

Current-induced magnetic vortex motion by spin-transfer torque

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We investigate the dynamics of a magnetic vortex driven by spin-transfer torque due to spin current in the adiabatic case. The vortex core represented by collective coordinate experiences a transverse force proportional to the product of spin current and gyrovector, which can be interpreted as the geometric force determined by topological charges. We show that this force is just a reaction force of Lorentz-type force from the spin current of conduction electrons. Based on our analyses, we propose analytically and numerically a possible experiment to check the vortex displacement by spin current in the case of single magnetic nanodot.

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Manipulation of nanoscale magnetization by electric current is one of the most attractive subjects in both basic physics and technological applications. After its theoretical prediction,^{1,2} it has been widely recognized that spin-polarized current (spin current) plays a crucial role in magnetization dynamics. The spin current exerts a torque on localized spins by transferring the spin angular momenta of electrons through the exchange interaction between conduction electrons and localized spins, which is called the spin-transfer torque. The key understanding of the effect is that the spin current favors magnetic configurations with spatial gradient, or more precisely, with finite Berry-phase curvature along the current. Such spatial gradient by spin current indeed gives rise to the motion of domain wall,^{3,4} spin-wave instability in a uniform ferromagnet,⁵⁻⁷ and domain nucleation.⁸

Recent experiments^{9,10} and numerical simulation¹¹ for current-induced domain wall motion have shown that there is a vortexlike configuration in magnetic nanowire. Also, magnetic vortices in nanodots have drawn much attention since the magnetic force microscopy (MFM) observation of a vortex core.¹² However, an effective force on the vortex and its dynamics by spin-transfer torque due to the spin current have not been clarified.

In this Rapid Communication, we present a microscopic theory of vortex dynamics in the presence of spin current by using the collective coordinate method. In the adiabatic approximation, we derive an effective force exerted on the vor-

tex core due to spin-transfer torque. It is shown that the vortex core experiences a transverse force, which compensates the Magnus-type force derived from the so-called Berry's phase term. This specific force is topologically invariant, which is characterized by topological charges. Unlike the case of domain wall,⁴ we show that there is no threshold current to induce the vortex motion in the absence of an external pinning. It is of great interest to the vortex-based devices for application. To verify the existence of current-induced transverse force on the vortex, we propose a possible experiment for the current-induced vortex displacement in a single magnetic nanodot.

We consider the Lagrangian of the localized spins in the continuum approximation. The spins are assumed to have an easy plane taken to be the x - y plane, and are described by the Lagrangian

$$L_S = \hbar S \int \frac{d^3x}{a^3} \dot{\phi} (\cos \theta - 1) - H_S, \quad (1)$$

$$H_S = \frac{S^2}{2} \int \frac{d^3x}{a^3} \{J(\nabla \mathbf{n})^2 + K_{\perp} \cos^2 \theta\}, \quad (2)$$

where $\mathbf{S}(\mathbf{x}, t) = S\mathbf{n}(\mathbf{x}, t)$ represents the localized spin vector with unit vector $\mathbf{n} = \sin \theta \cos \phi \mathbf{e}_x + \sin \theta \sin \phi \mathbf{e}_y + \cos \theta \mathbf{e}_z$, and the magnitude of spin S ; $\mathbf{e}_i (i=x, y, z)$ are unit vectors of Cartesian frame. The J and K_{\perp} are, respectively, the ex-

change and the hard-axis anisotropy constants and a is the lattice constant. The first term of the right-hand side in Eq. (1) is the so-called Berry-phase term, which in general determines the dynamical property of the localized spins.

Let us denote the spin configuration of a vortex centered at the origin by a vector field $\mathbf{n}_v(\mathbf{x})$ with unit modulus. As a vortex profile, we take an out-of-plane vortex; $\mathbf{n}_v(\mathbf{x} \rightarrow \mathbf{0}) = p\mathbf{e}_z$, where $p = \pm 1$ is the polarization, which refers to the spin direction of the vortex core center and $\mathbf{n}(|\mathbf{x}| \gg \delta_v) = \cos(q\varphi + C\pi/2)\mathbf{e}_x + \sin(q\varphi + C\pi/2)\mathbf{e}_y$, where δ_v is the vortex core radius, $\varphi = \tan^{-1}(y/x)$, $q = \pm 1, \pm 2, \dots$, is the vorticity, which describes the number of windings of the spin vector projected on the order-parameter space and $C = \pm 1$ is the chirality, which refers to the counterclockwise ($C=1$) or the clockwise ($C=-1$) rotational direction of the spin in the plane. We here introduce a collective coordinate $\mathbf{X}(t) = X(t)\mathbf{e}_x + Y(t)\mathbf{e}_y$, which represents the vortex core center, and assume that a moving vortex can be written as $\mathbf{n}(\mathbf{x}, t) = \mathbf{n}_v[\mathbf{x} - \mathbf{X}(t)]$ at least as a first approximation; that is, ignoring the spin-wave excitation. Substituting this into Eq. (1), we obtain the Lagrangian for the collective coordinate as

