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Quantum state tomography with quantum shot noise

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We propose a scheme for a complete reconstruction of one- and two-particle orbital quantum states in mesoscopic conductors. The conductor in the transport state continuously emits orbital quantum states. The orbital states are manipulated by electronic beam splitters and detected by measurements of average currents and zero frequency current shot-noise correlators. We show how, by a suitable complete set of measurements, the elements of the density matrices of the one- and two-particle states can be directly expressed in terms of the currents and current correlators.

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According to the standard interpretation of quantum mechanics, the wave function, or more generally the density matrix, determines the probabilities for the possible outcomes of any measurement on the quantum state. To completely characterize the wave function of the state is therefore of fundamental interest.[1] A complete characterization of an unknown state requires an ensemble of identically prepared states and the measurement of a complete set of observables on the state.[2] A reconstruction of the quantum state wave function via such a series of measurements is known as quantum state tomography (QST).[3]

Initially, QST was performed experimentally on the discrete angular momentum state of an electron in a hydrogen atom.[4] During the last decade QST has been performed on the quantum state of squeezed light,[5] the vibrational state of a molecule,[6] and the motional states of trapped ions,[7] and of atomic wave packets.[8] Recently there has been an interest in QST of two-particle states in the context of quantum information processing. The entanglement of a quantum state, a potential resource for quantum information processing, is characterized by the density matrix of the state. The quantum state of polarization entangled pairs of photons has been reconstructed using QST.[9]

To date, no QST has been performed on quantum states in solid state systems. Very recently a theoretical scheme[10] was developed for solid-state two-levels systems, qubits, appropriate for, e.g., the macroscopic superposition state in superconducting qubits and the spin state of electrons in quantum dots. The set of measurements necessary to reconstruct the state involves controlled rotations and detection of the individual qubits. For coupled qubits, where entanglement between the qubits is of interest, such measurements are highly involved and have not been demonstrated.

In this paper we take a different approach and present a scheme for QST of discrete single and two-particle orbital quantum states in mesoscopic conductors. The key point is that our proposal can be implemented with existing experimental technics. In mesoscopic conductors one typically measures electrical currents and current correlators, shot noise.[11,12] Since orbital quantum states[13–15] are continuously emitted from the conductor during transport, a long time measurement is equivalent to an average over an ensemble of states. Orbital states can be manipulated by electronic beam splitters[16,17] and detected by shot-noise measurements. In addition QST demands phase sensitive rotations of the qubit and below we show how these can be implemented.

The QST procedure is most directly illustrated for orbitally entangled states. In several recent works,[13–15,18,19] it has been shown theoretically that quantum correlations, entanglement, between two spatially separated particles can be investigated via current correlation measurements. In particular, in Ref. 15 it was shown how entangled orbital quasiparticle states could be generated, manipulated, and detected in a quantum Hall system[17,20] by violating a Bell inequality.[13–15,19] In contrast to a Bell test, QST allows for a complete reconstruction of the two-particle density matrix and consequently a complete characterization of the entanglement. Importantly, QST also demonstrates that the maximum possible information one can infer about the two particle properties of a mesoscopic conductor can be obtained from current and shot-noise measurements.

A generic setup for orbital QST is shown in Fig. 1. A mesoscopic conductor S acts as a source for orbital quantum states. The source is connected via four single mode leads $A1, A2, B1$, and $B2$ to two regions, A and B, where the emitted state is manipulated and detected. The mesoscopic source has one or more reservoirs biased at eV and an arbitrary number of reservoirs kept at ground. We note that two-particles effects are only present for two or more biased reservoirs.[11] The temperature is taken to be zero. It is assumed that the scattering in the conductor is elastic; however, we can account for arbitrary dephasing inside the conductor.

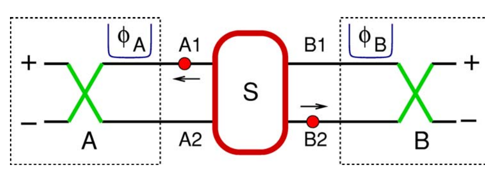

FIG. 1. (Color online) Schematic of the setup. A mesoscopic conductor acting as a source S [thick red (dark gray) box] is connected via four leads, $A1, A2, B1$, and $B2$, to regions A and B (dashed boxes) each containing two reservoirs + and −, a beam splitter [green (gray) cross] and a side gate (thin open box), which induces a phase shift ϕ_A or ϕ_B. Two particles [red (dark gray) dots] emitted from the source are shown.

