Penetration depth anisotropy due to the proximity effect in a *d*-density-wave scenario

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We calculate the penetration depth λ along the *a* and *b* directions in the presence of the pseudogap and superconducting phases for a simple model that incorporates two layers—a CuO_2 plane and a CuO chain per unit cell. The CuO chains become superconducting due to the proximity to the planes below the critical temperature. The pseudogap phase has been considered to be a result of the *d*-density-wave (DDW) phase. The temperature dependence of λ_a is always different from λ_b as it depends on the induced gap in the chains. The DDW phase plays a vital role not only to distinguish the temperature dependence of λ_b from that of λ_a but also to identify the mixed phase from the pure phase of these superconductors.

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The search for the mechanism which causes the pseudogap phenomena in the high- T_c cuprates has been an ongoing effort and is yet to reach a general consensus. Several experimental measurements and theoretical works are being carried out to elucidate this suppression of the spectral weight below a characteristic temperature T^* in the underdoped cuprates. Hence we observe the pseudogap phase below T^*. Recently there have been various proposals to understand the nature of the pseudogap. One concrete proposal has been the broken symmetry of the $d_{x^2-y^2}$ type in the particle-hole channel.[1] Various theoretical works have been performed on the $d_{x^2-y^2}$ density waves (DDW) to model the pseudogap.[2–5] In this scenario the temperature T^* is not a crossover but a transition temperature. A large Nernst effect is a clear signature of the DDW. The measured Nernst effect in the pseudogap phase was successfully described in underdoped LSCO and Bi-2212.[5] There have also been various other scenarios for this phenomena based on the crossover picture, such as the singlet pairing of the spinons,[6] phase fluctuations,[7] etc. In Ref. 6 the pairing of the spinons occur at T^* which describes the magnetic properties within an extended *t-J* model in the slave-boson representation. The mechanism suggested by Emery and Kivelson[7] indicates a phase fluctuation of the superconducting order parameter in the pseudogap regime. The temperature scale of phase fluctuations is controlled by the zero-temperature superfluid stiffness. Unlike the conventional superconductors, the transition is controlled by phase ordering and explains the Uemura scaling.[8] It is important to mention that in the regime of interest, it is the vanishing of the superfluid density that controls the transition and determines T_c and not the collapse of the gap. The pseudogap phase has also been studied as a precursor of superconductivity in Ref. 9 where the pseudogap state has been described as a natural consequence of local, dynamic pairing correlations in the normal state of low-density small pair-size superconductors. In this scenario they have shown that unlike the BCS (Bardeen-Cooper-Schrieffer theory) case the pairing and the phase coherence of the $q=0$ pair state takes place at two temperatures. The phase coherence is lost at the critical temperature T_c but the pairs dissociate only at a much higher temperature T^*.

In our previous work,[2] we considered the DDW as the pseudogap along with $d_{x^2-y^2}$-wave superconducting phase (DSC) and studied the phase diagram to understand this new phase in the underdoped region of the cuprates. Various other properties were also calculated. The essence of this idea is that the pseudogap observed in underdoped cuprates is a real gap in the one-particle excitation spectrum at the wave vector $(\pi,0)$. However, it is "pseudo" in experiments due to the difficulty in its detection. It is widely believed that the source of the pairing interaction which is responsible for the superconducting transition lies in the CuO_2 planes and hence could be explained by the two-dimensional model. The only active pieces of the crystal are the CuO_2 planes and the remaining ions act as charge reservoirs. But in some materials, such as the $YBa_2Cu_3O_7$ (Y-123) and $YBa_2Cu_4O_8$ (Y-124), there is clear evidence of the fact that CuO_2 planes are not the only active portions. In these materials we have the quasi one-dimensional CuO chain structures. Experiments on the dc resistivity,[10,11] the polarized reflectance,[12,13] and penetration depth[13–15] have found large anisotropies between the *a* and the *b* directions which suggest that currents also flow along the chains in both the normal and superconducting states. But little is known about the coupling of the planes and the chains and how the planar superconductivity affects the physics in the chains. Recently, the proximity model[16] has been demonstrated to explain consistently the scanning tunneling microscopy, angle-resolved photoemission, and infrared-spectroscopy experiments.

