

Subwavelength imaging: Resolution enhancement using metal wire gratings

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An experimental evidence of subwavelength imaging with a “lens,” which is a uniaxial negative permittivity wire medium slab, is reported. The slab is formed by gratings of long, thin, parallel conducting cylinders. Taking into account the anisotropy and spatial dispersion in the wire medium we theoretically show that there are no usual plasmons that could be excited on surfaces of such a slab, and there is no resonant enhancement of evanescent fields in the slab. The experimentally observed clear improvement of the resolution in the presence of the slab is explained as filtering out the harmonics with small wave numbers. In other words, the wire gratings (the wire medium) suppress strong traveling-mode components increasing the role of evanescent waves in the image formation. This effect can be used in near-field imaging and detection applications.

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I. INTRODUCTION

Recently, much effort has been devoted to studies of sub-wavelength focusing and imaging systems. Sir John Pendry first suggested employing slabs of materials with negative permittivity and permeability to overcome the Rayleigh limit in imaging.¹ Since then there have been many papers published confirming Pendry’s prediction both in computer simulations and experimentally.^{2–10} Such imaging becomes possible due to the resonant excitation of surface plasmons on the slab surfaces by evanescent waves existing near an object. These surface plasmons subsequently excite evanescent waves reproducing the object’s near fields in the image plane. In other words, the super-resolution is achieved by amplification of near fields by plasmon resonances.^{11–14} The known realization of the effect in the microwave region is based on the use of a composite material that combines an array of thin conducting wires (wire medium) and an array of split rings.¹⁵ Due to the inductances of wires forming the medium, the wire medium behaves like an anisotropic material with negative permittivity for the electric field component parallel to the wires and positive (close to unity) permittivity for the electric field component perpendicular to the wires.^{13,16,17,20} Near the fundamental resonance of the currents induced in split rings, the induced magnetic moments can be out of phase with the incident magnetic field, realizing negative effective permeability.^{21,22} A slab of a material with only negative permittivity can also be used for focusing and imaging waves of *P* polarization.¹ There are many materials exhibiting negative permittivity in optics, but at microwaves there are no low-loss natural materials that would provide a negative real part of permittivity. To realize the effect at microwaves, one might try to use the same wire medium as a negative-epsilon metamaterial,⁶ as in the known experiments with double-negative metamaterials.^{18,19} The use of only a wire medium could have the advantages of a wideband frequency response, because there are no resonant split rings. Actually, even single or double wire gratings could be used for this purpose.

In this paper we provide experimental data on imaging by means of wire media and explain the phenomena observed in

our experiment. The experiment shows that a dense wire grating or a pair of such gratings put in an arrangement similar to that of Ref. 7 can provide subwavelength imaging. We use dipole antennas as the field sources and detectors. In our experiment the radiated by antenna field is of *S*-polarization [transversal-electric (TE)], rather than *P*-polarization [transversal-magnetic (TM)]. According to Ref. 1, only fields of TM polarization take part in subwavelength imaging by a negative-epsilon slab. Thus, the experiment is not a simple confirmation of the theory of Ref. 1 but shows other phenomena. Since it is known that subwavelength imaging is due to the excitation of surface waves (plasmons) we carefully study what kind of surface waves can exist on an interface between free space and a wire medium formed by infinite ideally conducting cylinders (wires) placed in free space. The wires are assumed to be thin so that the relative transversal dielectric permittivity is close to unity. The interface is parallel to the wires.

We show that when the spatial dispersion in wire medium²³ is properly taken into account, there are no usual surface plasmons of TM polarization. Instead, there are waves that behave like modes of a multiwire transmission line. In a multiwire transmission line composed of *N* wires there are exactly *N* modes. Although these modes differ in the distribution of currents among the wires, they all propagate with the speed of light along the wires. In a system of an infinite number of wires (wire medium) there is an infinite number of similar transmission-line modes. By superimposing these modes one can get an arbitrary distribution of wire currents. On the macroscopic scale (on the whole medium scale) that means that for such modes the wave vector component orthogonal to the wires can be arbitrary, however, the component along the wires always equals ω/c . We show that unlike usual plasmons in metals, these modes of a uniaxial wire medium do not exhibit a resonant interaction with incident evanescent fields. Therefore, there is no resonant enhancement of evanescent fields performed by a slab of such medium. However, our experiments show a clear improvement of the resolution in the presence of wire medium slabs. We explain this as filtering out the harmonics with small

