J}_1(\text{eV})\cos(\Delta\theta + 2\text{eV}t) + \tilde{J}_2(\text{eV})\sin(\Delta\theta + 2\text{eV}t)]. \tag{30}$$

where

$$\tilde{J}_1(\text{eV}) = 2\mu_B|T|^2\sum_{\mathbf{k},\mathbf{p}}\frac{|\Delta_\mathbf{q}|^2|h_\mathbf{q}|^2}{\sqrt{\varepsilon_{\mathbf{k}s}^2 + |\Delta_\mathbf{q}|^2}\sqrt{\varepsilon_{\mathbf{p}s}^2 + |\Delta_\mathbf{q}|^2}\sqrt{\varepsilon_{\mathbf{k}a}^2 + h_\mathbf{q}|^2}\sqrt{\varepsilon_{\mathbf{p}a}^2 + h_\mathbf{q}|^2}} \times\left[\frac{1}{\text{eV} + E_{i\mathbf{k}} + E_{i\mathbf{p}}} - \frac{1}{\text{eV} - E_{i\mathbf{k}} - E_{i\mathbf{p}}} - \left(\frac{1}{\text{eV} + E_{i\mathbf{k}} + E_{j\mathbf{p}}} - \frac{1}{\text{eV} - E_{i\mathbf{k}} - E_{j\mathbf{p}}}\right)\right] \tag{31}$$

and

$$\tilde{J}_2(\mathrm{eV}) = 2\pi\mu_B|T|^2\sum_{\mathbf{k},\mathbf{p}} \frac{|\Delta_\mathbf{q}|^2|h_\mathbf{q}|^2}{\sqrt{\varepsilon_{\mathbf{k}s}^2+|\Delta_\mathbf{q}|^2}\sqrt{\varepsilon_{\mathbf{p}s}^2+|\Delta_\mathbf{q}|^2}\sqrt{\varepsilon_{\mathbf{k}a}^2+ h_\mathbf{q}|^2}\sqrt{\varepsilon_{\mathbf{p}a}^2+ h_\mathbf{q}|^2}} \times[\delta(\mathrm{eV}-E_{i\mathbf{k}}-E_{i\mathbf{p}})-\delta(\mathrm{eV}+E_{i\mathbf{k}}+E_{i\mathbf{p}})-\delta(\mathrm{eV}-E_{i\mathbf{k}}-E_{j\mathbf{p}})+\delta(\mathrm{eV}+E_{i\mathbf{k}}+E_{j\mathbf{p}})] \quad (32)$$

At zero voltage the spin Josephson current becomes

$$I_J^{\mathrm{spin}}(0) = \tilde{J}_1(0)\cos\Delta\theta\sin\Delta\varphi. \quad (33)$$

We see that the term proportional to $\sin\Delta\varphi\cos\Delta\theta$ vanishes for zero voltage because $\tilde{J}_2(0)=0$, similarly to the charge Josephson current case. This result also agrees with the corresponding one in Ref. 13. Note once more the crucial role played by the voltage in this system.

V. EFFECT OF AN EXTERNAL MAGNETIC FIELD

Interesting results follow from Eqs. (29) and (33) in the presence of an external magnetic field $\mathbf{H}$ perpendicular to the current direction, say x direction, and in the plane of the junction. By assuming that the external field points in the z direction, it is straightforward to derive the results

$$I_J^{\mathrm{charge}}(0) = [J_1(0)+J_2(0)\cos\Delta\varphi]\sin\left(\Delta\theta+\frac{2\pi Hyl}{\Phi_0}\right), \quad (34)$$

$$I_J^{\mathrm{spin}}(0) = J(0)\cos\left(\Delta\theta+\frac{2\pi Hyl}{\Phi_0}\right)\sin\Delta\varphi, \quad (35)$$

where $l=2\lambda+d$, with λ being the penetration depth and d the junction thickness. Φ_0 is the elementary flux quantum. Indeed, the magnetic field can only couple to the phase of the superconducting order parameter. The helimagnetic order parameter is *neutral* and for this reason its phase cannot couple to the external magnetic field. If the junction has a cross section of area L_yL_z, the total currents $I_{J,\mathrm{tot}}^{\mathrm{charge}}$ and $I_{J,\mathrm{tot}}^{\mathrm{spin}}$ flowing through the junction are obtained by integrating the y variable over the interval $[0,L_y]$

$$I_{\mathrm{J,tot}}^{\mathrm{charge}} = (I_{1c}+I_{2c}\cos\Delta\varphi)\frac{\Phi_0}{\pi\Phi}\sin\left(\frac{\pi\Phi}{\Phi_0}\right)\sin\left(\Delta\theta+\frac{\pi\Phi}{\Phi_0}\right), \quad (36)$$

$$I_{\mathrm{J,tot}}^{\mathrm{spin}} = I_c\frac{\Phi_0}{\pi\Phi}\sin\left(\frac{\pi\Phi}{\Phi_0}\right)\cos\left(\Delta\theta+\frac{\pi\Phi}{\Phi_0}\right)\sin\Delta\varphi, \quad (37)$$

where $I_{1c}=J_1(0)L_yL_z$, $I_{2c}=J_2(0)L_yL_z$, $I_c=J(0)L_yL_z$, and $\Phi=HL_yl$.

From Eqs. (36) and (37) we see that the phase difference $\Delta\theta$ can be adjusted in such a way to vanish the spin Josephson current (37). Remarkably, the opposite situation is also possible, i.e., the vanishing of the charge Josephson current by adjusting the phase difference $\Delta\theta$. This constitutes an example of a system with a spin current but no charge current.

VI. CONCLUSION

We have shown that dissipationless dc and ac Josephson spin currents exist between two nonuniform superconductors with helimagnetic order. For the spin current the nonzero average $\langle c_\uparrow^\dagger c_\downarrow\rangle$ plays a crucial role. We expect that effects similar to the ones discussed here may also occur in some superconductor/ferromagnet/insulator/ferromagnet/superconductor heterostructures.[13,21,22]

Measuring a spin current is presently a considerable challenge. One way could be to detect the electric fields induced by such a current.[5,23] However, in our case the signature of the spin current would be a corresponding modulation of the charge Josephson current as follows from Eq. (28).

One of the main results of our analysis was discussed in Sec. IV, namely, the possibility of using an external magnetic field to tune the charge Josephson current to zero, while the spin Josephson current remains nonvanishing. In such a situation the resulting spin Josephson effect is very similar to the one discussed recently in the context of ferromagnet/ferromagnet tunnel junctions.[5] However, here we have a much better control of the system through the external magnetic field. This result presumably holds also in the case of a helimagnetic superconductor with a uniform order parameter.[13]

The model we have studied here assumes the coexistence of a FFLO state with helimagnetism. In order to fully confirm the validity of this scenario, further theoretical and experimental investigation is necessary.

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