Current and voltage noise spectrum due to generation and recombination fluctuations in semiconductors

Fabio E. Zocchi*

Media Lario Technologies, Località Pascolo, 23842 Bosisio Parini (LC), Italy

(Received 3 August 2005; revised manuscript received 26 October 2005; published 9 January 2006)

The power spectral densities of both current and voltage noise due to generation-recombination fluctuations in finite-length semiconductors are calculated taking into account space-charge interactions through the Poisson equation and the dynamics of both charge carriers and trap levels through their coupled continuity equations in the drift-diffusion approximation. Proper boundary conditions for the charge-carrier density include the effects of the finite length of the semiconductor sample. The frequency dependence of the power spectral density, the asymptotic behavior of the current noise for high and low applied voltages, and the dependence of the voltage noise from the sample length are studied in detail.

DOI: 10.1103/PhysRevB.73.035203 PACS number(s): 78.40.Fy, 85.30.−z, 72.70.+m

I. INTRODUCTION

Spontaneous random transitions of charge carriers between different energy levels in semiconductors are the sources of generation-recombination noise that modulates the electrical conductivity of the material and can be measured as fluctuations in the electric current flowing through the sample or in the voltage at its electrical terminals. Although several experimental and theoretical studies[1–8] have investigated the subject, analytical expressions for the power spectral density of generation-recombination noise have been derived only by assuming some remarkable simplifications of the physical model, such as for example, neglecting space-charge interactions or diffusion currents or ignoring the effects of boundary conditions at the electrical terminals of finite-length semiconductor samples. More recently the low frequency value of the power spectral density of the current noise due to generation-recombination fluctuations has been calculated by Gomila and Reggiani[9] in a more complete model that takes into account all of the physical effects mentioned above in the drift-diffusion approximation. However, since the approach in Ref. 9 neglects the time derivatives of the electron and trap densities, it only allows for the calculation of the zero-frequency value of the power spectrum of the current noise.

Much less attention has been drawn on the theoretical analysis of the voltage noise across a semiconductor sample due to generation-recombination fluctuations. It should be noted that the power spectral density of the voltage noise is not simply obtained by multiplying the current spectral density by the square of the sample resistance since the resistance itself is fluctuating as a result of the fluctuation of the carrier density. Instead, the voltage noise must be independently determined by taking into account the actual electrical boundary conditions with which a voltage measurement would be performed.

In the present paper, the power spectral density of both current and voltage noise in finite-length semiconductors is calculated at all frequencies in the framework of a model that includes space-charge interactions through the Poisson equation and the dynamics of both charge carriers and trap levels through their coupled continuity equations in the drift-diffusion approximation. The details of the theoretical model for the calculation of charge fluctuations are described in Sec. II. In particular, the model equations are linearized in the assumption of small fluctuations, which amounts to neglecting Coulomb interactions between charge carriers. The power spectral density of the current and the voltage noise are derived and studied in Secs. III and IV, respectively. In particular, the asymptotic behavior of the current noise for high and low applied voltages and the dependence of the voltage noise from the sample length are studied in detail.

II. CHARGE FLUCTUATIONS IN THE DRIFT-DIFFUSION APPROXIMATION

Assuming a one-dimensional geometry in the x direction, we consider a uniform n-doped semiconductor of length L in steady state conditions, ideally terminated by two metallic ohmic contacts at $x=0$ and $x=L$. We further suppose that the free electron density $N(x,t)$ in the semiconductor is controlled by a single trap level through stochastic generation and recombination rates $g(N,N_T)$ and $r(N,N_T)$ per unit length and time. Both $g(N,N_T)$ and $r(N,N_T)$ are assumed to be explicitly independent from both position and time but dependent on $N(x,t)$ and on the ionized trap density $N_T(x,t)$. When an electric field $E(x,t)$ is applied to the semiconductor, the free electron density $N(x,t)$ and the ionized trap density $N_T(x,t)$ are obtained as solutions of the coupled continuity equations that, in the drift-diffusion approximation,[6,9,10] are given by

$$\frac{\partial N(x,t)}{\partial t} = D\frac{\partial^2 N(x,t)}{\partial x^2} + \mu\frac{\partial(E(x,t)N(x,t))}{\partial x} + g(N,N_T) - r(N,N_T), \tag{1}$$

$$\frac{\partial N_T(x,t)}{\partial t} = g(N,N_T) - r(N,N_T). \tag{2}$$

In Eq. (1), μ and D are the electron mobility and the diffusion coefficient, respectively, again assumed independent from position and time. The terms $g(N,N_T)$ and $r(N,N_T)$ are random source and sink for $N(x,t)$ and $N_T(x,t)$ and give the character of stochastic Langevin equations[6,11,12] to (1) and (2). When space-charge effects are included in the model, the electric field $E(x,t)$ in (1) is the sum of the uniform and constant external field E_e and a term $E_i(x,t)$ due to charge fluctuations. Thus, to solve (1) and (2), the Poisson equation is also needed,

$$\frac{\partial E(x,t)}{\partial x} = \frac{q}{A\varepsilon}(N_T(x,t) - N(x,t)), \tag{3}$$

where A is the cross section of the semiconductor sample, q is the absolute value of the electron charge, and ε is the dielectric constant, the latter assumed uniform and constant in the semiconductor. Since in steady state conditions $\langle E_i(x,t)\rangle = 0$, averaging Eqs. (1)–(3), gives

$$\langle N_T(x,t)\rangle = \langle N(x,t)\rangle = N_0, \tag{4}$$

$$\langle g(N_0,N_0)\rangle = \langle r(N_0,N_0)\rangle. \tag{5}$$

In the following $g(N_0,N_0)$ and $r(N_0,N_0)$ are taken to be two stochastic independent Poisson processes with equal averages, $\langle g(N_0,N_0)\rangle = \langle r(N_0,N_0)\rangle = g_0$, and equal power spectral density $2g_0$.

