Energetics of magnetic ring and disk elements: Uniform versus vortex state

C. A. F. Vaz, C. Athanasiou, and J. A. C. Bland
Cavendish Laboratory, University of Cambridge, Cambridge, CB3 0HE, United Kingdom

G. Rowlands
Department of Physics, The University of Warwick, Coventry CV4 7AL, United Kingdom
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Magnetic energy expressions for the uniform and vortex states of ring elements are derived and compared with the results of micromagnetic simulations. In particular, the effect of roughness on the energy of the vortex state is considered and an expression for the magnetometric demagnetizing factor of rings is found. These energy expressions allow us to calculate the phase diagram separating the uniform from the vortex state. Our results suggest that the roughness contribution to the magnetic energy is sizable in the vortex state and is magnetostatic in origin and its effect is to increase the energy of the vortex state. For the case of rings, the vortex state is favoured with respect to the uniform state on account of the extra magnetostatic energy arising from the inner surface of the ring.

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I. INTRODUCTION

Recently much work has been devoted to the study of small magnetic elements, motivated both by the study of fundamental magnetic phenomena, such as magnetization dynamics[1–3] and the spin configuration of equilibrium states,[4–8] and by the potential for applications (e.g., magnetic RAM and magnetic sensors[9–13]). Particular attention has been given to high symmetry geometries, such as disks,[6–8] rings,[14–18] and squares,[19,20] since for these elements one expects high symmetry spin configurations, with the prospect of yielding simple and reproducible memory states. In this context, the onset of *quasi-uniform* magnetic states as a function of the element size and geometry is of particular interest as is the determination of the boundary that separates the stability regions of such states, since this allows one to controllably select a given magnetic state by choosing adequate geometrical parameters.[21–27] However, such a boundary depends on a whole range of intrinsic and extrinsic factors, such as the physical dimensions, material, edge and surface roughness, defects, temperature, etc., and the extent to which a given magnetic state may depend on these factors is key. For example, the equilibrium states of rings and disks are now understood to depend strongly on the presence of defects which act as pinning sites for the magnetization, thereby stabilizing magnetic states which are not the lowest in energy;[28] this is case with the "onion" state in rings[14,15] and the diamond and triangle state in disks.[7] While opening up a whole new range of possibilities, these additional extrinsic factors also make the analytical calculation of the boundary between different equilibrium states more complicated. One possible approach consists of combining the results of micromagnetic simulations with the results of the analytical expressions; while the intrinsic magnetic behavior may be obtained from analytical expressions, the results of micromagnetic simulations can be used to gain insights into the contribution of extrinsic parameters (such as edge roughness, defects or local shape deformations) to the magnetic energy and to the magnetic behavior of the element. Here, we report the results of our attempt to tackle such a problem for the case of the vortex and uniform states of disk and ring elements. We consider first the separate energy contributions to the magnetic energy of the vortex state in Sec. III, which are compared with the results of micromagnetic simulations, followed by the calculation of the demagnetization factor for rings and disks in the uniform state, in Sec. IV. The energy expressions thus obtained are used to estimate the boundary separating the regions in parameter space for the states of lowest energy (Sec. V) and in Sec. VI we summarize our main results.

II. MICROMAGNETIC SIMULATIONS

The micromagnetic simulations were performed using the OOMMF package[29] on permalloy ($Ni_{80}Fe_{20}$) ring and disk elements with the following values for the magnetic parameters: $A=1.3\times10^{-11}$ J/m for the exchange constant, $M_s=8.6\times10^5$ A/m for the saturation magnetization, and zero magnetic anisotropy. For the simulations we assumed a uniform component for the out of plane magnetization, and a cell size of $(4\text{ nm})^2$ in the plane of the element. This is a good approximation for thin films with no perpendicular anisotropies and for thicknesses up to several times the exchange length. This is due to a combination of the strong dipolar interactions, which tend to keep the magnetization in the plane of the film, and the exchange energy, which favours uniform spin configurations. In small elements this is valid as long as the aspect ratio of the element (height over lateral dimension) is much smaller than unity. While we used permalloy for the sake of concreteness, we shall present our results in terms of energy densities normalized by $\mu_0 M_s^2/2$ and distances normalized by the exchange length $l_{\text{ex}}=(2A/\mu_0 M_s^2)^{0.5}$ where relevant, so that our results are material independent. The OOMMF program implements the Landau-Lifshitz equation of motion of the magnetization, but since we consider only equilibrium states, the dynamics of the magnetization during the relaxation process need not concern us here. To determine the energy of the vortex state, we allowed the magnetization to relax from a state of perfect circular magnetization to the lowest energy state, which involves only the realignment of the edge spins, as will be discussed below. (We refer to the state of circular magnetization in rings as the vortex state for simplicity, even though there is no vortex core in such structures.) For the uniform state we consider the energy values as provided by the first iteration of the simulation, since for the larger structures, the uniform state is not that of the lowest energy; this, however, is sufficient to provide information about the magnetostatic energy, since this energy term scales with the size of the element. While we expect the calculated energy terms to depend on the cell size, we have checked that the changes are minimal for cell sizes in the range 2.5–4 nm (except for the contribution to the magnetostatic energy of the vortex state due to edge discretisation). We shall argue in Sec. III B that the presence of edge roughness can be used to determine the energy contribution due to the presence of edge irregularities in real magnetic elements.

