Counting statistics and decoherence in coupled quantum dots

G. Kießlich,[1] P. Samuelsson,[2] A. Wacker,[2] and E. Schöll[1]
[1]*Institut für Theoretische Physik, Technische Universität Berlin, D-10623 Berlin, Germany*
[2]*Department of Physics, University of Lund, Box 118, SE-22100 Lund, Sweden*
(Received 9 November 2005; published 17 January 2006)

We theoretically consider charge transport through two quantum dots coupled in series. The corresponding full counting statistics for noninteracting electrons is investigated in the limits of sequential and coherent tunneling by means of a master equation approach and a density matrix formalism, respectively. We clearly demonstrate the effect of quantum coherence on the zero-frequency cumulants of the transport process, focusing on noise and skewness. Moreover, we establish the continuous transition from the coherent to the incoherent tunneling limit in all cumulants of the transport process and compare this with decoherence described by a dephasing voltage probe model.

DOI: 10.1103/PhysRevB.73.033312 PACS number(s): 73.23.−b, 72.70.+m, 73.63.Kv, 74.40.+k

I. INTRODUCTION

The analysis of current fluctuations in mesoscopic conductors provides detailed insight into the nature of charge transfer.[1,2] The complete information is available by studying the full counting statistics (FCS), i.e., by the knowledge of all cumulants of the distribution of the number of transferred charges.[2,3] As a crucial achievement, the measurement of the third-order cumulant of transport through a single tunnel junction was recently reported.[4] To what extent one can extract informations from current fluctuations about quantum coherence and decoherence is the subject of intense theoretical investigations: e.g., dephasing in mesoscopic cavities and Aharonov-Bohm rings[5] and decoherence in a Mach-Zehnder interferometer.[6]

Quantum dots (QDs) constitute a representative system for mesoscopic conductors. Real time tunneling of individual electrons, an important step towards experimental detection of the FCS, has recently been observed in various QD systems.[7] Very recently, Gustavsson *et al.*[8] presented the first measurements of the FCS in a single QD. Their result is in good agreement with theory,[9,10] predicting that no effects of the coherence are displayed in the FCS.

In contrast, in serially coupled double QDs[11] the superposition between states from both dots causes prominent coherent effects. Noise properties have been studied theoretically both in the low[12] and finite frequency range[13,14] for these structures but no FCS studies are available yet. Experimentally, the low-frequency noise has been investigated very recently in related double-well junctions.[15]

In this work we show that detailed information about quantum coherence in double QD systems can be extracted from the zero-frequency current fluctuations. For this purpose we elaborate on the FCS in the limits of coherent and incoherent transport through the QD system by means of a density matrix (DM) and master equation (ME) description. We demonstrate a smooth transition between these approaches by decoherence originating from coupling the QDs to a charge detector. The results are compared to a scattering approach, where decoherence is introduced via phenomenological voltage probes.

II. MODEL

The central quantity in the FCS is $P(N,t_0)$, the distribution function of the number N of transferred charges in the time interval t_0. The associated cumulant generating function (CGF) $F(\chi)$ is[2]

$$\exp[-F(\chi)] = \sum_N P(N,t_0)\exp[iN\chi]. \tag{1}$$

Here we consider the zero-frequency limit, i.e., t_0 much longer than the time for tunneling through the system. From the CGF we can obtain the cumulants $C_k = -(-i\partial_\chi)^k F(\chi)|_{\chi=0}$ which are related to, e.g., the average current $\langle I\rangle = eC_1/t_0$ and to the zero-frequency noise $S=2e^2C_2/t_0$. The Fano factor is defined as C_2/C_1. The skewness of the distribution of transferred charges is given by the third-order cumulant C_3.

The setup of the coupled QD system is shown as the inset of Fig. 1: QD1 is connected to the emitter with a tunneling rate Γ_e and QD2 to the collector contact with rate Γ_c. Mutually they are coupled by the tunnel matrix element Ω. One level in each dot, at energies ε_1 and ε_2 respectively, is assumed. We consider zero temperature and work in the limit

