One-particle conductance of an open quasi-two-dimensional Fermi system: Evidence of the parallel-magnetic-field-induced mode reduction effect

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The conductance of an open quench-disordered two-dimensional (2D) electron system subject to an in-plane magnetic field is calculated within the framework of conventional Fermi liquid theory actually applied to a three-dimensional system of spinless electrons confined to a highly anisotropic (planar) near-surface potential well. Using the calculation method suggested earlier [Phys. Rev. B **71**, 125112 (2005)], the magnetic field piercing a finite range of an infinitely long laterally confined system of carriers is treated (technically) as introducing the additional highly nonlocal scattering region which separates the circuit thereby modeled into three parts—the system as such and two perfect leads. The transverse quantization spectrum of the inner part of the electron waveguide thus constructed can be effectively tuned by means of the magnetic field, even though the least transverse dimension of the waveguide is small compared to the magnetic length. The initially finite (metallic) value of the conductance, which is attributed to the existence of extended modes of the transverse quantization, decreases rapidly as the magnetic field grows. This decrease is due to the mode number reduction effect produced by the magnetic field. The closing of the last current-carrying mode, which is slightly sensitive to the disorder level, is suggested as the conceivable origin of the magnetic-field-driven metal-to-insulator transition widely observed in 2D systems.

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I. INTRODUCTION

The conduction properties of low-dimensional electron and hole systems with the disorder of different origin have long been the subject of active research. Investigations into such objects of mesoscopic size have currently become particularly intensive in view of their applied importance (semiconductor heterostructures, quantum dot devices, etc.), on the one hand, and due to the intriguing uncommonness of the obtained results, on the other. One of the most puzzling phenomenon which has not as yet been understood in full measure is the existence of conducting phase of two-dimensional (2D) systems of carriers (the extensive bibliography on the subject can be found in Refs. 1–3). The apparently observed transition of such systems from insulating to conducting state evidently contradicts the common view stemming from the well-known scaling theory of localization.[4]

Of particular concern among other observed intriguing phenomena is the behavior of the conductance of two-dimensional systems subjected to the parallel magnetic field. Such a field of moderate strength was systematically observed to result in an unexpectedly large reduction of the conductance of 2D heterostructures, driving ultimately the test system from conducting to insulating state.[5–9] It is currently still debatable which physical mechanism is mainly responsible for this surprising effect.

Since nowadays many of the researchers adhere to the opinion that the one-parameter scaling approach is not justified in the experiments (see, e.g., Refs. 3 and 10), it seems opportunely to apply to the approaches based on the assumptions different from those intrinsical to the scaling theory. Specifically, instead of thinking of the initial states of carriers to be Anderson localized, with possible next delocalization due to some dephasing factors, one can start from the initially clean (perfect) system whose states, being fully coherent, are extended (if any). The evolution of the conductance or other parameters should then be appropriately traced, assuming some random and/or regular fields to be incorporated perturbatively.

The above formulated approach was recently employed for calculating the conductance of strictly 2D electron gas in restricted geometry,[11] being afterwards improved for the case of realistic quasi-2D (Q2D) open systems of carriers.[12] It was shown that in the confined Fermi system of waveguide configuration the conducting (metallic) state is a natural ground state for noninteracting electrons provided that the confinement potential possesses more than one extended mode in such an electron waveguide.

Subsequently, the mode approach employed in Refs. 11 and 12 for 2D systems not subjected to the external magnetic field was applied to realistic quasi-two-dimensional systems of carriers placed in such a field oriented parallel to the reference 2D plane.[13] Despite the fact that many of the researchers (see, e.g., Refs. 14–16) adhere to the position that it was essentially the spin polarization that should be considered as a physical origin of 2D metal-to-insulator transition (MIT) in a parallel magnetic field, it was shown in Ref. 13 that the orbital coupling of Q2D electrons to the in-plane magnetic field of quite moderate strength suffices to modify substantially the spectrum of an open side-confined planar system of carriers. The most important effect of the magnetic field is that it makes a significant impact upon the mode content of the electron waveguide, thereby resulting in an appreciable change in the number of current-carrying modes (identified as the open conducting channels[17]) as well as in the spectral width of their energy levels. In the confined system of a highly anisotropic cross section the number of conducting channels was shown to decrease strongly as the magnetic field grows, being simultaneously accompanied by the gain in coherence of the electron states. Basing on the obtained results, it was suggested therein that such a magnetic-field-caused reduction in the number of extended modes of the electron waveguide should be taken as an essential ingredient when searching for the physical origin of the transition of Q2D electron system from conducting to insulating state.