Concerning the effect of magnetocrystalline anisotropy, we expect that for small values it adds a constant term to the energy of the vortex state; for rings this will be mostly correct, but for disks and very wide rings, the magnetic anisotropy will induce changes in the spin structure of the vortex state and the analytical expression for the energy of this state will lose the simplicity of the isotropic case. The effect of cubic anisotropy on the magnetic states of disks showing this effect has been studied experimentally by Vaz *et al.*[6,30] while the effect of uniaxial anisotropy has been considered numerically by Jubert and Allenspach.[27] For the uniform state, its effect is that of setting a preferential direction for the magnetization along the easy axes directions, but otherwise does not contribute to the magnetic energy of the equilibrium state. Therefore, the presence of magnetocrystalline anisotropy tends to favour the uniform state with respect to the vortex state.

III. VORTEX STATE

A. Exchange energy

We consider first the expression for the energy of the vortex state; in this case the magnetostatic energy is zero (assuming perfect flux closure of the magnetic induction), and the only energy term is the exchange energy, which in this geometry has cylindrical symmetry. The expression for the energy density (normalized to $\mu_0 M_s^2/2$) is then given by[31–33]

$$e_{\mathrm{ex}}^{(0)} \equiv \epsilon_{\mathrm{ex}}^{(0)}/(\mu_0 M_s^2/2) = \frac{2l_{\mathrm{ex}}^2}{R_0^2 - R_1^2} \ln \frac{R_0}{R_1}, \qquad (1)$$

where l_{ex} is the exchange length and R_0 and R_1 are the outer and inner radius, respectively. We see that this expression diverges for $R_1=0$ (disks), but this is an artefact of our approximations (namely, an in-plane magnetization distribution);[33,34] we may overcome this problem by setting the lower integration limit to a certain cut-off radius, $R_1=r_0$, of the order of the exchange length (or of the lattice constant, at most).[31] We compare in Fig. 1 the values for the exchange energy term as derived from the previous expression with the results of micromagnetic simulations as a function of the ring width for several outer ring diameters and for two selected thickness values, 2 and 20 nm. We see that while the agreement is very good for the 2 nm rings (and in general for thin rings and disks), for the 20 nm thick structures the above expression largely underestimates the value of the exchange energy for the narrower rings. This is attributed to the effect of edge roughness and the magnetostatic energy, which acts to make the edge spins to point in directions away from the azimuthal direction; this contribution is larger for narrow rings, since in this case the edge contribution to the total energy is more significant, and for thicker structures, since again the magnetostatic energy density increases with thickness. While this energy contribution would not be present in a perfect structure (i.e., with no edge roughness or edge imperfections), it becomes sizable once edge roughness is present, here introduced by the cell discretization. This is illustrated in Fig. 2, which shows the average value of the effective field amplitude in a ring element with $R_0 = 1000$ nm, $w/R_0=0.5$, and thickness $t=10$ nm as obtained from micromagnetic simulations; the largest effective field amplitude is attained at the edges, as discussed above.

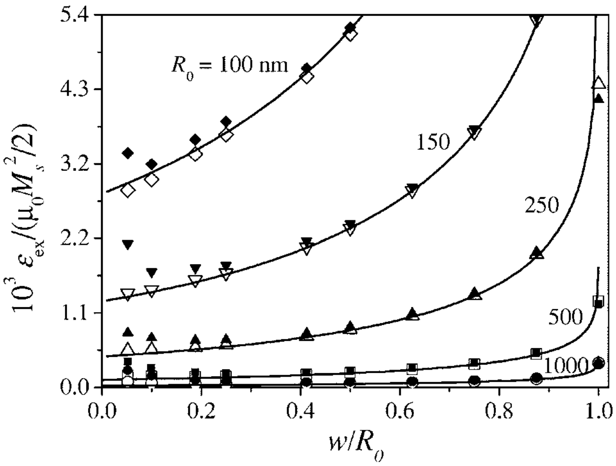

FIG. 1. Variation of the exchange energy of the vortex state against the reduced ring width, w/R_0, for various values of the outer diameter, R_0 (as labelled) and two thickness values, 2 and 20 nm (open and closed symbols, respectively) as obtained from the micromagnetic simulations. The full curves correspond to the values given by the analytical expression (which has no thickness dependence).