The regions A and B each contain an electronic beam splitter[16,17] and an electrostatic side gate (see e.g. Ref. 20) to induce a phase shift, ϕ_A or ϕ_B, by modifying the length of the lead. The beam splitters, taken to be reflectionless, are further connected to two grounded reservoirs + and − where the current is measured. The combined beam splitter–side gate structure can be characterized by a scattering matrix, for, e.g., A given by

$$S_A = \begin{pmatrix} \sqrt{R_A}e^{i\varphi_{A2}} & \sqrt{T_A}e^{i(\varphi_{A3}-\phi_A)} \\ \sqrt{T_A}e^{i(\varphi_{A1}+\varphi_{A2})} & -\sqrt{R_A}e^{i(\varphi_{A1}+\varphi_{A3}-\phi_A)} \end{pmatrix}. \tag{1}$$

The transmission probability $T_A=1-R_A$ can be controlled via electrostatic gating.[16,17,20] The phases $\varphi_{Ai}, i=1, 2, 3$ picked up when scattering at the beam splitter are, however, assumed to be uncontrollable but fixed during the measurement.

In the general case, the quantum state emitted by the mesoscopic source is a many-body state. It is a linear superposition of states with different number of particles.[21] This is different from, e.g., the true two-particle states investigated in optics.[9] Importantly, the states with more than one (two) particle(s) contribute to one- (two-) particle observables, such as current (noise). However, the contribution of such many-particle states can be incorporated in effective one- and two-particle states that completely characterize any one- and two-particle observables. It is thus these effective states, quantified by the reduced density matrix, which are the objects of interest. Only in some special cases, in conductors typically only in the tunneling limit,[13–15] are the emitted states true one- or two-particle states. In the presence of dephasing, the emitted state is mixed. Moreover, even an emitted pure many-body state generally gives rise to a mixed reduced one- or two-particle state. It is therefore appropriate to discuss the state in terms of density matrices.

To simplify the discussion we consider a spin-polarized system with scattering amplitudes independent on energy on the scale of the applied bias eV, i.e., the linear voltage regime. The emitted state then has only orbital degrees of freedom. We first consider the single-particle orbital state emitted, e.g., towards A (the same considerations hold for B). Introducing operators $b_{An}^\dagger$ creating electrons in lead An, with $n=1, 2$, propagating out from the source, the 2×2 density matrix (not normalized) is by definition given by

$$\rho_A = \sum_{n,m=1}^{2} \rho_{nm} b_{An}^\dagger |0\rangle\langle 0| b_{Am} = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}, \tag{2}$$

where we work in the basis $\{|1\rangle_A, |2\rangle_A\}$, with $b_{An}^\dagger|0\rangle=|n\rangle_A$, formed by the lead indices (see Fig. 1). The matrix elements $\rho_{nm}=\langle b_{Am}^\dagger b_{An}\rangle$. The Hermitian density matrix, $\rho_A=\rho_A^\dagger$, has four independent parameters and can be written as follows:

$$\rho_A = \frac{1}{2}\sum_{i=0}^{3} c_i\sigma_i = \frac{1}{2}\begin{pmatrix} c_0+c_3 & c_1-ic_2 \\ c_1+ic_2 & c_0-c_3 \end{pmatrix}, \tag{3}$$

where $\{\sigma_i\}=[\mathbf{1},\sigma_x,\sigma_y,\sigma_z]$. A normalized density matrix is obtained by dividing all elements by c_0. In the same way, the two-particle density matrix is given by

$$\rho_{AB} = \sum_{n,m,k,l=1}^{2} \rho_{nm}^{kl} b_{An}^\dagger b_{Bk}^\dagger |0\rangle\langle 0| b_{Bl} b_{Am} \tag{4}$$

with the matrix elements $\rho_{nm}^{kl}=\langle b_{Am}^\dagger b_{Bl}^\dagger b_{Bk} b_{An}\rangle$. The two-particle density matrix has 16 independent parameters and can be written

$$\rho_{AB} = \frac{1}{4}\sum_{i,j=0}^{3} c_{ij}\sigma_i \otimes \sigma_j \tag{5}$$

with $\otimes$ the direct product. Expressing the real coefficients c_i and c_{ij} in terms of outcomes of ensemble averaged measurements thus gives a complete reconstruction of the emitted state. The accessible measurements are average current and zero frequency current correlations. Importantly, in the transport state the source continuously emits quantum states. As a consequence, the long time measurements automatically provide an ensemble average measurement. At A the average currents at contacts $\alpha=\pm$ are

$$I_A^\alpha = \frac{e^2V}{h}\langle n_A^\alpha\rangle, \quad n_A^\alpha = b_{A\alpha}^\dagger b_{A\alpha}. \tag{6}$$