Following a perturbative approach,[4,8,13] the free electron and ionized trap densities are written as $N(x,t)=N_0+n(x,t)$ and $N_T(x,t)=N_0+n_T(x,t)$, respectively, where $n(x,t)$ and $n_T(x,t)$ are the fluctuations of $N(x,t)$ and $N_T(x,t)$. Substituting these expressions in (1)–(3) and expanding $g(N,N_T)$ and $r(N,N_T)$ in a power series up to the first order with respect to $N(x,t)$ and $N_T(x,t)$, the following equations for $n(x,t)$, $n_T(x,t)$, and $E_i(x,t)$ are obtained:

$$\frac{\partial n(x,t)}{\partial t} = D\frac{\partial^2 n(x,t)}{\partial x^2} + \mu E_e \frac{\partial n(x,t)}{\partial x} + \mu \frac{\partial E_i(x,t)}{\partial x} N_0 - \frac{n(x,t)}{\tau_N} - \frac{n_T(x,t)}{\tau_T} + \Delta gr, \tag{6}$$

$$\frac{\partial n_T(x,t)}{\partial t} = -\frac{n_T(x,t)}{\tau_T} - \frac{n(x,t)}{\tau_N} + \Delta gr, \tag{7}$$

$$\frac{\partial E_i(x,t)}{\partial x} = \frac{q}{A\varepsilon}(n_T(x,t) - n(x,t)), \tag{8}$$

where

$$\frac{1}{\tau_N} = \left(\frac{\partial r(N,N_T)}{\partial N} - \frac{\partial g(N,N_T)}{\partial N}\right)_{N=N_0,N_T=N_0}, \tag{9}$$

$$\frac{1}{\tau_T} = \left(\frac{\partial r(N,N_T)}{\partial N_T} - \frac{\partial g(N,N_T)}{\partial N_T}\right)_{N=N_0,N_T=N_0} \tag{10}$$

are the electron and trap lifetimes, respectively. In Eqs. (6) and (7), $\Delta gr = g(N_0,N_0) - r(N_0,N_0)$ is a Poisson process with a zero average and power spectral density $4g_0$. The term $\mu\partial(E_i(x,t)n(x,t))/\partial x$ in (6) has been neglected since it is of second order with respect to $\mu E_e \partial n(x,t)/\partial x$ and $\mu N_0 \partial E_i(x,t)/\partial x$, provided $E_i(x,t) \ll E_e$ and $n(x,t) \ll N_0$. This approximation amounts to neglecting the Coulomb interaction among the electrons.

Equations (6)–(8) allow the calculation of small charge fluctuations in semiconductors in the drift-diffusion approximation. To solve such equations, proper boundary conditions must be set, depending on the specific noise measurement performed on the semiconductor. Current and voltage noise power spectral density will be considered in Sec. III and Sec. IV, respectively.

III. POWER SPECTRAL DENSITY OF CURRENT NOISE

We assume that a voltage of absolute value $V=E_eL$ is applied to the semiconductor sample and we calculate the spectrum of the noise associated to the current flowing through the sample. In this case, Eqs. (6)–(8) must be solved with the boundary conditions $n(0,t)=n(L,t)=0$,[9,13–15] for the electron density. No boundary conditions are required for the trap density since no spatial derivatives of $n_T(x,t)$ are involved in Eqs. (6)–(8). One condition on the electric field $E_i(x,t)$ is needed instead. Since the total applied voltage across the semiconductor sample is fixed, the voltage associate to $E_i(x,t)$ must vanish,

$$\int_0^L E_i(x,t)dx = 0. \tag{11}$$

The current flowing through the semiconductor sample is given by[9,16]

$$i(t) = \frac{1}{L}\int_0^L \left(-qj(x,t) + A\varepsilon \frac{\partial E_i(x,t)}{\partial t}\right)dx, \tag{12}$$

where $j(x,t)$ is the current density,

$$j(x,t) = -D\frac{\partial n(x,t)}{\partial x} - \mu E_e n(x,t) - \mu E_i(x,t)N_0. \tag{13}$$

However, taking into account relation (11) and the boundary conditions for $n(x,t)$, relation (12) simplifies to

$$i(t) = \frac{q\mu E_e}{L}\int_0^L n(x,t)dx. \tag{14}$$

To evaluate the power spectral density associated with the fluctuations of $i(t)$, Eqs. (6)–(8) and (14) are interpreted as the differential description of a linear system with stochastic input Δgr. Thus, the spectrum of the current noise is obtained by the application of the standard linear system theory,

$$S_I(f) = 4g_0\int_0^L |I_{x_0}(2\pi i f)|^2 dx_0 = 4g_0\left(\frac{q\mu E_e}{L}\right)^2 \int_0^L dx_0 \left| \int_0^L \mathcal{N}_{x_0}(x,2\pi i f)dx \right|^2, \tag{15}$$

where $\mathcal{N}_{x_0}(x,s)$ and $I_{x_0}(s)$ are the transfer functions of the linear system described by (6)–(8) and (14) when the stochastic generation-recombination event takes place at the position $x=x_0$. In other terms, $\mathcal{N}_{x_0}(x,s)$ is the Laplace transform of $n(x,t)$ when the homogenous equations associated with to (6)–(8) are solved with the initial conditions $n(x,0)=\delta(x-x_0)$, and $n_T(x,0)=\delta(x-x_0)$, and $I_{x_0}(s)$ is the corresponding Laplace transform of $i(t)$. The integration over x_0 follows from the assumption that the generation events at different positions are statistically independent.

To calculate (15), we take the Laplace transform of Eqs. (6)–(8), taking into account the above initial conditions. Then, eliminating the Laplace transforms of $E_i(x,t)$ and $n_T(x,t)$ using (7) and (8), the following equation for $\mathcal{N}_{x_0}(x,s)$ is obtained:

$$-\frac{\tau_T}{\tau_\varepsilon}\frac{1+s\tau_\varepsilon}{1+s\tau_T}\delta(x-x_0)=D\frac{\partial^2\mathcal{N}_{x_0}(x,s)}{\partial x^2}+\mu E_e\frac{\partial\mathcal{N}_{x_0}(x,s)}{\partial x}-\mathcal{N}_{x_0}(x,s)\frac{\tau_T}{\tau_\varepsilon}\frac{1+s\tau_\varepsilon}{1+s\tau_T}\frac{1+s\tau}{\tau},\quad(16)$$

where

$$\frac{1}{\tau}=\frac{1}{\tau_N}+\frac{1}{\tau_T}\ \text{and}\ \frac{1}{\tau_\varepsilon}=\mu\frac{q}{A\varepsilon}N_0.\quad(17)$$

In (17), τ can be considered an effective lifetime and τ_ε is the dielectric relaxation time in the semiconductor.