$$L_V = \frac{1}{2} \mathbf{G} \cdot (\dot{\mathbf{X}} \times \mathbf{X}) - U(\mathbf{X}). \quad (3)$$

Here \mathbf{G} is the gyrovector defined by

$$\mathbf{G} = \mathbf{e}_z \hbar S \int \frac{d^3x}{a^3} \mathbf{n} \cdot (\partial_x \mathbf{n} \times \partial_y \mathbf{n}) = \frac{\hbar S}{a^3} 2\pi L p q \mathbf{e}_z, \quad (4)$$

with L being the thickness of the system, and $U(\mathbf{X})$ is a potential energy of a vortex evaluated from the Hamiltonian H_S . The gyrovector \mathbf{G} is topologically invariant corresponding to polarization p and vorticity q and the number covering the mapping space D . In the case of $\mathbf{n}_v(\mathbf{x})$, this mapping number is $1/2$ in units of surface area 4π .

The first term of the right-hand side in Eq. (3), which comes from the Berry-phase term, represents that X and Y are essentially canonically conjugate each other. This term provides a transverse force $-\mathbf{G} \times \dot{\mathbf{X}}$ on the moving vortex, the so-called Magnus force, perpendicular both to the gyrovector and to the vortex velocity, whose term has been derived and discussed by many workers¹⁵⁻¹⁹ in various systems.

Let us investigate the force acting on the vortex by spin current of conduction electrons. The Lagrangian of the electrons is given by

$$L_{\text{el}}^0 = \int d^3x c^\dagger(\mathbf{x}, t) \left\{ i\hbar \frac{\partial}{\partial t} + \frac{\hbar^2}{2m} \nabla^2 \right\} c(\mathbf{x}, t) - H_{\text{sd}}, \quad (5)$$

where $c(c^\dagger)$ is annihilation (creation) operator of conduction electrons. The last term H_{sd} represents the exchange interaction between localized spins and conduction electrons given by $H_{\text{sd}} = -\Delta \int d^3x \mathbf{n} \cdot (c^\dagger \boldsymbol{\sigma} c)$, where 2Δ is the energy splitting, and $\boldsymbol{\sigma}$ is a Pauli-matrix vector. Here we perform a local gauge transformation^{4,13} in electron spin space so that the quantization axis is parallel to the localized spins $\mathbf{n}(\mathbf{x}, t)$ at each point of space and time; $c(\mathbf{x}, t) = U(\mathbf{x}, t) a(\mathbf{x}, t)$, where $a(\mathbf{x}, t)$ is the two-component electron operator in the rotated frame, and $U(\mathbf{x}, t) = \mathbf{m}(\mathbf{x}, t) \cdot \boldsymbol{\sigma}$ is an SU(2) matrix with

$\mathbf{m} = \sin(\theta/2) \cos \phi \mathbf{e}_x + \sin(\theta/2) \sin \phi \mathbf{e}_y + \cos(\theta/2) \mathbf{e}_z$. The Lagrangian is now given by

$$L_{\text{el}} = \int d^3x a^\dagger \left[i\hbar (\partial_0 + iA_0) + \frac{\hbar^2}{2m} (\partial_i + iA_i)^2 \right] a + \Delta \int d^3x a^\dagger \sigma_z a, \quad (6)$$

