Magnon wave forms in the presence of a soliton in two-dimensional antiferromagnets with a staggered field

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In this paper we study the interactions between magnons and a soliton, in a classical and isotropic two-dimensional Heisenberg antiferromagnet in the presence of a staggered field applied perpendicularly to the *xy* plane. We obtained the exact solutions to the magnons in the presence of the soliton. As a consequence we obtain the exact phase shifts, which were compared to the ones obtained by Born approximation. The quantum corrections of the energy of the soliton were also encountered. Our results can be applied to study the thermodynamics and generalized for two-dimensional isotropic ferromagnets with an axial magnetic field.

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The study of the interaction between magnons and topological excitations in classical magnetic systems is of fundamental importance, as it is the starting point to a thermodynamic analysis of the system, as shown by Currie *et al.*[1] As pointed out by Zaspel *et al.*[2] this is also important in the study of the dynamics of vortices. As is well known, topological excitations contribute to a central peak in the dynamical relaxation function and this peak is hard to detect in an unambiguous way. Nevertheless, the signature of the topological excitations can be seen in the electron paramagnetic resonance (EPR) line width.[2] The EPR linewidth is the temporal integral of the four-spin correlation function,[2,3] and the determination of this function and its dependence on the soliton excitation will probably have to be done numerically and is beyond the scope of this paper. The topological excitations are also responsible for the Kosterlitz-Thouless phase transition.[4]

Solitons interacting with magnons have been studied, in the two-dimensional (2D) nonlinear sigma models (isotropic[5] and anisotropic[6]) and in the 2D anisotropic ferromagnets.[7] It has been found that the quantum corrections to the classical soliton, or vortex energy, given by the zero-point energy of the spin waves measured with respect to the vacuum can change strongly the classical picture, introducing interactions between solitons[5] as well as an internal degree of freedom.[6]

Vortex-magnon interaction in discrete lattices has been studied in easy-plane ferromagnets by Wysin and Völkel[8] using numerical diagonalization on small systems. In particular, it has been shown how the spin-wave modes are related to the instability of vortices.[8,9]

As is well known, the classical easy-plane ferromagnet in the presence of a magnetic field applied in the *z* direction and the isotropic antiferromagnet in the presence of a staggered field have some similarities. They are similar from the thermodynamic point of view, although the dynamic behavior is different. Many papers have been dedicated to the study of materials in the presence of external fields.[10–12] This study is very important because, in general, experiments use external fields. Interest in staggered fields has appeared in the literature after the paper by Oshikahawa and Affleck.[13]

Therefore, our purpose in this paper is to study the interaction between spin waves and a soliton, in a two-dimensional antiferromagnet with a uniform staggered field applied perpendicularly to the plane. Since the lowest-order effect of an inhomogeneous soliton is to produce an elastic scattering center for the spin waves, we obtain the exact solutions and the respective phase shifts to the scattered spin waves. The quantum corrections with relation to the zero-point energy vacuum will be also calculated.

We start by considering the model, described by the following Hamiltonian

$$H = \sum_{\langle ij\rangle} [J\mathbf{S}_{i,j} \cdot (\mathbf{S}_{i+1,j} + \mathbf{S}_{i,j+1}) + g_0\mu_0\mathbf{B} \cdot (-1)^i \mathbf{S}_{i,j}], \quad (1)$$