Since the additional exchange energy contribution arises from the edge roughness, we may expect it to scale with the ratio between perimeter and the volume of the ring,

$$\epsilon_{\mathrm{ex}}^{(r)} \propto \frac{2\pi(R_0+R_1)t\sigma}{\pi(R_0^2-R_1^2)t} = \frac{2\sigma}{w}, \qquad (2)$$

where $w=R_0-R_1$ is the ring width, and σ the roughness amplitude. This relationship holds very well for the roughness exchange energy contribution (difference between simulated and the analytical values) for $R_0 \geqslant 150$ nm and $w/R_0 \geqslant 0.1$, as illustrated in Fig. 3 for the 20 nm thickness structures. We find that there is only a weak dependence of the exchange

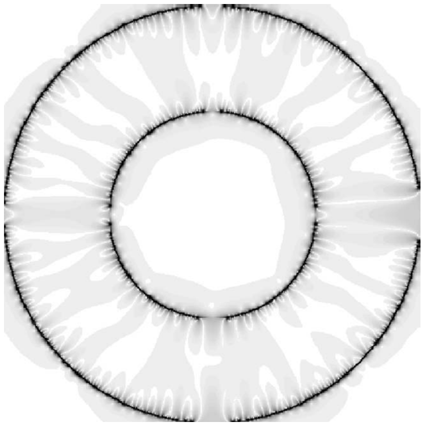

FIG. 2. Micromagnetic simulation showing the averaged effective field amplitude in a ring element with $R_0=1000$ nm, $w/R_0=0.5$, and $t=10$ nm in the vortex state showing that the large contribution arises from edge roughness due to cell discretization.

roughness energy variation with w/R_0 on R_0, and that the largest deviation arises for small and wide rings, but since the total exchange energy is then very large, such deviations are small relative to the total exchange energy. For such small structures, the magnetostatic energy becomes very small compared with the exchange energy so that twisting of the spins at the edges becomes unimportant. The extent to which the exchange roughness energy contribution varies with outer radius R_0 and thickness can be appreciated by plotting the coefficient affecting the $1/w$ dependence against thickness (inset to Fig. 3). We see that there is only a weak dependence of $e_{\mathrm{ex}}^{(r)}/(R_0/w)$ with thickness, and we therefore approximate this variation by a linear t dependence crossing the origin; for the different R_0/w values (Fig. 3, main graph), we see that the data points cluster relatively close together, and we may assume that the "exchange roughness energy" depends only on the R_0/w ratio. This is to say that the exchange roughness energy contribution may be expressed as

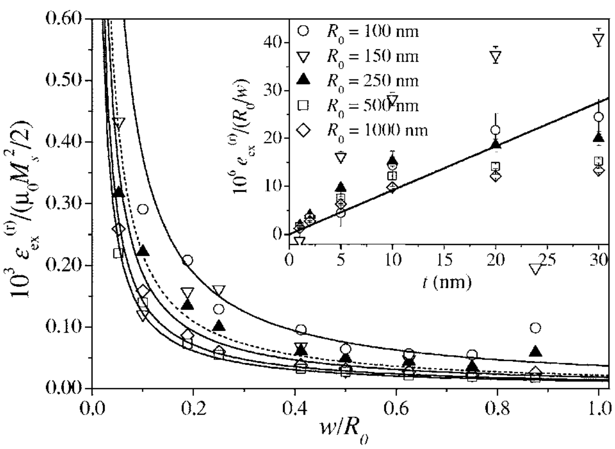

FIG. 3. Difference between simulated and the analytical values for the exchange energy of ring elements ($t=20$ nm) in the vortex state [$e_{\mathrm{ex}}^{(r)}$]. The curves in the main graph correspond to linear R_0/w fits constrained to cross the origin; the dotted line corresponds to $R_0=100$ nm. The inset shows the variation of $e_{\mathrm{ex}}^{(r)}/(R_0/w)$ with thickness.

$$e_{\mathrm{ex}}^{(r)} \equiv \epsilon_{\mathrm{ex}}^{(r)}/(\mu_0 M_s^2/2) \approx e_0^{\mathrm{ex}} \frac{R_0 t}{l_{\mathrm{ex}}^2} \frac{2\sigma}{w} \tag{3}$$

with $e_0^{\mathrm{ex}}=(9.2\pm 0.9)\times 10^{-6}$ a characteristic energy for these type of structures.

As mentioned before, Eq. (1) breaks down for the case of disks, due to the assumption that the magnetization remains in-plane; in fact, it is known that the vortex core singularity is overcome by making the magnetization point in the out-of-plane direction,[35,36] a fact confirmed by recent experimental observations.[37–41] Although some suggestions have been made for the analytical expression of the magnetic energy of the vortex state in disks based on variational methods,[35,42] we suggest an empirical way to obtain the correct result for the exchange energy of the vortex state of disks, which recovers the results obtained by other methods.[31–33,35] This consists of introducing a cutoff radius in Eq. (1), by setting the lower integration limit of the radial coordinate to the value $r_0=R_1$ which gives the correct energy as obtained by the simulations (we neglect here the relatively small edge roughness contribution to the exchange energy of wide elements, as discussed above). We find that the value $r_0=2.2\pm 0.4$ nm thus found is close to half the exchange length of permalloy, $l_{\mathrm{ex}}=5.3$ nm. It is also coincidentally very close to half the in-plane cell size, but simulations with cell sizes of 2.5 and 3 nm (for 5 and 10 nm thick disks) also yield a value of r_0 identical to those with cell size of 4 nm (these simulations also show that the exchange energy is largely insensitive to the cell size in the range from 2.5 to 4 nm). This result is in agreement with the calculated value of the vortex core radius in disks ($\approx l_{\mathrm{ex}}$–$2l_{\mathrm{ex}}$ in the limit of small thicknesses, depending on the definition; see Sec. III B).[35,42,43] We also find that r_0 increases slightly with thickness and decreases with outer radius; these variations (included in the error bars given above for r_0) are likely related with the details of the vortex core and the additional magnetostatic energy contribution; in particular, one expects these variations to be more pronounced for thicker disks (where the magnetostatic energy becomes more important) and for smaller radius, for which the vortex core contributes comparatively more to the total energy; this is consistent with the trends observed for r_0 as a function of R_0 and t.