where $A_\nu = \mathbf{A}_\nu \cdot \boldsymbol{\sigma} = -iU^\dagger \partial_\nu U$ ($\nu = 0, x, y, z$) is an SU(2) gauge field determined by the time and spatial derivative of the localized spins. For slowly varying magnetic configurations, the electron spins can mostly follow adiabatically the localized spins. This is justified for the condition $k_F \lambda \gg 1$, where k_F is the Fermi wave number of conduction electrons, and λ is the characteristic length scale of the spin texture of the localized spins. In this adiabatic approximation, taking the expectation value of L_{el} for the current-carrying nonequilibrium state, we can obtain the following interaction Hamiltonian of the first-order contribution to the localized spins;⁸

$$H_{\text{ST}} = \int d^3x \frac{\hbar}{2e} \mathbf{j}_s \cdot \nabla \phi \cdot (1 - \cos \theta), \quad (7)$$

where \mathbf{j}_s is the spin-current density, which is written by using the distribution function $f_{k\sigma} = \langle a_{k\sigma}^\dagger a_{k\sigma} \rangle$ in the rotated frame specifying the current-carrying nonequilibrium state as $\mathbf{j}_s = (1/V) \sum_{k,\sigma} \sigma (\hbar \mathbf{k} / m) f_{k\sigma}$. As seen from Eq. (7), the spin current favors a finite Berry-phase curvature along the current. Indeed, the Hamiltonian H_{ST} leads to the spin-transfer torque in the case of domain wall.⁸

From Eq. (7), we can derive the effective Hamiltonian represented by the collective coordinate for the vortex core. Taking the variation $\delta \mathbf{n} = -\partial_j \mathbf{n}_v(\mathbf{x} - \mathbf{X}) \delta X_j$, where repeated roman indices imply sum over the in-plane spatial direction $j = x, y$, we obtain

$$\delta H_{\text{ST}} = \frac{\hbar S}{a^3} 2\pi L p q (v_{s,x} \delta Y - v_{s,y} \delta X), \quad (8)$$

where $\mathbf{v}_s = (a^3/2eS) \mathbf{j}_s$ represents the drift velocity of electron spins. By integrating Eq. (8), we obtain

$$H_{\text{ST}} = \mathbf{G} \cdot (\mathbf{v}_s \times \mathbf{X}). \quad (9)$$

Thus, a force acting on the vortex core is given by

$$\mathbf{F}_{\text{ST}} = - \frac{\partial H_{\text{ST}}}{\partial \mathbf{X}} = - \mathbf{G} \times \mathbf{v}_s. \quad (10)$$

This current-induced transverse force has been previously derived by Berger¹⁴ based on a phenomenological treatment in the case of Bloch line. Here we have derived this force from microscopic theory. It is noted that this force does not depend on the chirality $C = \pm 1$ of the vortex in contrast to a force produced by a magnetic field. Since it is hard to control the chirality, this fact would be a great advantage in application. A microscopic derivation of a general relation between force and torque in the Landau-Lifshitz-Gilbert (LLG) equation is presented in Ref. 20.

Before proceeding, we here briefly remark that this force can be interpreted as a reaction force of a Lorentz-type force

from the spin current of conduction electrons. The interaction Hamiltonian, Eq. (7), can be rewritten by using the the U(1) gauge field, $\mathbf{A}(\mathbf{x}) = (\hbar/2e) \nabla \phi(\mathbf{x}) [\cos \theta(\mathbf{x}) - 1]$, interacting with the spin-current density. Thus the magnetic field $\mathbf{B}(\mathbf{x})$ is given by

$$\mathbf{B} = (\partial_x A_y - \partial_y A_x) \mathbf{e}_z = -\frac{\hbar}{2e} \mathbf{n} \cdot (\partial_x \mathbf{n} \times \partial_y \mathbf{n}) \mathbf{e}_z, \quad (11)$$

which corresponds to the so-called topological field.²¹ It is noted that the SU(2) field intensity, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$, vanishes, since the original Lagrangian L_{el} in Eq. (5) does not include the local gauge field. The finite magnetic field is a consequence of the projection from SU(2) to the U(1) by taking the adiabatic approximation. Under this magnetic field, the spin current of the conduction electrons may experience the following Lorentz-type force as:

$$\mathbf{F}_L = \int d^3x \mathbf{j}_s \times \mathbf{B} = \mathbf{G} \times \mathbf{v}_s = -\mathbf{F}_{\text{ST}}, \quad (12)$$

which can be interpreted as the reaction force acting on the vortex.

