Out-of-plane fluctuation conductivity of layered superconductors in strong electric fields

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The non-Ohmic effect of a high electric field on the out-of-plane magnetoconductivity of a layered superconductor near the superconducting transition is studied in the frame of the Langevin approach to the time-dependent Ginzburg-Landau equation. The transverse fluctuation conductivity is computed in the self-consistent Hartree approximation for an arbitrarily strong electric field and a magnetic field perpendicular to the layers. Our results indicate that high electric fields can be effectively used to suppress the out-of-plane fluctuation conductivity in high-temperature superconductors and a significant broadening of the transition induced by a strong electric field is predicted. Extensions of the results are provided for the case when the electric field is applied at an arbitrary angle with respect to the layers, as well as for the three-dimensional anisotropic regime of a strong interlayer coupling.

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I. INTRODUCTION

Outside the critical region above T_c, in the absence of magnetic field and for small electric fields, the excess conductivity due to fluctuations of the superconducting order parameter can be explained by the Aslamazov-Larkin[1] theory, subsequently extended by Lawrence and Doniach[2] for two-dimensional layered superconductors, a situation very much resembling the crystal structure in high-temperature superconductors (HTSC). In the presence of a magnetic field, the fluctuation transport properties of superconductors were initially treated[3–6] in the noninteracting (Gaussian) fluctuation approach, which predicted a divergence at $T_c(H)$ that is, however, not observed. The physical reason is the motion of vortices providing dissipation and hence a finite flux-flow conductivity. Ikeda *et al.*[7] and Ullah and Dorsey[8] showed that the theoretical divergence can be eliminated by using the Hartree approximation, which treats self-consistently the quartic term in the Ginzburg-Landau (GL) free-energy expansion. This approach was applied for the longitudinal[7,8] and Hall conductivity,[8,9] as well as for the out-of-plane conductivity,[10] in the linear-response approximation for a layered superconductor under perpendicular magnetic field.

The fluctuation transport properties can be calculated in the linear-response approximation only for sufficiently weak electric fields that do not perturb the fluctuation spectrum.[11] At reasonably high values of the electric field, the acceleration of the paired electrons is so large, that on a distance of the order of the coherence length they change their energy by a value corresponding to the fluctuation Cooper pair binding energy.[12] This results in an additional, field dependent, decay mechanism, and leads to deviation of the current-voltage characteristics from the Ohm's law. In connection with the low-temperature superconductors, the non-Ohmic fluctuation conductivity in the absence of magnetic field has been studied theoretically for the isotropic case[13,14] and found experimentally on thin aluminum films.[15,16] For a layered superconductor the issue has been more recently addressed for the in-plane conductivity, starting from a microscopic approach[17] and subsequently in the frame of the time-dependent Ginzburg-Landau (TDGL) theory, in the Gaussian[18] as well as in the self-consistent Hartree approximation.[19] Several experimental investigations of the fluctuation suppression effect of high electric fields in HTSC were performed for the in-plane paraconductivity in zero magnetic field,[20–24] and a good agreement with the theoretical models[17,19] was proven.

The nonlinear effect of a strong electric field under the simultaneous application of a perpendicular magnetic field on the in-plane fluctuation conductivity and Hall effect was recently addressed by the authors of the present paper,[25,26] for a layered superconductor in the Hartree approximation of the TDGL theory. It has been revealed that the simultaneous application of the two fields results in a slightly stronger suppression of the superconducting fluctuation conductivity, compared to the case when the fields are applied individually, while the relative suppression of the excess Hall conductivity turns out to be stronger than for the longitudinal one. Experimental investigations of the fluctuation suppression effect of strong electric fields under simultaneous application of a magnetic field are however lacking so far.

Also the *out-of-plane* conductance of a layered superconductor in the non-Ohmic regime of high electric fields has been, to our knowledge, neither experimentally nor theoretically investigated up to the present. The purpose of this paper is to provide a theoretical approach to this issue, in the frame of the TDGL equation solved in the Hartree approximation, when both the magnetic and the electric field are applied perpendicular to the layers. We shall thus be able to find the expression of the Aslamazov-Larkin (AL) contribution to the out-of-plane fluctuation conductivity at any values of the magnetic and electric fields, and predict for the AL term a significant supplementary suppression induced by a strong electric field in layered HTSC. Based on the proportionality between the Cooper pair concentration and the density-of-states (DOS) part of the out-of-plane conductivity, known from the microscopic theory in the linear response approximation,[6,12] we shall also give an estimate of the non-Ohmic effect on the DOS contribution. The work will be completed by extending the calculation of the non-Ohmic fluctuation conductivity to the case when the electric field is applied at an arbitrary angle with respect to the layers. This calculation, particularly useful for investigations on vicinal thin films,[27] is necessary because the non-Ohmic current produced by a tilted electric field cannot be calculated as the superposition of the non-Ohmic currents produced separately by the in-plane and out-of-plane field components, as it was the case in the linear response approximation. Eventually, the results of the paper will be extended also to the three-dimensional (3D) anisotropic regime of a strong interlayer coupling.

II. NON-OHMIC OUT-OF-PLANE FLUCTUATION CONDUCTIVITY

In order to calculate the non-Ohmic out-of-plane conductivity under the presence of a magnetic field perpendicular to the layers, we shall adopt the Langevin approach to the gauge-invariant relaxational TDGL equation[8,13] governing the critical dynamics of the superconducting order parameter in the lth superconducting plane:

$$\Gamma_0^{-1}\left(\frac{\partial}{\partial t} - i\frac{e_0 sl}{\hbar}E\right)\psi_l + a\psi_l + b|\psi_l|^2\psi_l - \frac{\hbar^2}{2m}\left[\partial_x^2 + \left(\partial_y - \frac{ie_0}{\hbar}xB\right)^2\right]\psi_l + \frac{\hbar^2}{2m_c s^2}(2\psi_l - \psi_{l+1} - \psi_{l-1}) = \zeta_l(x,y,t), \quad (1)$$

where m and m_c are effective Cooper pair masses in the ab plane and along the c axis, s is the distance between superconducting planes, and the pair electric charge is $e_0=-2e$. The order parameter ψ_l has the same physical dimension as in the three-dimensional case, and SI units are used. The GL potential $a=a_0\varepsilon$ is parametrized by $a_0=\hbar^2/2m\xi_0^2 =\hbar^2/2m_c\xi_{0c}^2$ and $\varepsilon=\ln(T/T_0)$, with T_0 being the mean-field transition temperature, while ξ_0 and ξ_{0c} are, respectively, the in-plane and out-of-plane coherence lengths extrapolated to $T=0$. The order parameter relaxation time Γ_0^{-1} is given by[28,29] $\Gamma_0^{-1}=\pi\hbar^3/16m\xi_0^2k_BT$. The magnetic field $\mathbf{B}$, perpendicular to the layers, is generated by the vector potential in the Landau gauge $\mathbf{A}=(0,xB,0)$, with x and y the in-plane coordinates. Since we are interested in the out-of-plane conductivity, we consider the electric field $\mathbf{E}$ as being applied along the z axis, and generated by the scalar potential $\varphi_l =-Esl$. The Langevin white-noise forces $\zeta_l(x,y,t)$ that describe the thermodynamical fluctuations must satisfy the fluctuation-dissipation theorem, $\langle\zeta_l(x,y,t)\zeta_{l'}^*(x',y',t')\rangle =2\Gamma_0^{-1}k_BT\delta(x-x')\delta(y-y')\delta(t-t')\delta_{ll'}/s$.

The quartic term in the thermodynamical potential will be treated in the Hartree approximation,[8,30] which results in a linear problem with a modified (renormalized) reduced temperature

$$\tilde{\varepsilon} = \varepsilon + b\langle|\psi_l|^2\rangle/a_0. \quad (2)$$

The out-of-plane fluctuation conductivity $\Delta\sigma_{zz}$ will be eventually found by calculating the Josephson current density between the lth and $(l+1)$th layers, which in the chosen gauge writes

$$\langle\Delta j_z^{(l)}\rangle = -\frac{i\hbar e_0}{2m_c s}[\langle\psi_l^*\psi_{l+1}\rangle - \langle\psi_l\psi_{l+1}^*\rangle], \quad (3)$$

and further

$$\Delta\sigma_{zz} = \langle\Delta j_z^{(l)}\rangle/E. \quad (4)$$

It is worth mentioning, at this point, that a slight modification of the GL free-energy functional for the layered superconductors was recently proposed,[31] which, besides the BCS-type Josephson coupling, allows for additional interlayer interactions that can contribute to the condensation energy and give rise to energy savings that enhance T_0. The fluctuation spectrum of the proposed functional and consequently the fluctuation-induced observables like in-plane paraconductivity and magnetoconductivity were found to be the same as for the usual Lawrence-Doniach free-energy, once the transition temperature of each bare layer is renormalized to its value enhanced through the interlayer energy savings.[31] It can be easily verified that this fact is valid also for the out-of-plane fluctuation conductivity, since the additional terms in the GL functional of Ref. 31 do not involve the phase of the superconducting parameter, and consequently do not influence the definition (3) of the transversal current density. The results of the present paper will be thus applicable also for the kind of interlayer coupling proposed in Ref. 31.