II. EXPERIMENT

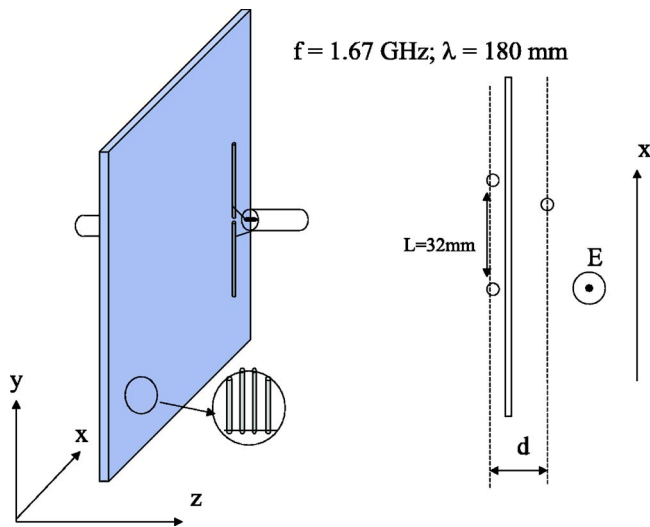


FIG. 1. (Color online) Experimental configuration. The sources are two vertical dipole antenna positioned behind the grid at a distance of $L=32$ mm from each other. The near-field distribution is measured by a probe dipole antenna moving along the grid so that the distance between the source plane and the probe d is maintained constant in each measurement.

wave numbers performed by wire gratings of which the slabs are composed. In other words, wire gratings suppress strong traveling-mode components increasing the role of evanescent waves in the formation of the image.

The experimental setup is shown in Fig. 1. One or two planar wire grids formed by round copper wires of $8 \mu\text{m}$ in diameter and 30 cm in length were excited by two parallel half-wavelength dipoles. The source dipoles were parallel to the wires (this arrangement is similar to that of Lagarkov *et al.*,⁷ who used an artificial double-negative slab formed by wires and split-ring resonators). The grid period (distance between the wires) was 2 mm. The values of the other quantities are shown in Fig. 1. The working frequency ($f = 1.67$ GHz) was determined experimentally as the one that provided the maximum signal in the probe antenna in the image plane in the absence of wire grids (Fig. 2).

In Figs. 3–5 the intensity measured by the probe antenna behind the grid as a function of the position along the axis x is shown. We can see that without the grid we have a single maximum, whereas in the system with one or two grids we can see two maxima.

In the near-field zone, the electric fields of the two antennas have nonzero components along any direction. The electric field component along the wires excites the grid currents. Because the distance between the source dipoles is about $\lambda/6$, the experimental results show that the device is capable to resolve subwavelength details.

In the next two sections we will give a theoretical explanation of the observed phenomena.