Let us derive the equation of motion for the collective coordinate of vortex in the presence of spin current based on the Euler-Lagrange equation,

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{\mathbf{X}}} - \frac{\partial L}{\partial \mathbf{X}} = -\frac{\partial W}{\partial \mathbf{X}}, \quad (13)$$

where $L = L_V - H_{\text{ST}}$ is the total Lagrangian, W is the so-called dissipation function given by

$$W = \alpha \frac{\hbar S}{2} \int \frac{d^3x}{a^3} \dot{\mathbf{n}}^2(\mathbf{x}, t) = \frac{\alpha}{2} D \dot{\mathbf{X}}^2, \quad (14)$$

with α being the Gilbert damping constant. The constant

$$D = \frac{\hbar S}{a^3} L \int_D dx dy \{(\partial_t \theta)^2 + \sin^2 \theta (\partial_t \phi)^2\}, \quad (15)$$

generally includes a factor $\ln(R_V/\delta_V)$, where R_V is the system size. Here we assume that system has rotational invariance along the z axis. The concrete expression in Eq. (13) is given by

$$\mathbf{G} \times (\mathbf{v}_s - \dot{\mathbf{X}}) = -\frac{\partial U(\mathbf{X})}{\partial \mathbf{X}} - \alpha D \dot{\mathbf{X}}. \quad (16)$$

This is the equation of motion for the vortex dynamics in the presence of spin current. If the right-hand side in Eq. (16) is absent, we obtain $\dot{\mathbf{X}} = \mathbf{v}_s$, where the vortex core moves along the spin current perpendicular to the transverse force \mathbf{F}_{ST} . This situation can be seen from the current-induced domain wall motion.^{4,8} On the other hand, the damping term $-\alpha D \dot{\mathbf{X}}$ acts as a deviation from the orbital direction of the moving vortex along the current. Importantly, there is no intrinsic pinning in the dynamics of vortex unlike the case of a domain wall. This leads to a vanishing threshold current for the vortex motion in the absence of an external pinning. This is because, in the translationally invariant system, there is no pinning on X and Y . This is in contrast with the case of the

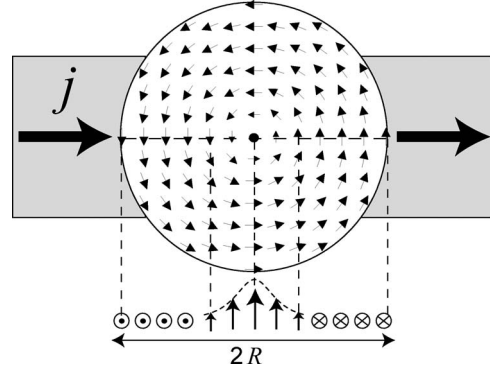


FIG. 1. Schematic illustration of an experimental setup for current-induced vortex displacement. The topological charges are chosen to be $p=1$, $q=1$, and $C=1$ in the above vortex.

domain wall, where ϕ_0 is pinned by the hard-axis magnetic anisotropy even in the translationally invariant system.⁴ Thus vortex-based devices would have great advantages in low-current operations.

To verify the existence of current-induced force on the vortex, we propose the vortex displacement by spin current in a single magnetic nanodot,²² where an out-of-plane vortex with vorticity $q=1$ is stabilized. We assume the electric current is uniform in the nanodot, and flowing in the positive x direction; that is, $\mathbf{v}_s = (a^3/2eS) \mathbf{j}_s = v_s \mathbf{e}_x$. We assume the full spin polarization of the current, $P=1$, for simplicity (Fig. 1). The potential energy $U(\mathbf{X})$ is modeled by a harmonic one $U(\mathbf{X}) = \kappa \mathbf{X}^2/2$, where κ is a force constant. In Ref. 23, κ is evaluated in detail, which depends on the aspect ratio $g=L/R$, where R is the dot radius. From Eq. (16), the equation of motion is given by

$$(1 + i\tilde{\alpha}) \dot{Z} = -i\omega Z + v_s, \quad (17)$$

where $Z = X + iY$, $\tilde{\alpha} = \alpha D/G$, and $\omega = \kappa/G$. For an initial condition $Z(0)=0$, the solution is given by

$$Z(t) = i \frac{v_s}{\omega} \left\{ \exp\left(-\frac{i\omega t}{1 + i\tilde{\alpha}}\right) - 1 \right\}. \quad (18)$$

