Generating spin-entangled electron pairs in normal conductors using voltage pulses

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We suggest an operating scheme for the deliberate generation of spin-entangled electron pairs in a normal-metal mesoscopic structure with fork geometry. Voltage pulses with associated Faraday flux equal to one flux unit $\Phi_0 = hc/e$ drive individual singlet pairs of electrons toward the beam splitter. The spin-entangled pair is created through a post-selection in the two branches of the fork. We analyze the appearance of entanglement in a Bell inequality test formulated in terms of the number of transmitted electrons with a given spin polarization.

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I. INTRODUCTION

Quantum entanglement of electronic degrees of freedom in mesoscopic devices has attracted a lot of interest recently. Early proposals for structures generating streams of entangled particles exploit the interaction between electrons as a resource for producing entanglement, the pairing interaction in superconductors[1] or the repulsive Coulomb interaction in confined geometries.[2,3] Recently, another class of devices has been suggested that avoids direct interparticle interaction; instead, the entanglement originates from a proper post-selection of orbital[4–6] or spin[7–9] degrees of freedom. The majority of these proposals deal with the situation where the entangled particles are emitted in a random and uncontrolled fashion, while entanglement on demand is implicit in the scheme proposed in Ref. 2.

Current interest concentrates on setups which are capable of producing pairs of entangled electrons "on demand." Such controlled entanglement is an essential step toward the realization of quantum computing devices for which electronic orbital or spin degrees of freedom may serve as qubits.[10] In addition, prospects of converting electronic entanglement into a photonic one with high efficiency look promising;[11] this may open opportunities for the manipulation of entangled photons with an enhanced efficiency.

While entanglement on demand is implicit in the work of Ionicioiu *et al.*,[2] a detailed discussion of the controlled production of entanglement in a mesoscopic device has only been given recently by Samuelsson and Büttiker;[12] they proposed a scheme for the dynamical generation of orbitally entangled electron-hole pairs where a time-dependent harmonic electric potential is applied between two spatially separated regions of a Mach–Zehnder interferometer operating in the quantum Hall regime. This (perturbative) analysis concentrated on the limit of a weak pumping potential generating only a small fraction of entangled electron-hole pairs per cycle. Later on, several schemes have been suggested producing entangled pairs on demand with a high efficiency: In their setup, Beenakker *et al.*[13] make use of a ballistic two-channel conductor driven with a strong oscillating potential. In their nonperturbative analysis they demonstrate that this device can pump up to one (spin- or orbital-) entangled Bell pair per two cycles. A different proposal based on spin resonance techniques acting on electrons trapped in a

double quantum dot structure and subsequently released into two quantum channels has been suggested by Blaauboer and DiVincenzo[14]; their detailed analysis of the manipulation and measurement schemes demonstrates that the production and measurement of entangled pairs via an optimal entanglement witness can be performed with present days experimental technology.

In the present paper, we discuss an alternative scheme generating pulsed spin-entangled electron pairs in a normal-metal mesoscopic structure arranged in a fork geometry, see Fig. 1. In this device, spin-entangled electron pairs are generated via the injection of spin-singlet pairs into the source lead from the reservoir.[8] This entanglement is made accessible by splitting the pair into the two leads "*u*" and "*d*" and subsequent projection (through the Bell measurement) to that
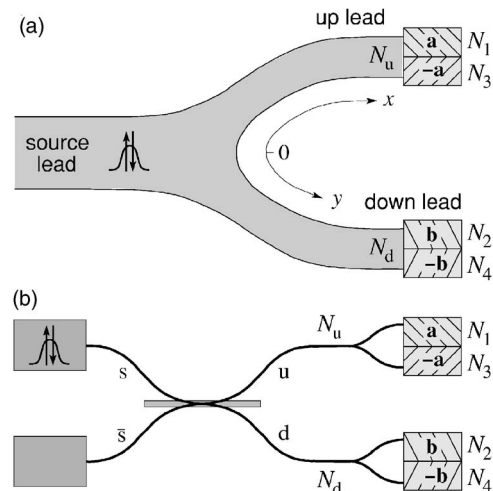