The explicit calculation of the power spectral density of the current noise for both the long and short semiconductor sample is performed separately in the next two sections, respectively.

A. Long semiconductors

The time constant τ gives an estimation of the lifetime of the electron in the semiconductor. Thus, if the length L of the semiconductor sample is much greater then the distance traveled by the electron in a time interval τ, or $L\gg\mu E_e\tau+\sqrt{2D\tau}$, the electron dynamics can be studied in an approximation in which the semiconductor has an infinite length, extending from $-\infty$ to $+\infty$. In this assumption, Eq. (16) must be solved with the boundary conditions $\lim_{x\to\pm\infty}\mathcal{N}_{x_0}(x,s)=0$, corresponding to the vanishing electron density at infinity. Actually, it is not necessary to explicitly determine the solution of (16). Indeed, integrating (16) over x between $-\infty$ and $+\infty$, taking into account the boundary conditions, and noting that the space slope of the carrier density must also vanish, $\lim_{x\to\pm\infty}\partial\mathcal{N}_{x_0}(x,s)/\partial x=0$ [since the integral of $\mathcal{N}_{x_0}(x,s)$ over x must be convergent] we get

$$\int_{-\infty}^{+\infty}\mathcal{N}_{x_0}(x,s)dx=\frac{\tau}{1+s\tau}.\quad(18)$$

This quantity is, as expected, independent from x_0. Thus, to calculate the power spectral density, large but finite values for L must be considered again. Taking the absolute value of (18), we obtain

$$S_I^\infty(f)=4g_0\frac{(q\mu E_e)^2}{L}\frac{\tau^2}{1+4\pi^2f^2\tau^2}.\quad(19)$$

This is the original result by van Vliet,[2,4] independent from both the drift-diffusion dynamics of the electrons and the space-charge effects. Indeed, since the current noise is related to the total charge in the semiconductor, the electron motion is relevant only if it contributes to the total charge by modifying the rate with which electrons are captured by the electrodes. However, in long samples the boundary effect of the electrodes can be neglected, thus justifying (19).

B. Short semiconductors

When the semiconductor has a finite length, Eq. (16) must be solved with the boundary conditions $\mathcal{N}_{x_0}(0,s)=\mathcal{N}_{x_0}(L,s)=0$. In this case the solution of (16) is given by

$$\mathcal{N}_{x_0}(x,s)=\frac{1}{D(\alpha_+-\alpha_-)}\frac{\tau_T}{\tau_\varepsilon}\frac{1+s\tau_\varepsilon}{1+s\tau_T}\Bigg[\frac{e^{\alpha_+(L-x_0)}-e^{\alpha_-(L-x_0)}}{e^{\alpha_+L}-e^{\alpha_-L}}\times(e^{\alpha_+x}-e^{\alpha_-x})-(e^{\alpha_+(x-x_0)}-e^{\alpha_-(x-x_0)})\times\text{step}(x-x_0)\Bigg],\quad(20)$$

where

$$\alpha_\pm=\alpha_\pm(s)=\frac{1}{L}\left(-\frac{V}{2V_T}\pm\sqrt{\frac{V^2}{4V_T^2}+\xi^2(s)}\right)$$

with

$$\xi^2(s)=\frac{L^2\tau_T}{D\tau_\varepsilon\tau}\frac{(1+s\tau_\varepsilon)(1+s\tau)}{1+s\tau_T}\quad(21)$$

and we have exploited Einstein's relation $D=V_T\mu$, $V_T=kT/q$ being the thermal voltage. The integral of $\mathcal{N}_{x_0}(x,s)$ over x is given by

$$\int_0^L\mathcal{N}_{x_0}(x,s)dx=\frac{\tau}{1+s\tau}\left[\frac{e^{\alpha_+(L-x_0)}(e^{\alpha_-L}-1)-e^{\alpha_-(L-x_0)}(e^{\alpha_+L}-1)}{e^{\alpha_+L}-e^{\alpha_-L}}+1\right].\quad(22)$$

By taking the integral over x_0 of the square modulus of (22), we get, according to (15),

$$S_I(f)=4g_0\frac{(q\mu E_e)^2}{L}\frac{\tau^2}{1+4\pi^2f^2\tau^2}\Bigg\{1+\frac{|A|^2}{L}\frac{e^{(\alpha_++\alpha_+^*)L}-1}{\alpha_++\alpha_+^*}+\frac{|B|^2}{L}\frac{e^{(\alpha_-+\alpha_-^*)L}-1}{\alpha_-+\alpha_-^*}+\frac{2}{L}\,\text{Re}\left[A\frac{e^{\alpha_+L}-1}{\alpha_+}\right]+\frac{2}{L}\,\text{Re}\left[B\frac{e^{\alpha_-L}-1}{\alpha_-}\right]+\frac{2}{L}\,\text{Re}\left[AB^*\frac{e^{(\alpha_++\alpha_-^*)L}-1}{\alpha_++\alpha_-^*}\right]\Bigg\},\quad(23)$$