Although the analysis given in Ref. 13 at a spectral level provides a specific insight into the plausible physical mechanism of the observed magnetic-field-driven MIT, it is necessary to bear out the conclusions by calculating the observable quantities, e.g., the conductance. To accomplish this task, it would be inconvenient to use the Landauer approach[18] since it does not look into the coherence properties of the *internal* spectrum of the system in question, operating it as a scattering object integrally. Therefore, for calculating the magnetoconductance we apply in this study the linear response theory[19,20] which permits us to carry out all calculations at a microscopic level.

II. CHOOSING THE MODEL

Two-dimensional electron and hole systems in use, in view of their open property in the direction of current and the resemblance of the master equation to that of the classical wave theory, can be simulated as planar quantum waveguides whose transverse design is governed by the lateral confinement potentials. Although near-surface potential wells in Si metal-oxide-semiconductor field-effect transistors (MOSFETs) and GaAs/AlGaAs heterostructures are close in shape to triangular or parabolic form,[21,22] this fact is of minor importance for its principal application, which is to restrict electron transport in the direction normal to heterophase areas, thus resulting in *transverse* quantization of the electron spectrum. With this consideration in mind, in order to simplify calculations we assume the Q2D system of carriers having the form of a three-dimensional planar "electron waveguide" of a rectangular cross section, which occupies the coordinate region

$$x \in (-L/2, L/2),\qquad y \in [-W/2, W/2],\qquad z \in [-H/2, H/2]. \tag{1}$$

The length L, the width W, and the height H of the waveguide will be regarded as arbitrary, within the restrictions imposed below. In practice the change of a quantum waveguide thickness implies alteration of the width of the near-surface potential well and, as a consequence, of sheet density of the carriers. Inasmuch as this density is known to follow the variation in depletion voltage under the simple law,[23,24] the results obtained below as a function of the waveguide thickness can be easily related to the experiment.

We will examine the $T=0$ magnetoresistance of a Q2D electron system by expressing the dimensionless (in units of $e^2/\pi\hbar$) conductance g_{xx} in terms of one-particle propagators. Taking the system of units with $\hbar=2m=1$ (m is the electron effective mass), the static conductance is given by

$$\begin{aligned} g_{xx}(L,\mathbf{B}) = \frac{2}{L^2}\int\int d\mathbf{r}\, d\mathbf{r}'\left[\frac{\partial}{\partial x} - \frac{ie}{\hbar c}A_x(\mathbf{r})\right][G^a_{\varepsilon_F}(\mathbf{r},\mathbf{r}') \\ - G^r_{\varepsilon_F}(\mathbf{r},\mathbf{r}')]\left[\frac{\partial}{\partial x'} - \frac{ie}{\hbar c}A_x(\mathbf{r}')\right][G^a_{\varepsilon_F}(\mathbf{r}',\mathbf{r}) \\ - G^r_{\varepsilon_F}(\mathbf{r}',\mathbf{r})], \end{aligned} \tag{2}$$

where $G^{r(a)}_{\varepsilon_F}(\mathbf{r},\mathbf{r}')$ is the retarded (advanced) Green's function of the electrons of Fermi energy ε_F, $A_i(\mathbf{r})$ is the i-th component of the external vector potential $\mathbf{A}(\mathbf{r})$. Integration in Eq. (2) is carried out over region (1) occupied by the quantum waveguide, spin degeneracy is taken into account by the factor of 2. Note that throughout this paper the Fermi energy (or the chemical potential, as the situation requires) will be considered to have a constant value, regardless of the confinement potential. This is undoubtedly true on the metallic side of the MIT discussed below. Moreover, on the just-dielectric side of the transition this assertion is also valid since the "expanded" electron system, which includes the attached leads, in the static limit is in a homogeneous equilibrium state provided that its segment of interest is open, at least, partially.

Within the model of isotropic Fermi liquid, the retarded Green's function of the electrons subjected to a static magnetic field obeys the equation (all indices at the function G are omitted for brevity)

$$\left[\left(\nabla - \frac{2\pi i}{\Phi_0}\mathbf{A}(\mathbf{r})\right)^2 + k_F^2 + i0 - V(\mathbf{r})\right]G(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}'), \tag{3}$$

where $\Phi_0=hc/e$ is the magnetic flux quantum, $k_F^2\equiv\varepsilon_F$, $V(\mathbf{r})$ is the random potential due to impurities or the roughness of the confining well boundaries. This potential is assumed to have zero mean value, $\langle V(\mathbf{r})\rangle=0$, and the binary correlation function $\langle V(\mathbf{r})V(\mathbf{r}')\rangle=\mathcal{Q}\mathcal{W}(\mathbf{r}-\mathbf{r}')$. The angular brackets stand for configurational averaging, $\mathcal{W}(\mathbf{r})$ is the function normalized to unity and falling off its maximal value at $\mathbf{r}=0$ over the characteristic length r_c (the correlation radius).