In this work, we calculate the penetration depth of the mixed (DDW+DSC) and the pure DSC states using a similar proximity model for the chain-plane system. The model considered in this work is different from that of Refs. 1 and 2 as we consider the CuO_2 plane, and the CuO chains along with the effect of the proximity coupling of chains and planes which is of the utmost importance for some of the cuprates as discussed above. However, in Ref. 3 they considered a bilayer lattice model which includes pair hopping and next-neighbor repulsion. So with this in view, we study the penetration depth along the *a* and *b* directions to observe the effect of the anisotropy due to the chains and the characteristic due to the presence of the DDW wave which is the pseudogap phase in the cuprates.

The chains become superconducting by a proximity effect

and the size of the induced gap depends on the strength of the coupling across the junction. We consider the pairing interaction to be solely localized to the CuO_2 planes. We neglect the k dependence of the plain-chain coupling, since this term is important in a very small k space region. Hence, omitting the k_z dependence, we can write down the Hamiltonian as

$$H = \sum_{\mathbf{k}\sigma\lambda} (\varepsilon_{\lambda\mathbf{k}} - \mu_\lambda) c^\dagger_{\lambda\mathbf{k}\sigma} c_{\lambda\mathbf{k}\sigma} + \sum_{k\sigma} t(k) c^\dagger_{1\mathbf{k}\sigma} c_{2\mathbf{k}\sigma} + \sum_{\mathbf{k}\sigma} W_{\mathbf{k}} c^\dagger_{1\mathbf{k}\sigma} c_{1\mathbf{k}+\mathbf{Q}\sigma} + \sum_{\mathbf{k}} \Delta_{\mathbf{k}} c^\dagger_{1\mathbf{k}\uparrow} c^\dagger_{1-\mathbf{k}\downarrow} + \text{H.c.}, \quad (1)$$

where $c^\dagger_{1\mathbf{k}\sigma}$ is the fermion creation operator for wave vector $\mathbf{k}$ and spin σ in the plane and $c^\dagger_{2\mathbf{k}\sigma}$ is the same for the chain band. $W_{\mathbf{k}}$ and $\Delta_{\mathbf{k}}$ are the density wave gap and superconducting gap, respectively. $\mathbf{Q}$ is the nesting vector (π,π). The energy dispersion relations are given by $\epsilon_{1\mathbf{k}} = -2t_1[\cos(k_x a) + \cos(k_y b)]$ and $\epsilon_{2\mathbf{k}} = -2t_2 \cos(k_y b)$ where a and b are the lattice constants in the planes. μ_λ is the chemical potential for the plane and the chain bands. $t(k)$ is the coupling between the planes and the chains. λ corresponds to the chain and/or plane bands.

We shall calculate the penetration depth which is given by

$$\lambda^{-2}_{ij} = \frac{8\pi e^2}{\hbar^2 c^2} \frac{1}{\Omega} \sum_{\mathbf{k},\alpha\beta} \hat{\gamma}^{(i)}_{\mathbf{k}\alpha\beta} \hat{\gamma}^{(j)}_{\mathbf{k}\beta\alpha} (\hat{G}_{\mathbf{k}\alpha\beta}|_{\Delta=0} - \hat{G}_{\mathbf{k}\alpha\beta}), \quad (2)$$

where

$$\hat{G}_{\mathbf{k}\alpha\beta} \equiv \frac{\partial f(E_{\mathbf{k},\alpha})}{\partial E_{\mathbf{k},\alpha}} \delta_{\alpha\beta} + \frac{f(E_{\mathbf{k},\alpha}) - f(E_{\mathbf{k},\beta})}{E_{\mathbf{k},\alpha} - E_{\mathbf{k},\beta}} (1 - \delta_{\alpha\beta}),$$

$$\gamma^{(i)}_{\mathbf{k}\alpha\beta} \equiv \sum_{\gamma\delta} \hat{U}^\dagger_{\mathbf{k}\alpha\gamma} \frac{\partial \varepsilon_{\mathbf{k},\gamma\delta}}{\partial k_i} \hat{U}_{\mathbf{k}\delta\beta}.$$

$E_{\mathbf{k},\alpha}$ are the eigenvalues, $f(x)$ is the Fermi function, $\delta_{\alpha\beta}$ is a Kronecker delta, and $\hat{U}_{\mathbf{k}\alpha\gamma}$ is the unitary matrix which diagonalizes the Hamiltonian.

