Scaling of cross-over currents in current-voltage characteristics of $YBa_2Cu_3O_{7-\delta}$ films

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We investigate current-voltage measurements of a superconductor in a magnetic field for a continuous superconducting transition. Existence of such a transition in the mixed state has been the subject of recent controversy due to flexibility in the conventional scaling analysis. To address this, we analyze current-voltage data using scaling forms based on the crossover current. One of these scaling forms, based on the logarithmic derivative of current-voltage isotherms, is a stringent test for a superconducting transition. Applying this derivative scaling test to the data shows marked disagreement with a superconducting transition, which indicates that one does not occur within the mixed state.

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There has been recent controversy surrounding the experimental determination of continuous superconducting-phase transitions according to the scaling theories of Fisher, Fisher, and Huse.[1] The common test of scaling is a data collapse of current-voltage (I-V) measurements relating the electric field E to an applied current density J,

$$E\xi^{2+z-D}/J = \rho(J,T)\xi^{2+z-D} = \chi_\pm(J\xi^{D-1}/T), \tag{1}$$

where D is the dimensionality, T is the temperature, $\xi \sim |1-T/T_g|^{-\nu}$ is the diverging correlation length near the transition temperature (T_g), ν is the static critical exponent, z is the dynamic exponent, $\rho(J,T)$ is the nonlinear resistivity, and $\chi_\pm$ are scaling functions.

Some have argued that this scaling analysis is too flexible and not conclusive evidence for a phase transition.[2–5] However, these arguments do not address the apparent scaling of a crossover current,[6–8] which has been argued as crucial to determining a superconducting transition,[7,8] with debate on this issue still ongoing.[9] One such crossover-current determination is to locate where the value of the logarithmic derivative $(\partial \log E/\partial \log J)_T$ is an arbitrarily chosen constant.[6] A second method, which is limited to the regime $T > T_g$, defines the crossover current as the location where $\rho(J,T)$ divided by the linear resistivity $\rho_L(T)$ (measured at low J) is an arbitrarily chosen constant.[7,8]

A limitation of these methods is that they depend on arbitrarily chosen constants. We show that these crossover currents stem from general scaling forms for $(\partial \log E/\partial \log J)_T$ and $\rho(J,T)/\rho_L(T)$. We find that the crossover scaling for $\rho(J,T)/\rho_L(T)$ shows flexibility reminiscent of other scaling tests;[2–5] i.e., agreement is found for widely different critical exponents and transition temperatures. In contrast, the scaling test of $(\partial \log E/\partial \log J)_T$ does not agree with a phase transition over the same range of temperatures, which supports the view that a superconducting transition does not exist within the mixed state.[2–5,10]

A crossover-current scaling form for $(\partial \log E/\partial \log J)_T$ is made by taking the natural log of Eq. (1) and then taking the partial derivative with respect to $\ln E$,[11]

$$\ln E = (D-2-z)\ln \xi + \ln J + \ln \chi_\pm(J\xi^{D-1}/T), \tag{2}$$

$$\left(\frac{\partial \ln E}{\partial \ln J}\right)_T = 1 + \frac{\partial[\ln \chi_\pm(J\xi^{D-1}/T)]}{\partial[J\xi^{D-1}/T]}\frac{\partial[e^{\ln J}\xi^{D-1}/T]}{\partial[\ln J]}, \tag{3}$$

$$\left(\frac{\partial \log E}{\partial \log J}\right)_T = \left(\frac{\partial \ln E}{\partial \ln J}\right)_T = F_\pm(J\xi^{D-1}/T), \tag{4}$$

where the explicit z dependence has dropped out. Equation (4) reduces to the crossover-current scaling of Ref. 6 by setting the left side equal to an arbitrary constant. This requires that the argument of $F_\pm$ on the right side is also a constant that yields (assuming $J\xi^{D-1}/T \approx J\xi^{D-1}/T_g$ near the transition),