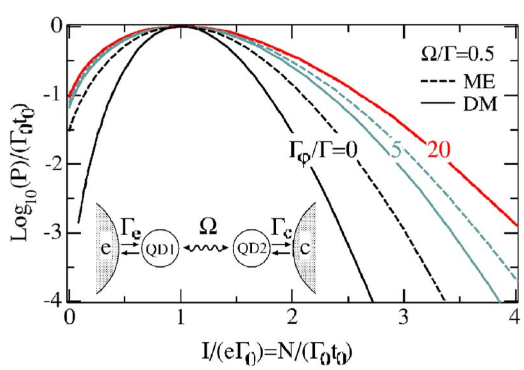

FIG. 1. (Color online) Current statistics for $\Omega/\Gamma=0.5$ and for various dephasing rates $\Gamma_\varphi/\Gamma=0$, 5, 20; dashed lines: master equation (ME) approach; solid lines: density matrix (DM) formalism; on-resonance $\Delta\varepsilon=0$; symmetric contact coupling: $\Gamma=\Gamma_e=\Gamma_c$. $\Gamma_0 \equiv (2\Gamma\Omega^2)/[4\Omega^2+\Gamma(\Gamma+\Gamma_\varphi)]$. Inset: Setup of the coupled QD system with (e)mitter and (c)ollector contact and mutual coupling Ω.

of large bias applied between the collector and emitter, with the broadened energy levels well inside the bias window. To compare DM/ME and scattering approaches we consider noninteracting electrons (spin degrees of freedom decouple, we give all results for a single spin direction) throughout this work. We note, however, that a strong Coulomb blockade can be treated within the DM/ME approaches along the same lines.

A. Coherent tunneling

The FCS for coherent tunneling through coupled QDs can be obtained from the approach developed by Gurvitz and coworkers in a series of papers[12,16] (for related work, see, e.g., Ref. 17). Starting from the time dependent Schrödinger equation one derives a modified Liouville equation, a system of coupled first-order differential equations for DM elements $\rho^N_{\alpha\beta}(t_0)$ at a given number N of electrons transferred through the QD system at time t_0. Here $\alpha,\beta\in\{a,b,c,d\}$, where a, b, c, and d denote the Fock states $|00\rangle$, $|10\rangle$, $|01\rangle$, $|11\rangle$ of the system, i.e., no electrons, one electron in the first dot, one in the second dot, and one in each dot, respectively. The probability distribution is then directly given by $P(N,t_0)=\rho^N_{aa}(t_0)+\rho^N_{bb}(t_0)+\rho^N_{cc}(t_0)+\rho^N_{dd}(t_0)$. The FCS is formally obtained by first Fourier transforming the DM elements as $\rho_{\alpha\beta}(\chi,t_0)=\Sigma_N\rho^N_{\alpha\beta}(t_0)e^{iN\chi}$. This gives the Fourier transformed equation $\dot{\rho}=\mathcal{L}_c(\chi)\rho$, with

$$\mathcal{L}_c(\chi)=\begin{pmatrix} -\Gamma_{\rm e} & 0 & \Gamma_{\rm c}e^{i\chi} & 0 & 0 & 0\\ \Gamma_{\rm e} & 0 & 0 & \Gamma_{\rm c}e^{i\chi} & 0 & 2\Omega\\ 0 & 0 & -2\Gamma & 0 & 0 & -2\Omega\\ 0 & 0 & \Gamma_{\rm e} & -\Gamma_{\rm c} & 0 & 0\\ 0 & 0 & 0 & 0 & -\Gamma & -\Delta\varepsilon\\ 0 & -\Omega & \Omega & 0 & \Delta\varepsilon & -\Gamma \end{pmatrix} \quad (2)$$

and $\rho\equiv(\rho_{aa},\rho_{bb},\rho_{cc},\rho_{dd},\mathrm{Re}[\rho_{bc}],\mathrm{Im}[\rho_{bc}])^T$, $\Gamma\equiv(\Gamma_{\rm e}+\Gamma_{\rm c})/2$, $\Delta\varepsilon\equiv\varepsilon_1-\varepsilon_2$.

Note that the counting field χ enters the matrix elements in (2), where an electron jumps from QD2 into the collector contact. The CGF is then obtained as the eigenvalue of $\mathcal{L}_c$ which goes to zero for $\chi=0$, as required by probability conservation [see Eq. (1)],

$$F_c(\chi)=\frac{t_0}{2}\Big[2\Gamma-\Big(p_1+2\sqrt{p_2^2+16\Gamma^2\Omega^2(e^{i\chi}-1)}\Big)^{1/2}\Big], \quad (3)$$

with $p_1=2(\Gamma^2-4\Omega^2+\Delta\varepsilon^2)$ and $p_2=\Gamma^2+4\Omega^2-\Delta\varepsilon^2$ for symmetric contact coupling $\Gamma_{\rm e}=\Gamma_{\rm c}=\Gamma$.