Equation (3) must be supplemented with the appropriate boundary conditions (BC). We will regard the electrons to be confined by infinitely high potential walls at side boundaries of the region (1) and specify this fact by the Dirichlet conditions

$$G(\mathbf{r},\mathbf{r}')\Big|_{\substack{y=\pm W/2\\ z=\pm H/2}} = 0. \tag{4}$$

As far as open ends of the system are concerned, the BC problem is resolved somewhat less trivially. The mere fact that the system is open, even partially, implies non-Hermicity of the operator in square brackets of Eq. (3) in the domain (1). This may cause some vagueness regarding the applicability of formula (2), whose derivation relies essentially on Hermitian property of the Hamilton operator. In the case of a finite-length system, the hermitizing BC at $x=\pm L/2$ would, in fact, correspond to its closeness (or periodicity) in the x direction, which does not conform with the requirement for current overflow between independent reservoirs.

In this study, in view of the chosen Green's function formalism, to specify the openness of the quantum system we employ the method based on the analogy between the problem (3) and that of the monochromatic point source radiation in a classical waveguide. When solving the latter problem, Sommerfeld's radiation conditions are normally used,[25,26] which imply, for the source positioning at some finite coordinate, the existence of solely outgoing waves at infinity. In order to adapt these conditions to the system under consideration it is necessary to prolong the disordered and magnetic-field-biased segment (1) of the electron waveguide with semi-infinite ideal leads, in which the electron waves generated at some point inside the segment could propagate freely to infinity, not being subjected to any kind of backscattering. Joining of the solutions to Eq. (3) at interfaces $x=\pm L/2$ within the electron waveguide results eventually in the complete solution corresponding to the infinite open system, thereby giving rise to the correct BC at the ends of the segment of interest. This somewhat troublesome procedure was accomplished in Ref. 13, and here we will use the results of that derivation.

Apart from the openness BC, yet another problem is to be resolved before we proceed to the conductance calculation. Specifically, this is the choice of the vector potential gauge corresponding to the in-plane magnetic field $\mathbf{B}=(B_x,B_y,0)$. Commonly used symmetric and Landau gauges appear to be not quite convenient as regards the calculation technique being applied. It was shown previously[13] that the gauge

$$\mathbf{A}(\mathbf{r})=(B_y z,-B_x z,0), \tag{5}$$

is more convenient when handling the system of waveguide configuration, because it enables one to avoid undesirable inhomogeneity of the Hamiltonian in the direction of current (the axis x). Given this gauge, Eq. (3) assumes the form

$$\left[\nabla^2+k_F^2+i0-V(\mathbf{r})-\frac{4\pi i}{\Phi_0}\left(B_y z\frac{\partial}{\partial x}-B_x z\frac{\partial}{\partial y}\right)-\left(\frac{2\pi}{\Phi_0}\right)^2\mathbf{B}^2 z^2\right]G(\mathbf{r},\mathbf{r}')=\delta(\mathbf{r}-\mathbf{r}'). \tag{6}$$

which is well adapted for solving it in mode representation.

Equation (6) contains the potentials of two kinds. The first one is represented by the random function $V(\mathbf{r})$. This potential will be assumed to result in weak disorder-induced scattering (WDS) of the waveguide modes. The weakness implies the fulfillment of the pair of inequalities

$$k_F^{-1},\quad r_c \ll \ell\ , \tag{7}$$

where ℓ stands for the electron mean free path at zero magnetic field. For the reference purpose, this path evaluated from the model of the white-noise Gaussian-distributed potential, whose binary correlation function is $\langle V(\mathbf{r})V(\mathbf{r}')\rangle=\mathcal{Q}\delta(\mathbf{r}-\mathbf{r}')$, equals $4\pi/\mathcal{Q}$.