We would like to address the question of anisotropy within the scenario of the proximity model with the inclusion of the DDW wave. This problem was dealt with in the papers of Ref. 17 but they did not consider the pseudogap phase. We see that with the consideration of the pseudogap, the proximity model can explain the anisotropy for the underdoped, optimally doped as well as the overdoped systems with changes in the value of t and other parameters. Moreover the proximity model[16] are consistent with the recent scanning tunneling microscopy, angle-resolved photoemission, and infrared spectroscopy experimental results.

We determine the phase diagram in the presence of the DDW state by numerically diagonalizing the Hamiltonian and then solving simultaneously for the gap and μ_1. We find our results to be in accord with our previous findings.[2] The parameters of the model are $V_{PG}=0.9$ and $V_{SC}=0.6$ which represent the interaction for the density-wave channel and the d-wave pairing, respectively. For the chain layer we have $t_2=0.6$ so that we obtain the $\lambda^2_a(0)/\lambda^2_b(0) \sim 1.8$ as observed

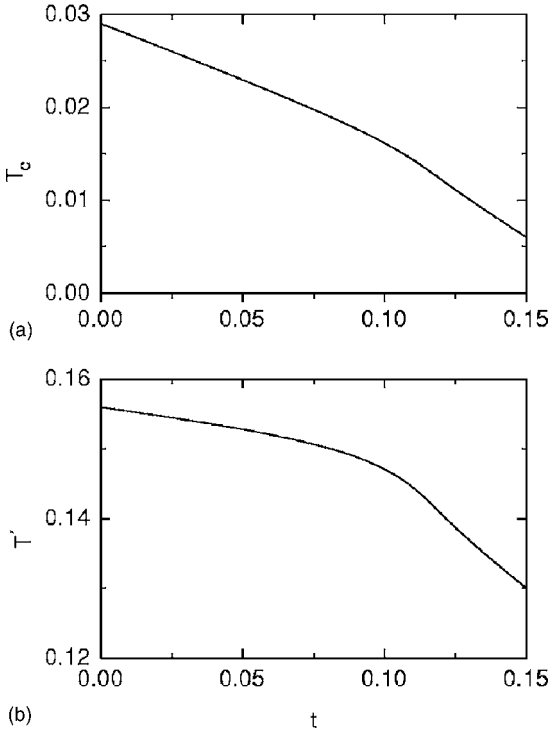

FIG. 1. (a) T_c vs t, (b) T^*_t vs t.

experimentally in Ref. 13. The chemical potential for the chains $\mu_2=0.4$. All the parameters in this work are in terms of t_1 unless otherwise stated.

In Fig. 1 we demonstrate the dependence of T_c and T^* on t. We see that the critical temperature decreases rapidly with increasing t and approaches zero at $t=0.15$. At $t=0$ we have $T_c=0.02$ in the presence of the DDW wave. But the T^* does not seem to be affected by t at such a rapid rate. In the absence of the pseudogap we find the T_c to be much higher; and hence a much higher value of the coupling between the chain and the plane bands can be applied before the disap-