The *zero frequency* correlator between currents fluctuations ΔI in reservoirs $A\alpha$ and $B\beta$, given by $S_{AB}^{\alpha\beta} = (1/2)\int dt \langle \Delta I_{A\alpha}(0)\Delta I_{B\beta}(t) + \Delta I_{B\beta}(t)\Delta I_{A\alpha}(0)\rangle$ can be written[11]

$$S_{AB}^{\alpha\beta} = \frac{2e^3V}{h}\left[\langle n_A^\alpha n_B^\beta\rangle - \langle n_A^\alpha\rangle\langle n_B^\beta\rangle\right] \tag{7}$$

with $n_B^\alpha=b_{B\alpha}^\dagger b_{B\alpha}$. The operators $b_{A\alpha}$ and $b_{B\beta}$ in the reservoirs at A and B are related to operators b_{An} and b_{Bk} in the leads An, Bk, with $n,k=1, 2$ (see Fig. 1) via the scattering matrix S_A of the beam splitters at A as

$$\begin{pmatrix} b_{A+} \\ b_{A-} \end{pmatrix} = S_A \begin{pmatrix} b_{A1} \\ b_{A2} \end{pmatrix} \tag{8}$$

and similarly at B.

We start with the reconstruction of the one-particle state at A, accessible via the average current (the same procedure holds for the state at B). Here a formal approach is taken that can be extended to the investigation of the two-particle state. We note that the reconstruction approach is similar to QST schemes for qubits in other systems (see, e.g., Refs. 10 and 22). There are, however, a number of important special features for mesoscopic systems, making a detailed investigation important. It is desirable to minimize both the type and number of experiments having to be carried out. As is clear from the following, for a complete reconstruction it is sufficient to consider only measurements of currents in one reservoir in A. Here we consider the current at $A+$. Using the relation between operators, Eq. (8) and Eq. (6), we have

$$I_A^+/(e^2V/h) = \langle n_A^+\rangle = \mathrm{tr}\,(\rho_A \mathcal{A}) \tag{9}$$

with the matrix

Setting	Parameters
I	$T_A=0$, ϕ_A arb.
II	$T_A=1$, ϕ_A arb.
III	$T_A=1/2$, $\phi_A=0$
IV	$T_A=1/2$, $\phi_A=\pi/2$

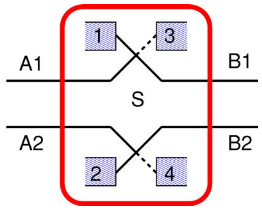

FIG. 2. (Color online) Left: Table with scattering parameters at A for the four different settings. Right: Schematic of an elementary mesoscopic source S [thick red (dark gray) box, see Fig. 1]. Two reservoirs biased at eV, 1 and 2, and two grounded reservoirs, 3 and 4, are connected via beam splitters to the four leads going out towards A and B. Scattering between upper and lower leads, e.g., $A1$ and $A2$, is not possible.

$$\mathcal{A} = \begin{pmatrix} R_A & \sqrt{T_A R_A} e^{-i(\phi_A+\varphi_A)} \\ \sqrt{T_A R_A} e^{i(\phi_A+\varphi_A)} & T_A \end{pmatrix}. \tag{10}$$

The phase $\varphi_A = \phi_{A2} - \phi_{A3}$ contains all the information about uncontrollable phases of the beam splitter. From Eqs. (9) and (10) it is clear that the phase φ_A can be included in ρ_A by a change of local basis $\rho_A \to U_A \bar{\rho}_A U_A^\dagger$ with $U_A = \mathrm{diag}\,[\exp(-i\varphi_A/2), \exp(i\varphi_A/2)]$. Below we consider the reconstruction of $\bar{\rho}_A$, parametrized by coefficients $\bar{c}_i$ [see Eq. (3)], thus working with $\mathcal{A}(\varphi_A=0)$ in Eq. (10). This yields ρ_A up to an unknown local basis rotation.