B. Sequential tunneling

For incoherent tunneling the FCS can be obtained along similar lines from a ME[10] for the diagonal elements of ρ as $\dot{\bar{\rho}}=\mathcal{L}_s\bar{\rho}$, with $\bar{\rho}=(\rho_{aa},\rho_{bb},\rho_{cc},\rho_{dd})$. The coefficient matrix is

$$\mathcal{L}_s(\chi)=\begin{pmatrix} -\Gamma_{\rm e} & 0 & \Gamma_{\rm c}e^{i\chi} & 0\\ \Gamma_{\rm e} & -Z & Z & \Gamma_{\rm c}e^{i\chi}\\ 0 & Z & -(2\Gamma+Z) & 0\\ 0 & 0 & \Gamma_{\rm e} & -\Gamma_{\rm c} \end{pmatrix}, \quad (4)$$

with the coupling between the single-particle states given by Fermi's golden rule: $Z\equiv(2|\Omega|^2/\Gamma)L(\Delta\varepsilon,2\Gamma)$ with the normalized Lorentzian $L(x,w)\equiv[1+(2x/w)^2]^{-1}$.[19] The CGF corresponds to the eigenvalue of the matrix (4) which goes to zero for $\chi=0$ and reads

$$F_s(\chi)=\frac{t_0}{6}[(1+i\sqrt{3})q_1+(1-i\sqrt{3})q_2+6\Gamma+4Z],$$

$$q_{1/2}=[-u\pm\sqrt{u^2-v^3}]^{1/3}, \quad (5)$$

with $u=8Z^3+9Z\Gamma^2(1-3e^{i\chi})$ and $v=4Z^2+3\Gamma^2$.

III. RESULTS

The probability distributions for coherent and incoherent tunneling obtained from the CGFs (3) and (5), respectively, in a saddle-point approximation are plotted in Fig. 1 for $\Omega/\Gamma=0.5$, where the effect of coherence is most pronounced. We see that the fluctuations are smaller in the coherent limit, i.e., decoherence generally enhances current fluctuations. In the limits of small interdot coupling $\Omega\ll\Gamma$ one obtains a Poissonian transfer of unit elementary charges and for large coupling $\Omega\gg\Gamma$ the FCS of a single QD is recovered.[9,10] In these limits the statistics for sequential and coherent tunneling are indistinguishable.

The CGF for coherent (3) and sequential (5) tunneling yield the same expression for the average current through the coupled QD system[18–20]

$$\langle I\rangle=e\left[\frac{1}{\Gamma_{\rm e}}+\frac{1}{\Gamma_{\rm c}}+\frac{1}{\Gamma_i}\right]^{-1}L\left(\Delta\varepsilon,2\Gamma\sqrt{1+\frac{4|\Omega|^2}{\Gamma_{\rm e}\Gamma_{\rm c}}}\right), \quad (6)$$

with $\Gamma_i\equiv2\Omega^2/\Gamma$. The higher order cumulants C_k with $k\geq2$ deviate for intermediate Ω reflecting their sensitivity to quantum coherence in the transport process. For $\Gamma=\Gamma_e=\Gamma_c$ and $\Delta\varepsilon=0$ we have the Fano factors[12,13]

$$\frac{S_c}{2e\langle I\rangle}=\frac{\Gamma^4-2\Gamma^2\Omega^2+8\Omega^4}{(\Gamma^2+4\Omega^2)^2} \quad (7)$$

for the coherent case and

$$\frac{S_s}{2e\langle I\rangle}=\frac{\Gamma^4+2\Gamma^2\Omega^2+8\Omega^4}{(\Gamma^2+4\Omega^2)^2} \quad (8)$$

for the sequential, incoherent case. Clearly, coherence suppresses the noise.[13,14] The noise and the Fano factors are shown in Fig. 2 (results for $\Gamma_\varphi=0$). The noise for coherent tunneling shows a local minimum at $2\Omega=\Gamma$. At this coupling the normalized skewness has a local maximum, as can be seen in Fig. 2, and a close inspection reveals a FCS identical to a Poissonian transfer of quarter elementary charges: $F(\chi)=t_0\Gamma(e^{i\chi/4}-1)$.