We proceed further by introducing the Fourier transform with respect to the in-plane coordinate y, the layer index l, and also the Landau level (LL) representation with respect to the x-dependence, through the relation

$$\psi_l(x,y,t) = \int\frac{dk}{2\pi}\int_{-\pi/s}^{\pi/s}\frac{dq}{2\pi}\sum_{n\geq 0}\psi_q(n,k,t)e^{-iky}e^{-iqls} \times u_n\left(x - \frac{\hbar k}{2eB}\right), \quad (5)$$

where the functions $u_n(x)$ with $n\in\mathbb{N}$ build the orthonormal eigenfunction system of the harmonic oscillator Hamiltonian, so that $(-\hbar^2\partial_x^2+4e^2B^2x^2)u_n(x)=2\hbar eB(2n+1)u_n(x)$. The TDGL equation (1) will write in the new variables:

$$\left[\Gamma_0^{-1}\frac{\partial}{\partial t} + \chi\frac{\partial}{\partial q} + a_0\tilde{\varepsilon}_n + a_0\frac{r}{2}(1-\cos qs)\right]\psi_n(k,q,t) = \zeta_n(k,q,t), \quad (6)$$

where the new noise terms $\zeta_n(k,q,t)$ are delta-correlated as

$$\langle\zeta_n(k,q,t)\zeta_{n'}^*(k',q',t')\rangle = 2\Gamma_0^{-1}k_BT(2\pi)^2\delta(k-k') \times\delta(q-q')\delta(t-t')\delta_{nn'}.$$

We have also introduced the notations:

$$\tilde{\varepsilon}_n = \tilde{\varepsilon} + (2n+1)h, \quad h = \frac{B}{B_{c2}(0)} = \frac{2e\xi_0^2 B}{\hbar},$$

$$r=\left(\frac{2\xi_{0c}}{s}\right)^2,\quad \chi=\frac{2eE}{\hbar\Gamma_0}, \tag{7}$$

with h denoting the reduced magnetic field and r the anisotropy parameter.

Equation (6) can be solved with the aid of the Green function technique and has the solution

$$\psi_n(k,q,t)=\Gamma_0\int_0^\infty d\tau\zeta_n\left(k,q-\frac{2eE}{\hbar}\tau,t-\tau\right)\times\exp\left\{-\Gamma_0 a_0\left[\left(\tilde{\varepsilon}_n+\frac{r}{2}\right)\tau-\frac{r\hbar}{4seE}\left(\sin qs-\sin\left(q-\frac{2eE}{\hbar}\tau\right)s\right)\right]\right\}, \tag{8}$$

so that the correlation function between the order parameter in two layers l and l' will be given by

$$\langle\psi_l(\mathbf{x},t)\psi_{l'}^*(\mathbf{x},t)\rangle=\frac{k_BT}{2\pi a_0\xi_0^2}h\int_0^\infty du\sum_{n=0}^{N_c}\int_{-\pi/s}^{\pi/s}\frac{dq}{2\pi}e^{-iqs(l-l')}\times\exp\left[-u\left(\tilde{\varepsilon}_n+\frac{r}{2}\right)+\frac{r}{2}\frac{\sin pu}{p}\cos(qs-pu)\right], \tag{9}$$

where the electric field enters the parameter

$$p=\frac{\pi es}{16k_BT}E=\frac{s\sqrt{3}}{\xi_0}\frac{E}{E_0}, \tag{10}$$

with $E_0=16\sqrt{3}k_BT/\pi e\xi_0$ being the characteristic electric field defined as in Refs. 17 and 18.

We point out that the sum over the LL in Eq. (9) must be cut off at some index N_c, reflecting the inherent UV divergence of the Ginzburg-Landau theory. The classical[13,30] procedure is to suppress the short wavelength fluctuating modes through a *momentum* (or, equivalently, *kinetic energy*) *cutoff* condition, which, in terms of the LL representation writes[8,30] $(2e\hbar B/m)\left(n+\frac{1}{2}\right)\leqslant ca_0=c\hbar^2/2m\xi_0^2$, with the cutoff parameter c of the order of unity. A *total energy cutoff* was also recently proposed,[32] whose physical meaning was shown to follow from the uncertainty principle, and whose importance is revealed especially at high reduced temperatures and magnetic fields close to $B_{c2}(0)$.[33] Formally, the total energy cutoff can be obtained from the momentum cutoff by replacing c with $c-\varepsilon$. However, for low magnetic fields with respect to $B_{c2}(0)$ and in the critical fluctuation region, the two cutoff conditions almost coincide quantitatively, so that we shall apply for simplicity the momentum cut-off procedure. In terms of the reduced magnetic field h, it writes thus $h\left(N_c+\frac{1}{2}\right)=c/2$.

Now we are able to apply expression (9) in Eqs. (2) and (3) in order to write, respectively, the self-consistent Hartree equation and the out-of-plane current density. Thus, after performing the q integral in the correlation function (9) taken for $l=l'$, one obtains the renormalizing equation for the reduced temperature $\tilde{\varepsilon}$,

$$\tilde{\varepsilon}=\ln\frac{T}{T_0}+2gTh\sum_{n=0}^{N_c}\int_0^\infty du\, e^{-u(\tilde{\varepsilon}_n+r/2)}I_0\left(\frac{r}{2p}\sin(pu)\right), \tag{11}$$

where $I_0(x)$ is the modified Bessel function and the parameter

$$g=\frac{2\mu_0\kappa_{\mathrm{GL}}^2e^2\xi_0^2k_B}{\pi\hbar^2 s} \tag{12}$$

was introduced according to the expression of the quartic term coefficient[8] $b=\mu_0\kappa_{\mathrm{GL}}^2e_0^2\hbar^2/2m^2$, with κ_{GL} being the in-plane Ginzburg-Landau parameter $\kappa_{\mathrm{GL}}=\lambda_0/\xi_0$.

In an analogous manner, after computing the correlation function (9) for $l'=l+1$, and using the current density definition (3), one can eventually obtain the out-of-plane fluctuation conductivity under arbitrary magnetic and electric fields,

$$\Delta\sigma_{zz}^{\mathrm{AL}}(E,B)=\frac{e^2srh}{32\hbar\xi_0^2}\sum_{n=0}^{N_c}\int_0^\infty du\, e^{-u(\tilde{\varepsilon}_n+r/2)}\frac{\sin(pu)}{p}\times I_1\left(\frac{r}{2p}\sin(pu)\right), \tag{13}$$

with $I_1(x)$ the modified Bessel function of first order. The integrals in Eqs. (11) and (13) are convergent provided $\tilde{\varepsilon}+r/2+h>0$, so that $\tilde{\varepsilon}_n+r/2>0$ for any LL index n. This condition is however assured while solving Eq. (11) for the parameter $\tilde{\varepsilon}$ at any temperature T.

It is useful to write Eq. (13) also when the cutoff is neglected, i.e., for c, $N_c\to\infty$:

$$\Delta\sigma_{zz}^{\mathrm{AL-no\ cut}}(E,B)=\frac{e^2sr}{64\hbar\xi_0^2}\int_0^\infty du\frac{2hue^{-uh}}{1-\exp(-2hu)}\times e^{-u(\tilde{\varepsilon}+r/2)}\frac{\sin(pu)}{pu}I_1\left(\frac{r}{2p}\sin(pu)\right). \tag{14}$$

This slightly simpler formula provides a good approximation for Eq. (13) in the temperature region close to the transition (where $\tilde{\varepsilon}\ll c$) and for small magnetic fields [for which $N_c=(c-h)/2h$ is already high]. The cutoff procedure remains however essential for calculating the Cooper pairs density $\langle|\psi_l(\mathbf{x},t)|^2\rangle$ contained in the renormalization equation (11), since the u integral would be divergent for c, $N_c\to\infty$. As we shall see in Sec. VII, Eq. (14) can also be directly transformed in order to find the three dimensional limit (i.e., for $s\to 0$) of the non-Ohmic fluctuation conductivity $\Delta\sigma_{zz}^{\mathrm{AL}}|_{\mathrm{no\ cut}}^{(3D)}$ parallel to the magnetic field, when the cutoff is neglected.