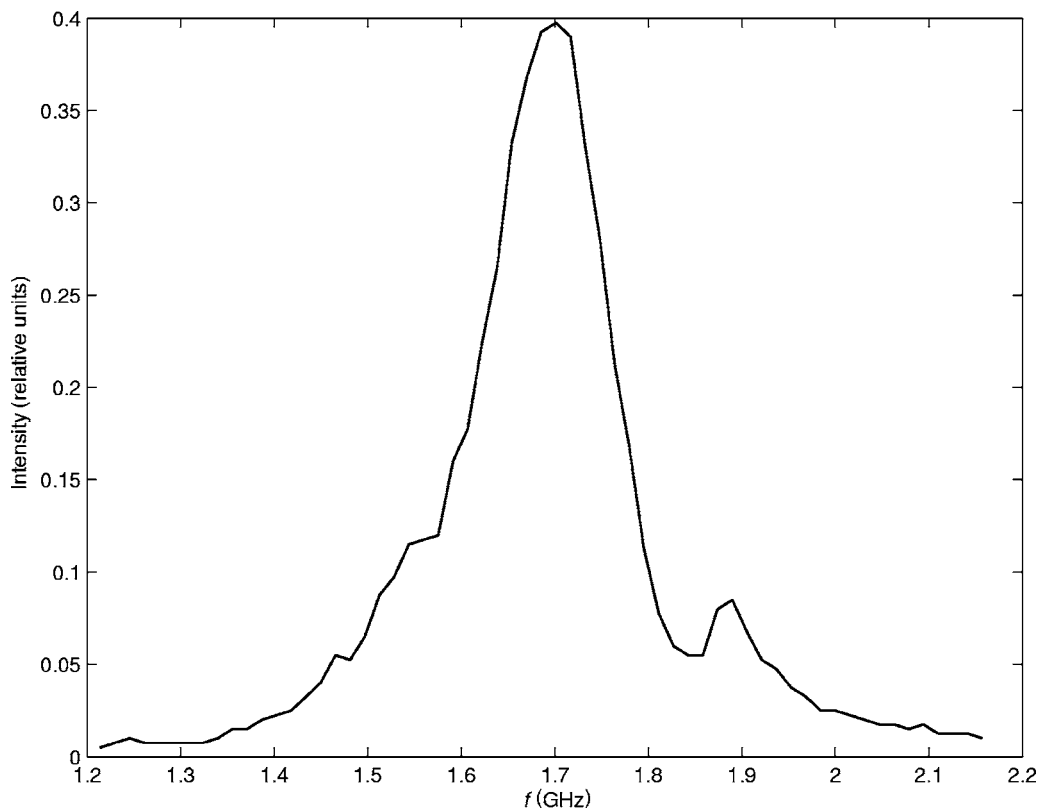


FIG. 2. Signal in the probe antenna as a function of frequency.

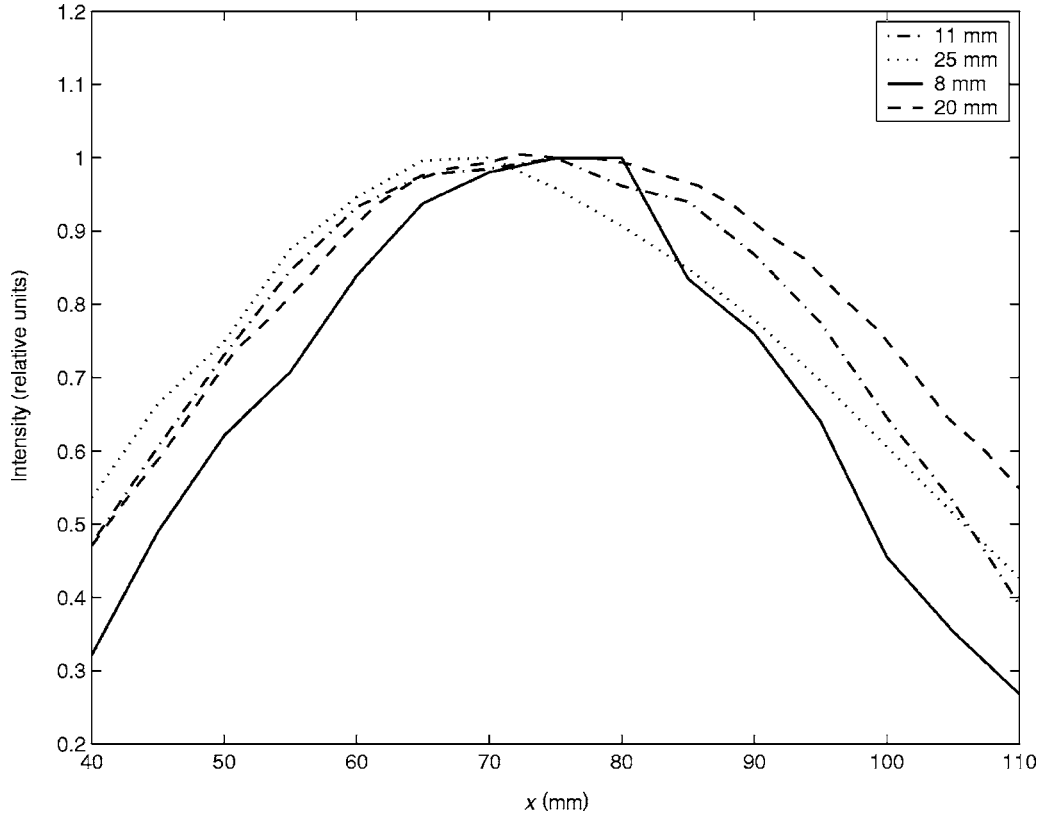


FIG. 3. The measured intensity as a function of the position of the receiving antenna on the x axis. The grid is absent. Different curves correspond to different distances from the grid to the receiving antenna (these distances are indicated as curve labels).