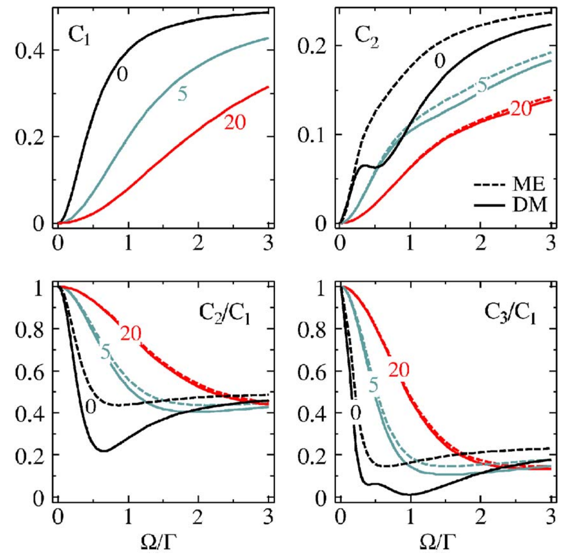

FIG. 2. (Color online) Average current C_1, noise C_2 in units of $t_0\Gamma$, Fano factor C_2/C_1, normalized skewness C_3/C_1 vs coupling Ω for various dephasing rates $\Gamma_\varphi/\Gamma=0$, 5, 20; master equation approach (ME): dashed lines; density matrix formalism (DM): solid lines; on-resonance: $\Delta\varepsilon=0$; symmetric contact coupling: $\Gamma=\Gamma_e=\Gamma_c$.

A. Decoherence—charge detector

In order to connect the limits of coherent and incoherent charge transport through the QD system we consider the exponential damping of the off-diagonal elements in the modified Liouville equation with rate Γ_φ: i.e., in the last two rows of the coefficient matrix (2), Γ is replaced by $\Gamma+\Gamma_\varphi$. This apparent phenomenological treatment of decoherence can be substantiated, e.g., by the introduction of a quantum point contact close to one of the QDs: whenever an electron enters the QD the transmission through the quantum point contact changes. This charge detection leads to the exponential damping of the off-diagonals, as microscopically derived in Ref. 21. Due to the finite coupling Ω, it also leads to an exponential relaxation of the diagonal density matrix elements. Its effect on the FCS is presented in Fig. 1 and its effect on the current and noise in Fig. 2.

For comparison with the sequential tunneling cumulants, the broadening of the resonance due to the coupling to the quantum point contact has to be considered and therefore the replacement $\Gamma\rightarrow\Gamma+\Gamma_\varphi$ in Z of the coefficient matrix (4) is carried out. Then, the currents C_1 in both treatments agree for any Γ_φ (Fig. 2). The higher order cumulants merge for $\Gamma_\varphi \gg \Omega$ as shown for the noise C_2, the Fano factor C_2/C_1, and for the normalized skewness C_3/C_1 in Fig. 2.

B. Decoherence—voltage probe model

The coherent FCS in Eq. (3) can also be obtained from the scattering formula of Levitov and coworkers,[3] $F(\chi) =(t_0/\hbar)\int d\varepsilon \ln[1+T(\varepsilon)(e^{i\chi}-1)]$, where $T(\varepsilon)$ is the transmission probability through the QD system (see, e.g., Ref. 12). This makes it interesting to compare dephasing within the DM approach with dephasing in a scattering formalism. This

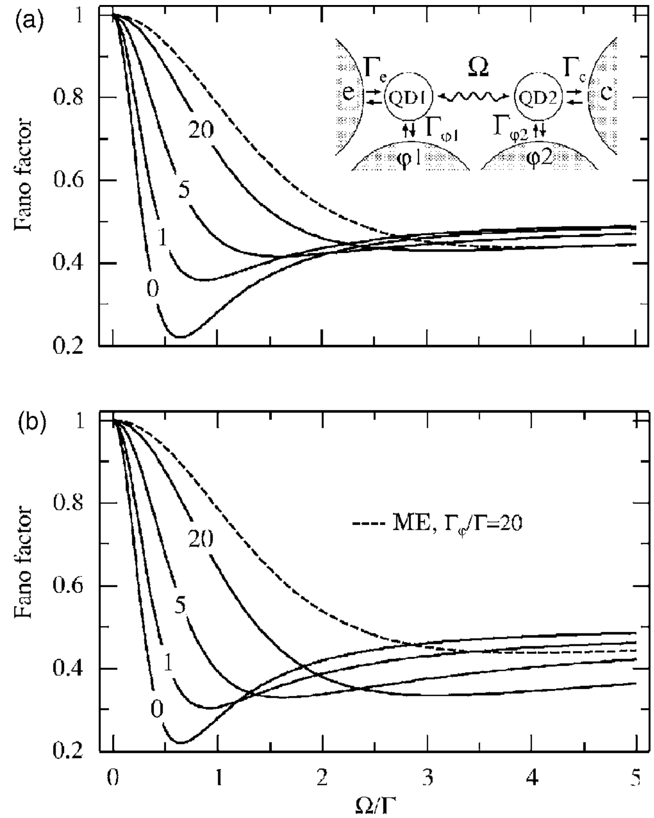

FIG. 3. Fano factor vs inter-QD coupling Ω for various dephasing rates Γ_φ. (a) elastic voltage probe, (b) inelastic voltage probe in scattering formalism (solid curves); dashed curves: master equation (ME) Fano factor for $\Gamma_\varphi/\Gamma=20$; on-resonance: $\Delta\varepsilon=0$; symmetric coupling: $\Gamma=\Gamma_e=\Gamma_c$.