In Eq. (13) we have explicitly specified that the out-of-plane fluctuation conductivity $\Delta\sigma_{zz}^{\mathrm{AL}}$ corresponds to the Aslamazov-Larkin (AL) fluctuation process, since the phenomenological Ginzburg-Landau theory cannot account for indirect contributions like the density-of-states (DOS) and Maki-Thompson (MT) terms, which can be found only from a microscopical approach. However, whereas the DOS and MT contributions to the in-plane paraconductivity and Hall effect coefficient are known to be negligible near the superconducting transition with respect to the AL term due to the more singular behavior of the latter, an investigation of the out-of-plane conductivity needs taking into account also the DOS term, which can compete with the AL one especially for highly anisotropic materials, as pointed out by Larkin and Varlamov.[12] The reason is that the DOS contribution to the out-of-plane fluctuation conductivity turns out to be proportional to a lower order of the interlayer transparency than the AL one, as shown by the microscopical approach in the linear response approximation.[6,12] We shall tentatively give in Sec. IV an estimate of the DOS contribution to $\Delta\sigma_{zz}$ also for an arbitrarily strong electric field, after discussing the limit values of expressions (11) and (13) in the cases of a vanishing magnetic or electric field.

III. LIMIT CASES $B\to 0$ AND/OR $E\to 0$

The Ohmic out-of-plane fluctuation conductivity in the presence of a magnetic field but in an infinitesimally small electric field can be easily obtained by taking the limit $p\to 0$ in Eq. (13), which acquires thus the form

$$\Delta\sigma_{zz}^{\mathrm{AL}}(B)|_{E\to 0}=\frac{e^2sr^2}{64\hbar\xi_0^2}h\sum_{n=0}^{N_c}[\tilde{\varepsilon}_n(\tilde{\varepsilon}_n+r)]^{-3/2}. \quad (15)$$

The expression (15) thus matches the result previously obtained within the diagrammatic microscopic approach, for Gaussian fluctuations (i.e., with $\tilde{\varepsilon}=\varepsilon$), in the linear response approximation.[6] Analogously, the self-consistent equation (11) becomes, in the same limit,

$$\tilde{\varepsilon}|_{E\to 0,B>0}=\ln\frac{T}{T_0}+2gTh\sum_{n=0}^{N_c}[\tilde{\varepsilon}_n(\tilde{\varepsilon}_n+r)]^{-1/2}, \quad (16)$$

which represents the Hartree renormalization equation in the linear response approximation under an applied magnetic field found in Ref. 8.

The other limit case, namely for vanishing magnetic field but under a finite applied electric field, needs taking the limit $h\to 0$ in Eqs. (11) and (13) after performing the sum over Landau levels and taking into account the cutoff condition, so that the Hartree self-consistent renormalization (11) and the out-of-plane fluctuation conductivity (13) become, respectively,

$$\tilde{\varepsilon}|_{E>0,B=0}=\ln\frac{T}{T_0}+gT\int_0^\infty du\frac{1-e^{-cu}}{u}e^{-u(\tilde{\varepsilon}+r/2)}I_0\left(\frac{r}{2p}\sin(pu)\right), \quad (17)$$

$$\Delta\sigma_{zz}^{\mathrm{AL}}|_{E>0,B=0}=\frac{e^2sr}{64\hbar\xi_0^2}\int_0^\infty du(1-e^{-cu})e^{-u(\tilde{\varepsilon}+r/2)}\frac{\sin(pu)}{pu}\times I_1\left(\frac{r}{2p}\sin(pu)\right). \quad (18)$$

It can be easily verified, by using the integral identities of the Bessel functions I_0 and I_1, that in the further limit $E\to 0$, Eq. (17) becomes

$$\tilde{\varepsilon}|_{E=0,B=0}=\ln\frac{T}{T_0}+2gT\ln\frac{\sqrt{\tilde{\varepsilon}+c}+\sqrt{\tilde{\varepsilon}+c+r}}{\sqrt{\tilde{\varepsilon}}+\sqrt{\tilde{\varepsilon}+r}}, \quad (19)$$

as also found in Ref. 30, while expression (18) takes, respectively, the form

$$\Delta\sigma_{zz}^{\mathrm{AL}}|_{E=0,B=0}=\frac{e^2s}{32\hbar\xi_0^2}\left[\frac{\tilde{\varepsilon}+\frac{r}{2}}{\sqrt{\tilde{\varepsilon}(\tilde{\varepsilon}+r)}}-\frac{\tilde{\varepsilon}+c+\frac{r}{2}}{\sqrt{(\tilde{\varepsilon}+c)(\tilde{\varepsilon}+c+r)}}\right]. \quad (20)$$

If one neglects the cutoff procedure (i.e., $c\to\infty$), expression (20) matches the result obtained in Ref. 12 for the AL out-of-plane fluctuation conductivity in the linear response limit and in the absence of magnetic field, with the difference that in Ref. 12, based on the Gaussian approximation, the reduced temperature $\varepsilon=\ln(T/T_0)$ is present instead of our Hartree renormalized $\tilde{\varepsilon}$.

The Hartree renormalization procedure consists in using the reduced temperature parameter $\tilde{\varepsilon}$ instead of $\varepsilon=\ln(T/T_0)$, by solving Eq. (19). This procedure causes the critical temperature to shift downwards with respect to the bare mean-field transition temperature T_0. In analogy with the Gaussian fluctuation case, we shall adopt as definition for the critical temperature the vanishing of the reduced temperature, $\tilde{\varepsilon}=0$, where the fluctuation conductivity, given by Eq. (20), diverges. In practice, one knows experimentally the actual critical temperature T_{c0} measured at very low electrical field and with zero magnetic field, so that the relationship between T_{c0} and T_0 will be found by putting $\tilde{\varepsilon}=0$ in Eq. (19). It writes[19]

$$T_0=T_{c0}[\sqrt{c/r}+\sqrt{1+(c/r)}]^{2gT_{c0}}. \quad (21)$$

Now, having T_0 one can use Eq. (11) for any temperature T and fields E and B in order to find the actual renormalized $\tilde{\varepsilon}(T,E,B)$.

IV. ESTIMATION OF THE NON-OHMIC DOS CONTRIBUTION

In the microscopic approach of Ref. 6, valid in the linear response approximation (i.e., for vanishing electric field) and for noninteracting (Gaussian) fluctuations, the DOS contribution to the out-of-plane fluctuation conductivity under a magnetic field is found to amount, in our notations, to

$$\Delta\sigma_{zz}^{\mathrm{DOS}}|_{E\to 0,B>0}^{\mathrm{Gaussian}}=-\frac{e^2s\kappa r}{8\hbar\xi_0^2}h\sum_{n=0}^{N_c}[\tilde{\varepsilon}_n(\tilde{\varepsilon}_n+r)]^{-1/2}, \quad (22)$$

where the parameter κ depends on the impurity scattering time τ and temperature:[6]

$$\kappa=\frac{1}{\pi^2}\frac{-\psi'\left(\frac{1}{2}+\frac{1}{4\pi}\frac{\hbar}{\tau k_BT}\right)+\frac{1}{2\pi}\frac{\hbar}{\tau k_BT}\psi''\left(\frac{1}{2}\right)}{\psi\left(\frac{1}{2}+\frac{1}{4\pi}\frac{\hbar}{\tau k_BT}\right)-\psi\left(\frac{1}{2}\right)-\frac{1}{4\pi}\frac{\hbar}{\tau k_BT}\psi'\left(\frac{1}{2}\right)}, \tag{23}$$

with $\psi(x)$ the Euler digamma function. One can notice that Eq. (22) can be written as

$$\Delta\sigma_{zz}^{\mathrm{DOS}}|_{E\to 0,B>0}^{\mathrm{Gaussian}}=-\frac{e^2\pi\hbar\kappa}{2m_ck_BT}\langle|\psi|^2\rangle|_{E\to 0,B>0}^{\mathrm{Gaussian}}, \tag{24}$$

where

$$\langle|\psi|^2\rangle|_{E\to 0,B>0}^{\mathrm{Gaussian}}=\frac{mk_BT}{\pi\hbar^2 s}h\sum_{n=0}^{N_c}[\tilde{\varepsilon}_n(\tilde{\varepsilon}_n+r)]^{-1/2} \tag{25}$$

is the Coooper pairs density for vanishing electric field and in the Gaussian approximation (i.e., with $\tilde{\varepsilon}=\varepsilon$), as one can infer from the general correlation function (9) in the $E\to 0$ limit. The proportionality between the DOS fluctuation conductivity and the Cooper pair concentration in Eq. (24) is qualitatively easy to be grasped, since the DOS contribution means in fact the reduction of the normal state conductivity due to the decrease of the one-electron density of states, which reduction is in turn proportional to the superfluid density.[12]