We may therefore express the exchange energy of ring and disk elements with edge roughness energy contribution as

$$e_{\mathrm{ex}} \approx \frac{2l_{\mathrm{ex}}^2}{R_0^2 - R_1^2} \ln \frac{R_0}{R_1 + l_{\mathrm{ex}}/2} + e_0^{\mathrm{ex}} \frac{R_0 t}{l_{\mathrm{ex}}^2} \frac{2\sigma}{w}. \tag{4}$$

B. Magnetostatic energy: Effect of roughness

It has been pointed out before that the cell discretization in micromagnetic simulations introduces an extra magnetostatic energy contribution to the total energy.[27,44–46] However, it is usually the case that irregularities are inevitably present in small structures,[28,47–51] either as a consequence of the spatial resolution of the pattern writing or due to the different steps involved during the lithography process or, in the case of structures fabricated by deposition using a pre-

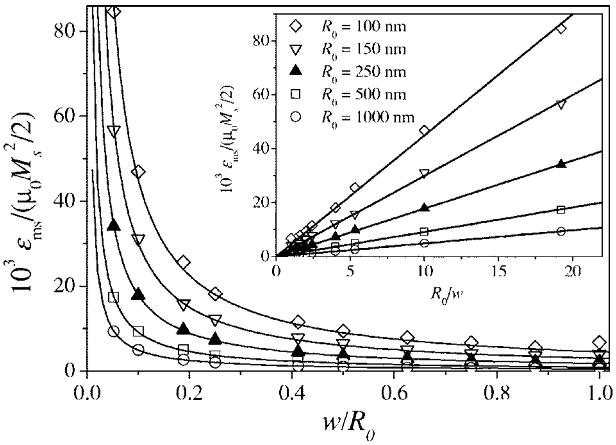

FIG. 4. Variation of the magnetostatic energy of the vortex state with the reduced ring width, w/R_0, for various values of the outer diameter, R_0, for the 10 nm structures, as obtained from the micromagnetic simulations; full curves correspond to $1/w$ fits, as shown in the figure inset.

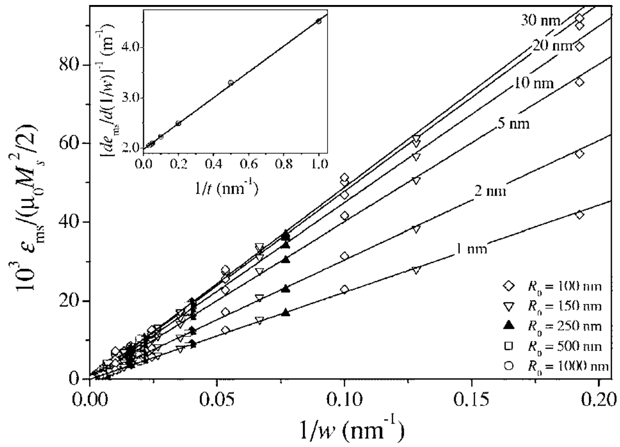

FIG. 5. Variation of the magnetostatic energy of the vortex state with the inverse ring width for various values of the outer diameter (different symbols) and thicknesses (numbers next to the data points); full curves correspond to $1/w$ fits. In the inset we plot the variation of the slope of the magnetostatic energy curves shown in the main graph against the thickness; the line is a linear fit to the data points.

defined mask, by mask irregularities or even due to the size of the crystallites in the case of polycrystalline elements[52] (which are of the order of 5–20 nm, depending on film thickness,[53–56] substrate,[56,57] growth conditions,[58] growth rate,[59,60] deposition technique and thermal treatments[61]). In addition, it is widely acknowledged that the edge roughness has an important role in the magnetic behavior of small elements, in particular in determining the nucleation field for magnetization reversal,[28,52,62–68] the dynamics of the magnetic reversal,[69] the effective magnetic energy of small elements[70] and in the stabilization of metastable equilibrium states.[14,71,72] Therefore, we may consider the presence of roughness introduced by cell discretization as an opportunity for studying its effect on the magnetic energy, in the present case, of the vortex state; we suggest that our conclusions can be extended for magnetic states for which the magnetostatic energy contribution dominates.