where the summation extends over all sites of a square lattice, J is the positive exchange constant, $\mathbf{S}_{i,j}$ is the spin vector at site (i,j), g_0 is the gyromagnetic ratio, $\mu_0=e/2mc$ is the Bohr magneton divided by the Planck constant, and $\mathbf{B}$ is the magnetic field which will be taken to point in the third direction $\mathbf{B}=B\hat{z}$. In the limit of zero temperature, the continuum version for this model can be obtained in the usual way[14] defining normalized vectors of magnetization $\mathbf{m}_n=(\mathbf{S}_{2n}+\mathbf{S}_{2n+1})/2S$ and the vectors of sublattice magnetization $\mathbf{n}_n=(\mathbf{S}_{2n}-\mathbf{S}_{2n+1})/2S$, where the subscripts refer to the different sublattices. Vectors $\mathbf{m}_n$ and $\mathbf{n}_n$ satisfy the relations $\mathbf{m}_n^2+\mathbf{n}_n^2=1$ and $\mathbf{m}_n\cdot\mathbf{n}_n=0$. In the critical region, the condition $|\mathbf{m}_n| \ll |\mathbf{n}_n| \approx 1$ is satisfied and the Hamiltonian can be expressed in terms of $\mathbf{n}_n$ only.[15] Then, the Hamiltonian can be written as

$$H = \frac{J}{2}\int [(\partial_0\mathbf{n}_n)^2 - (\partial_\alpha\mathbf{n}_n)^2 + hn_3]d^2x, \quad \alpha = 1,2, \quad (2)$$

where $h=g_0\mu_0\mathbf{B}/(4JS)$. It is useful to resolve the constraint $\mathbf{n}_n^2=1$ explicitly using the spherical parametrization $\mathbf{n}_n=S(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ in terms of which

$$H = \frac{J}{2}\int\left[\frac{1}{c^2}\left(\frac{\partial\theta}{\partial t}\right)^2 - (\vec{\nabla}\theta)^2 + \sin^2\theta\left[\frac{1}{c^2}\left(\frac{\partial\phi}{\partial t}\right)^2 - (\vec{\nabla}\phi)^2\right] + h\cos\theta\right]d^2x, \quad (3)$$

where $c=2aJS$ is the spin-wave velocity. The parameter a is the lattice spacing. The equations of motion following from Eq. (3) are

$$\nabla^2\theta - \frac{1}{c^2}\frac{\partial^2\theta}{\partial t^2} = \sin\theta\cos\theta\left[(\vec{\nabla}\phi)^2 - \frac{1}{c^2}\left(\frac{\partial\phi}{\partial t}\right)^2\right] + h\sin\theta, \quad (4)$$

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = -2\cot\theta\left[(\vec{\nabla}\theta\cdot\vec{\nabla}\phi) - \frac{1}{c^2}\frac{\partial\phi}{\partial t}\frac{\partial\theta}{\partial t}\right]. \quad (5)$$

The static solution ϕ_s for Eqs. (4) and (5) is $\phi_s = q\arctan(y/x)$, where the parameter $q=0,1,2,\ldots$ plays the role of the topological charge of the soliton. We can write the localized solutions of Eqs. (4) and (5) in polar coordinates in the form

$$\theta = \theta_s(r), \quad \theta_s(0) = \theta_s(\infty) = 0, \quad \phi_s(\varphi,t) = q\varphi - \Omega t. \quad (6)$$

Here, Ω is the internal precession frequency of the soliton and can be determinate, through the number of bound magnons N. The necessity to consider solitons with internal precession is caused by the fact that, according to the Derrick-Hobart theorem, static solitons in models like Eq. (6) are unstable. Substituting Eq. (6) into Eqs. (4) and (5), we notice that Eq. (5) is automatically satisfied. Then, from Eq. (4) we obtain

$$\frac{1}{k_0^2}\left(\frac{d^2\theta_s}{dr^2} + \frac{1}{r}\frac{d\theta_s}{dr}\right) + \left(1 - \frac{q^2}{k_0^2 r^2}\right)\sin\theta_s\cos\theta_s - \frac{h}{k_0^2}\sin\theta_s = 0, \quad (7)$$