III. TRANSMISSION LINE MODES INSTEAD OF PLASMONS

We will look for plasmons on the surface of the wire medium formed by thin parallel ideally conducting cylinders densely (in terms of the wavelength) packed in free space. This artificial material can be modeled by a uniaxial relative permittivity dyadic whose longitudinal (along the wire direction) component depends on the frequency and the longitudinal wave number²³

$$\bar{\bar{\epsilon}}_r = \epsilon_{\parallel} \mathbf{u}_0 \mathbf{u}_0 + \epsilon_{\perp} (\bar{\bar{I}} - \mathbf{u}_0 \mathbf{u}_0), \quad (1)$$

where \mathbf{u}_0 is the unit vector along the wires, and $\bar{\bar{I}}$ is the unit dyadic. The components of the dielectric permittivity dyadic are ϵ_{\parallel} and ϵ_{\perp} (for thin wires):

$$\epsilon_{\parallel} = \left(1 - \frac{k_p^2}{k_0^2 - k_{\parallel}^2} \right), \quad \epsilon_{\perp} \approx 1. \quad (2)$$

Here k_p is the plasma wave number,²³ k_{\parallel} is the propagation constant component along the wires, and k_0 is the free-space wave number.

If the interface between the wire medium (seen as a composition of many parallel ideally conducting cylindrical wires) and free space is parallel to the direction of the wires, the whole system is regular along that direction. This means that all the eigenmodes of such a system (including surface modes) split into ordinary and extraordinary modes.¹³ The ordinary modes have a zero electric field component along

the wires (TE modes with respect to the direction along the wires), and the extraordinary modes have the corresponding magnetic field component equal to zero (TM modes with respect to the wires). Obviously, the ordinary (TE) modes are the same as the waves in free space because they are not affected by thin wires. Therefore, there are no surface modes (surface plasmons) among the ordinary (TE) eigenmodes. There are also modes with both electric and magnetic field components vanishing along the wires (TEM modes with respect to the wires). We may consider them as a limiting case of the extraordinary (TM) modes.

To find the dispersion relation for the surface plasmons of extraordinary polarization one has to equate the tangential electric and magnetic fields at the interface of the wire medium and free space. We denote $E_{\parallel} = \mathbf{u}_0 \cdot \mathbf{E}$ and $H_{\perp} = (\mathbf{u}_0 \times \mathbf{n}) \cdot \mathbf{H}$, where \mathbf{n} is the unit normal. In these notations $H_{\parallel} = 0$ (we are interested only in extraordinary modes). The same notation is used for the components of the wave vector \mathbf{k} . Then the following relation between E_{\parallel} and H_{\perp} can be derived from the Maxwell equations:

$$\begin{aligned} \frac{E_{\parallel}}{H_{\perp}} &= \pm \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{k_0^2 - k_{\parallel}^2}{k_0 [(k_0^2 - k_{\parallel}^2) \epsilon_{\parallel} - k_{\perp}^2]^{1/2}} \\ &= \pm \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{k_0^2 - k_{\parallel}^2}{k_0 [k_0^2 - k_p^2 - k_{\perp}^2 - k_{\parallel}^2]^{1/2}}. \end{aligned} \quad (3)$$

Different signs in this formula correspond to two possible directions of propagation (decay) with respect to \mathbf{n} . The