where

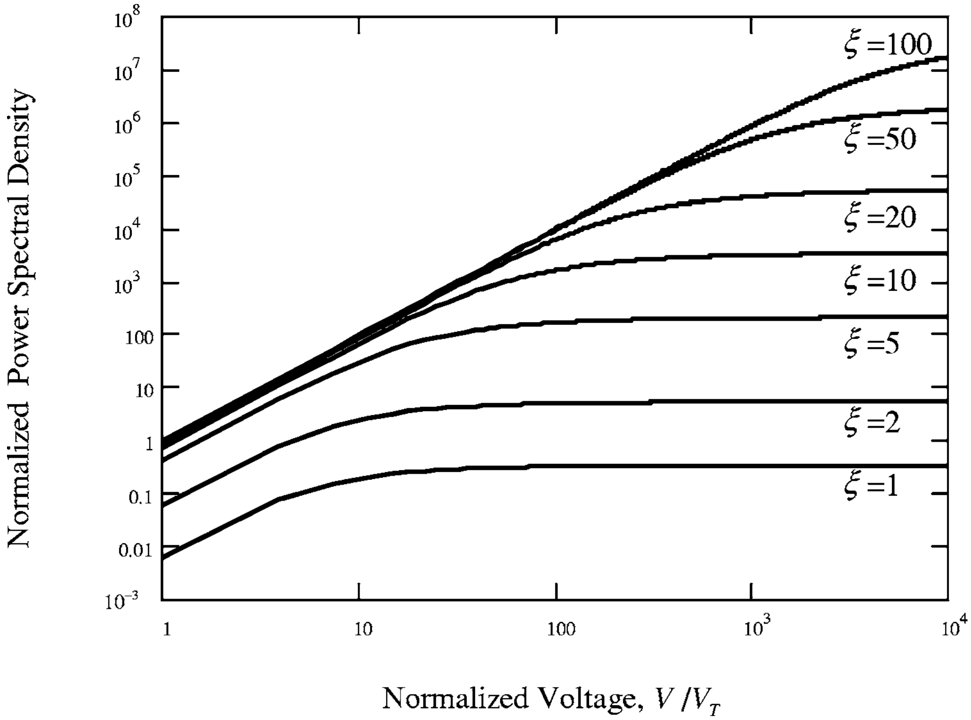

FIG. 1. Low frequency behavior of the normalized power spectral density of current noise, $S_I(0)L^3/4q^2g_0\tau^2D^2$, as a function of the normalized voltage V/V_T for different values of the adimensional parameter $\xi^2=L^2\tau_T/D\tau_\varepsilon\tau$.

$$A=\frac{e^{\alpha_-L}-1}{e^{\alpha_+L}-e^{\alpha_-L}}\text{ and }B=-\frac{e^{\alpha_+L}-1}{e^{\alpha_+L}-e^{\alpha_-L}}.$$

The first term in the curly brackets of (23) provides the result (19) for the long semiconductors, whereas the other terms give the correction due to the finite length of the sample.[9] It is expected that the long sample limit (19) is recovered from (23) for large values of L at fixed applied voltage. Indeed, by letting L approach infinity in (23), we get $\exp(\alpha_-L)\to 0$, $\exp(\alpha_+L)\to\infty$, $A\to 0$, and $B\to -1$, leading to $S_I^\infty(f)$.

The power spectral density (23) can be studied in the limit of high and low applied voltages. At high electric fields, when the inequality $V/V_T \gg 2|\xi(2\pi \mathrm{i}f)|$ holds, then $\alpha_-(s)\cong -E_e/V_T$ and the exponential of $\alpha_-(s)L$ is negligible with respect to 1 if, in addition, the magnitude of the applied voltage V is much greater than V_T. Developing $S_I(f)$ in a power series of $\alpha_+(2\pi \mathrm{i}f)L\cong\xi^2(2\pi \mathrm{i}f)V_T/V$ to the second order and neglecting also $\alpha_+(2\pi \mathrm{i}f)$ with respect to $\alpha_-(2\pi \mathrm{i}f)$, we find that $S_I(f)$ saturates at

$$\begin{aligned}S_I(f)&\cong\frac{4g_0}{3}\frac{(q\mu E_e)^2}{L}\frac{\tau^2}{1+4\pi^2f^2\tau^2}\frac{V_T^2}{V^2}|\xi^4(2\pi \mathrm{i}f)|\\&=\frac{4q^2g_0L}{3}\frac{\tau_T^2}{\tau_\varepsilon^2}\frac{1+4\pi^2f^2\tau_\varepsilon^2}{1+4\pi^2f^2\tau_T^2}.\end{aligned}\tag{24}$$

At low electric fields, $V/V_T \ll 2|\xi^2(2\pi \mathrm{i}f)|$, then $\alpha_+(s)\cong-\alpha_-(s)\cong\xi(s)/L$. If $|\xi(2\pi \mathrm{i}f)|L\gg 1$ we get again, as expected, Eq. (19), whereas, if $|\xi(2\pi \mathrm{i}f)|L\ll 1$, developing $S_I(f)$ in a power series of $\alpha_+(2\pi \mathrm{i}f)L\cong\xi(2\pi \mathrm{i}f)$ to the fourth order, we get

$$\begin{aligned}S_I(f)&\cong\frac{g_0}{30}\frac{(q\mu E_e)^2}{L}\frac{\tau^2}{1+4\pi^2f^2\tau^2}|\xi^4(2\pi \mathrm{i}f)|\\&=\frac{q^2g_0L}{30}\frac{\tau_T^2}{\tau_\varepsilon^2}\frac{1+4\pi^2f^2\tau_\varepsilon^2}{1+4\pi^2f^2\tau_T^2}\frac{V^2}{V_T^2}.\end{aligned}\tag{25}$$

Thus, the low voltage and short length limit is just scaled by a factor $V^2/120\cdot V_T^2$ with respect to the high voltage limit.

The frequency behavior of the spectrum of the current noise is studied by assuming that the inequalities $\tau_\varepsilon\ll\tau_N$ and $\tau_\varepsilon\ll\tau_T$ hold. This is justified by noting, for example, that the value of τ_ε in intrinsic silicon at room temperature is 0.32 μs and decreases at increasing donor density. Under the above assumption, three frequency ranges can be considered: low frequencies, $f<0.1/2\pi\tau_T$, $0.1/2\pi\tau_N$, midfrequencies, $0.1/2\pi\tau_T$, $0.1/2\pi\tau_N<f<0.1/2\pi\tau_\varepsilon$, and high frequencies, $f>0.1/2\pi\tau_\varepsilon$. A factor of ten below the critical frequencies $1/2\pi\tau_N$, $1/2\pi\tau_T$, and $1/2\pi\tau_\varepsilon$ is assumed to be large enough for neglecting their contribution to the noise power spectrum.