Another kind of potential is presented by the sum of all the terms in the left-hand side of Eq. (6), which contain the dependence on the magnetic field. These terms can be regarded as introducing the regular potential barrier into the infinitely long quantum waveguide, whose length along the x axis is equal to the entire sample length L. We will refer to this barrier as the magnetically biased region of the waveguide. The scattering produced by this region will be assumed to be in some sense weak as well. By the weakness of the magnetic-field-induced scattering (WMS) we mean small entanglement of the waveguide modes due to the presence of the magnetically biased region. It was shown in Ref. 13 that this implies the inequality

$$\left(\frac{H}{R_c}\right)^2 \ll 1 \tag{8}$$

to hold, where $R_c=k_F l_B^2$ is the maximal classical cyclotron radius, $l_B=\sqrt{\Phi_0/2\pi B}$ is the *total* magnetic length.

III. CALCULATION OF THE MAGNETOCONDUCTANCE

By substituting the Green's functions into Eq. (2) in the form of expansion in series over transverse Hamiltonian eigenfunctions we obtain the following mode representation of the conductance:

$$\begin{aligned}g_{xx}(L,\mathbf{B})=\frac{2}{L^2}\int\int_L dx\,dx'\sum_{\boldsymbol{\nu}_1,\boldsymbol{\nu}_2\boldsymbol{\nu}_3,\boldsymbol{\nu}_4}&\left(\frac{\partial}{\partial x}\delta_{\boldsymbol{\nu}_1\boldsymbol{\nu}_2}-i\varsigma_y\frac{\hat{z}_{\boldsymbol{\nu}_1\boldsymbol{\nu}_2}}{l_y^2}\right)\\ &\times\left[G^a_{\boldsymbol{\nu}_2\boldsymbol{\nu}_3}(x,x')-G^r_{\boldsymbol{\nu}_2\boldsymbol{\nu}_3}(x,x')\right]\\ &\times\left(\frac{\partial}{\partial x'}\delta_{\boldsymbol{\nu}_3\boldsymbol{\nu}_4}-i\varsigma_y\frac{\hat{z}_{\boldsymbol{\nu}_3\boldsymbol{\nu}_4}}{l_y^2}\right)\\ &\times\left[G^a_{\boldsymbol{\nu}_4\boldsymbol{\nu}_1}(x',x)-G^r_{\boldsymbol{\nu}_4\boldsymbol{\nu}_1}(x',x)\right].\end{aligned} \tag{9}$$

Here, $\boldsymbol{\nu}=(n,m)$ is the vector mode index (with $n,m\in\mathbb{N}$) conjugate to the transverse radius-vector $\mathbf{r}_\perp=(y,z)$, $\delta_{\boldsymbol{\nu}_i\boldsymbol{\nu}_j}$ is the vector-argument Kronecker delta, $\hat{z}_{\boldsymbol{\nu}_i\boldsymbol{\nu}_j}$ is the intermode matrix element of the z-coordinate operator, $l_y=\sqrt{\Phi_0/2\pi|B_y|}$ is the *partial* (or directional) magnetic length. Given the model of the confining potential, matrix element $\hat{z}_{\boldsymbol{\nu}_i\boldsymbol{\nu}_j}$ assumes the value

$$\hat{z}_{\boldsymbol{\nu}\boldsymbol{\nu}'}=-H\delta_{nn'}(1-\delta_{mm'})\frac{8mm'}{\pi^2(m^2-m'^2)^2}\sin^2\left[\frac{\pi}{2}(m-m')\right]. \tag{10}$$

If the scattering in the waveguide is weak in general, which implies inequalities (7) and (8) to be held simultaneously, expression (9) can be substantially simplified. In this case all intermode (i.e., nondiagonal in mode indices) propagators are parametrically small as compared to the intramode ones,[13] so they can be omitted. Also, the terms enclosed in parenthesis, which are proportional to the z-coordinate matrix elements $\hat{z}_{\boldsymbol{\nu}_i\boldsymbol{\nu}_j}$, can be omitted owing to their comparative smallness over the WMS parameter (8). In this way we arrive at the relatively uncomplicated form of the magnetoconductance expression, viz.

$$g_{xx}(L,\mathbf{B}) \approx \frac{2}{L^2}\sum_{\boldsymbol{\nu}} \int\int_L dx dx' \frac{\partial}{\partial x}\Big[G^a_{\boldsymbol{\nu\nu}}(x,x') - G^r_{\boldsymbol{\nu\nu}}(x,x')\Big]\frac{\partial}{\partial x'}\Big[G^a_{\boldsymbol{\nu\nu}}(x',x) - G^r_{\boldsymbol{\nu\nu}}(x',x)\Big], \tag{11}$$

which is asymptotically valid under WS (=WDS+WMS) condition.