is done by introducing phenomenological voltage probes[1] coupled with strength $\Gamma_\varphi=\Gamma_{\varphi 1}=\Gamma_{\varphi 2}$ to the QDs [see inset of Fig. 3(a)]. The probes absorb and subsequently re-emit electrons, thereby randomizing their phases. Here we focus on the current and the noise; higher cumulants can be investigated with a modified version of the stochastic path-integral technique in Ref. 22, but this is beyond the scope of the present paper.

The scattering matrix $\mathbf{s}$ for the four-terminal QD-probe system is given by

$$\mathbf{s}=1-iW^T G W, \quad G=[\varepsilon-H+iWW^T]^{-1},$$

$$H=\begin{pmatrix}\varepsilon_1 & \Omega \\ \Omega & \varepsilon_2\end{pmatrix}, \quad W=\begin{pmatrix}\sqrt{\Gamma_\varphi} & \sqrt{\Gamma_c} & 0 & 0 \\ 0 & 0 & \sqrt{\Gamma_\varphi} & \sqrt{\Gamma_e}\end{pmatrix}. \tag{9}$$

The average current in lead $\alpha=e,c,\varphi_1,\varphi_2$ is given by[1,23]

$$\langle I_\alpha\rangle=\frac{e}{h}\sum_\beta \int d\varepsilon A^{\alpha}_{\beta\beta}(\varepsilon) f_\beta(\varepsilon), \tag{10}$$

with $A^{\alpha}_{\beta\gamma}(\varepsilon)=\delta_{\alpha\beta}\delta_{\alpha\gamma}-s^{\dagger}_{\alpha\beta}(\varepsilon)s_{\alpha\gamma}(\varepsilon)$ and the distribution function $f_\alpha(\varepsilon)$ of terminal α. The zero-frequency noise between terminal α and β reads[1,23]

$$S_{\alpha\beta} = \frac{2e^2}{h} \sum_{\gamma\delta} \int d\varepsilon A^{\alpha}_{\gamma\delta}(\varepsilon) A^{\beta}_{\delta\gamma}(\varepsilon) f_{\gamma}(\varepsilon)[1 - f_{\delta}(\varepsilon)]. \quad (11)$$

We first consider an elastic, purely dephasing voltage probe,[24] where the average current as well as the low-frequency current fluctuations into the probe is zero at each energy. The conservation of average current gives the average distribution functions $f_{\varphi 1/\varphi 2}$. From the conservation of the current fluctuations one obtains the fluctuating part of the distribution functions $\delta f_{\varphi 1/\varphi 2}$ in terms of the bare current fluctuations.[1] The total noise is then obtained as a weighted sum of the bare current correlations in Eq. (11). It is found that both current and noise qualitatively reproduce the DM result. The Fano factor is plotted in Fig. 3(a); however, there is a quantitative difference. Since in the DM approach, the electrons in the dots can exchange energy with electrons at the quantum point contacts, the dephasing is inelastic and a quantitative agreement with an elastic scattering dephasing approach is not to be expected.

To account for inelastic dephasing we next consider inelastic voltage probes which conserve only total, energy-integrated current and fluctuations. Trying to mimic the effect of the point contacts in the DM approach, we assume the distribution functions in the probes to be constant, independent of energy in the entire bias window. The average current and noise are then obtained along the same lines as for the purely dephasing probe. We find that the average current coincides with the DM result; the noise, however, again differs quantitatively but not qualitatively. The Fano factor is plotted in Fig. 3(b). We thus conclude that in double QD systems, dephasing in a scattering and a DM approach yield qualitatively similar but in general quantitatively different results.

IV. CONCLUSIONS

Within density matrix and master equation approaches, we have examined the FCS for coherent and sequential charge transport through coupled QDs. While the average currents in the two cases coincide, all higher cumulants differ, clearly demonstrating the sensitivity of the charge transport to quantum coherence which generally suppresses the fluctuations. Coupling the QDs to a charge detector introduces decoherence, which results in a continuous transition from coherent to sequential tunneling. A scattering approach, where decoherence is introduced via phenomenological voltage probes, gives qualitatively similar results.

ACKNOWLEDGMENTS

We acknowledge helpful discussions with S. Pilgram and M. Büttiker. This work was supported by Deutsche Forschungsgemeinschaft in the framework of Sfb 296 and the Swedish Research Council.

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