As mentioned in the Introduction, the low frequency value $S_I(0)$ of the current noise spectrum has been studied in Ref. 9. The main results, which can be also derived from (23)–(25), are summarized for completeness. The normalized power spectral density $S_I(0)L^3/4q^2g_0\tau^2D^2$ as a function of V/V_T depends only on the adimensional parameter $\xi(0)^2=L^2\tau_T/D\tau_\varepsilon\tau$ and is plotted in Fig. 1.[9] At a high electric field, for $V/V_T\gg 2\xi(0)$ and $V/V_T\gg 1$, relation (24) gives[9,17]

$$S_I(0)\cong\frac{4q^2g_0L}{3}\frac{\tau_T^2}{\tau_\varepsilon^2},\tag{26}$$

whereas at low electric fields, for $V/V_T\ll 2\xi(0)$ and $\xi(0)\ll 1$, we get, from (25),[9]

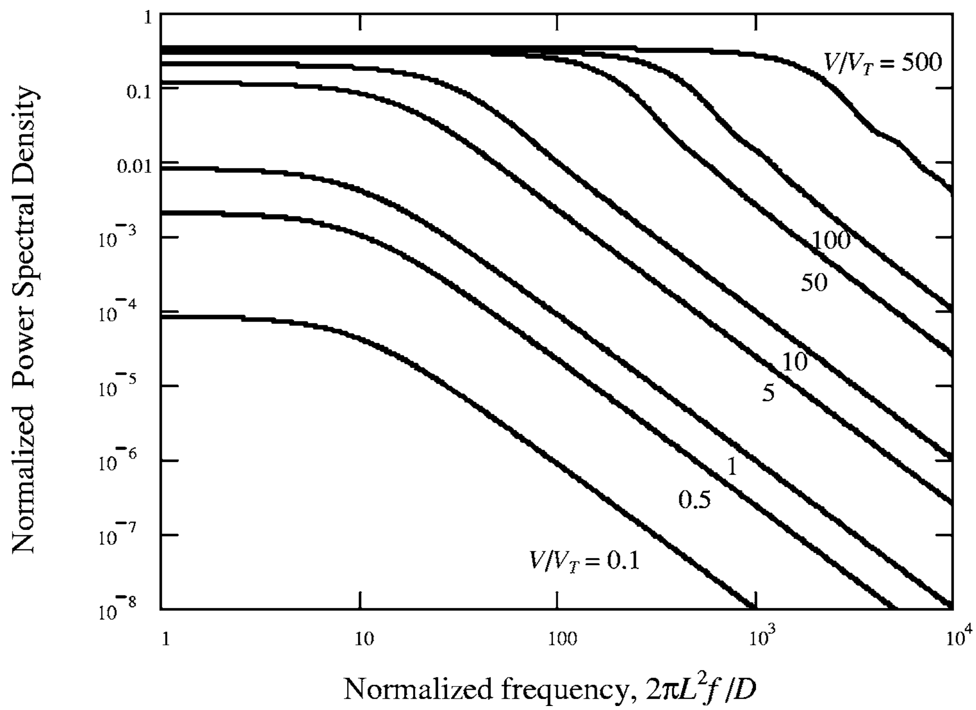

FIG. 2. High frequency behavior of the normalized power spectral density of current noise, $S_I(f)/4q^2g_0L$, as a function of the normalized frequency $2\pi L^2 f/D$ for different values of the normalized voltage V/V_T.

$$S_I(0) \cong \frac{q^2 g_0 L}{30}\frac{\tau_T^2}{\tau_\varepsilon^2}\frac{V^2}{V_T^2}. \tag{27}$$

In the midfrequency range, since $\tau \cong \min(\tau_N,\tau_T)$, two opposite cases can be distinguished,

$$\xi^2(s) \cong \begin{cases} \dfrac{L^2\tau_T}{D\tau_\varepsilon\tau} & \text{for } \tau_N > \tau_T \\ \dfrac{L^2\tau_T}{D\tau_\varepsilon\tau}\dfrac{1+s\tau_N}{1+s\tau_T} & \text{for } \tau_N < \tau_T. \end{cases} \tag{28}$$

For $\tau_N > \tau_T$, $\xi(s) \cong \xi(0)$ and the midfrequency behavior of the noise spectrum is just scaled as the low frequency one, $S_I(f) \cong S_I(0)/(1+4\pi^2 f^2\tau^2) \cong S_I(0)/(1+4\pi^2 f^2 \tau_T^2)$. Instead, for $\tau_N < \tau_T$, the frequency dependence is more complex. However, in both cases the high and low voltage limits are the same, as given by (24) and (25). At high electric fields, for $V/V_T \gg 2|\xi(2\pi i f)|$ and $V/V_T \gg 1$, the noise power spectrum $S_I(f)$ saturates at

$$S_I(f) \cong \frac{4q^2 g_0 L}{3}\frac{\tau_T^2}{\tau_\varepsilon^2}\frac{1}{1+4\pi^2 f^2\tau_T^2}, \tag{29}$$

whereas, at low applied voltages, for $V/V_T \ll 2|\xi(2\pi i f)|$ and $\xi(2\pi i f) \ll 1$, we obtain

$$S_I(f) \cong \frac{q^2 g_0 L}{30}\frac{\tau_T^2}{\tau_\varepsilon^2}\frac{1}{1+4\pi^2 f^2\tau_T^2}\frac{V^2}{V_T^2}. \tag{30}$$

At high frequencies, above about $0.1/2\pi\tau_\varepsilon$, $\xi^2(s) \cong L^2(1+s\tau_\varepsilon)/D\tau_\varepsilon$. At even higher frequencies, for $f > 10/2\pi\tau_\varepsilon$ the normalized power spectral density $S_I(f)/4q^2g_0L$ is a function of the adimensional frequency $2\pi L^2 f/D$ only and is plotted in Fig. 2 for different values of the ratio V/V_T. The asymptotic behavior, for high and low electric fields, is again given by (24) and (25). At high voltage, when the inequalities $V/V_T \gg 2|\xi(2\pi i f)|$ and $V/V_T \gg 1$ hold, $S_I(f)$ saturates at

$$S_I(f) \cong \frac{4q^2 g_0 L}{3}\frac{1+4\pi^2 f^2\tau_\varepsilon^2}{4\pi^2 f^2\tau_\varepsilon^2} \underset{f>10/2\pi\tau_\varepsilon}{\rightarrow} \frac{4q^2 g_0 L}{3}. \tag{31}$$

At low electric fields, for $V/V_T \ll 2|\xi(2\pi i f)|$ and $|\xi(2\pi i f)| \ll 1$, we get

$$S_I(f) \cong \frac{q^2 g_0 L}{30}\frac{1+4\pi^2 f^2\tau_\varepsilon^2}{4\pi^2 f^2\tau_\varepsilon^2}\frac{V^2}{V_T^2} \underset{f>10/2\pi\tau_\varepsilon}{\rightarrow} \frac{q^2 g_0 L}{30}\frac{V^2}{V_T^2}. \tag{32}$$

It should be noted that the above analysis can be extended to a regime of saturated drift velocity. In this case, the quantity μE_e in Eq. (6) must be substituted with the saturated carrier velocity ν_s and the term containing $E_i(x,t)$ can be neglected. Thus, the Poisson equation (8) is no longer needed to derive the electron density from (6) and (7). This amounts to letting τ_ε approach infinity, $\tau_\varepsilon \rightarrow \infty$, and to making the formal substitution $V/V_T \rightarrow \nu_s L/D$ in all relations of this section.