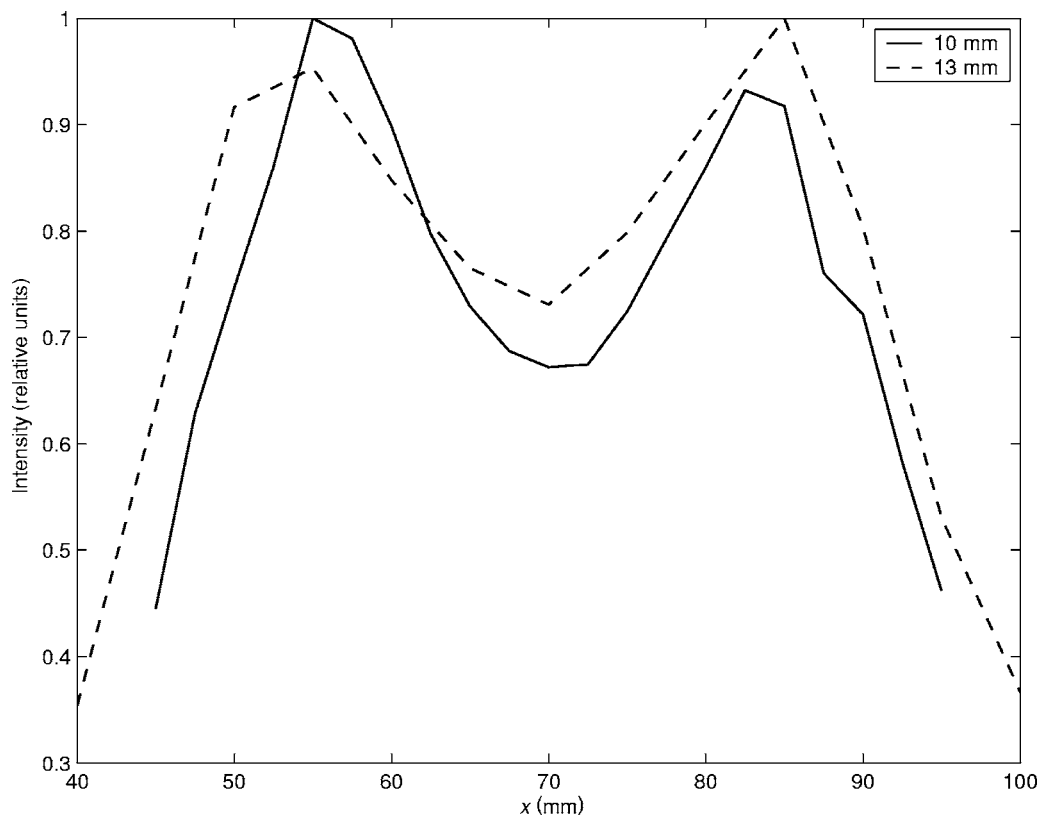


FIG. 4. The same as in Fig. 3 but with one grid present.