where $k_0^2 = \Omega^2/c^2$. By convenience we have introduced $l_0^2 = 1/k_0^2$ and $H = h/k_0^2$. The solution of Eq. (7) describes the axisymmetrical distribution of magnetization with a fixed circulation of the phase gradient $\nabla\varphi$, i.e., with a given circulation of spin (magnon) flux round the z axis. The magnetization field momentum is related to the number of magnons by the relationship $K_z = -\hbar qN$. Each soliton corresponds to a set of integrals of motion E, Z_z, and N. Dynamical solitons exist as stationary states due to the conservation of the mechanical integral of motion. If small perturbations that disturb these integrals of motion are inserted into the equations of motion, the dynamical solitons can be reduced to a homogeneous magnetization by a way of continuous deformations. Sheka *et al.*[16] have studied magnon scattering on topological solitons in 2D easy-axis ferromagnets. They have encountered another equation similar to Eq. (7). Although the static soliton solution is similar, in some aspects, to our equation, the dynamics is quite different.

The magnetization in equilibrium is $[\theta_s(r) = \theta_0]$ far from the soliton ($r \to \infty$). The soliton axis is limited and therefore $\theta_s(r) = 0$ for $r = 0$. It follows from the latter condition that, as $r \to 0$,

$$\theta_s(r) = (r/r_0)^{|q|}, \quad r_0 = \text{const}. \quad (8)$$

Notice that $\theta_s(r)$ does not depend on H; however, the behavior for $r \to \infty$ depends on H. For $H \neq 0$ and 1 the solution has the following behavior at infinity:

$$\theta_s(r) = \theta_0 - \frac{q^2 H^2}{\sqrt{1-H^2}}\left(\frac{l_0}{r}\right)^2, \quad H \neq 0,1, \quad (9)$$

and for $H=0$, the magnetization approaches the equilibrium direction at infinity exponentially,

$$\theta_s(r) = \frac{\pi}{2} + \frac{constant}{\sqrt{r}}e^{-r/l_0}, \quad H = 0. \quad (10)$$

When $H=1$, we have $\theta=0$ at infinity and the coordinate dependence of the magnetization vector at $r=\infty$ vanishes. In this case the magnetic vortex-type solitons ($q \neq 0$) cannot exist.

The magnetization distribution near a magnetic vortex for $q=1$ was calculated numerically by Kosevich *et al.*[17] When $H \sim 1$ the behavior $\theta_s(r)$ is qualitatively similar to the density distribution of a superfluid component within the vortex described by Pitaevskii.[18] For $H \to 1$ their similarity becomes more prominent. Since $\theta_s \lesssim \sqrt{1-H}$, for $H \to 1$ Eqs. (4) and (5) can be replaced by their θ_s power series expansions. Then, we have

$$l_0^2\nabla^2\theta_s + (1-H)\theta_s - \frac{1}{2}\theta_s^3 = 0. \quad (11)$$

Using the variables $\theta_s(r) = \sqrt{2(1-H)}\psi(\xi)$ and $\xi = \sqrt{1-H}(r/l_0)$, Eq. (11) can be reduced to the Gross-Pitaevskii equation of the function $\psi(\xi)$.[18,19]

The main macroscopic characteristic of the soliton is its energy. It is well known that soliton energy in an infinity crystal diverges logarithmically. Hence, with a logarithmical accuracy the soliton energy is

$$E_s \approx 2\pi q^2 J\frac{M_0^2}{2}a\int_{l_0}^{R}\sin^2\theta_s(r)\frac{dr}{r} \approx \pi J q^2(1-H^2)aM_0^2\ln(R/l_0), \quad (12)$$

where a is lattice parameter, M_0 is the z magnetization, and R is a cutoff soliton radius. The cutoff is necessary because the soliton center is a singularity in the continuum limit. The exact energy value is different from Eq. (12) by a term which is finite with $R \to \infty$. If the solution of Eq. (7) is known for different values of h, then Eq. (12) can be improved,

$$E_s = \pi J q^2(1-H^2)aM_0^2\ln[RA(H)/l_0]. \quad (13)$$

The function $A(H)$ can be found by numerical methods. As was shown by Kosevich *et al.*,[16] this function varies between 0.2 for ($H=1$) and 4.2 for ($H=0$).