IV. POWER SPECTRAL DENSITY OF VOLTAGE NOISE

The power spectral density of the voltage noise across the semiconductor sample of length L can be studied according to a relation analogous to (11),

$$S_V(f) = 4g_0 \int_0^L |\mathcal{V}_{x_0}(2\pi i f)|^2 dx_0, \tag{33}$$

where $\mathcal{V}_{x_0}(s)$ is the Laplace transform of the voltage due to a generation-recombination event taking place at $x = x_0$ at the instant $t = 0$. In its most general, Eq. (33) should be studied for an arbitrary fixed current flowing through the semicon-

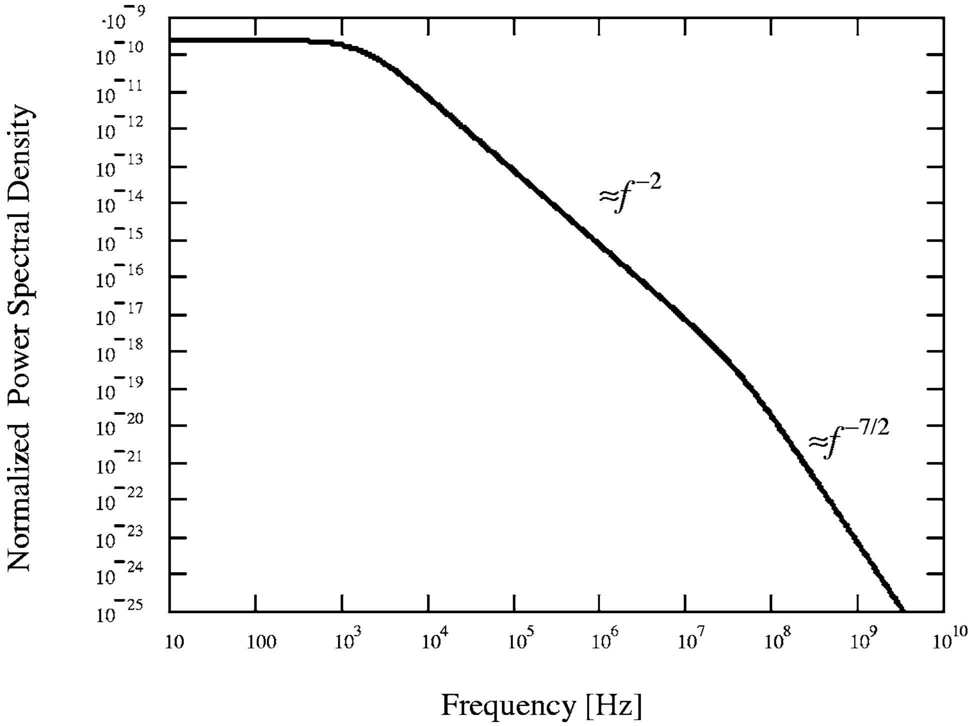

FIG. 3. An example of the normalized power spectral density of voltage noise, $S_V(f)\varepsilon^2 A^2/4q^2 g_0 \tau_T^2 L^3$ for a low-doped silicon sample 1 cm long.

ductor, as much as the spectrum of the current noise has been analyzed for an arbitrary applied voltage. However, the treatment here is limited to the simpler and probably most interesting case in which the external electric field is zero and the two electrical terminals of the semiconductor sample are left open. In this case the equation for the Laplace transform of the charge density is

$$-\frac{\tau_T}{\tau_\varepsilon}\frac{1+s\tau_\varepsilon}{1+s\tau_T}\delta(x-x_0) = D\frac{\partial^2 \mathcal{N}_{x_0}(x,s)}{\partial x^2} - \mathcal{N}_{x_0}(x,s)\frac{\tau_T}{\tau_\varepsilon}\frac{1+s\tau_\varepsilon}{1+s\tau_T}\frac{1+s\tau}{\tau} \quad (34)$$

and must be solved with the boundary conditions $\mathcal{J}(0,s) = \mathcal{J}(L,s)=0$, where $\mathcal{J}(x,s)$ is the Laplace transform of the current density

$$\mathcal{J}(x,s) = -D\frac{\partial \mathcal{N}_{x_0}(x,s)}{\partial x} - \mu \mathcal{E}_i(x,s)N_0, \quad (35)$$

and $\mathcal{E}_i(x,s)$ is the Laplace transform of the internal electric field $E_i(x,t)$.

A boundary condition is also needed for $\mathcal{E}_i(x,s)$, for which we take $\mathcal{E}_i(0,s)=0$. This implies that the electric field is also zero at the other end of the semiconductor sample, or $\mathcal{E}_i(L,s)=0$. To show that, we first note that by eliminating the Laplace transform of $N_T(x,t)$ between (7) and (8), $\mathcal{E}_i(x,s)$ is related to $\mathcal{N}_{x_0}(x,s)$ by the relation

$$\frac{\partial \mathcal{E}_i(x,s)}{\partial x} = \frac{q}{A\varepsilon}\frac{\tau_T}{1+s\tau_T}\delta(x-x_0) - \mathcal{N}_{x_0}(x,s)\frac{q}{A\varepsilon}\frac{\tau_T}{\tau}\frac{1+s\tau}{1+s\tau_T}. \quad (36)$$

Integrating Eq. (34) over x between 0 and L and taking into account the boundary conditions through relations (35) and (36) gives

$$\int_0^L \mathcal{N}_{x_0}(x,s)dx = \frac{\tau}{1+s\tau}. \quad (37)$$

If we now perform the integration of (36) over x between 0 and L we finally see that $\mathcal{E}_i(L,s)=0$. Thus, the boundary conditions for the current density reduce to

$$\left.\frac{\partial \mathcal{N}_{x_0}(x,s)}{\partial x}\right|_{x=0} = \left.\frac{\partial \mathcal{N}_{x_0}(x,s)}{\partial x}\right|_{x=L} = 0. \quad (38)$$