FIG. 1. Mesoscopic normal-metal structures in a fork geometry generating spin-correlated electrons in the two arms of the fork. The Bell-type setups detect the number of transmitted particles N_i, $i = 1, 3$, with spin projected onto the directions $\pm\mathbf{a}$ in the upper arm and correlates them with the number N_j, $j = 2, 4$, of particles with a spin projected onto the directions $\pm\mathbf{b}$ in the lower arm. (a) Fork with a simple splitter with particles injected along the single source lead. (b) Fork in the geometry of a four-terminal beam splitter with particles injected along one of the incoming channels only. Quenching the transmission T_{ud} between the upper and lower leads allows elimination of equilibrium fluctuations spoiling the entanglement.

part of the wave function describing separated electrons traveling in different leads.[8,9] Rather then quantum pumping with a cyclic potential as in Refs. 12 and 13, our proposal makes use of definite voltage pulses generating spin-entangled electron pairs. A pulsed sequence of ballistic electrons is implicitly assumed in the generation of orbital entanglement by Ionicioiu *et al.*,[2] however, no description has been given for how such (single-electron) pulses are generated in practice. Below we discuss a scheme where voltage pulses of specific form accumulating one unit of flux $\Phi_0 = -c \int dt V(t)$ and applied to the source lead "s" generate pairs of spin-entangled electrons which then are distributed between the two outgoing leads of the fork, the upper and lower arms denoted as "u" and "d." These spin-entangled electron states are subsequently analyzed in a Bell experiment[15] involving the measurement of cross correlations[16] between the number of electrons transmitted through the corresponding spin filters in the two arms of the fork, see Fig. 1. Using time-resolved correlators, we are in a position to analyze arbitrary forms of voltage pulses and determine the resulting degree of violation in the Bell setup. We find that Lorentzian shaped pulses generate spin-entangled pairs with 50% probability, corresponding in efficiency to the optimal performance of one entangled pair per two cycles as found by Beenakker *et al.*[13] The reduction in efficiency to 50% is due to the competing processes where the spin-entangled pair generated by the voltage pulse propagates into only one of the two arms. In order to make use of this structure as a deterministic entangler, the Bell measurement setup has to be replaced through a corresponding projection device (post-) selecting that part of the wave function with the two electrons distributed between the two arms; alternatively, this post-selection may be part of the application device itself, as is the case in the Bell inequality measurement.

In the following, we first derive (Sec. II) an expression for the Bell inequality involving the particle-number cross correlators appropriate for a pulse-driven experiment. We proceed with the calculation of the particle-number correlators for a single voltage pulse associated with an arbitrary Faraday flux (Sec. III). The results are presented in Sec. IV: We find the Bell inequalities violated for single pulses carrying one Faraday flux, corresponding to one pair of electrons with opposite spin. Although the Bell inequality appears to be violated for weak pulses (producing less than one pair) too, we argue that this violation is unphysical and that its appearance is due to a misconception in the original derivation of the Bell inequality arising in the weak pumping limit. We also generalize the discussion to the situation with more complex drives (multipulse case and alternating pulse sequences) and demonstrate that our Bell inequalities again are violated only for single-pair pulses flowing in either direction through the device. Our analysis of an alternating signal produces an apparent violation of the Bell inequality, which, however, again appears to be an artifact resulting from an improper derivation of the Bell inequality for the alternating signal. In both cases of failure, weak pulses and alternating pulse sequences, we encounter backflow phenomena that spoil the proper derivation of the Bell inequality for our setup.

II. BELL INEQUALITY WITH NUMBER CORRELATORS

The Bell inequality we are going to use here has been introduced by Clauser and Horne[17]; it is based on the Lemma saying that, given a set of real numbers x, $\bar{x}$, y, $\bar{y}$, X, Y with $|x/X|$, $|\bar{x}/X|$, $|y/Y|$, and $|\bar{y}/Y|$ restricted to the interval $[0,1]$, the inequality $|xy - x\bar{y} + \bar{x}y + \bar{x}\bar{y}| \leq 2|XY|$ holds true. We define the operator of electric charge $\hat{N}_i(t_{ac})$ transmitted through the ith spin detector during the time interval $[0, t_{ac}]$, where $t_{ac} > 0$ is the accumulation time. The charge operator $\hat{N}_i(t_{ac})$ can be expressed via the electric current $\hat{I}_i(t)$ flowing through the ith detector, $\hat{N}_i(t_{ac}) = \int_0^{t_{ac}} dt' \hat{I}_i(t')$. In the Bell test experiment, see Fig. 1, one measures the number of transmitted electrons with a given spin polarization, N_i, $i = 1, \ldots, 4$, and defines the quantities $x = N_1 - N_3$, $y = N_2 - N_4$, $X = N_1 + N_3$, and $Y = N_2 + N_4$ for fixed orientations **a** and **b** of the polarizers (and similar for $\bar{x}$ and $\bar{y}$ for the orientations $\bar{\mathbf{a}}$ and $\bar{\mathbf{b}}$), see Ref. 16. Our Bell setup measures the correlations