In order to determine the behavior of magnons in the presence of a soliton, we assume that the spin-polar angle is given by $\theta(\vec{r},t) = \theta_s(r) + \eta(\vec{r},t)$. Here, $\eta(\vec{r},t)$ are assumed to be a small quantity, i.e., $\eta(\vec{r},t) \ll 1$, which reduce to magnon solutions if no solitons are present. In the presence of a soliton, $\eta(\vec{r},t)$ gives the change in the soliton configuration as a result of the soliton-magnon interaction. Considering that the asymptotic comportment $\theta_s(r)=0$ [Eqs. (6), (8), and (9)], since $\theta_0 \approx 0$, we can substitute $\theta(\vec{r},t)$ in Eq. (4), neglecting quadratics terms in $\eta(\vec{r},t)$, and we obtain the equation of motion for magnons in the presence of a soliton as

$$\nabla^2 \eta - \frac{1}{c^2}\frac{\partial^2 \eta}{\partial t^2} = \eta\left[(\vec{\nabla}\phi_s)^2 - \frac{1}{c^2}\left(\frac{\partial \phi_s}{\partial t}\right)^2 + h\right]. \quad (14)$$

The solutions for Eq. (14) represent the out-of-plane spin waves. Note that Eq. (14) contains the potential term $V(r) = (\vec{\nabla}\phi_s)^2 = (q/r)^2$ due to the interaction between out-of-plane spin waves and the soliton. Substituting Eq. (6) into Eq. (14), in the limit $r \to \infty$, Eq. (14) admits pure plane-wave solutions in the form $\eta(\vec{r},t) = \exp[i(\vec{k}\cdot\vec{r} - \omega t)]$, with the dispersion law

$$\omega^2 = k^2c^2 - \Omega^2 + hc^2. \quad (15a)$$

In the rotating frame the magnons frequency is

$$\tilde{\omega}^2 = k^2c^2 + hc^2. \quad (15b)$$

Here, $k = |\vec{k}|$, where $\vec{k}$ is the wave vector. In a classical theory we can choose any sign of the frequency; but in order to make contact with quantum theory, with a positive frequency and energy $\varepsilon_k = \hbar|\omega_k|$, we will discuss the case $\omega > 0$ only. Of course, the potential in Eq. (14) is cylindrically symmetric because we are dealing with a unique center problem. Therefore, the phase-shift matrix is diagonal. Now, we can write the solution $\eta(\vec{r},t)$ as $\eta(\vec{r},t) = B(r,\varphi)\exp(-i\omega t)$. Thus, Eq. (14) can be rewritten as

$$\nabla^2 B + \left[\frac{\omega^2}{c^2} - (\vec{\nabla}\phi_s)^2 + \frac{1}{c^2}\left(\frac{\partial \phi_s}{\partial t}\right)^2 - h\right]B = 0. \quad (16)$$

The exact solutions of Eq. (16) represent the out-of-plane spin waves in the presence of the soliton and can be written as

$$\eta(\vec{r},t) = C_1 J_\mu(kr) e^{-i[n\varphi + \omega t]}, \quad (17)$$

where $J_\mu(kr)$ is the Bessel's function, $\mu = \sqrt{n^2 + q^2}$, and $n = 0,1,2,3,\ldots$ represents a quantum number of angular momentum for the out-of-plane spin waves. The parameter k in Eq. (15) is the respective wave vector. The constant C_1 is determined through the normalization of the eigenfunctions; however, we have not encountered it in this manuscript.