$$J_c \propto |1 - T/T_g|^{\nu(D-1)}. \tag{5}$$

Another crossover-current method is based on $\rho(J,T)$ in Eq. (1). The linear resistivity is defined as $\rho_L(T) \equiv \rho(J,T)_{\lim J\to 0} \propto \xi^{D-2-z}$ for $T > T_g$. The ratio of $\rho(J,T)$ over $\rho_L(T)$ yields another scaling form,

$$\frac{\rho(J,T)}{\rho_L(T)} = G_+(J\xi^{D-1}/T), \tag{6}$$

where again the explicit z dependance has dropped out. Eq. (6) is similar to Eq. (4) in that the ξ dependance is limited to the argument of the scaling function. Thus, the same arguments used to determine a crossover current via Eq. (4) are valid, and Eq. (6) also leads to Eq. (5) for an arbitrarily chosen constant. This is the crossover current scaling for an arbitrarily chosen ratio $\rho(J,T)/\rho_L(T)$ that is investigated in Refs. 7 and 8.

Equations (4) and (6) are useful because they probe the crossover current behavior without requiring arbitrarily chosen constants. These relations test all possible constants simultaneously. The logarithmic derivative of Eq. (4) is also useful because it is valid at all temperatures near a transition; whereas the scaling of $\rho(J,T)/\rho_L(T)$ in Eq. (6) is limited to the regime high enough above T_g such that $\rho_L(T)$ can be measured.

Another issue with Eq. (6) is that $\rho(J,T)/\rho_L(T)$ will trivially scale at low J. Since I-V curves above T_g have Ohmic

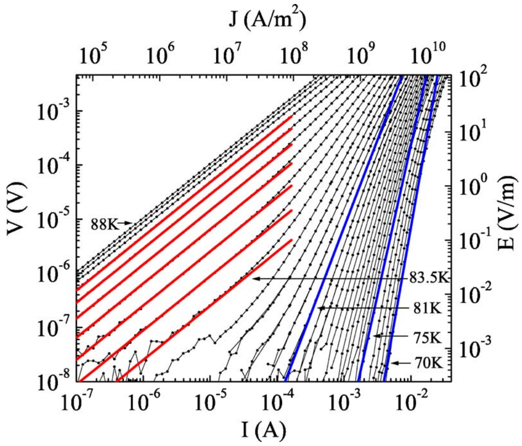

FIG. 1. (Color online) I-V isotherms for a 2200 Å $YBa_2Cu_3O_{7-\delta}$ film at 4 T (data from Ref. 2). The linear fits for $T \geqslant 83.5$ K determine $\rho_L(T)$ at low currents. The lines at 81, 75, and 70 K are power-law fits.

tails, $\rho(J,T)/\rho_L(T)=1$ as $J\rightarrow 0$, and the ratio will always satisfy Eq. (6) at low currents. To circumvent this trivial scaling we subtract it (i.e., 1) away from Eq. (6), yielding

$$\frac{\rho(J,T)}{\rho_L(T)} - 1 = \mathcal{G}_+(J\xi^{D-1}/T), \tag{7}$$

which is in terms of a new scaling function $\mathcal{G}_+$. The fact that the scaling in Refs. 7 and 8 was achieved with a choice for $\rho(J,T)/\rho_L(T)=1.1\approx 1$ attests to the relevance of this issue.[12]

We now investigate the crossover-current scaling using Eqs. (4) and (7) with I-V measurements (Fig. 1) for a 2200 Å thick film in 4 Tesla from Ref. 2. It has already been shown that these I-V measurements can be scaled according to Eq. (1) using a large range of transition temperatures and exponents—demonstrating the difficulty in determining the existence of a transition through this method.[2] Specifically, the data can be scaled about the apparent power-law behaviors at 81, 75, and 70 K (see the plots in Ref. 2).