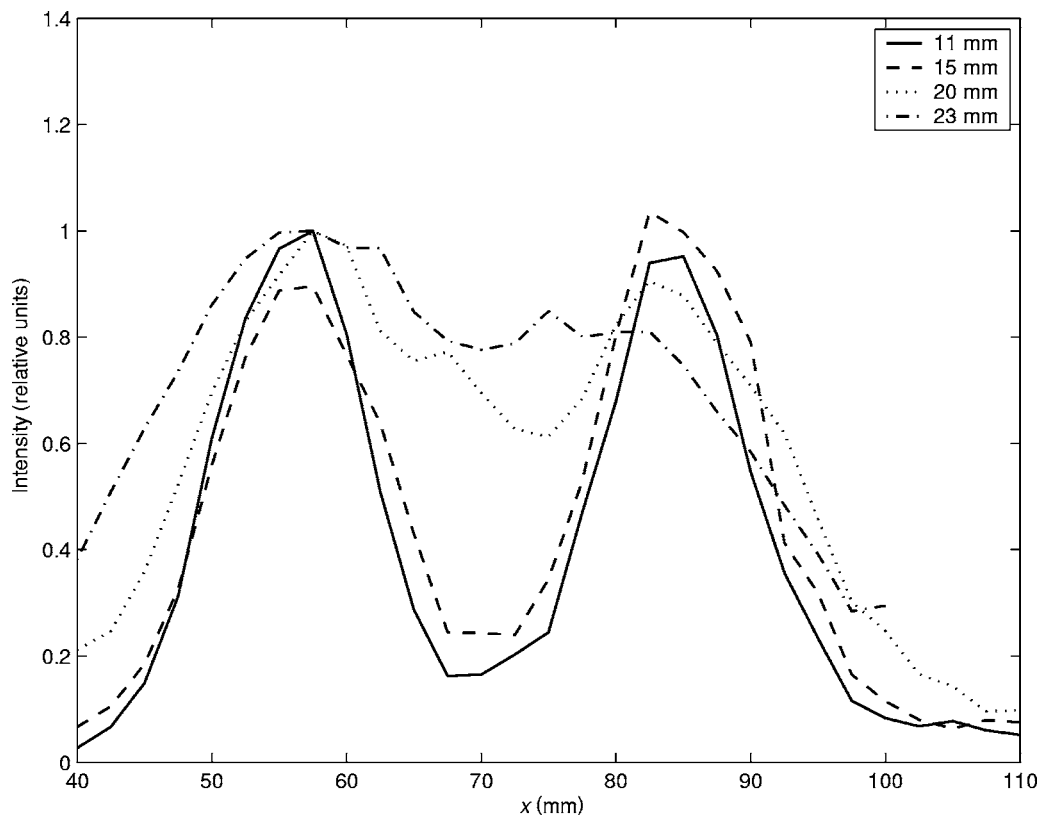


FIG. 5. The same as in Fig. 3 but with two grids present.

square root in the denominator of this formula is the magnitude of the normal component of the wave vector. When dealing with surface modes, the argument of the square root is negative and the normal component of the wave vector is imaginary. In this case the following branch of the square root must be used:

$$[k_0^2 - k_p^2 - k_\perp^2 - k_\parallel^2]^{1/2} = -j\sqrt{k_p^2 + k_\perp^2 + k_\parallel^2 - k_0^2}. \quad (4)$$

Here we also give the dispersion equation of the extraordinary modes in an *unbounded* wire medium. This is obtained in the same derivation procedure along with (3):

$$k^2 = k_0^2 - k_p^2. \quad (5)$$

The waves in the free space region can also be formally separated into ordinary and extraordinary modes of the same definition. Then, for the extraordinary modes we obtain

$$\frac{E_\parallel}{H_\perp} = \pm \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{k_0^2 - k_\parallel^2}{k_0[k_0^2 - k_\perp^2 - k_\parallel^2]^{1/2}}. \quad (6)$$

In the general case $E_\perp \neq 0$. The following relation holds for an extraordinary (TM) wave

$$\frac{E_\perp}{E_\parallel} = -\frac{k_\parallel k_\perp}{k_0^2 - k_\parallel^2}. \quad (7)$$

This relation is the same in the wire medium and in free space (in the case when $\varepsilon_\perp = 1$) and, therefore, it is not necessary for the derivation of the dispersion relation.

The dispersion relation of the surface plasmons is obtained by equating (3) and (6) for the modes both decaying from the interface

$$(k_0^2 - k_\parallel^2)(\sqrt{k_p^2 + k_\perp^2 + k_\parallel^2 - k_0^2} + \sqrt{k_\perp^2 + k_\parallel^2 - k_0^2}) = 0. \quad (8)$$

The arguments of these two square roots are positive (we are interested in surface waves), that means that the only possible solution is $k_\parallel^2 = k_0^2$.

The perpendicular wave-vector component k_\perp disappears from the dispersion equation, which means that it can be *arbitrary*. When $k_\perp = 0$ this solution corresponds to a simple plane wave propagating along the wires with the speed of light. It does not decay from the interface, therefore it is not a surface wave. However, when $k_\perp \neq 0$ the wave propagation factor in the interface plane is $\sqrt{k_\parallel^2 + k_\perp^2} > k_0$ which gives a wave bounded to the surface both in free space and in the medium. Indeed, in the medium, using the dispersion relation (5) we have for the normal component of the wave vector (remember that $k_\parallel^2 = k_0^2$)

$$k_n^2 = k^2 - k_\parallel^2 - k_\perp^2 = -k_p^2 - k_\perp^2. \quad (9)$$

Perhaps, it is worth noting that the specific solution we have just discussed is not a surface plasmon in the usual meaning. Instead, it results from the fact that a system of many parallel wires can support transmission-line type modes (TL modes). In our case they appear as the limit of the extraordinary (TM) modes (which have $H_\parallel = 0$) when $k_\parallel \rightarrow \pm k_0$. Indeed, from (6) it is seen also that $E_\parallel \rightarrow 0$, and we obtain a TL mode as a limit of a TM mode. TL modes of a multiwire transmission-line may have arbitrary phases of

separate wire currents which corresponds to the arbitrary k_\perp in our case of the wire crystal. Still, the propagation factor along the wires is the same, it does not depend on k_\perp , and equals k_0 .