Since the lowest-order effect of the soliton is to produce an elastic scattering center for the magnons, we will calculate the phase shifts of the waves scattered out-of-plane. So we will study the behavior of $J_\mu(kr)$ in the limit $r \to \infty$. The functions $J_\mu(kr)$ can be written as

$$J_\mu(kr) = \frac{1}{2}[H_\mu^{(1)}(kr) + H_\mu^{(2)}(kr)], \quad (18)$$

where $H_\mu^{(1)}(kr)$ and $H_\mu^{(2)}(kr)$ are the Hankell's functions. The Hankell's functions' first and second types corresponds to a circular wave entering and a circular wave leaving the origin in $r \to \infty$, respectively. In these regions we have plane waves, so that the waves in these regions should be described by an overlap the Hankell's function.

The difference between $\eta(kr) = J_\mu(kr)$ (solution in the presence of a soliton) and $J_n(kr)$ (solution in the absence of the a soliton) is physically clear. As an incoming spin wave approaches the zone of influence of the potential, it is more and more perturbed by this potential. When, after turning back, it is transformed into an outgoing spin wave, it has accumulated a phase shift of $2\Delta_n(k)$ relative to the free outgoing spin wave that would have resulted if the potential had been identically zero. In fact, the factor $e^{-2i\Delta_n(k)}$ summarizes the total effect of the potential on a magnon of angular momentum n, so that we can write

$$J_n(kr) = \frac{1}{2}[H_{|n|}^{(1)}(kr) + e^{-2i\Delta_n(k)} H_{|n|}^{(2)}(kr)] \text{ for } (r \to \infty). \quad (19)$$

By comparing the solution (19) with the exact solution (18) at $r \to \infty$, we obtain the phase shifts

$$\Delta_n(k) = (n - \sqrt{n^2 + q^2})\frac{\pi}{2} \text{ for } (n \geqslant 0), \quad (20)$$

$$\Delta_n(k) = -(n + \sqrt{n^2 + q^2})\frac{\pi}{2} \text{ for } (n \leqslant 0). \quad (21)$$

Notice that $\Delta_{|n|}(k) = \Delta_{-|n|}(k)$ and that the phase shift of the partial wave $\eta_n(kr)$ does not depend on k, that is, on the energy. The scattering on continuum states can contribute to the correlation function. Soliton motion results in a central peak at zero frequency, which is far removed from EPR. To analyze it better we will calculate the phase shifts $\Delta_n(k)$ using the Born approximation and then compare the approximation with the exact result given by Eqs. (20) and (21). We can use our exact result to check the Born approximation and it may help to do the calculations using this technique as in Refs. 2, 20, 5, and 7.

The first-order Born terms for the phase shifts are given, in general, by

$$\Delta_n^{(1)}(k) = -\frac{\pi}{2}\int_0^\infty r dr \langle J_{|n|}(kr) e^{-in\varphi} V(r) e^{in\varphi} J_{|n|}(kr)\rangle_\varphi, \quad (22)$$

where the symbol $\langle\ldots\rangle\varphi$ denotes an angular average. Since the out-of-plane spin waves "feel" a potential of the soliton $V(r) = (q/r)^2$, we get for $n \neq 0$.

$$\Delta_n^{(1)} = -\frac{\pi}{2}\frac{q^2}{2|n|} \text{ for } (n \neq 0). \quad (23)$$

Here, we use $q=1$ which represents low energy solitons, consequently appear more easily in the system. For $n=0$, the integral (22) diverges because the vortex core is a singularity in a continuum limit. Usually this means that a short-distance cutoff,[4,21] must be applied *ad hoc* to integrals over the spin field, but the cutoff radius itself is not well known. The agreement between the approximation Eq. (23) and exact results Eqs. (20) and (21) for a large angular momentum channels $|n| \geqslant 1$ is presented in Table I. For $|n| = 1$ the error is 21 percent.