Formula (11) would enable one to reduce the initially stated three-dimensional dynamic problem to a set of strictly 1D problems provided the equations for all intramode propagators are separated. Fortunately, this indeed can be done, at least in the case of confined systems. The mathematical procedure was suggested in Ref. 11 first for strictly 2D waveguide systems. Subsequently, in Refs. 12 and 13, the method was generalized onto the case of 3D quantum waveguides including those subjected to magnetic field. Referring the reader to those papers for mathematical details, here we merely quote some of the results necessary for further analysis.

The exact form of the equation for intramode propagator in a confined system is quite complicated, see Ref. 13. It was shown there that the solution to this equation is governed substantially by the parameter

$$\varkappa^2_{\boldsymbol{\nu}} = k^2_{\boldsymbol{\nu}} - \frac{H^2}{12 l_B^4}\left(1 - \frac{6}{\pi^2 m^2}\right), \tag{12}$$

where $k^2_{\boldsymbol{\nu}}=\varepsilon_F-(\pi n/W)^2-(\pi m/H)^2$ is the conventional mode energy in the absence of the magnetic field. The effect of this field, which is described by the second term in the right-hand side of Eq. (12), reduces to lessening of the lengthwise energies of the modes. In the case where mode energy (12) has a negative sign, intramode propagator $G_{\boldsymbol{\nu\nu}}$ is to the parametric accuracy given by

$$G_{\boldsymbol{\nu\nu}}(x,x') \approx \frac{-1}{2|\varkappa_{\boldsymbol{\nu}}|}\exp(-|\varkappa_{\boldsymbol{\nu}}||x-x'|), \tag{13}$$

which corresponds to strongly localized *evanescent* modes. If the mode energy assumes a positive value, the equation for $G_{\boldsymbol{\nu\nu}}(x,x')$ has the (asymptotic) form

$$\left(\frac{\partial^2}{\partial x^2} + \varkappa^2_{\boldsymbol{\nu}} + i/\tau^{(\varphi)}_{\boldsymbol{\nu}}\right) G_{\boldsymbol{\nu\nu}}(x,x') = \delta(x-x'). \tag{14}$$

The imaginary addend to the mode energy in this equation arises due to the pair of factors, namely, the openness of the system, on the one hand, and the scattering between *extended* modes, on the other. In the particular case where the correlation function of the disorder potential is chosen so as

$$\mathcal{W}(\mathbf{r}-\mathbf{r}') = \mathcal{W}(x-x')\,\delta(\mathbf{r}_\perp - \mathbf{r}'_\perp), \tag{15}$$

the dephasing term in Eq. (14) is given by

$$\frac{1}{\tau^{(\varphi)}_{\boldsymbol{\nu}}} = \frac{\mathcal{Q}}{4S}\overline{\sum_{\boldsymbol{\nu}'\neq\boldsymbol{\nu}}}\frac{1}{\varkappa_{\boldsymbol{\nu}'}}\Big[\widetilde{\mathcal{W}}(\varkappa_{\boldsymbol{\nu}} - \varkappa_{\boldsymbol{\nu}'}) + \widetilde{\mathcal{W}}(\varkappa_{\boldsymbol{\nu}} + \varkappa_{\boldsymbol{\nu}'})\Big]. \tag{16}$$

Here $\widetilde{\mathcal{W}}(\varkappa)$ is the Fourier transform of the function $\mathcal{W}(x)$ from Eq. (15), the bar over the summation index signifies the summation over extended modes only, if any. In the case where the electron waveguide has no or only one extended mode, scattering rate (16) is exactly equal to zero, which implies the entire set of the mode states of the system under consideration to be coherent.

The solution to Eq. (14) for the open system considered here, which is valid under WS conditions, is written as

$$G_{\boldsymbol{\nu\nu}}(x,x') \approx \frac{1}{2i\varkappa_{\boldsymbol{\nu}}}\exp[(i\varkappa_{\boldsymbol{\nu}} - 1/l^{(\varphi)}_{\boldsymbol{\nu}})|x-x'|], \tag{17}$$

where $l^{(\varphi)}_{\boldsymbol{\nu}}=2\varkappa_{\boldsymbol{\nu}}\tau^{(\varphi)}_{\boldsymbol{\nu}}$ is the parameter which may be interpreted as the length of the mode $\boldsymbol{\nu}$ phase coherence. With function (17) substituted into Eq. (11), the average magnetoconductance is given ultimately as

$$\langle g_{xx}(L,\mathbf{B})\rangle = \overline{\sum_{\boldsymbol{\nu}}}\frac{l^{(\varphi)}_{\boldsymbol{\nu}}}{L}\left[1 - \frac{l^{(\varphi)}_{\boldsymbol{\nu}}}{L}\exp\left(-\frac{L}{l^{(\varphi)}_{\boldsymbol{\nu}}}\right)\sinh\frac{L}{l^{(\varphi)}_{\boldsymbol{\nu}}}\right]. \tag{18}$$