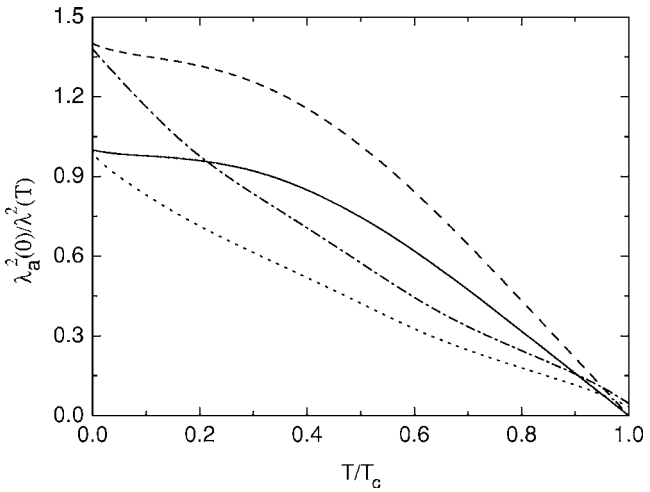

FIG. 2. $\lambda^2_a(0)/\lambda^2(T)$ vs T/T_c for $t=0.0$. The solid line (dashed line) indicates the λ_a (λ_b) for the DDW+DSC phase whereas the dotted (dashed-dotted) line indicates the λ_a (λ_b) for the pure DSC phase.