As mentioned earlier, the edge roughness gives rise to a large magnetostatic energy contribution, which we may expect, on the basis of a simple physical argument, to have a geometrical dependence of the type described by Eq. (2). In analogy with the case for the exchange energy, we have plotted the value of the magnetostatic energy term as a function of the inverse width, as shown in Fig. 4 for the case of the 10 nm thick structures, where for convenience, w has been normalized by R_0 (when plotted against w, all data points fall into a single curve). We see, in particular, that the energy of the disks ($R_0/w=1$) is systematically slightly above the value expected from the trend predicted by Eq. (2), again due to the additional magnetostatic energy contribution from the vortex core; this will be considered below. That the magnetostatic energy term is independent of the outer radius can be seen in Fig. 5, where we plot the magnetostatic energy as a function of the inverse width for the thicknesses studied. The slope is found to increase steadily for small thicknesses and to saturate for larger thicknesses, in a $t/(t+a)$ fashion; this is expected from simple physical arguments: in analogy with a uniformly magnetized element, one expects a linear increase of the magnetostatic energy for small thicknesses, while for large thicknesses, of the order of the lateral dimension of the element, we expect the magnetostatic energy to become largely independent of the thickness. That this expression turns out to fit extremely well the variation of the magnetostatic energy with the thickness (see inset to Fig. 5) may be fortuitous, but more likely it is a trend characteristic of uniformly magnetized elements; for example, the magnetostatic energy of an elongated ellipsoid along the long axis direction varies linearly with t for small thicknesses and as $t/(w+t)$ as the thickness approaches the lateral dimension of the ellipsoid.[73] For the parameter a we obtain $a=1.3$ nm, which can be identified as the average roughness amplitude (of the order of $\sqrt{2}c/4$, where c is the cell size).

We have therefore for the energy expression of the roughness contribution to the magnetostatic energy of the vortex state,

$$e_{\mathrm{ms}} \approx e_0^{\mathrm{ms}} \xi \frac{2\sigma}{w} \frac{t}{t+\sigma}, \tag{5}$$

with $e_0^{\mathrm{ms}}=0.36\pm0.01$, $\xi=2\sigma/\lambda$, λ the roughness correlation length (of the order of $\sqrt{2}c$ in our case) and σ is the roughness amplitude. This empirical expression should describe the edge roughness contribution of magnetic states for which the magnetostatic energy term dominates; in such case, the magnetization should adopt configurations which avoid the creation of "pole charges" at the edges, i.e., where the magnetization is parallel to the geometrical edge of the element, thus giving rise to an edge roughness energy contribution due to edge irregularities of the type described by Eq. (5). It should be valid when the roughness amplitude is identical to the lateral correlation length, which tends to be the case with edge imperfections found in patterned elements, as discussed above.

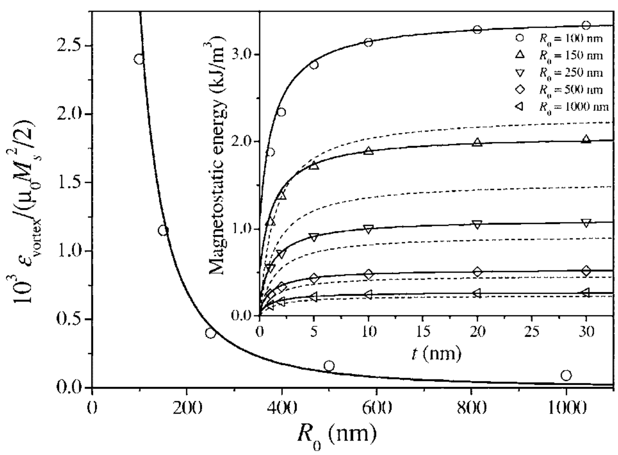

FIG. 6. Variation of the (magnetostatic) vortex core energy of the vortex state against disk radius; the full line corresponds to the curve $(5.3/R_0)^2$. The inset shows the variation of the magnetostatic energy of disks against thickness (symbols) while the dashed lines correspond to the edge roughness contribution, as extrapolated from Eq. (5); when shifted to coincide with the numerical values (full lines), we obtain the vortex energy (main graph).

For the case of disks, we need to account for the extra magnetostatic energy arising from the out of plane magnetization at the vortex core. This energy contribution corresponds to the energy difference between the total magnetostatic energy of the vortex state minus the roughness energy, which can be determined by extrapolation from the case of rings (with no vortex core). Analytical expressions for the vortex core energy have been made available which are valid in the thin film range ($t \ll l_{ex}$),[35] but the general case is not so easy to tackle. A rough estimate for this energy contribution may be obtained by assuming that the vortex core width does not change much with the radius and thickness of the disk; in that case, for $R_0 > l_{ex}$, the magnetostatic energy contribution to the total energy is given approximately by the ratio of the volume of the vortex core to that of the whole disk, $e^{vc} = (l_{ex}/R_0)^2$, independent of thickness. We find this indeed to be the case, and the previous relation applies with no need for scaling factors for all studied thickness and radii. This is shown in Fig. 6, where we plotted the (normalized) vortex core energy against the disk radius, while the inset shows the total magnetostatic energy of the disk, which consists of a roughness contribution (dashed lines) plus a term constant in thickness corresponding to the vortex core energy contribution. This suggests that for $R_0 > l_{ex}$ the vortex core radius is approximately equal to l_{ex} and depends weakly on both the disk outer radius and thickness; this is in good agreement with published results for the radius of the vortex core in thin films [$\sim 1.2 l_{ex}$ defined at $m_z = 0.5$,[27] $1.32 l_{ex}$ if defined as the derivative of the polar angle with radius at the origin[35] or $\sim 2 l_{ex}$ for nonzero m_z (Refs. 35, 42, and 43)].[74] Since this energy term only appears for disks (or rings with radii smaller than the exchange length), its inclusion in Eq. (5) can be made, formally, by means of a Kronecker symbol,

$$e_{ms} \approx e_0^{ms} \xi \frac{2\sigma}{w} \frac{t}{t+\sigma} + \delta_{R_1,0} (l_{ex}/R_0)^2. \tag{6}$$