IV. FILTERING INSTEAD OF A RESONANCE

In this section we will show that the amplitude of excitation of the TL modes considered above remain finite in structures with infinitely long wires. This is because the incident field spectral components, which are in phase synchronism with the TL modes, are also TEM with respect to the wires. The electric field of such components vanishes along the wires. We will show that the susceptibility of a wire grating to an external electric field grows with the same proportion as the longitudinal electric field decays when an incident wave transforms from TM to TEM. Because of this the induced grating currents remain finite and do not exhibit a resonance.

Diffraction of plane waves by wire gratings has been well studied in the literature. We will use the formulas from Refs. 13 and 24 which are applicable to thin wires. The current I_w induced in a chosen reference wire of a grating can be expressed in terms of the external field complex amplitude on the wire axis E_\parallel , the frequency, the wave vector, and the parameters of the grating

$$I_w(k_\perp, k_\parallel) = Y(k_\perp, k_\parallel) E_\parallel(k_\perp, k_\parallel), \quad \text{where} \quad (10)$$

$$Y(k_\perp, k_\parallel) = -j \sqrt{\frac{\varepsilon_0}{\mu_0}} \frac{2b}{(1 - k_\parallel^2/k_0^2) \gamma(k_\perp, k_\parallel)}. \quad (11)$$

We use time dependence of the form $\exp(+j\omega t)$. In Eq. (11) b is the grating period, and $\gamma(k_\perp, k_\parallel)$ can be calculated as follows (r_0 is the wire radius):

$$\gamma(k_\perp, k_\parallel) = \frac{k_0 b}{\pi} \left[\ln \frac{b}{2\pi r_0} + \frac{1}{2} \sum_{n=-\infty}^{\infty} \left(\frac{2\pi}{\sqrt{(2\pi n + k_\perp)^2 + (k_\parallel^2 - k_0^2)b}} - \frac{1}{|n|} \right) \right]. \quad (12)$$

From (11) it is seen that the grating susceptibility to the incident electric field has a pole when k_\parallel approaches $\pm k_0$. Because we are interested only in the extraordinary modes, it is suitable to express the incident electric field as $E_\parallel = Z^e H_\perp$, where $Z^e = E_\parallel/H_\perp$ is given by (6) (with a plus sign). Then, we get for the wire current,

$$I_w(k_\perp, k_\parallel) = \frac{2k_0 b H_\perp(k_\perp, k_\parallel)}{\gamma(k_\perp, k_\parallel) \sqrt{k_\perp^2 + k_\parallel^2 - k_0^2}}. \quad (13)$$

This expression remains finite at $k_\parallel = \pm k_0$.

It is of practical interest to find the field produced by the induced grating currents. Due to the discrete nature of the grating, it will produce all possible Floquet harmonics with spatial frequencies $2\pi n/b$, $n \in Z$. However, the most interesting is the case of a dense grid, $|k_\perp| < 2\pi/b$. In this region