We may assume that the proportionality (24) will hold also in the case of an arbitrarily strong electric field and in the Hartree approximation, so that the DOS contribution to the out-of-plane fluctuation conductivity can be generally written

$$\Delta\sigma_{zz}^{\mathrm{DOS}}(E,B)=-\frac{e^2s\kappa r}{8\hbar\xi_0^2}h\sum_{n=0}^{N_c}\int_0^\infty du\, e^{-u(\tilde{\varepsilon}_n+r/2)}I_0\left(\frac{r}{2p}\sin(pu)\right), \tag{26}$$

where Eqs. (2) and (11) are to be compared in order to reveal the Cooper pair density $\langle|\psi|^2\rangle$. Analogous with Eq. (18) we can consequently write the DOS contribution also in the vanishing magnetic field limit,

$$\Delta\sigma_{zz}^{\mathrm{DOS}}|_{E>0,B=0}=-\frac{e^2s\kappa r}{16\hbar\xi_0^2}\int_0^\infty du\frac{1-e^{-cu}}{u}e^{-u(\tilde{\varepsilon}+r/2)}\times I_0\left(\frac{r}{2p}\sin(pu)\right). \tag{27}$$

Equations (26) and (27) remain however to be confirmed or refuted by a microscopic approach.

V. EXAMPLE: OPTIMALLY DOPED $YBa_2Cu_3O_{6+x}$

In order to illustrate the main features of our model, we take as example a common HTSC material, like the optimally doped $YBa_2Cu_3O_{6+x}$ (YBCO). Typical characteristic parameters are then: $s=1.17$ nm for the interlayer distance, $\xi_0=1.2$ nm and $\xi_{0c}=0.14$ nm for the zero-temperature-extrapolated in-plane and out-of-plane coherence lengths, respectively, $\kappa_{\mathrm{GL}}=70$ for the Ginzburg-Landau parameter, $T_{c0}=92$ K for the critical temperature under very small electric and zero magnetic field, and the parameter $\kappa=3.57$ that corresponds to a scattering time $\tau\simeq 30$ fs in Eq. (23). It must be stated that the form of the normal-state background chosen for the temperature region masked by the onset of the superconductivity can be crucial for an eventual comparison with the experiment. There is however no consensus whether the peculiarities of the out-of-plane resistivity, namely its peak and its non-metallic character just above T_{c0}, as observed in the oxygen-deficient YBCO and the more anisotropic $Bi_2Sr_2CaCu_2O_{8+x}$, are mainly due to the competition between the fluctuation AL and DOS contributions, as illustrated in Ref. 10 by succesfull fits at different magnetic fields in the Ohmic regime while assuming a metallic linear extrapolation for the normal-state resistivity, or to the inherent behavior of the normal-state itself, as suggested by analysis of conductivity in incoherent layered crystals.[34] Since in this paper we focus on the fluctuation conductivity in the non-Ohmic regime and need the normal-state background only for illustration purposes of the resistivity characteristics, we shall further assume, for simplicity, an out-of-plane normal state resistivity almost constant near the transition, with a typical value $\rho_c^{\mathrm{N}}=4$ mΩ cm for optimally doped YBCO.[35]

In Fig. 1(a) the out-of-plane resistivity is presented, when both AL and DOS contributions are taken into account, while Fig. 1(b) shows the effect of the AL term alone, if the same normal state background is assumed. One can notice the supplementary broadening of the transition induced by a strong electric field, and also the relative reduction of the non-Ohmic effect when a magnetic field is simultaneously applied. The effect of various electric fields, at a fixed magnetic field, on the AL and DOS fluctuation conductivities is detailed in Figs. 1(c) and 1(d), respectively. It turns out that the non-Ohmic effect is important only for the AL term, while the DOS one is little affected by an electric field of experimentally accessible strength. This behavior stems from the peculiar dependence of Eq. (26) on the electric field only in the argument of the Bessel function.

It is worth mentioning that for $Bi_2Sr_2CaCu_2O_{8+x}$, for which the DOS contribution as such competes stronger with the AL one due to the higher anisotropy, the estimate of the non-Ohmic effect on the DOS term turns out however to be even more insignificant than for $YBa_2Cu_3O_{6+x}$, and thus almost undiscernable in the same range of electric fields. The reason is the much smaller anisotropy parameter r which reduces the effect of the Bessel function factor in Eq. (26).

VI. NON-OHMIC CONDUCTION FOR A TILTED ELECTRIC FIELD

In the linear response approximation, the current produced by an arbitrarily oriented electric field can be simply obtained by superposing the currents generated separately by its components. However, this is not anymore the case in the non-Ohmic regime of a strong electric field. As we shall see below, the current components will depend now on all the field components, and not only to the particular one corresponding to the respective axis. This case requires therefore a

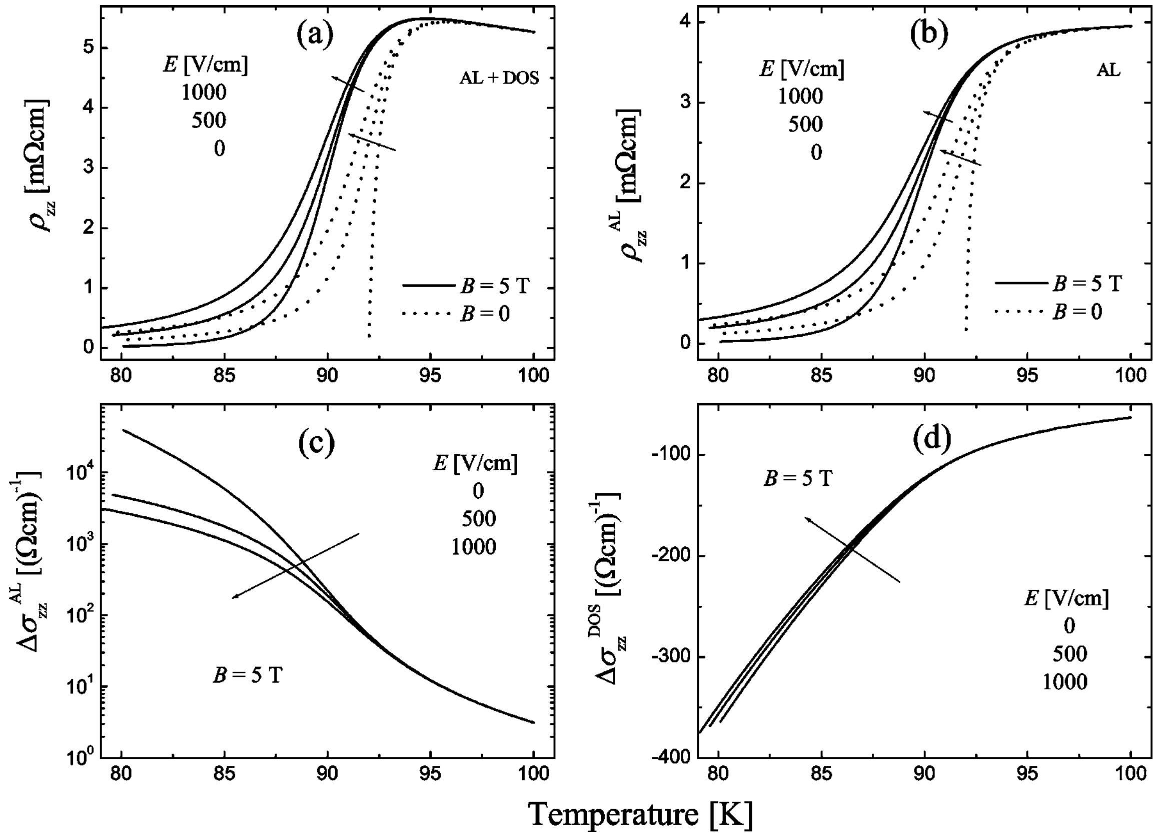

FIG. 1. (a) Out-of-plane resistivity as a function of temperature in $YBa_2Cu_3O_{6+x}$, for two values of the magnetic field, at several magnitudes of the electric field, when both AL and DOS fluctuation contributions are taken into account. The material parameters are given in the text. The arrows show the increasing electric field direction. (b) Same as (a), when only the AL part is considered. (c) The out-of-plane AL conductivity, at different magnitudes of the electric field, at $B=5$ T. (d) The DOS contribution to the out-of-plane conductivity, at different electric fields and fixed magnetic field.

special treatment, where both the in-plane and out-of-plane electric field components are to be included from the beginning in the TDGL equation. This calculation would be particularly useful if the investigation of the out-of-plane non-Ohmic conduction were performed on vicinal thin films,[27] where a mixture of the in-plane and out-of-plane transport properties is assessed, since the injected current has a slanted direction with respect to the crystallographic axes.