The solution of (34) with the boundary conditions (38) is given by

$$\mathcal{N}_{x_0}(x,s) = \frac{1}{2D\alpha}\frac{\tau_T}{\tau_\varepsilon}\frac{1+s\tau_\varepsilon}{1+s\tau_T}\left[\frac{e^{\alpha(L-x_0)}+e^{-\alpha(L-x_0)}}{e^{\alpha L}-e^{-\alpha L}}(e^{\alpha x}+e^{-\alpha x}) - (e^{\alpha(x-x_0)}-e^{-\alpha(x-x_0)})\mathrm{step}(x-x_0)\right], \quad (39)$$

where

$$\alpha = \alpha(s) = \sqrt{\frac{\tau_T}{D\tau_\varepsilon}\frac{1+s\tau_\varepsilon}{1+s\tau_T}\frac{1+s\tau}{\tau}}. \quad (40)$$

To calculate the voltage across the sample, we compare Eqs. (34) and (36) and find the following relation:

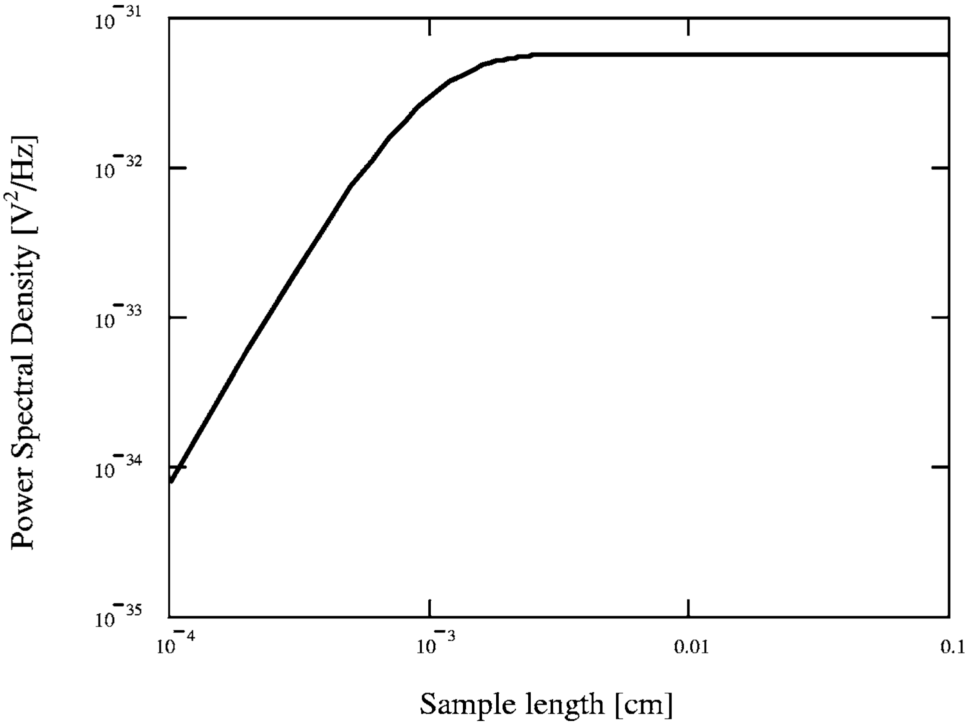

FIG. 4. An example of the low frequency value of the power spectral density of voltage noise in low-doped silicon as a function of the sample length.

$$\frac{1+s\tau_\varepsilon}{\tau_\varepsilon}\frac{\partial \mathcal{E}_i(x,s)}{\partial x} = -\frac{qD}{A\varepsilon}\frac{\partial^2 \mathcal{N}_{x_0}(x,s)}{\partial x^2}. \tag{41}$$

Finally, integrating (41) over x and taking into account the boundary conditions (38), we get

$$\frac{1+s\tau_\varepsilon}{\tau_\varepsilon}\mathcal{E}_i(x,s) = -\frac{qD}{A\varepsilon}\frac{\partial \mathcal{N}_{x_0}(x,s)}{\partial x}, \tag{42}$$

so that the voltage across the sample is given by

$$\begin{aligned}\mathcal{V}_{x_0}(s) &\equiv -\int_0^L \mathcal{E}_i(x,s)dx = \frac{qD}{A\varepsilon}\frac{\tau_\varepsilon}{1+s\tau_\varepsilon}[\mathcal{N}_{x_0}(L,s)-\mathcal{N}_{x_0}(0,s)]\\ &= \frac{q}{A\varepsilon}\frac{\tau_T}{1+s\tau_T}\frac{e^{\alpha(L-x_0)}(e^{-\alpha L}-1)+e^{-\alpha(L-x_0)}(e^{\alpha L}-1)}{\alpha(e^{\alpha L}-e^{-\alpha L})}.\end{aligned} \tag{43}$$

According to (33), the power spectral density of the voltage noise is obtained by taking the integral over x_0 of the square modulus of (43),

$$\begin{aligned}S_V(f) = \frac{4q^2g_0L\tau_T^2}{A^2\varepsilon^2}\frac{1}{1+4\pi^2f^2\tau_T^2}\frac{1}{|\alpha|^2}\Bigg\{&\frac{|A|^2}{L}\frac{e^{(\alpha+\alpha^*)L}-1}{\alpha+\alpha^*}\\ &-\frac{|B|^2}{L}\frac{e^{-(\alpha+\alpha^*)L}-1}{\alpha+\alpha^*}-\frac{2}{L}\,\mathrm{Re}\left[AB^*\frac{e^{(\alpha-\alpha^*)L}-1}{\alpha-\alpha^*}\right]\Bigg\},\end{aligned} \tag{44}$$

where

$$A = \frac{e^{-\alpha L}-1}{e^{\alpha L}-e^{-\alpha L}} \text{ and } B = -\frac{e^{\alpha L}-1}{e^{\alpha L}-e^{-\alpha L}}.$$