$$\mathcal{K}_{ij}(\mathbf{a}, \mathbf{b}) = \langle \hat{N}_i(t_{ac})\hat{N}_j(t_{ac}) \rangle = \int_0^{t_{ac}} dt_1 dt_2 \langle \hat{I}_i(t_1)\hat{I}_j(t_2) \rangle \quad (1)$$

between the number of transmitted electrons N_i, $i = 1, 3$, in the lead "u" with spin polarization along $\pm\mathbf{a}$ and their partners N_j, $j = 2, 4$, in lead "d" with spin polarization along $\pm\mathbf{b}$. Using the above definitions for x, y, X, and Y, we obtain the normalized particle-number difference correlator

$$E(\mathbf{a}, \mathbf{b}) = \frac{\langle [\hat{N}_1 - \hat{N}_3][\hat{N}_2 - \hat{N}_4] \rangle}{\langle [\hat{N}_1 + \hat{N}_3][\hat{N}_2 + \hat{N}_4] \rangle} = \frac{\mathcal{K}_{12} - \mathcal{K}_{14} - \mathcal{K}_{32} + \mathcal{K}_{34}}{\mathcal{K}_{12} + \mathcal{K}_{14} + \mathcal{K}_{32} + \mathcal{K}_{34}}, \quad (2)$$

and evaluating the correlators for the four different combinations of directions **a**, $\bar{\mathbf{a}}$ and **b**, $\bar{\mathbf{b}}$, we arrive at the Bell inequality

$$E_{BI} = |E(\mathbf{a}, \mathbf{b}) - E(\mathbf{a}, \bar{\mathbf{b}}) + E(\bar{\mathbf{a}}, \mathbf{b}) + E(\bar{\mathbf{a}}, \bar{\mathbf{b}})| \leq 2. \quad (3)$$

We proceed further by separating the current correlators in Eq. (1) into irreducible parts $C_{ij}(\mathbf{a}, \mathbf{b}; t_1, t_2) = \langle \delta\hat{I}_i(t_1)\delta\hat{I}_j(t_2) \rangle$ with $\delta\hat{I}_i(t) = \hat{I}_i(t) - \langle \hat{I}_i(t) \rangle$ and products of average currents and rewrite $E(\mathbf{a}, \mathbf{b})$ in the form

$$E(\mathbf{a}, \mathbf{b}) = \frac{K_{12} - K_{14} - K_{32} + K_{34} + \Lambda_-}{K_{12} + K_{14} + K_{32} + K_{34} + \Lambda_+}, \quad (4)$$

where we have defined $\Lambda_\pm = [\langle \hat{N}_1 \rangle \pm \langle \hat{N}_3 \rangle][\langle \hat{N}_2 \rangle \pm \langle \hat{N}_4 \rangle]$ with the irreducible particle number correlator

$$K_{ij}(t_{ac}) = \langle \delta\hat{N}_i(t_{ac})\delta\hat{N}_j(t_{ac}) \rangle = \int_0^{t_{ac}} dt_1 dt_2 C_{ij}(\mathbf{a}, \mathbf{b}; t_1, t_2). \quad (5)$$

The average currents are related via $\langle \hat{I}_1(t) \rangle = \langle \hat{I}_3(t) \rangle = \langle \hat{I}_u(t) \rangle / 2$ and $\langle \hat{I}_2(t) \rangle = \langle \hat{I}_4(t) \rangle = \langle \hat{I}_d(t) \rangle / 2$ and thus $\Lambda_- = 0$, $\Lambda_+ = \langle \hat{N}_u \rangle \langle \hat{N}_d \rangle$. The irreducible current-current correlator factorizes into a product of spin and orbital parts, $C_{ij}(\mathbf{a}, \mathbf{b}; t_1, t_2) = |\langle \mathbf{a}_i | \mathbf{b}_j \rangle|^2 C_{ud}(t_1, t_2)$ with $\mathbf{a}_{1,3} = \pm\mathbf{a}$ and $\mathbf{b}_{2,4} = \pm\mathbf{b}$. The spin projections involve the angle θ_{ab} between the directions **a**