We shall consider in the following the case of an electric field $\mathbf{E}$ applied on a layered superconductor at an angle θ with the c axis, having thus the components ($E_x=E\sin\theta, E_y=0, E_z=E\cos\theta$), and generated by the scalar potential $\varphi=-E_x x-E_z sl$. Since the presence of a magnetic field at arbitrary direction would overcomplicate the calculations, we shall consider in the following the case of a zero magnetic field. The TDGL equation analogous to Eq. (6) will write in this case

$$\left[\Gamma_0^{-1}\frac{\partial}{\partial t}+\chi\sin\theta\frac{\partial}{\partial k}+\chi\cos\theta\frac{\partial}{\partial q}+a_0(\tilde{\varepsilon}+\xi_0^2k^2+\xi_0^2k_y^2)+a_0\frac{r}{2}(1-\cos qs)\right]\psi(k,k_y,q,t)=\zeta(k,k_y,q,t), \qquad (28)$$

where the Fourier transformed order parameter $\psi(k,k_y,q,t)$ is given by

$$\psi_l(x,y,t)=\int\frac{dk}{2\pi}\int\frac{dk_y}{2\pi}\int_{-\pi/s}^{\pi/s}\frac{dq}{2\pi}e^{-ikx}e^{-ik_yy}e^{-iqls}\psi(k,k_y,q,t), \qquad (29)$$

and the noise terms are delta-correlated such as

$$\langle\zeta(k,k_y,q,t)\zeta^*(k',k_y',q',t')\rangle=2\Gamma_0^{-1}k_BT(2\pi)^3\delta(k-k')\times\delta(k_y-k_y')\delta(q-q')\delta(t-t').$$

It can be verified that with the aid of the Green function in the three variables (k,q,t) for Eq. (28), the solution for the Fourier-transformed order parameter writes

$$\psi(k,k_y,q,t)=\Gamma_0\int_0^\infty d\tau\,\zeta\left(k-\frac{2eE_x}{\hbar}\tau,k_y,q-\frac{2eE_z}{\hbar}\tau,t-\tau\right)\times\exp\left\{-a_0\Gamma_0\tau\left[\left(\tilde{\varepsilon}+\xi_0^2k_y^2+\xi_0^2k^2+\frac{r}{2}\right)-\xi_0^2k\frac{2eE_x}{\hbar}\tau+\frac{\xi_0^2}{3}\left(\frac{2eE_x}{\hbar}\right)^2\tau^2\right]\right\}\times\exp\left[\frac{a_0\Gamma_0\hbar r}{2eE_zs}\sin\frac{eE_zs\tau}{\hbar}\cos\left(qs-\frac{eE_zs\tau}{\hbar}\right)\right]. \qquad (30)$$

Consequently, the general order parameter correlation function that will allow us to calculate the Cooper pair density as well as the current density along the x and z axes will write

$$\langle \psi_l(x,y,t)\psi^*_{l'}(x',y,t)\rangle = \frac{k_BT}{a_0}\int_0^\infty du\left(\int \frac{dk}{2\pi}\int\frac{dk_y}{2\pi}\right)_{\xi_0^2(k_y^2+k^2)\leqslant c}\int_{-\pi/s}^{\pi/s}\frac{dq}{2\pi}e^{-ik(x-x')}e^{-iqs(l-l')}$$
$$\times\exp\left[-u\left(\tilde{\varepsilon}+\xi_0^2k_y^2+\xi_0^2k^2+\frac{r}{2}\right)-4\left(\frac{E\sin\theta}{E_0}\right)^2u^3\right]\exp\left(2\sqrt{3}\xi_0k\frac{E\sin\theta}{E_0}u^2\right)$$
$$\times\exp\left[\frac{r}{2}\frac{\sin(pu\cos\theta)}{p\cos\theta}\cos(qs-pu\cos\theta)\right],$$

where E_0 and p are defined in Eq. (10). One should also notice the cutoff condition for the in-plane momentum $\xi_0^2(k_y^2+k^2)\leqslant c$.

We are now able to summarize the results obtained for the Hartree renormalization equation and for the current density components as being, respectively,

$$\tilde{\varepsilon}_{(E_x,E_z)} = \ln\frac{T}{T_0}+gT\int_0^\infty du\, e^{-u(\tilde{\varepsilon}+r/2)-4(E_x/E_0)^2u^3}I_0\left(\frac{r}{2}\frac{\sin(p_zu)}{p_z}\right)\times\int_0^c dw\, e^{-uw}I_0\left(2\sqrt{3}\sqrt{w}\frac{E_x}{E_0}u^2\right), \quad (31)$$

$$\Delta j_x(E_x,E_z) = \frac{e^2}{16\hbar s}E_x\int_0^\infty du\, u^2e^{-u(\tilde{\varepsilon}+r/2)-4(E_x/E_0)^2u^3}\times I_0\left(\frac{r}{2}\frac{\sin(p_zu)}{p_z}\right)\times\int_0^c dw\, we^{-uw}(I_0-I_2)_{(2\sqrt{3}\sqrt{w}(E_x/E_0)u^2)}, \quad (32)$$

$$\Delta j_z(E_x,E_z) = \frac{e^2}{16\hbar s}\gamma_a^2E_z\int_0^\infty du\, e^{-u(\tilde{\varepsilon}+r/2)-4(E_x/E_0)^2u^3}\frac{\sin(p_zu)}{p_z}\times I_1\left(\frac{r}{2}\frac{\sin(p_zu)}{p_z}\right)\int_0^c dw\, e^{-uw}I_0\left(2\sqrt{3}\sqrt{w}\frac{E_x}{E_0}u^2\right), \quad (33)$$

where $p_z=p\cos\theta=(s\sqrt{3}/\xi_0)(E_z/E_0)$, $\gamma_a=\xi_{0c}/\xi_0=\sqrt{m/m_c}$ is the anisotropy factor, and I_0, I_1 and I_2 are the modified Bessel functions. The current density components can be written in a simpler form if the cutoff procedure is neglected ($c\rightarrow\infty$), namely

$$\Delta j_x^{\text{no cut}}(E_x,E_z) = \frac{e^2}{16\hbar s}E_x\int_0^\infty du\, e^{-u(\tilde{\varepsilon}+r/2)-(E_x/E_0)^2u^3}\times I_0\left(\frac{r}{2}\frac{\sin(p_zu)}{p_z}\right), \quad (34)$$

$$\Delta j_z^{\text{no cut}}(E_x,E_z) = \frac{e^2}{16\hbar s}\gamma_a^2E_z\int_0^\infty du\, e^{-u(\tilde{\varepsilon}+r/2)-(E_x/E_0)^2u^3}\frac{\sin(p_zu)}{p_zu}\times I_1\left(\frac{r}{2}\frac{\sin(p_zu)}{p_z}\right). \quad (35)$$

Besides the superconducting fluctuation contribution $(\Delta j_x,\Delta j_z)$, also the normal state conduction must be considered in the total current density, which will thus be given by

$$j_x = \Delta j_x(E_x,E_z)+\sigma_{ab}^{\text{N}}E_x,$$
$$j_z = \Delta j_z(E_x,E_z)+\sigma_c^{\text{N}}E_z, \quad (36)$$

if one supposes Ohmic in-plane and out-of-plane normal state conductivities σ_{ab}^{N} and σ_c^{N}, respectively. As it is generally the case for an anisotropic conductor, the current is not parallel to the field, unless the latter is applied along one of the principal axes of the material. Moreover, unlike the Ohmic regime, the current densities components Δj_x and Δj_z in Eqs. (32) and (33) or in Eqs. (34) and (35) depend generally on both E_x and E_z, so that the non-Ohmic effect of an arbitrarily oriented electric field cannot be reduced to the superposition of the non-Ohmic currents produced separately by the in-plane and out-of-plane field components.