This is in contrast to the logarithmic derivative form (Eq. (4)), which shows in Fig. 2(a) marked deviations from scaling when using the conventional choice of critical parameters. Figures 2(b) and 2(c) demonstrate that this is also the case for the other combinations of exponents and transition temperatures that successfully scale the data in Ref. 2 over identical ranges. This suggests that the crossover-current scaling is not satisfied for arbitrary choices of constant $(\partial \log E/\partial \log J)_T$, despite the fact that a convenient choice for a constant can give apparent agreement with a transition. This also supports the view that I-V measurements do not represent a superconducting transition within the mixed state.

Moving on to the scaling of $\rho(J,T)/\rho_L(T)$, Fig. 3 shows the data according to Eq. (7) and Eq. (6) (insets), with the same exponents used in Fig. 2. The data ranges are the same as in Fig. 2 (and in Ref. 2) except for being limited to $T \geqslant 83.5$ K, a temperature regime with well determined $\rho_L(T)$ from the linear fits in Fig. 1.[13] These scaling plots show notably much better agreement with a transition than those of

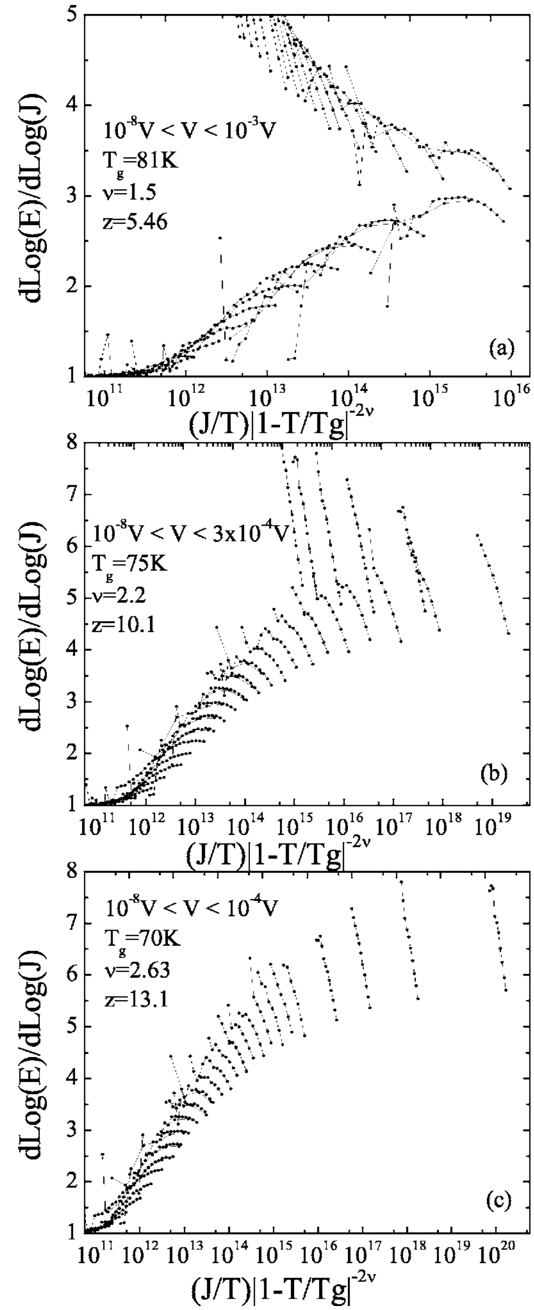

FIG. 2. Attempts at scaling the logarithmic derivative according to Eq. (4) using the critical parameters that collapsed the same data to the conventional I-V scaling form in Ref. 2.

Fig. 2—indicating that it is a more lenient test of a transition. The deviations to scaling at low values of $\rho(J,T)/\rho_L(T)-1$ in Fig. 3 result from the finite precision of the resistance measurements, as the scatter in the isotherms is much larger in this regime.