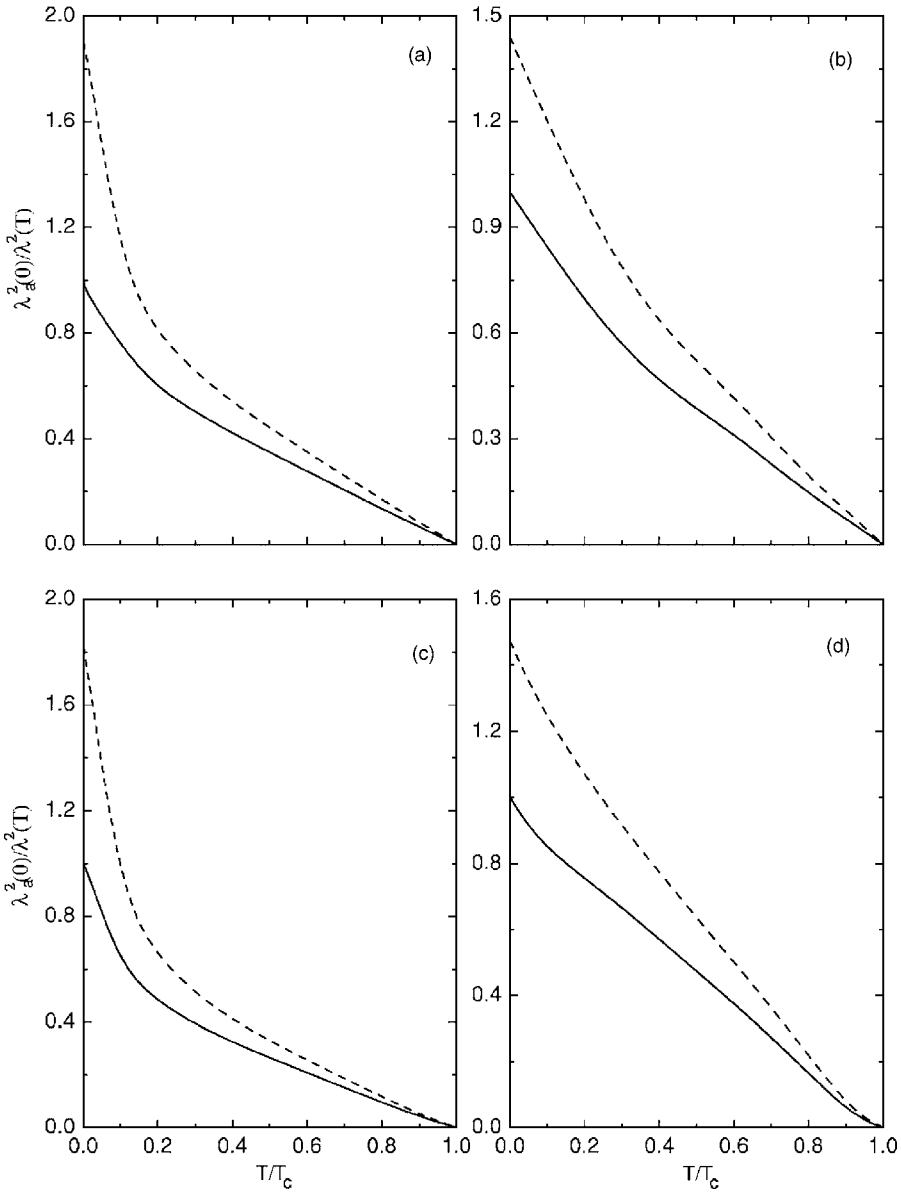

FIG. 3. $\lambda_a^2(0)/\lambda^2(T)$ vs T/T_c for $t=0.1$. (a) Pure DSC state and (b) mixed DDW+DSC phase. $\lambda_a^2(0)/\lambda^2(T)$ vs T/T_c for $t=0.12$, (c) pure DSC state, and (d) mixed DDW+DSC phase. The solid line (dashed line) indicates the λ_a (λ_b).

pearance of the superconductivity. The competition between the DDW and the DSC decreases T_c in the presence of the pseudogap as expected.

Next we concentrate on the penetration depth calculation on the plane along the a and b directions to study the anisotropy due to the consideration of the chain bands in the DDW model. In Fig. 2 we plot the results of numerical calculations of λ_a and λ_b in the mixed phase, i.e., where we have the DDW and the DSC coexisting in the underdoped region for $\mu_1=-0.39$ with $T_c=0.021$, $T^*=0.16$, and the pure superconducting wave after the DDW has already disappeared in the overdoped regime for $\mu_1=-0.6$ and $T_c=0.12$. The values of μ_1 are chosen according to our phase diagram where we observe the pure DDW, mixed DDW+DSC, and the pure DSC phases with doping. The above plots are done for $t=0.0$. We observe an anisotropy in the penetration depth in the mixed phase and also in the pure superconducting phase even in the absence of the interaction between the two layers. The change in the nature of the graph from underdoped to the overdoped case is similar to that of the results of Ref. 18 where they make measurements of the planar penetration depth for various doping which should qualitatively depict the a-axis penetration depth of our model. The anisotropy in the chains and its values are an clear indication of the dissipation in the chains. So we see that in the DDW model incorporating the chains and the plane we can explain the penetration depth of these superconductors.

The anisotropy observed in the λ results also throws light on the understanding of the universal correlation between T_c and the superfluid density as suggested by Uemura *et al.*[19] in materials with copper chains.

Next we introduce the effect of the interaction between the layers and it can be observed in Fig. 3. In Fig. 3(a) we plot λ_a and λ_b in the pure DSC phase for $t=0.10$ with $T_c=0.12$ and in Fig. 3(b) we obtain the penetration depth for the mixed DDW+DSC phase in the underdoped regime. In the mixed phase we have $T_c=0.017$ and $T_*=0.15$. The ratio of $\lambda_a^2(0)/\lambda_b^2(0)=1.8$ at $T=0.0$ which is close to the value observed in experimental measurements for the pure DSC state.[13] The temperature dependence has a distinct character

when compared to the mixed state and is very similar to the calculation in the pure state in Ref. 17. Also we see that introduction of the coupling decreases in the value of λ_a and λ_b irrespective of the doping. In Figs. 3(c) and 3(d) for $t=0.12$ we observe similar traits as observed in Figs. 3(a) and 3(b). Hence we see that our model can make some insight into the penetration depth results as we go from the underdoped to the overdoped region of the phase diagram along the *a* axis and also along the chains of the superconductor to understand the effect of the DDW and the chains on the values of λ. Both the pseudogap phase and *t* defines the nature of temperature dependence.

In the range of temperatures close to T_c in which the critical phase fluctuations dominate the low-frequency electromagnetic response and a low temperature regime in which the superfluid density have an anomalous temperature dependence as a result of the phase fluctuations to first order in *T*. However away from the critical points we expect our calculation to be correct as already observed in the calculation of superfluid density[20] and its agreement with experimental measures.[21]

In conclusion we see that the anisotropy in the penetration depth occurs due to the pseudogap as well as the proximity effect between the chains and the planes, but the temperature dependence can decide which region of doping we are concerned about. It will also be interesting to study the *c* axis and the planar penetration depth to make a comparative study with the experimental results which will be done in our future work.

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