It must be also mentioned that the superconducting fluctuation current $(\Delta j_x,\Delta j_z)$ in Eq. (36), as inferred in the GL approach, regards only the AL fluctuation process.

VII. RESULTS IN THE 3D LIMIT

A. General case of nonzero magnetic and electric fields

The results obtained in the previous sections for a layered superconductor cannot be transformed for the three-dimensional case by directly imposing the 3D condition $s\rightarrow 0$ (or $r\rightarrow\infty$), because in the layered model it was assumed that a cutoff in the z direction is not necessary, since the out-of-plane momentum q was already confined to the interval $[-\pi/s,\pi/s]$. However, in the 3D case, a cutoff condition is necessary for all the three momentum components, (k_x,k_y,q). Assuming isotropy in the xy plane, the 3D cutoff condition for the total kinetic energy will write thus, in the absence of a magnetic field,

$$\frac{\hbar^2 k_x^2}{2m}+\frac{\hbar^2 k_y^2}{2m}+\frac{\hbar^2 q^2}{2m_c}\leqslant ca_0 \quad \text{or} \quad (k_x^2+k_y^2)\xi_0^2+q^2\xi_{0c}^2\leqslant c. \tag{37}$$

If a magnetic field B is applied along the z-axis, the cutoff condition will be written though in terms of the Landau level n, as

$$a_0 h(2n+1)+\frac{\hbar^2 q^2}{2m_c}\leqslant ca_0 \quad \text{or} \quad h(2n+1)+q^2\xi_{0c}^2\leqslant c. \tag{38}$$

The kinetic energy $\hbar^2 q^2/2m_c$ will replace therefore the interlayer coupling energy $a_0 r(1-\cos qs)/2$ from the layered case. This can be formally performed by taking the cosine function in the small-q limit, such as

$$\frac{r}{2}(1-\cos qs)\approx\frac{rq^2 s^2}{4}=q^2\xi_{0c}^2\equiv q'^2, \tag{39}$$

which can be interpreted by saying that the 3D behavior is approached when the size of the Cooper pairs along the z axis is so large that the peculiarities of the layered structure do not play a role any more, meaning that only small values of the out-of-plane momentum q are important in the integrations. This regime indeed occurs for the layered superconductors in the near vicinity of the transition temperature, where the fluctuations acquire anisotropic 3D character.

In the presence of an electric field E along the z axis, one must solve now the TDGL equation in the form

$$\left[\Gamma_0^{-1}\frac{\partial}{\partial t}+\chi\frac{\partial}{\partial q}+a_0\tilde{\varepsilon}_n+a_0 q^2\xi_{0c}^2\right]\psi_n(k,q,t)=\zeta_n(k,q,t), \tag{40}$$

so that the solution is found to be

$$\psi_n(k,q,t)=\Gamma_0\int_0^\infty d\tau\,\zeta_n\left(k,q-\frac{2eE}{\hbar}\tau,t-\tau\right)\exp\left\{-\Gamma_0\tau\left[a_0\tilde{\varepsilon}_n+\frac{e^2E^2}{6m_c}\tau^2+\frac{\hbar^2}{2m_c}\left(q-\frac{eE}{\hbar}\tau\right)^2\right]\right\}. \tag{41}$$

The current density along the z direction in the 3D case has in the chosen gauge the usual form

$$\langle j_z\rangle=-\frac{ie_0\hbar}{2m_c}(\partial_z-\partial_{z'})\langle\psi(x,y,z,t)\psi^*(x,y,z',t)\rangle\Big|_{z=z'}, \tag{42}$$

where

$$\psi(x,y,z,t)=\int\frac{dk}{2\pi}\int\frac{dq}{2\pi}\sum_{n\geqslant 0}\psi_q(n,k,t)e^{-iky}e^{-iqz}u_n\left(x-\frac{\hbar k}{2eB}\right), \tag{43}$$

and the correlation function is, according to Eq. (41),

$$\begin{aligned}\langle\psi(x,y,z,t)\psi^*(x,y,z',t)\rangle &=\frac{mk_BT}{\pi\hbar^2\xi_{0c}}h\int_0^\infty du\left(\int\frac{dq'}{2\pi}\sum_n\right)\Bigg|_{h(2n+1)+q'^2\leqslant c}\\ &\times\exp\left[-i\frac{q'}{\xi_{0c}}(z-z')-\frac{4p'^2u^3}{3}\right]\\ &\times\exp\{-u[\tilde{\varepsilon}+(2n+1)h+q'^2-2q'p'u]\},\end{aligned} \tag{44}$$

where

$$p'=\frac{\pi e\xi_{0c}}{16k_BT}E=\frac{\xi_{0c}\sqrt{3}}{\xi_0}\frac{E}{E_0}. \tag{45}$$

After careful evaluation of the q' integral and the LL sum, with consideration of the cutoff condition (38), one obtains eventually the Hartree renormalization equation with both the magnetic and electric fields applied along the z direction, as

$$\tilde{\varepsilon}^{(3D)}_{(E=E_z,B=B_z)}=\ln\frac{T}{T_0}+\frac{g^{(3D)}T}{\pi}\int_0^\infty du\frac{2uhe^{-uh}}{1-\exp(-2uh)}e^{-u\tilde{\varepsilon}-(4p'^2u^3/3)}\int_0^{c+h}dw\,e^{-uw}\frac{\sinh\left(2p'u^2\sqrt{w}\sqrt{\frac{c-h}{c+h}}\right)}{2p'u^2}, \tag{46}$$

$$g^{(3D)}=\frac{2\mu_0\kappa_{GL}^2e^2\xi_0^2k_B}{\pi\hbar^2\xi_{0c}}. \tag{47}$$

One should note that κ_{GL} in Eq. (47) is the Ginzburg-Landau parameter in the xy plane (and therefore proportional to the Cooper pair mass m in the xy plane, which in turn is proportional to $1/\xi_0^2$), so that the $g^{(3D)}$ parameter is in fact proportional with the product $1/\xi_0^2\xi_{0c}$, symmetric with respect to the coherence lengths along the three principal axes of the material.

The general expression of the fluctuation conductivity (the AL contribution) in a 3D anisotropic superconductor, when both the electric and magnetic fields are applied along the symmetry axis, will write in turn

$$\Delta\sigma_{zz}^{\mathrm{AL};(3\mathrm{D})}(E,B) = \frac{e^2\xi_{0c}}{8\pi\hbar\xi_0^2}\left(\frac{c-h}{c+h}\right)^{3/2}\int_0^\infty du\, u^2\frac{2uhe^{-uh}}{1-\exp(-2uh)}\exp\left[-u\tilde{\varepsilon}-\frac{4p'^2u^3}{3}\right]$$
$$\times\int_0^{c+h} dw\, w^{3/2}e^{-uw}\left[\frac{\cosh\left(2p'u^2\sqrt{w}\sqrt{\frac{c-h}{c+h}}\right)}{\left(2p'u^2\sqrt{w}\sqrt{\frac{c-h}{c+h}}\right)^2}-\frac{\sinh\left(2p'u^2\sqrt{w}\sqrt{\frac{c-h}{c+h}}\right)}{\left(2p'u^2\sqrt{w}\sqrt{\frac{c-h}{c+h}}\right)^3}\right]. \quad (48)$$

The above equation (48) takes a simpler form if one neglects the cutoff (i.e., for $c\to\infty$), namely,

$$\Delta\sigma_{zz}^{\mathrm{AL}}\big|_{\text{no cut}}^{(3\mathrm{D})}(E,B) = \frac{e^2\xi_{0c}}{32\sqrt{\pi}\hbar\xi_0^2}\int_0^\infty du\frac{2uhe^{-uh}}{1-\exp(-2uh)}\frac{1}{\sqrt{u}}\times\exp\left(-u\tilde{\varepsilon}-\frac{1}{3}p'^2u^3\right). \quad (49)$$

This expression can be also directly inferred from the corresponding result of the layered model, if one takes the 3D limit $s\to 0$, $r\to\infty$ in Eq. (14). In this case $p\to 0$, so that $\sin(pu)/pu\to 1$, while the sine function can be expanded up to the cubic term in u in the argument of the modified Bessel function, which becomes thus $I_1(ru/2-\xi_{0c}^2E'^2u^3/\xi_0^2)\simeq(\pi r)^{-1/2}\exp(ru/2-p'^2u^3/3)$, if one takes the asymptotic expression of the modified Bessel functions $I_n(z)\simeq e^z/\sqrt{2\pi z}$ for large arguments $z\to\infty$. It must be reminded, however, that the general expression (48), where the cutoff is considered, can not be directly obtained from the corresponding result (13) of the layered model, since for the latter the cutoff procedure is not applied for the out-of-plane momentum.