Thus the vortex center exhibits a spiral motion, whose rotational direction depends on the sign of the core polarization $p = \pm 1$ [Fig. 2(a)]. The final displacement of the vortex core is perpendicular to the current direction and given by

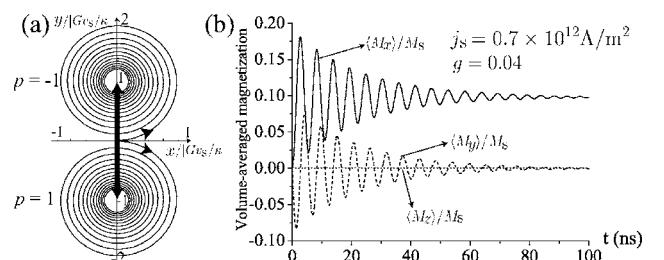


FIG. 2. (a) Spiral motion of vortex center under spin current obtained from the analytical calculations. We took $\tilde{\alpha}=0.02$. (b) Numerical results of the time evolution of the volume-averaged magnetization components.

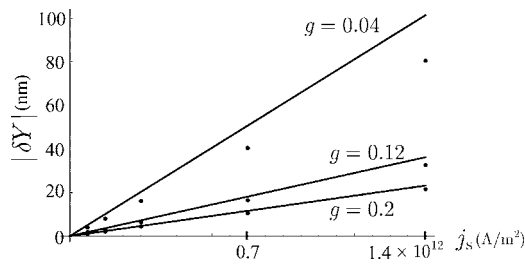


FIG. 3. Displacement of the vortex center as a function of the spin-current density j_s in various aspect ratios g . The solid lines and dots represent the analytical and numerical results, respectively.

$\delta Y = -Gv_s/\kappa$, which also depends on p , where the transverse force $-\mathbf{G} \times \mathbf{v}_s$ balances the restoring force $-\kappa\mathbf{X}$. This result is consistent with the recent experimental one in Ref. 10, where a distorted vortex wall was shifted to the diagonal direction and stopped moving.

For comparison, the current-induced dynamics of a vortex was calculated by micromagnetic numerical simulations based on the LLG equation with spin-current terms,

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma_0 \mathbf{M} \times \mathbf{H}_{\text{eff}} + \frac{\alpha}{M_s} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t} - \mathbf{v}_s \cdot \nabla \mathbf{M}, \quad (19)$$

where $\mathbf{M}(\mathbf{x}, t)$ is the magnetization vector, γ_0 is the gyromagnetic constant, \mathbf{H}_{eff} is the effective magnetic field including the exchange and demagnetizing field, and M_s is the saturation magnetization. The last term represents the spin-transfer

torque.^{5–8} The sample is divided into identical cells, in each of which magnetization is assumed to be constant. The dimension of the cells is $4 \times 4 \times h \text{ nm}^3$ with dot thickness $h = 10, 20, 30 \text{ nm}$. The dot radius is taken to be $R = 500 \text{ nm}$. The computational material parameters are typical for permalloy: $M_s = 1.0 \text{ T}$ ($M_s/\mu_0 = 8.0 \times 10^5 \text{ A/m}$), the exchange stiffness constant $A = 1.0 \times 10^{-11} \text{ J/m}$, and $\gamma_0 = 1.8 \times 10^5 \text{ m/A s}$. We take $\alpha = 0.02$. The programming code is based on those of Refs. 11 and 24.

Figure 2(b) shows the time evolution of volume-averaged magnetization, which exhibits the spiral motion of vortex core. It is noted that the rotational direction is opposite to the case of analytical result because of the replacement $\hbar S/a^3 \rightarrow -M_s/\gamma_0$. Figure 3 shows that vortex displacement as a function of the spin-current density in various aspect ratios $g = L/R$. The numerical results are in good agreement with the analytical ones for small vortex displacement. It is found that the smaller g is more advantageous to the vortex displacement.

In conclusion, we have clarified the transverse force on the vortex and its dynamics by spin-transfer torque due to the spin current in the adiabatic regime. We have proposed analytically and numerically a possible experiment for the vortex displacement by spin current in the case of single magnetic nanodot. Finally, this vortex displacement may affect a magnetoresistance, which is probable to be detected, for example, by using planar Hall effect.²⁵

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