Here, the terms corresponding to extended modes only are kept since the evanescent-mode Green's functions (13), being real valued, cancel each other in Eq. (11). Formula (18) represents the general expression describing the system conductance in the presence of both the random scatterers and the weak in-plane magnetic field, which enters implicitly through the mode coherence lengths.

From the result (18), conventional limiting formulae for the conductance can readily be obtained. In particular, in ballistic limit $L \ll \ell$ the dimensionless conductance becomes nearly equal to the number of open conducting channels $N_c(\mathbf{B})$

$$\langle g_{xx}(L,\mathbf{B})\rangle \approx N_c(\mathbf{B}). \tag{19}$$

In our model this number is determined by both the geometric confinement of the electron system and the magnetic field.[13] The conductance of the perfect system is thus running in steps as a function of either depletion voltage or the value of the in-plane magnetic field.

In diffusion limit $L \gg \ell$, if the potential well is wide enough to contain the large number of quantization levels in the z direction, by replacing the sum in the right-hand side (r.h.s.) of Eq. (18) with the integral we arrive at

$$\langle g_{xx}(L,\mathbf{B})\rangle \approx \frac{4}{3}\frac{N_c(\mathbf{B})\ell}{L} \approx \langle g_{xx}(L,0)\rangle\left(1 - \frac{H^2}{12R_c^2}\right). \tag{20}$$

In the case of zero magnetic field this result is coincident in form with classical Drude conductance.[12] If $\mathbf{B}\neq 0$, the magnetoconductance is negative, being varied smoothly with the magnetic field, specifically, under the quadratic law. This is because the van Hove singularities in the mode density of states (MDOS) prove to be integrated out in such a rough calculation.

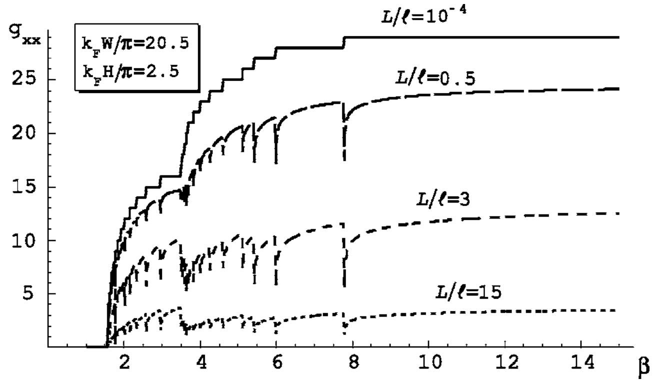

FIG. 1. The conductance versus inverse magnetic field at different disorder level in the quantum waveguide.

However, the singularities are actually contained in the mode coherence lengths, as they are determined using the dephasing rate formula (16). These singularities should appear both in Shubnikov-de Haas (i.e., magnetic-field-driven) oscillations of the conductance and in the conductance dependence on the quantum well width, which is normally tuned by the depletion voltage. In Fig. 1, the results numerically obtained from Eq. (18) at several values of the diffusion parameter L/ℓ are presented. The magnetic field, assumed to be codirectional with the axis of current flow, is scaled as the dimensionless parameter $\beta=(k_F l_B)^2=k_F R_c$. Numerical considerations reveal that in-plane rotation of the magnetic field slightly changes the picture presented. The MDOS singularities show themselves in the form of sharp dips placed close to those points where the number of conducting modes undergoes stepwise variations, i.e., close to the thresholds of the transverse subbands, no matter what the disorder level may be. The disorder appears to manifest itself, above all, through the absolute value of the conductance. Note that in the case of relatively large disorder (larger values of L/ℓ) the conductance develops nonmonotonically versus the magnetic field, even if the MDOS singularities are smoothed out. As the mean free path decreases, nonmonotonicity becomes so apparent that it appears to be inadmissible to disregard this effect in the experimental data. In particular, with regard to this analysis it would be tempting to revise the observed positive magnetoresistance which is frequently attributed to spin properties of 2D systems.[28,29]

To make a comparison with the magnetic-field run, in Fig. 2 the conductance is shown versus the width of the potential well forming a quasi-2D quantum waveguide. Here, the MDOS singularities are equally more pronounced, which naturally demonstrates the change in the number of conducting channels. Meanwhile, in the latter case, in contrast to the magnetic-field dependence depicted in Fig. 1, these singularities are completely anticipated from the very outset, since the number of channels is normally associated with size quantization.