For long semiconductors, $\exp(-\alpha L)\to 0$ and $\exp(\alpha L)\to\infty$, we get

$$S_V(f) \cong \frac{4q^2g_0\tau_T^2}{A^2\varepsilon^2}\frac{1}{1+4\pi^2f^2\tau_T^2}\cdot\frac{1}{|\alpha(2\pi \mathrm{i}f)|^2\,\mathrm{Re}[\alpha(2\pi\mathrm{i}f)]} \tag{45}$$

and the low frequency value $S_V(0)$ of the power spectral density (45) is equal to

$$S_V(0) \cong \frac{4q^2g_0\tau_T^2}{A^2\varepsilon^2}\frac{1}{\alpha^3(0)} = \frac{4q^2g_0\tau_T^{1/2}}{A^2\varepsilon^2}(D\tau_\varepsilon\tau)^{3/2}. \tag{46}$$

At midfrequencies, in long samples and in the assumption that $\tau\approx\tau_T$, $S_V(f)$ behaves like $(1+4\pi^2f^2\tau_T^2)^{-1}$ whereas at high frequency, for $f>10/2\pi\tau_\varepsilon$, $S_V(f)$ decrease as $f^{-7/2}$,

$$S_V(f) \cong \frac{q^2g_0}{2A^2\varepsilon^2}\frac{D^{3/2}}{\pi^{7/2}f^{7/2}}. \tag{47}$$

An example of this frequency dependence is shown in Fig. 3 in which Eq. (44) is plotted for a silicon sample 1 cm long. The electron mobility has been taken equal to $\mu=1400$ cm^2/V s (Ref. 10) at room temperature, $T=300$K. The diffusion coefficient D is related to the mobility μ by Einstein's relation $D=V_T\mu$. The time constants τ_N and τ_T are equal to 1 ms and the donor density N_0/A is 10^{12} cm^{-3}.

The $f^{-7/2}$ dependence is obtained at frequencies larger than $1/2\pi\tau_N$, $1/2\pi\tau_T$, and $1/2\pi\tau_\varepsilon$ when the only relevant physical process is diffusion and Eqs. (6)–(8) can be simplified to

$$\frac{\partial n(x,t)}{\partial t} \cong D\frac{\partial^2 n(x,t)}{\partial x^2} + \Delta gr, \tag{48}$$

$$\frac{\partial n_T(x,t)}{\partial t} \cong \Delta gr, \qquad (49)$$

$$\frac{\partial E_i(x,t)}{\partial x} = \frac{q}{A\varepsilon}\bigl(n_T(x,t) - n(x,t)\bigr). \qquad (50)$$

By solving the system of equations (48)–(50) in the long semiconductor approximation (in which one electrical terminal is at $x=0$ and the other one is at $x=\infty$), the solution (47) for the power spectral density of the voltage noise is recovered.

For short samples at low frequency, when $|\alpha(2\pi i f)|L \ll 1$ holds, by developing $S_V(f)$ in a power series to the second order, we get

$$S_V(f) \cong \frac{q^2 g_0 L^3 \tau_T^2}{3A^2\varepsilon^2}\frac{1}{1+4\pi^2 f^2 \tau_T^2}. \qquad (51)$$

While (46) is independent from sample length L, the low frequency value of (51) is proportional to L^3. This dependence is shown in Fig. 4, in which the low frequency value of (44) is plotted as a function of the sample length L for the same choice of the values of the physical parameters as above.

V. CONCLUSIONS

The analytical expression of the power spectral density of both current and voltage noise due to generation and recombination fluctuations has been derived in a theoretical model that includes the main physical effects in the semiconductor. The calculated functions allow the analysis of the frequency dependence of the noise spectrum and the influence of the physical parameters on the current and voltage noise. The low frequency value of the current noise spectrum is consistent with results already available in the literature[9] and general expressions for the high and low voltage limits of the current noise spectrum are provided. A dependence of the voltage noise spectrum to $f^{-7/2}$ at high frequency has been evidenced in long semiconductor samples.

The analysis performed in the paper does not take into consideration the effect of the single-particle autocorrelation associated to the diffusion of a charge carrier in the semiconductor.[13] Further work is needed to extend the above treatment to include such contributions.

*Email address: fabio.zocchi@media-lario.com

[1] R. E. Burgess, Physica (Amsterdam) **20**, 1007 (1954).
[2] K. M. van Vliet and J. Blok, Physica (Amsterdam) **22**, 231 (1956).
[3] J. E. Hill and K. M. van Vliet, J. Appl. Phys. **29**, 177 (1958).
[4] A. van der Ziel, *Fluctuation Phenomena in Semiconductors* (Butterworths, London, 1959).
[5] K. M. van Vliet and J. R. Fasset in *Fluctuation Phenomena in Solids*, edited by R E. Burguess (Academic Press, New York, 1965).
[6] K. M. van Vliet, A. Friedmann, R. J. J. Zijlstra, A. Gisolf, and A. van der Ziel, J. Appl. Phys. **46**, 1804 (1975).
[7] K. M. van Vliet, A. Friedmann, R. J. J. Zijlstra, A. Gisolf, and A. van der Ziel, J. Appl. Phys. **46**, 1814 (1975).
[8] A. Van der Ziel, *Noise in Solid-State Devices and Circuits* (Wiley, New York, 1986).
[9] G. Gomila and L. Reggiani, Appl. Phys. Lett. **81**, 4380 (2002).
[10] S. M. Sze, *Physics of Semiconductor Devices*, 2nd ed. (Wiley, New York, 1981).
[11] P. Langevin, C. R. Hebd. Seances Acad. Sci. **146**, 530 (1908).
[12] I. Karatsas and S. Shreve, *Brownian Motion and Stochastic Calculus*, 2nd ed. (Springer-Verlag, New York, 1997).
[13] M. Lazzaroni and F. E. Zocchi, J. Phys. D **38**, 2173 (2005).
[14] E. Gatti, M. Sampietro, and F. E. Zocchi, J. Appl. Phys. **74**, 5940 (1993).
[15] M. Lazzaroni and F. E. Zocchi, Nucl. Instrum. Methods Phys. Res. A **A523**, 126 (2004).
[16] E. Gatti and P. F. Manfredi, Riv. Nuovo Cimento **9**, 3 (1986).
[17] F. Bonani and G. Ghione, Solid-State Electron. **43**, 285 (1999).