IV. UNIFORM STATE

We consider here the case of perfectly uniformly magnetized circular elements, which simplifies the problem to that of calculating the magnetostatic self-energy of the system; this is often expressed in terms of a magnetometric demagnetizing factor, defined as the ratio between the magnetostatic energy to the maximum value attainable, $\mu_0 M_s^2/2$. This is not the true demagnetizing factor (which is, in general, a tensor relating the demagnetizing field to the magnetization) except in exceptional cases, such as that of the uniformly magnetized ellipsoid, since in general the demagnetizing field is nonuniform inside the element.[75,76] The assumption of uniform magnetization is also not a realistic one, at least for nonellipsoidal elements, since the corresponding nonuniform demagnetizing field tends to deviate the magnetization away from the direction of uniform magnetization[4,75,77] (such an assumption is a good approximation for single-domain particles, even though in nanometre size particles, surface spins are known to be noncollinear due to surface anisotropies[78]). However, the calculation of self-consistent expressions for the magnetostatic energy are prohibitively complicated, and the assumption of uniform magnetization is useful in that it allows analytical expressions to be calculated which provide, nevertheless, an upper limit to the magnetostatic energy.

It has already been noted in the literature that approximating the demagnetizing factor of a flat disk by that of an ellipsoid does not yield sufficiently accurate results,[79] a result which we also reproduce and that underlines the sensitivity of the magnetostatic energy to the shape of the element. Also, for rings, it is not sufficient to approximate it, locally, by a narrow wire with a local "shape" anisotropy of the form $t(t+w)^{-1} \cos^2 \theta$ (it underestimates the magnetostatic energy), even though for the narrowest rings, this expression yields a value which is close to the correct one. One must, therefore, turn to the exact expression of the demagnetizing factor of rings and disks.

Expressions for the demagnetizing factor of the cylinder have been considered in detail before,[80–85] including an approximate expression for the liming cases of wires and disks.[80,86] However, while for the case of disks the expression for the demagnetizing factor can be expressed analytically, for the case of rings only integral expressions or tabulated values are available.[81,82,84] Here we present an analytical expression for the demagnetizing energy of rings and we consider in particular its form in the limit of thin films (specifically, thickness much smaller than the outer radius and width); it reduces to the expression for disks in the case when the inner diameter vanishes. The magnetostatic energy of the uniform state with magnetization along the in-plane x direction is then written as

$$e_{ms}^{(u)} = D_{xx} \mu_0 M_s^2 / 2, \tag{7}$$

where D_{xx} is demagnetizing factor along the x direction, which in the case of elements with cylindric symmetry is identical for all directions perpendicular to the axis of symmetry; in particular, we have $2D_{xx} + D_{zz} = 1$ and the maximum value of D_{xx} is 0.5. The calculation of the magnetostatic en ergy of a magnetized body can be performed using various approaches,[87] and some formalisms for the calculation of the demagnetizing factors for arbitrary geometries have been discussed in the past[88] and need not be explained. Here, we start simply with the general expression given by Kaczér and Klem[81] for the demagnetizing factor of rings magnetized uniformly in-plane,

$$\pi t(R_0^2-R_1^2)D_{xx}=\pi t(R_0^2-R_1^2)/2+\pi R_0\int_{R_1}^{R_0} r\{I(R_0/r,1,t/r)-I(R_0,1,0)\}dr-\pi R_0\int_{R_1}^{R_0} r\{I(R_1/r,1,t/r)-I(R_1,1,0)\}dr, \tag{8}$$

where $\pi t(R_0^2-R_1^2)$ is the volume of the ring, and

$$I(a,b,c)=\int_0^\infty J_1(ax)J_1(bx)e^{-cx}x^{-1}dx, \tag{9}$$

where J_1 is the Bessel function of the first kind. The properties of these integrals have been studied in detail by Eason *et al.*[89] and Watson.[90] By means of a Gegenbauer transformation,[90] we can perform one of the integrations, leading to

$$t(R_0^2-R_1^2)D_{xx}=t(R_0^2-R_1^2)/2+R_0^3\int_0^\infty J_1^2(x)x^{-2}(e^{-tx/R_0}-1)dx+R_1^3\int_0^\infty J_1^2(x)x^{-2}(e^{-tx/R_1}-1)dx-2R_0^2R_1\int_0^\infty J_1(x)J_1(R_1x/R_0)x^{-2}\times(e^{-tx/R_0}-1)dx. \tag{10}$$