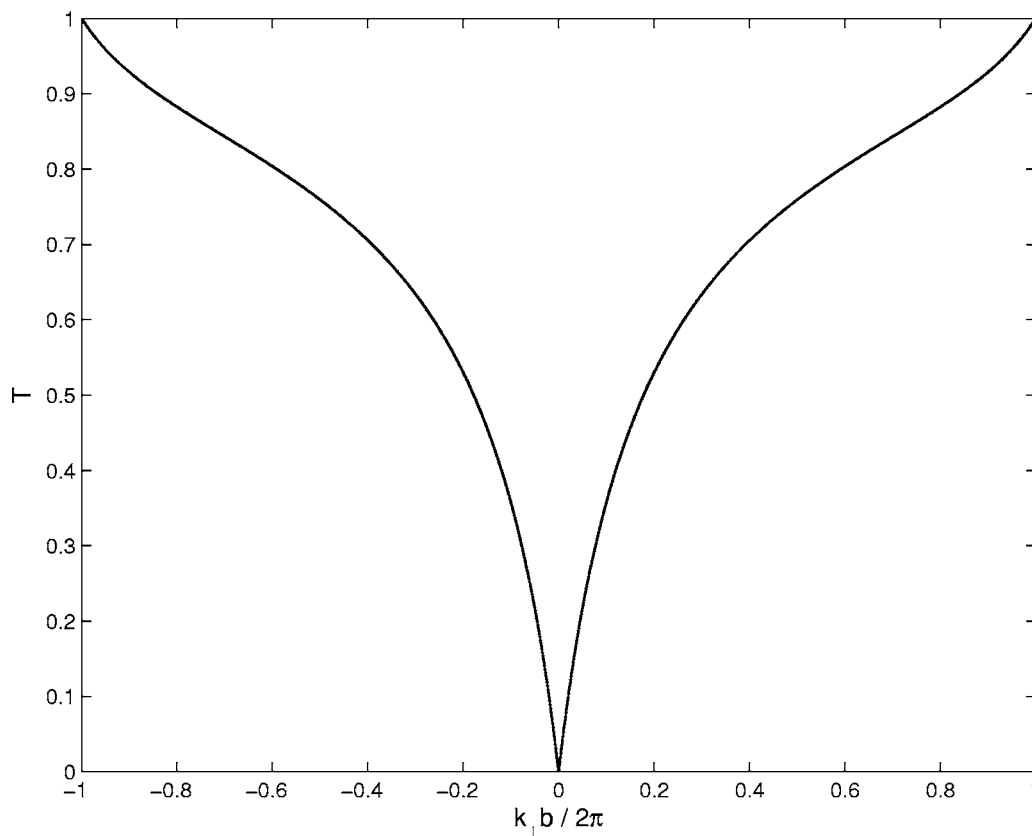


FIG. 6. The absolute value of the transmission coefficient for the main Floquet mode as a function of the normalized spatial frequency $k_{\perp} b / (2\pi)$. $k_{\parallel} = \pm k_0$ in this example.

we may consider only the main Floquet mode. The main Floquet mode of the scattered magnetic field at the grating plane can be found by the averaged boundary condition at the grating plane, $H_{\perp}^s = \pm I_w / (2b)$, therefore

$$H_{\perp}^s(k_{\perp}, k_{\parallel}) = \pm \frac{k_0 H_{\perp}(k_{\perp}, k_{\parallel})}{\gamma(k_{\perp}, k_{\parallel}) \sqrt{k_{\perp}^2 + k_{\parallel}^2 - k_0^2}}. \quad (14)$$

Here, plus corresponds to the illuminated side of the grating and minus corresponds to the opposite side. The transmission coefficient through the grating (for the main Floquet mode) reads

$$T(k_{\perp}, k_{\parallel}) = 1 - \frac{k_0}{\gamma(k_{\perp}, k_{\parallel}) \sqrt{k_{\perp}^2 + k_{\parallel}^2 - k_0^2}}. \quad (15)$$

The plot of the transmission coefficient as a function of k_{\perp} is given in Fig. 6.

It is seen that the grating is less transparent to the spatial harmonics with lower spatial frequencies. From here it becomes clear that the improving of the resolution in the presence of the wire grating can be explained as filtering out unwanted low-frequency components of the spatial spectrum. We have experimentally checked if the effect is due to possible excitation of transmission-line modes in *finite* gratings by bringing absorbers to both ends of the grating. The image was practically not changed, which proves that the effect is indeed due to the filtering of lower spatial harmonics.

It should be mentioned that the setup used in the experiment is tuned to the case given in Fig. 6. Indeed, in the near field of two $\lambda/2$ dipoles oriented along the wires of the grid the spatial harmonics with $k_{\parallel} = \pm k_0$ dominate. In the orthogonal direction the dipoles are seen almost as point objects, so that the k_{\perp} spectrum is wide and it contains components with high spatial frequency which are not blocked by the grating.

V. CONCLUSIONS

We have demonstrated, both theoretically and experimentally, that the resolution of a near-field imaging system can be enhanced by positioning a dense grating of thin parallel conductors before the image plane. The effect is due to filtering out the field spectral components with low spatial frequencies.

We have also shown that a direct comparison of silver films (at optical frequencies) and wire gratings (at microwaves) is not possible because of the spatial dispersion in wire media. One can note that the physical phenomena in the considered system of wires that lead to the resolution enhancement are rather different than in silver films, although silver films and wire media can both be modeled by negative effective permittivity. In wire gratings, the effect is simply due to the increasing of the grating transparency for the spectral components of high spatial frequencies, and not due to the excitation of surface plasmons as in the case of silver films.

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