Importantly, only four settings of the beam splitter are needed, both for the current and the current correlators, for a complete state reconstruction. The settings I to IV considered here are listed in the table in Fig. 2. By constructing suitable linear combinations $j_A(j)$ of the observables at the different settings, in the $\{|1\rangle_A, |2\rangle_A\}$ basis

$$j_A(0) = \mathcal{A}(\mathrm{I}) + \mathcal{A}(\mathrm{II}) = \mathbf{1},$$

$$j_A(1) = 2\mathcal{A}(\mathrm{III}) - [\mathcal{A}(\mathrm{I}) + \mathcal{A}(\mathrm{II})] = \sigma_x,$$

$$j_A(2) = 2\mathcal{A}(\mathrm{IV}) - [\mathcal{A}(\mathrm{I}) + \mathcal{A}(\mathrm{II})] = \sigma_y,$$

$$j_A(3) = \mathcal{A}(\mathrm{I}) - \mathcal{A}(\mathrm{II}) = \sigma_z, \tag{11}$$

we obtain a complete set[2] of measurements, since the Pauli matrices σ_j obey the relation $\mathrm{tr}\,(\sigma_i\sigma_j) = 2\delta_{ij}$. Here $\mathcal{A}(\mathrm{I})$ is the matrix $\mathcal{A}$ in Eq. (10) for the setting I, etc. From Eqs. (9) and (11) and the relation $\langle j_A(j)\rangle = \mathrm{tr}\,(\bar{\rho}_A\sigma_j) = \bar{c}_j$ we then directly obtain the coefficients $\bar{c}_j$, parametrizing ρ_A in Eq. (3)

$$\bar{c}_j = \sum_{k=0}^{3} Q_{jk}\langle n_A^+(k)\rangle, \quad Q = \begin{pmatrix} 1 & 1 & 0 & 0 \\ -1 & -1 & 2 & 0 \\ -1 & -1 & 0 & 2 \\ 1 & -1 & 0 & 0 \end{pmatrix} \tag{12}$$

in terms of the measured currents for the different settings, taking the index $\{k\} = [0,1,2,3] \equiv [\mathrm{I},\mathrm{II},\mathrm{II},\mathrm{IV}]$. This completes the one-particle state reconstruction.

We then turn to the two-particle state. In Eq. (7), the quantity that is directly linked to the density matrix elements ρ_{nm}^{kl} is the reducible correlator $\langle n_A^\alpha n_B^\beta\rangle$. This correlator is directly obtained from the measured noise and the average currents. In analogy to the current, it is sufficient to consider correlations between currents in one terminal in A and one in B. Considering here $A+$ and $B+$, one obtains from Eq. (7) and (8) the dimensionless correlator

$$\frac{S_{AB}^{++}}{2e^3V/h} + \frac{I_A^+ I_B^+}{(e^2V/h)^2} = \langle n_A^+ n_B^+\rangle = \mathrm{tr}\,(\rho_{AB}\mathcal{A}\otimes\mathcal{B}), \tag{13}$$

where the matrix $\mathcal{B}$ is given from $\mathcal{A}$ in Eq. (10) by changing indices $A \to B$ in the scattering amplitudes. Similar to the one-particle state, we note from Eq. (13) that both phases φ_A and φ_B can be included in ρ_{AB} by independent local rotations $\rho_{AB} \to (U_A\otimes U_B)\bar{\rho}_{AB}(U_A\otimes U_B)^\dagger$, with $U_B = \mathrm{diag}\,[\exp(-i\varphi_B/2), \exp(i\varphi_B/2)]$. Below we thus consider the reconstruction of $\bar{\rho}_{AB}$, parametrized as in Eq. (5) by the coefficients $\bar{c}_{ij}$, yielding ρ_{AB} up to a local basis rotation.[23]