The first two integrals have been solved exactly and correspond to the demagnetizing factor of disks; the full expression and the corresponding approximation for small values of the parameter t/R is given by Joseph.[80] An explicit expression for the last term is given in the Appendix in terms of elliptic integrals (which also includes the result for the other integrals as a particular case). This gives an exact, albeit a rather cumbersome, expression for the demagnetizing factor of rings. Since we are interested in the case of thin films, we find it useful to report here a simpler expression for D_{xx} valid when the thickness is much smaller that both width and outer radius. In such a limit,[80]

$$\int_0^\infty J_1^2(x)x^{-2}(e^{-cx}-1)dx=(c/2)\{(c/\pi)[\ln(8/c)-1/2]-1\},\quad (c\ll 1); \tag{11}$$

to calculate the last integral of Eq. (10) we expand the exponential in powers of tx/R_0. After some straightforward algebra and using the properties of integrals of the type (9) as detailed in Eason *et al.*,[89] we arrive at the following expression for the demagnetizing factor of rings valid for $t \ll w, R_0$:

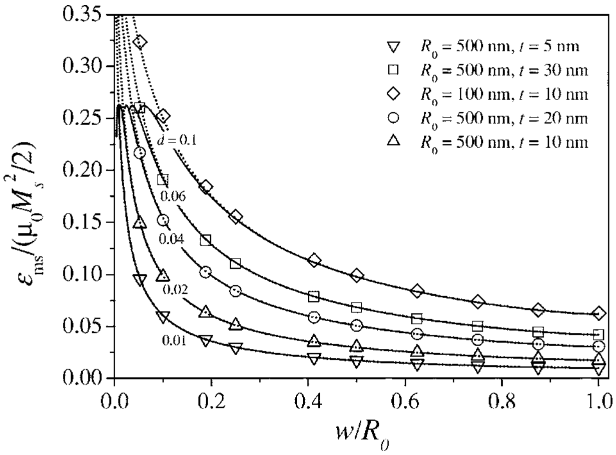

FIG. 7. Variation of the normalized magnetostatic energy of the uniform state with the reduced ring width, w/R_0, for various values of the parameter $d=t/R_0$. The symbols correspond to the values obtained from the micromagnetic simulations, the dotted lines to the exact values of the demagnetizing factor of rings [from expression (A1)] and the full lines correspond to the values obtained from the demagnetizing factor of rings for small thicknesses, expression (12) of the main text.

$$D_{xx}=\frac{d}{2\pi(1-r)}\left[\ln(8/d)-\frac{1}{2}+\frac{r}{r+1}\ln(r)-2\frac{1+r^2}{(1+r)^2}F(2\sqrt{r}/(1+r))+2E(2\sqrt{r}/(1+r))\right], \tag{12}$$

where $d=t/R_0$, $r=R_1/R_0$, and $F(k)$, $E(k)$ are the complete elliptic functions of the first and second kind (k being the modulus of the integral). We note that although we assumed also that t/R_1 is small so that Eq. (11) may be valid, there is a factor of R_1^3 multiplying this term (which is finite for all values of the parameter t/R_1), so that when R_1 is small this term becomes negligible (and goes to zero as $R_1\to 0$). The above expression is therefore accurate for d and R_1 small and in fact reduces to the expression for the demagnetizing factors of disks in the limit $r=0$. The third term of the expansion is zero, so that the expansion is correct to fourth order in t, except when r approaches 1, at which point the expansion breaks down. This is expected, since in such limit the thickness is no longer much smaller that the width, as assumed. Nevertheless, Eq. (12) is accurate over a wide range of r and t values, in fact as long as the demagnetizing factor does not exceed the value 0.25, as is suggested by the graph in Fig. 7, which shows the variation of D_{xx} for several values of d against $w/R_0=1-r$ obtained from Eq. (12), and which are compared with exact values obtained from Eq. (A1). We have plotted in the same graph the normalized magnetostatic energy of selected rings and disks in the uniform state (these values correspond to the first iteration of the micromagnetic simulations using a uniform initial configuration). We see that only for the extreme case of an aspect ratio $d=0.1$ does Eq. (12) break down for widths below $0.1R_0$. Such

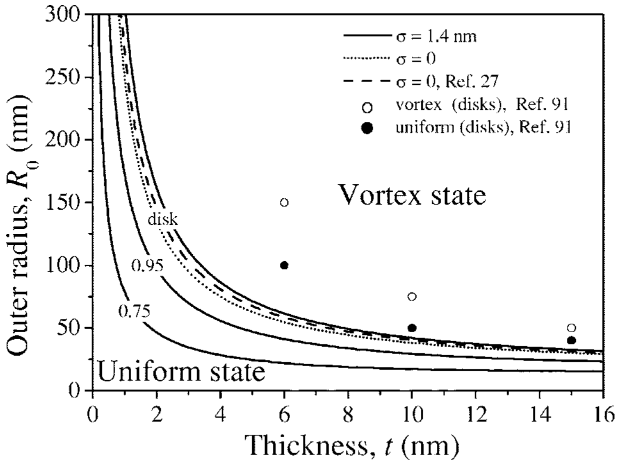

FIG. 8. Phase diagram for the equilibrium state of disk and ring elements as a function of outer radius and thickness. For rings, two width values are plotted, $w/R_0=0.75$ and 0.95, as labelled. The full curves correspond to a roughness amplitude $\sigma=1.41$ nm, broken curves to $\sigma=0$. The dashed curve correspond to the curve obtained in Ref. 27 while the open and closed symbols correspond to disks in the vortex and uniform state as determined experimentally in Ref. 91.

values correspond to rings whose width is comparable to the ring thickness, where the demagnetizing factor approaches 0.5 (maximum value of D_{xx}).