Slightly above this scattered regime, the scaling shows analytic behavior for small $\rho(J,T)/\rho_L(T)-1$. Expanding $\rho(J,T)$ for a fixed temperature about $J=0$ yields

$$\rho(J,T) = \rho_L(T) + \rho_2(T)J^2 + \rho_4(T)J^4 + \cdots . \tag{8}$$

This can be cast into the form of Eq. (7) as

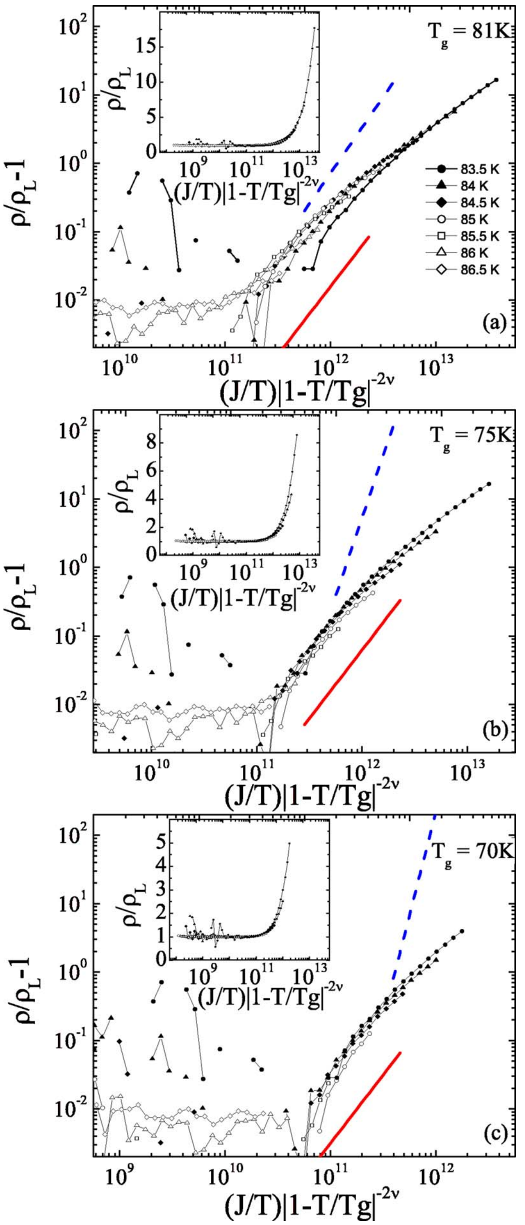

FIG. 3. (Color online) Data of Fig. 1 plotted according to scaling form of Eq. (7) with the insets in form of Eq. (6) by using the same ranges, exponents, and T_g's that collapse the data in Fig. 2 for $T \geqslant 83.5$ K.

$$\rho(J,T)/\rho_L(T) - 1 = \frac{\rho_2(T)}{\rho_L(T)}J^2 + \frac{\rho_4(T)}{\rho_L(T)}J^4 + \cdots , \qquad (9)$$

where we have set odd terms to zero since the resistance should be an even function of current. If the right-hand-side of the above expansion is to behave according to the scaling of Eq. (7), then the coefficients $\rho_n(T)/\rho_L(T)$ must behave as $C_n(\xi^{D-1}/T)^n$, where the C_n are unspecified constants. To lowest order the expansion becomes

$$\rho(J,T)/\rho_L(T) - 1 = C_2(J\xi^{D-1}/T)^2 + \cdots . \qquad (10)$$

The term in parentheses on the right side is the x-axis variable of Fig. 3, implying a power of 2 for small values on this plot. This is depicted in Fig. 3 with solid straight lines of slope 2 drawn below the data in the plots. The data show reasonable agreement with this power-law at small values, in accord with scaling.