The peculiar feature should be noted in Fig. 2 as against the dependence on the magnetic field. The curves in Fig. 2 tend to become nonmonotonic, on average, as the magnetic field grows. Evidently, this nonmonotonicity accounts for the nonmonotonic dependence on H of the mode eigenenergy (12).

It is instructive to dwell upon the physical nature of dips in Figs. 1 and 2. All of them are positioned in the vicinity of the points corresponding to opening/closing of the conducting channels. For one thing, as the electron waveguide gets thinner or the magnetic field grows, the closing of the channel must result in a benchlike fall of the conductance, since each of the conducting channels is expected to bring in exactly one conductance quantum. At the critical point, the marginal extended mode is transformed to the evanescent one, which is localized at a scale of the mode wavelength and in this way carry no current in an infinitely long system. Indeed, this picture is demonstrated by the upper, "ballistic," curve in Fig. 1.

For another thing, in approaching the transformation point (subband threshold) the MDOS of the marginal mode diverges whereas its mode velocity tends to zero. Being singularly capacious and slow, this mode serves as a destructive sink for dynamic electrons, leading to a decrease in the conductance. It is clear that the dips can only arise when we deal with an imperfect system of carriers, where scattering is allowed from all remaining extended modes to the slow critical mode. The conductance in the bottom of the dip may thus reach, hyperbolically, a nearly zero value, which can hardly be grasped in real experiments because of various unaccounted extra factors.

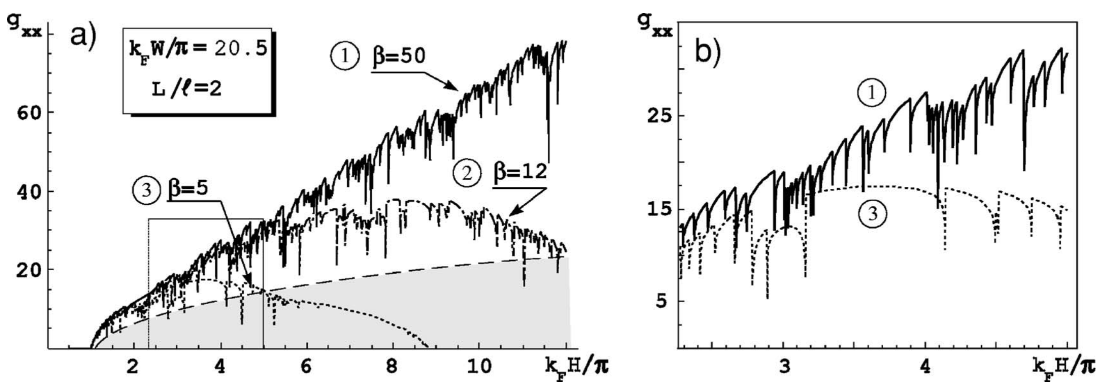

FIG. 2. The conductance versus the width of the near-surface potential well at different values of the in-plane magnetic field. In panel (b), the region selected by the rectangle in panel (a), with only two curves left, is zoomed in to make the dips due to MDOS singularities more clearly visible. The shading below the dashed curve signifies the region where criterion (8) is violated.

IV. DISCUSSION AND CONCLUDING REMARKS

We have demonstrated that the appreciable localizing effect produced by the relatively weak in-plane magnetic field on 2D electron and hole systems can be rationally interpreted in the context of Fermi liquid theory applied to spinless electrons which reside in an open near-surface potential well of finite rather than zero width. For a relatively weak magnetic field, its coupling to the carrier orbital degree of freedom, even though it is quite insignificant from a semiclassical point of view, proves to have a substantial influence on the carrier spectrum and, hence, on the conductance.