B. Limit cases of vanishing electric and/or magnetic field

The linear response limit for the results (46) and (48), i.e., the case of a vanishing electric field and a finite magnetic field, can be found directly from the correlation function (44), by performing the q' integral before the LL sum. They write

$$\tilde{\varepsilon}_{(E\to 0,B>0)}^{(3\mathrm{D})} = \ln\frac{T}{T_0}+\frac{2g^{(3\mathrm{D})}T}{\pi}h\sum_{n=0}^{N_c}\frac{1}{\sqrt{\tilde{\varepsilon}_n}}\arctan\sqrt{\frac{c+\tilde{\varepsilon}}{\tilde{\varepsilon}_n}-1}, \quad (50)$$

$$\Delta\sigma_{zz}^{\mathrm{AL}}\big|_{E\to 0,B>0}^{(3\mathrm{D})} = \frac{e^2\xi_{0c}h}{16\pi\hbar\xi_0^2}\sum_{n=0}^{N_c}\left[\frac{1}{\tilde{\varepsilon}_n^{3/2}}\arctan\sqrt{\frac{c+\tilde{\varepsilon}}{\tilde{\varepsilon}_n}-1}+\frac{(c+\tilde{\varepsilon}-\tilde{\varepsilon}_n)^{3/2}}{\tilde{\varepsilon}_n(c+\tilde{\varepsilon})^2}-\frac{(c+\tilde{\varepsilon}-\tilde{\varepsilon}_n)^{1/2}}{(c+\tilde{\varepsilon})^2}\right], \quad (51)$$

where the LL cutoff index is $N_c=(c-h)/2h$ and $\tilde{\varepsilon}_n=\tilde{\varepsilon}+(2n+1)h$.

In the other limit of a vanishing magnetic field ($B=0$) but under arbitrarily strong electric field ($E>0$), one obtains

$$\tilde{\varepsilon}_{(E=E_z,B=0)}^{(3\mathrm{D})} = \ln\frac{T}{T_0}+\frac{g^{(3\mathrm{D})}T}{\pi}\int_0^\infty du\,\exp\left(-u\tilde{\varepsilon}-\frac{4p'^2u^3}{3}\right)\times\int_0^c dw\, e^{-uw}\frac{\sinh(2p'u^2\sqrt{w})}{2p'u^2}, \quad (52)$$

which is the 3D equivalent of the renormalization equation (17), and

$$\Delta\sigma_{zz}^{\mathrm{AL}}\big|_{E>0,B=0}^{(3\mathrm{D})} = \frac{e^2\xi_{0c}}{8\pi\hbar\xi_0^2}\int_0^\infty du\, u^2e^{-u\tilde{\varepsilon}-4p'^2u^3/3}\int_0^c dw\, w^{3/2}e^{-uw}\times\left[\frac{\cosh(2p'u^2\sqrt{w})}{(2p'u^2\sqrt{w})^2}-\frac{\sinh(2p'u^2\sqrt{w})}{(2p'u^2\sqrt{w})^3}\right], \quad (53)$$

which is the 3D equivalent of the fluctuation conductivity (18) from the layered model. Taken in the isotropic case ($\xi_{0c}=\xi_0$), Eqs. (52) and (53) match the corresponding expressions already found in Ref. 19, where the calculations were performed in 3D limit in connection to the non-Ohmic *in-plane* conductivity for the layered model.

If one neglected the cutoff ($c\to\infty$), the right-hand side term in Eq. (52) would become divergent, while Eq. (53) would take the expression

$$\Delta\sigma_{zz}^{\mathrm{AL}}\big|_{\text{no cut}}^{(3\mathrm{D})} = \frac{e^2\xi_{0c}}{32\sqrt{\pi}\hbar\xi_0^2}\int_0^\infty\frac{du}{\sqrt{u}}e^{-\tilde{\varepsilon}u-(1/3)p'^2u^3}, \quad (54)$$

already known[14,18,36] for isotropic bulk superconductors ($\xi_{0c}=\xi_0$) and Gaussian fluctuations (i.e., with $\tilde{\varepsilon}=\varepsilon$).

Taking further the limit $E\to 0$, one obtains for the Hartree renormalization equation:

$$\tilde{\varepsilon}_{(E=0,B=0)}^{(3\mathrm{D})} = \ln\frac{T}{T_0}+\frac{2g^{(3\mathrm{D})}T}{\pi}\left(\sqrt{c}-\sqrt{\tilde{\varepsilon}}\arctan\sqrt{\frac{c}{\tilde{\varepsilon}}}\right), \quad (55)$$

and the fluctuation conductivity in the z direction,

$$\Delta\sigma_{zz}^{\mathrm{AL}}\big|_{E=0,B=0}^{(3\mathrm{D})} = \frac{e^2\xi_{0c}}{48\pi\hbar\xi_0^2}\left[\frac{3\arctan\sqrt{\frac{c}{\tilde{\varepsilon}}}}{\sqrt{\tilde{\varepsilon}}} - \frac{3\tilde{\varepsilon}\sqrt{c}}{(c+\tilde{\varepsilon})^2} - \frac{5c^{3/2}}{(c+\tilde{\varepsilon})^2}\right]. \tag{56}$$

The result (56) matches thus the expression found[37,38] for Gaussian fluctuations ($\tilde{\varepsilon}=\varepsilon$) in a 3D isotropic superconductor ($\xi_{0c}=\xi_0$).

Equation (55) taken for $\tilde{\varepsilon}=0$ gives the relation between the mean-field transition temperature T_0 and the actual critical temperature $T_{c0}=T_c|_{E=0,B=0}$ where the superconductivity is attained in the absence of external fields,

$$T_0 = T_{c0}\exp(2g^{(3\mathrm{D})}T_{c0}\sqrt{c}/\pi), \tag{57}$$

which represents the equivalent of Eq. (21) from the layered model.

C. Estimation of DOS term

Supposing that the same proportionality (24) between the DOS contribution to the fluctuation conductivity $\Delta\sigma_{zz}^{\mathrm{DOS}}$ and the Cooper pairs density $\langle|\psi|^2\rangle$ holds also in the 3D case, one can estimate for the different combinations of electric and magnetic fields,

$$\Delta\sigma_{zz}^{\mathrm{DOS}}(E,B)\big|^{(3\mathrm{D})} = -\frac{e^2\xi_{0c}\kappa}{4\pi\hbar\xi_0^2}\times\int_0^\infty du\frac{2uhe^{-uh}}{1-\exp(-2uh)}e^{-u\tilde{\varepsilon}-4p'^2u^3/3}\times\int_0^{c+h}dw\,e^{-uw}\frac{\sinh\left(2p'u^2\sqrt{w}\sqrt{\frac{c-h}{c+h}}\right)}{2p'u^2}, \tag{58}$$

$$\Delta\sigma_{zz}^{\mathrm{DOS}}(B)\big|_{E\to0}^{(3\mathrm{D})} = -\frac{e^2\xi_{0c}\kappa h}{2\pi\hbar\xi_0^2}\sum_{n=0}^{N_c}\frac{1}{\sqrt{\tilde{\varepsilon}_n}}\arctan\sqrt{\frac{c+\tilde{\varepsilon}}{\tilde{\varepsilon}_n}-1}, \tag{59}$$

$$\Delta\sigma_{zz}^{\mathrm{DOS}}(E)\big|_{B=0}^{(3\mathrm{D})} = -\frac{e^2\xi_{0c}\kappa}{4\pi\hbar\xi_0^2}\int_0^\infty du\exp\left(-u\tilde{\varepsilon}-\frac{4p'^2u^3}{3}\right)\times\int_0^c dw\,e^{-uw}\frac{\sinh(2p'u^2\sqrt{w})}{2p'u^2}, \tag{60}$$

$$\Delta\sigma_{zz}^{\mathrm{DOS}}\big|_{B=0,E\to0}^{(3\mathrm{D})} = -\frac{e^2\xi_{0c}\kappa}{2\pi\hbar\xi_0^2}\left(\sqrt{c}-\sqrt{\tilde{\varepsilon}}\arctan\sqrt{\frac{c}{\tilde{\varepsilon}}}\right). \tag{61}$$

Unlike the layered model, where, for instance, the AL contribution (15) was quadratic in the anisotropy parameter r while the DOS one (22) was linear, the corresponding 3D fluctuation contributions (51) and (59) are both proportional to the ratio ξ_{0c}/ξ_0^2, while the AL one is more singular in $\tilde{\varepsilon}$. The DOS contribution might thus be of less importance in the 3D case with respect to the layered model. However, the relations (58), (59), (60), and (61) remain to be confirmed or refuted by a microscopic approach.