As we could expect, the Born approximation in first order is not good for $n = 0, \pm 1$ angular momentum (s and p waves). In fact, in these cases the centrifugal barrier is given by approaching the zone of strong influence of the potential, where the Born approximation may fail. For $|n| \geqslant 2$, the centrifugal barrier is large and expels the spin waves from the

TABLE I. Comparison between exact results and Born approximation

Phase shift	Exact results	Born approximation
Δ_1	$-0.414(\pi/2)$	$-0.5(\pi/2)$
Δ_2	$-0.236(\pi/2)$	$-0.25(\pi/2)$
Δ_3	$-0.162(\pi/2)$	$-0.166(\pi/2)$
Δ_4	$-0.123(\pi/2)$	$-0.125(\pi/2)$
Δ_5	$-0.099(\pi/2)$	$-0.1(\pi/2)$

center of the vortex, where $V(r) \gg 1$. It is easy to see that, when $|n| \gg 1$, Eqs. (20) and (21) reduce to Eq. (23). This result is physically important for it implies that an out-of-plane spin wave in the state $\eta(\vec{r},t)$ is practically unaffected by what happens inside a circle centered at origin.

In order to improve the calculations for $\Delta_0^{(1)}(k)$ and $\Delta_1^{(1)}(k)$ we have to take in account the second-order Born approximation. However, these calculations present the same difficultly we have met in calculating $\Delta_0^{(1)}(k)$, since the cutoff radius is not well known. In using a value proportional to a (lattice constant), we get results that depend on this value and then it seems to be artificial.

By a generalization of arguments used in the semiclassical quantization of solitons in $(1+1)$ dimensions,[5] it can be shown that the quantum correction to the classical vortex energy, given by the zero-point energy of the spin waves measured with respect to the vacuum, is

$$E = E_s - \frac{\hbar}{2\pi}\int_0^{1/a} \frac{\partial\tilde{\omega}}{\partial k} \mathrm{tr}\, \Delta_n(k) dk, \qquad (24)$$

where E_s is the classical energy of the soliton [Eq. (13)]. The last term in the right-hand side of Eq. (24) represents the continuum states' energy contribution in the rotating frame. The trace is taken over the angular momentum indices. Here, we have assumed a Debye model for the spin-wave excitations, with $1/a$ as the cutoff momentum. The trace can be easily calculated, using an angular momentum cutoff given by[5] $n_D = R/a$. We find

$$\mathrm{tr}\, \Delta_n^{(1)}(k) = -\frac{\pi q^2}{2}\int_a^R \sum_{n=-\infty}^{\infty} \frac{J_{|n|}^2(kr)}{r} dr = -\frac{\pi q^2}{2} \ln\left(\frac{R}{a}\right). \qquad (25)$$

Substituting Eqs. (15) and (25) into Eq. (24) and calculating the integral, we obtain

$$E = E_s + \frac{\hbar q^2 c}{4} \ln(R/a)\left(\sqrt{(1/a^2)+h} - \sqrt{h}\right). \qquad (26)$$

The electron paramagnetic resonance (EPR) linewidth measurements provide an indirect method to experimentally detect solitons. As pointed out by Zaspel *et al.*[2] the soliton energy can be obtained from the linewidth. A large staggered field inhibits the quantum fluctuations and therefore quantum corrections to the soliton energy. Nevertheless, for low staggered fields, there are more quantum fluctuations, then the quantum corrections are more important. As happens in the other systems[22] the quantum corrections increase the classical soliton energy. It costs less energy to excite a "topological excitation" in the presence the spin waves (harmonic excitations or mesons, in general) than in the vacuum.

In this Brief Report we have studied the interactions between magnons and a soliton, in a classical 2D antiferromagnet model with a staggered magnetic field applied axially in the *xy* plane. We obtained the solutions in-plane and out-of-plane of the magnons in the presence of the soliton. The phase-shifts of the magnons were also found. The quantum correction of the classical energy of the soliton has been encountered. Finally, we would like to mention that the bound state(s) and scattering or continuum states may be of the fundamental importance not only for the spin dynamics but also for the statistical mechanics,[1] as well as quantizations procedures for soliton states[6] and perturbation theories[22] involving soliton responses to external perturbations.

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