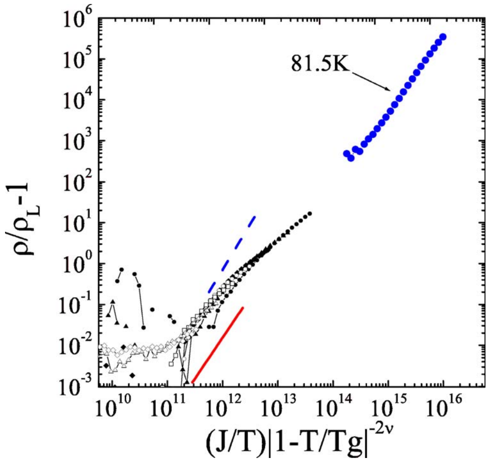

FIG. 4. (Color online) $\rho(J,T)/\rho_L(T)-1$ for the isotherm at 81 K plotted with the extrapolated value for $\rho_L(T)$.

At larger values of $\rho(J,T)/\rho_L(T)$—1 we also expect power-law behavior. Near a transition, $E \sim J^{(z+1)/(D-1)}$ in the high-current regime so that $\rho(J,T)/\rho_L(T) \sim f(T)J^{(z+2-D)/(D-1)}$ where $f(T)$ is a function of temperature. Maintaining consistency with Eq. (6), $f(T)$ must go as $(\xi^{D-1}/T)^{(z+2-D)/(D-1)}$ and

$$\rho(J,T)/\rho_L(T) - 1 \sim \rho(J,T)/\rho_L(T) \sim (J\xi^{D-1}/T)^{(z+2-D)/(D-1)}, \qquad (11)$$

for large J. This power law is depicted as the dashed line above the data in Fig. 3(a) for $D=3$ and $z=5.46$. Figures 3(b) and 3(c) also show high-current power-law fits for the corresponding dynamic exponents. In all plots of Fig. 3, the data deviate away from this power-law behavior at high J, which could be taken as a disagreement from scaling. Another possibility is that J has not attained a large enough value to make this behavior apparent.

Following this second line of reasoning, a more sensitive measurement of $\rho_L(T)$ closer to a transition temperature might probe regions with even larger $J\xi^{D-1}/T$ and yield agreement with the power-law dependence of Eq. (11). To see if this possibility is consistent with the scaling plot of Fig. 3(a) we determined a value for $\rho_L(T)$ by using the extrapolation procedure described in Ref. 2 for the isotherm at 81.5 K, a temperature far too low to determine the linear resistance experimentally. In Fig. 4 we plot the experimentally determined $\rho(J,T)$ and the extrapolated value of $\rho_L(T)$ in the form of $\rho(I,T)/\rho_L(T)-1$ versus $J\xi^{D-1}/T$ for the isotherm of 81.5 K. When plotted together with the data at higher temperatures, it appears consistent with the view that the data of Fig. 3(a) is not in the large-J regime, where the power-law behavior of Eq. (11) applies. The same holds for the other two data collapses in Fig. 3 with a T_g of 75 and 70 K. Thus, the discrepancies between the dashed lines and the data collapses at large J in Fig. 3 are not necessarily disagreements with scaling since the applied current may not be large enough.

In conclusion, we have proposed using two crossover-current scaling forms to test *I-V* characteristics for continuous superconducting transitions. Both these scaling forms, which are consistent with the original vortex-glass model,[1,14] eliminate the use of one scaling parameter in the analysis and, thus, reduce the flexibility of the analysis. One of these scaling forms, based on the logarithmic derivative, appears to be a much less lenient test of a phase transition. The Application of this logarithmic-derivative test to *I-V* characteristics of a YBCO film indicates that unambiguous scaling evidence for a continuous phase transition in the mixed state does not yet exist.[2,10]

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[13] The $\rho_L(T)$ used here and determined from the linear fits at low J in Fig. 1 are consistent with scaling of $\rho_L(T) \sim (T/T_g - 1)^{\nu(z-1)}$ shown in Ref. 2. We exclude $\rho_L(T)$ at 83 K in the analysis, as it has a large error that adversely affects the scaling in all plots of Fig. 3.
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