The conclusion about strong sensitivity of the carrier spectrum to the in-plane magnetic field is made from the analogy of tightly gated solid-state systems to classical waveguiding systems of planar, though three-dimensional, configuration. The mode content of these well-known objects is quite sensitive to the anisotropy in their cross section, that is to the applied gate voltage as far as electron devices are concerned. A remarkable feature of the latter type of systems is that provided that the magnetic field is applied to the finite length, these systems can be thought of as being subjected to both the random disorder potential, whose correlation length can be arbitrary, and the additive strongly nonlocal deterministic "magnetic" potential barrier. Scattering parameters of this barrier are specified by the magnetic field strength and orientation, on the one hand, and by the length of a magnetically biased section of the quantum waveguide, on the other. The effect produced by the barrier results from the mismatching of electron spectra in the inner and outer parts of the quantum well, the inner part representing the finite-length electron system under consideration.

Assuming the electron waveguide cross section to be highly anisotropic, the bulk of transverse modes in the electron spectrum can be efficiently transformed from extended to evanescent type as the magnetic field grows slightly.[13] This is because electron scattering from side boundaries of the confining potential well is, in the strict sense, specular if the boundaries are considered as completely inhibiting the transverse current flow. In view of this fact, the magnetic field effect on the electron system is improper to be assessed through the estimation of the electron trajectory deviation between successive collisions with quantum well boundaries. Rather, the mode phase coherence length seems to be the appropriate spatial scale, whatever the quantum waveguide transverse dimension.

It can be easily verified from Eq. (12) that the mode truncation effect of the magnetic field is the more significant the larger the aspect ratio of the waveguide cross section, given the dimension H. For H being small enough, such that only modes with quantization number $m=1$ in the corresponding direction can be regarded as extended, the total number of extended modes in the quantum waveguide is mainly determined by the larger cross-section dimension W. Owing to strong cross-section anisotropy, even a slight alteration of the magnetic field can transform a considerable number of modes from extended to evanescent type, thus leading to a significant reduction of the conductance, even though it might have a large (ballistic) value in the absence of the magnetic field.

The mode truncation effect of the in-plane magnetic field is quite similar to that of truly geometric confinement of the electron system. The closing of each of the current-carrying modes, which makes itself evident in the form of conductance jumps by precisely one conductance quantum in a perfect system at a zero temperature, should be regarded as a true quantum phase transition.[30] This statement is substantiated by the indisputable fact that there exists a well-defined correlation length in the vicinity of a closing point, whose role is played by the wavelength of the marginal extended mode. This correlation scale, as it must, tends to diverge as the critical point is approached. The closing of the last conducting mode by means of the in-plane magnetic field may thus be regarded as the magnetic-field-driven MIT.

It should be noted that in the proximity to the MIT the conductance is not quite accurately described by the present theory, since it is hard to satisfy the WMS conditions over the corresponding range of system parameters. Yet, closer examination of the magnetic-field-originated effective potentials in the decoupled equations for intramode propagators (see Ref. 13) indicates that the above-described mode truncation effect and, hence, the very fact of the existence of magnetic-field-driven MIT, is robust.

In conclusion, the remark should be made concerning the model of the confinement potential adopted in this study. In some papers where laterally confined electron systems are dealt with (see, e.g., Ref. 27), this potential is taken as a quadratic function of transverse coordinates. The confinement thus modeled seems to be beneficial from the technical point of view, as it enables one to account for the magnetic field nonperturbatively, the corresponding transverse eigenstates being known as Fock-Darwin levels.[31,32] It is quite natural that the precisely zero width of those levels in the absence of any disorder implies the entire lack of dephasing due to the magnetic field only. In this case, the magnetoconductance should not exhibit a dip structure because the latter results from MDOS singularities arising exclusively in the presence of the disorder.

Clearly, in the domain of weak magnetic fields [in a sense of inequality (8)] the quadratic confinement can hardly be substantiated. Therefore, the issue of the magnetic-field-induced mode entanglement and the related conceivable dephasing of the mode states might seem to be quite topical in this limiting case. Note, however, that although other kinds of confinement possess appreciable magnetic-field-induced intermode scattering, the rectangular one we choose in this study does not prove to result in widening the transverse quantization levels unless some kind of disorder is also taken into account.[13] One should bear in mind that the exact form of transverse eigenfunctions is of no fundamental significance for the development of transport theories in mode representation, but the mere fact of transverse energy quantization does matter. This prompts us to expect that the magnetic field alone cannot give rise to noticeable decoherence of electron states for any hard-wall model of the electron confinement. The specific form of the confinement potential can only rearrange the transverse quantization levels and, hence, to exert an influence upon the coherence properties of the electron system indirectly. Of primary value is the dephasing that results from scattering caused by some kind of a *random* (i.e., in a sense, uncontrolled) potential, no matter static or variable it may be.

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