V. PHASE DIAGRAM BETWEEN VORTEX AND UNIFORM STATE

Once energy expressions for the vortex and uniform states have been calculated, it is a trivial step to determine the boundary in parameter space that separates the regions where each of these states is that of lowest energy (numerically, if not explicitly). In Fig. 8 we show the phase diagram for the relative stability of the uniform and vortex state as a function of outer radius and thickness for disks and rings (with constant ring width), as obtained from our set of expressions (full and dotted lines). We see that the phase boundary for the ring structures is shifted to lower radius values; this is due to the extra magnetostatic energy in the uniform state, on account of the extra inner surface due to the hole. The phase boundary corresponding to the case of disks is shown for two roughness values, which are compared with another theoretical model[27] and experimental data for supermalloy disks by Cowburn *et al.*[91] [as deduced from magneto-optic Kerr effect (MOKE) measurements]. We see that the effect of roughness is to shift the boundary from uniform to vortex equilibrium state to higher radii, i.e., the effect of roughness is to increase the range of stability of the uniform state. The reason for this is that roughness leads to an extra magnetostatic energy cost in the vortex state, as discussed above. We see that this shift in the phase diagram is relatively small for larger thicknesses and more important in the thin thickness range. The curve as obtained from Jubert and Allenspach[27] is at slightly higher values than that obtained in this work, but the difference is relatively small. Although much work has been reported on the magnetic characterization of disk elements, most studies deal with submicrometer disks with thicknesses to the right of Fig. 8, thereby corresponding to elements which tend to fall into the vortex state (see however, Ref. 92); the data by Cowburn *et al.*[91] remain, to our knowledge, the only study to address experimentally the phase diagram for low-anisotropy disk elements; we have plotted in Fig. 8 those values that define the boundary between vortex and uniform state. These values should be used with caution; as already pointed out by Jubert and Allenspach,[27] such states are deduced from MOKE measurements and may correspond to metastable energy states;[75] in particular, it is possible that the uniform state, attained at saturation, may be stabilized by defects long enough to be interpreted as the uniform state in *M*-*H* loops. Such effects have been observed, albeit in larger disk elements.[7] We emphasize that the phase diagram presented here does not preclude the existence of other magnetic states which may be lower in energy than the vortex or uniform state; in particular, quasiuniform magnetic states, corresponding to small deviations from the uniform state, may be lower in energy than the uniform state near the boundary; multidomain states may be lower in energy for very large radius and small thicknesses and for elements with small aspect ratios ($t/R_0 \gtrsim 1$), perpendicular magnetized states may be lower in energy.[93] However, in the thickness and radius range considered in Fig. 8 we expect the uniform and vortex states to be the lowest in energy for typical 3*d* ferromagnetic transition metals.

VI. CONCLUSIONS

In conclusion, we have calculated analytical energy expressions for the vortex and uniform state of magnetic ring and disk elements. In particular, we suggest that edge roughness may contribute significantly to the magnetic energy and have suggested expressions that account for both exchange and magnetostatic contributions, which have been compared with the results of micromagnetic simulations. For the uniform state, we have calculated an explicit expression for the magnetostatic energy of ring elements in the thin film limit. Based on the energy expressions obtained, we have calculated the stability diagram for the vortex and uniform state for disk and ring elements (in rings, the presence of the inner hole is to increase the magnetostatic energy of the ring and therefore to increase the relative stability of the vortex state). The effect of roughness is to decrease the relative stability of the vortex state with respect to the uniform state, an effect which is more pronounced at small thicknesses.

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APPENDIX: CLOSE EXPRESSION FOR THE DEMAGNETISING FACTOR OF RINGS

The calculation of the last integral in Eq. (10) is rather long and we present here the final expression only:

$$\int_0^\infty J_1(x)J_1(rx)x^{-2}e^{-tx}dx = -\frac{rt}{2} + \frac{1}{6\pi r}\frac{1}{\sqrt{t^2+(1+r)^2}}\Bigg\{[t^4+2(1+r^2)t^2-2(1-r^2)^2]F(k)+[-t^4+(r-1)^2t^2 + 2(1+r^2)(1+r)^2]E(k) - 3t^2\left(\frac{1-r^2}{1+r}\right)^2\Pi(\alpha,k)\Bigg\}, \quad \text{(A1)}$$

where $r\leqslant 1$, $k^2=4r/[t^2+(1+r)^2]$, $\alpha=4r/(1+r)^2$, $F(k)$, $E(k)$, and $\Pi(\alpha,k)$ are the complete elliptic integrals of the first, second, and third kind respectively, and k is the modulus of the integral. From this expression all the terms of Eq. (10) can be calculated; in particular it yields the expression obtained by Joseph[80] for the left-hand side of Eq. (11).

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