D. Arbitrary orientation of the electric field

The 3D equivalents of Eqs. (31), (32), (33), (34), and (35), valid for an arbitrary orientation of the electric field with respect to the layers, can be obtained in a simpler manner, without having to solve again the TDGL equation, by using a special scaling transformation of the coordinates and field components that reduces the problem to the isotropic case.[12,39] In the isotropic system, the coordinate axes can be in turn freely rotated so that the electric field acquires again only one nonzero component, for which the solution is already known. For the anisotropic 3D model with axial symmetry treated in this section, having the zero-temperature-extrapolated coherence lengths ξ_{0c} in the z direction and ξ_0 in the xy plane, corresponding to the anisotropy factor

$$\gamma_a = \frac{\xi_{0c}}{\xi_0} = \sqrt{\frac{m}{m_c}} < 1. \tag{62}$$

the scaling transformation of the coordinates and vector potential[12,39] writes

$$\tilde{x}=x,\quad \tilde{y}=y,\quad \tilde{z}=\frac{z}{\gamma_a},$$

$$\widetilde{A_x}=A_x,\quad \widetilde{A_y}=A_y,\quad \widetilde{A_z}=\gamma_aA_z. \tag{63}$$

This transformation implies the rescaling of the fields as

$$\widetilde{B_x}=\gamma_aB_x,\quad \widetilde{B_y}=\gamma_aB_y,\quad \widetilde{B_z}=B_z,$$

$$\widetilde{E_x}=E_x,\quad \widetilde{E_y}=E_y,\quad \widetilde{E_z}=\gamma_aE_z, \tag{64}$$

and for the order parameter and current density as

$$|\tilde{\psi}|^2=\gamma_a|\psi|^2,$$

$$\widetilde{j_x}=\gamma_aj_x,\quad \widetilde{j_y}=\gamma_aj_y,\quad \widetilde{j_z}=j_z. \tag{65}$$

Thus the anisotropy is removed from the gradient terms in the GL equation and reintroduced into the magnetic energy term, whose fluctuation can however be usually neglected for hard type-II superconductors.[39]

In the case of a zero magnetic field and an electric field ($E_x=E\sin\theta, E_y=0, E_z=E\cos\theta$) applied in the anisotropic material at an angle θ with the z-axis, the problem can be reduced, according to Eqs. (64), to that of an electric field ($\widetilde{E_x}=\tilde{E}\sin\tilde{\theta}, \widetilde{E_y}=0, \widetilde{E_z}=\tilde{E}\cos\tilde{\theta}$) applied on an isotropic superconductor having the coherence length ξ_0, such as

$$\widetilde{E} = E(\sin^2\theta + \gamma_a^2\cos^2\theta)^{1/2},$$

$$\tan\,\widetilde{\theta} = \frac{1}{\gamma_a}\tan\,\theta. \tag{66}$$

Taking now into account Eqs. (52), (53), and (54) considered for $\xi_{0c}=\xi_0$, together with Eqs. (65), one obtains eventually

$$\widetilde{\varepsilon}^{(3D)}_{(E_x,E_z)} = \ln\frac{T}{T_0} + \frac{g^{(3D)}T}{\pi}\int_0^\infty du\,\exp\left[-u\widetilde{\varepsilon} - 4\left(\frac{\widetilde{E}}{E_0}\right)^2 u^3\right] \times \int_0^c dw\, e^{-uw}\frac{\sinh\left(2\sqrt{3}\frac{\widetilde{E}}{E_0}u^2\sqrt{w}\right)}{2\sqrt{3}\frac{\widetilde{E}}{E_0}u^2}, \tag{67}$$

$$\Delta j_x^{(3D)} = E_x\frac{e^2}{8\pi\hbar\xi_{0c}}\int_0^\infty du\, u^2 e^{-u\widetilde{\varepsilon}-4(\widetilde{E}/E_0)^2u^3}\int_0^c dw\, w^{3/2}e^{-uw} \times\left[\frac{\cosh\left(2\sqrt{3}\frac{\widetilde{E}}{E_0}u^2\sqrt{w}\right)}{\left(2\sqrt{3}\frac{\widetilde{E}}{E_0}u^2\sqrt{w}\right)^2} - \frac{\sinh\left(2\sqrt{3}\frac{\widetilde{E}}{E_0}u^2\sqrt{w}\right)}{\left(2\sqrt{3}\frac{\widetilde{E}}{E_0}u^2\sqrt{w}\right)^3}\right], \tag{68}$$

$$\Delta j_z^{(3D)} = E_z\frac{e^2\xi_{0c}}{8\pi\hbar\xi_0^2}\int_0^\infty du\, u^2 e^{-u\widetilde{\varepsilon}-4(\widetilde{E}/E_0)^2u^3}\int_0^c dw\, w^{3/2}e^{-uw} \times\left[\frac{\cosh\left(2\sqrt{3}\frac{\widetilde{E}}{E_0}u^2\sqrt{w}\right)}{\left(2\sqrt{3}\frac{\widetilde{E}}{E_0}u^2\sqrt{w}\right)^2} - \frac{\sinh\left(2\sqrt{3}\frac{\widetilde{E}}{E_0}u^2\sqrt{w}\right)}{\left(2\sqrt{3}\frac{\widetilde{E}}{E_0}u^2\sqrt{w}\right)^3}\right], \tag{69}$$

so that neglecting the cutoff,

$$\Delta j_x^{(3D)}|_{\text{no cut}} = E_x\frac{e^2}{32\sqrt{\pi}\hbar\xi_{0c}}\int_0^\infty \frac{du}{\sqrt{u}}\exp\left[-\widetilde{\varepsilon}u - \left(\frac{\widetilde{E}}{E_0}\right)^2u^3\right], \tag{70}$$

$$\Delta j_z^{(3D)}|_{\text{no cut}} = E_z\frac{e^2\xi_{0c}}{32\sqrt{\pi}\hbar\xi_0^2}\int_0^\infty \frac{du}{\sqrt{u}}\exp\left[-\widetilde{\varepsilon}u - \left(\frac{\widetilde{E}}{E_0}\right)^2u^3\right]. \tag{71}$$

VIII. CONCLUSION

In this work we have theoretically investigated the non-Ohmic effect of an arbitrarily strong electric field on the out-of-plane fluctuation magnetoconductivity of a layered superconductor. Our framework was provided by the Langevin approach to the TDGL equation, and the Hartree approximation was used in order to take into account the fluctuation interaction. The main general results of our work, valid when magnetic and electric fields of arbitrary magnitude are applied perpendicular to the layers, are the formulas (11) for the renormalized reduced temperature and, respectively, (13) for the AL contribution to the out-of-plane fluctuation conductivity, as well as the estimation (26) for the DOS term. In the limit case of a vanishing electric field, the results were found to reduce to the expressions already known from the linear response approximation (Sec. III). Extensions of the results have been provided for the case of a tilted electric field with respect to the crystallographic axes (Sec. VI), as well as for 3D anisotropic superconductors (Sec. VII).

In order to illustrate the predictions of the theoretical calculations, we have taken as example a typical HTSC material, like the optimally doped $YBa_2Cu_3O_{6+x}$, and evidenced the nonlinear effect of a strong electric field through comparison to the results obtained in the linear response approximation, in the presence of a finite or of a zero magnetic field. An important fluctuation suppression in the AL part of the out-of-plane conductivity has been predicted for electric fields of hundreds of V/cm, while the non-Ohmic effect on the DOS contribution turned out to be marginal in the same range (Fig. 1).

So far, the effect of a strong electric field on the transport properties of HTSC has been investigated and proven experimentally only for the in-plane fluctuation conductivity and only in the absence of magnetic field.[20–22,24] The difficulties lie in the high dissipation in cuprates (of the order of $\mathrm{GWcm^{-3}}$ at electric fields of hundreds V/cm) that can increase the sample temperature to values where the nonlinearity is no longer discernible. In this connection, applying short current pulses (tens of ns) at high current densities (a few $\mathrm{MA\,cm^{-2}}$), in combination with using very thin films (under 50 nm thick) in order to enhance the heat transport to the substrate,[24] seems to be a better alternative to the dc and ac measurements. This procedure, if applied on vicinal films in order to access also the c-axis conduction, will probably allow for the necessary accuracy necessary to detect the non-Ohmic behavior also for the out-of-plane fluctuation conductivity.

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