By considering the same four settings at B as at A, we can use the linear combination operators $j_A(j)$ in Eq. (11) and correspondingly $j_B(i)$ to construct a complete set of observables, in the basis $\{|1\rangle_A|1\rangle_B, |1\rangle_A|2\rangle_B, |2\rangle_A|1\rangle_B, |2\rangle_A|2\rangle_B\}$,

$$j_A(j)j_B(i) = \sigma_j\otimes\sigma_i \tag{14}$$

since the direct products of the σ matrices obey $\mathrm{tr}\,[(\sigma_j\otimes\sigma_i)(\sigma_k\otimes\sigma_l)] = 4\delta_{jk}\delta_{il}$. From Eq. (13) and the relation $\langle j_A(j)j_B(i)\rangle = \mathrm{tr}\,(\bar{\rho}_{AB}\sigma_j\otimes\sigma_i) = \bar{c}_{ji}$ we then directly obtain the coefficients $\bar{c}_{ji}$ as

$$\bar{c}_{ji} = \sum_{k,l=0}^{3} Q_{jk}Q_{il}\langle n_A^+(k)n_B^+(l)\rangle \tag{15}$$

in terms of the measured current correlators and averaged currents. We emphasize that all elements can be determined from 16 current correlations and eight average currents (four at A and four at B). We have so far assumed that the measurement process is ideal. In a real experiment there are, however, imperfections due to, e.g., fluctuations of the beam splitter gate potential or limited accuracy of the measurement electronics. In mesoscopics conductors,[12] such errors can roughly be estimated to lead to deviations of a couple of percent from the ideal result. This might lead to a reconstructed density matrix with negative eigenvalues, i.e., not positive semidefinite. Schemes to correct for this, maximum likelihood estimations, for one- and two-qubit states are discussed e.g., in Refs. 22 and 24.

In the context of two-particle entanglement, it is interesting to compare the QST scheme with a Bell inequality, recently discussed for mesoscopic system (see e.g., Refs. 13 and 19 and for a density matrix approach, Ref. 25). Both schemes require the same number of current correlation measurements. The density matrix reconstructed by QST, however, completely determines the entanglement. In contrast, a Bell inequality cannot be used to quantify the entanglement,[26] there are mixed entangled states[27] that do not lead to a violation of a Bell inequality.

It is clarifying to illustrate the above scheme with a simple example (see Fig. 2). We consider the Hanbury Brown–Twiss geometry of Ref. 15, where the number of nonzero elements of the one- and two-particle density matrices are reduced due to the topological properties of the conductor. Since no scattering between the upper, 1, and lower, 2, leads is physically possible due to the spatial separation, the one-particle density matrix $\bar{\rho}_A$ has only two nonzero elements, $\bar{\rho}_{11}$ and $\bar{\rho}_{22}$. These elements are parametrized by $\bar{c}_0$ and $\bar{c}_3$, obtained by measuring $\langle j_A(0)\rangle$ and $\langle j_A(3)\rangle$.

The two-particle density matrix $\bar{\rho}_{AB}$ has four nonzero elements, $\bar{\rho}_{11}^{22}, \bar{\rho}_{22}^{11}, \bar{\rho}_{21}^{12}$, and $\bar{\rho}_{12}^{21}$. Using the relation between the coefficients $\bar{c}_{ij}$ resulting from several matrix elements being zero, $\bar{\rho}_{AB}$ can then be parametrized as

$$\bar{\rho}_{AB} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \bar{c}_{00}+\bar{c}_{33} & \bar{c}_{11}-i\bar{c}_{21} & 0 \\ 0 & \bar{c}_{11}+i\bar{c}_{21} & \bar{c}_{00}-\bar{c}_{33} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \qquad (16)$$

Conseqently, only four correlations $\langle j_A(i) j_B(j)\rangle$ need to be measured to completely reconstruct $\bar{\rho}_{AB}$, reducing the number of actual current correlations needed to be carried out to 12 for the settings considered here [see Eq. (15)]. It is interesting to note that in the geometry in Fig. 2, considering the tunneling limit for the beam splitters in the source and changing to an electron-hole picture,[14] the emitted state is a true two-particle state.[15] Since the hole currents and current fluctuations are directly related to the electron ones, it is possible to employ our scheme to reconstruct an electron-hole state as well.

In this paper we have shown that a quantum state tomography procedure is possible in electrical conductors using only current and shot-noise measurements. In particular for orbital entanglement we have shown that a QST procedure is possible if the phase of qubits can be tuned with additional side gates. The fundamental advantage of QST over a Bell inequality test is the fact that QST allows a full determination of the entanglement content of a quantum state. The connection between QST and shot-noise measurement demonstrates that the maximum possible information one can infer about two-particle properties of a mesoscopic conductor can be gained from current and shot-noise measurements.

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