and **b** of the polarizers, $\langle \pm\mathbf{a} | \pm\mathbf{b}\rangle = \cos^2(\theta_{\mathbf{ab}}/2)$ and $\langle \pm\mathbf{a} | \mp\mathbf{b}\rangle = \sin^2(\theta_{\mathbf{ab}}/2)$, and the Bell inequality assumes the form

$$\left| \frac{K_{ud}[\cos\theta_{\mathbf{ab}} - \cos\theta_{\mathbf{\bar{a}b}} + \cos\theta_{\mathbf{a\bar{b}}} + \cos\theta_{\mathbf{\bar{a}\bar{b}}}]}{2K_{ud} + \langle \hat{N}_u\rangle\langle\hat{N}_d\rangle} \right| \le 1, \quad (6)$$

where $K_{ud}(t_{ac}) = \int_0^{t_{ac}} dt_1 dt_2 C_{ud}(t_1, t_2)$ is the (irreducible) number cross correlator between the upper and lower leads of the fork. The maximal violation of the Bell inequality is attained for the standard orientations of the detector polarizations $\theta_{\mathbf{ab}} = \theta_{\mathbf{\bar{a}b}} = \theta_{\mathbf{\bar{a}\bar{b}}} = \pi/4$, $\theta_{\mathbf{a\bar{b}}} = 3\pi/4$; the Bell inequality (6) then reduces to

$$E_{\mathrm{BI}} = \left| \frac{2K_{ud}}{2K_{ud} + \langle\hat{N}_u\rangle\langle\hat{N}_d\rangle} \right| \le \frac{1}{\sqrt{2}}. \quad (7)$$

III. NUMBER CORRELATORS FOR A SINGLE PULSE

The orbital part $C_{ud}(t_1, t_2)$ of the current cross correlator between the upper and lower leads can be calculated within the standard scattering theory of noise.[18–21] We assume that the time dependent voltage drop $V(t)$ at the splitter can be treated adiabatically (i.e., the voltage changes slowly during the electron scattering time). The electrons incident from the source lead "s" and scattered to the "up" or "down" lead then acquire an additional time-dependent phase $\phi(t) = \int_{-\infty}^t dt' eV(t')/\hbar$. The scattering states (for one spin component) describing the electrons in the upper and lower leads $\hat{\Psi}_u(x, t)$ and $\hat{\Psi}_d(x, t)$ take the form

$$\hat{\Psi}_u = \int \frac{d\epsilon}{\sqrt{hv_\epsilon}} [(t_{su}e^{i\phi(t-x/v_\epsilon)}\hat{c}_\epsilon + r_u\hat{a}_\epsilon + t_{du}\hat{b}_\epsilon)e^{ikx}$$
$$+ \hat{a}_\epsilon e^{-ikx}]e^{-i\epsilon t/\hbar}, \quad (8)$$

$$\hat{\Psi}_d = \int \frac{d\epsilon}{\sqrt{hv_\epsilon}} [(t_{sd}e^{i\phi(t-x/v_\epsilon)}\hat{c}_\epsilon + r_d\hat{b}_\epsilon + t_{ud}\hat{a}_\epsilon)e^{ikx}$$
$$+ \hat{b}_\epsilon e^{-ikx}]e^{-i\epsilon t/\hbar}, \quad (9)$$

where $v_\epsilon = \sqrt{2m\epsilon}$; $\hat{a}_\epsilon$, $\hat{b}_\epsilon$, and $\hat{c}_\epsilon$ denote the annihilation operators for spinless electrons at energy ϵ in leads "u", "d", and "s"; the scattering amplitudes $t_{su}(t_{du})$ and $t_{sd}(t_{ud})$ describe particle transmission from the source (down) lead into the upper lead and from the source (up) lead into the lower ("d") lead; r_u, r_d denote the reflection amplitudes into the leads "u" and "d." Such adiabatically deformed scattering states (8) and (9) have first been used in the calculation of the spectral noise power in an ac driven system[22]; the validity of this approach has been confirmed in several experiments.[23]

We substitute these expressions into the current operator $\hat{I}_{u(d)}(x, t)$ and drop all terms small in the parameter $|\epsilon - \epsilon'|/\epsilon_F$ (we assume a linear dispersion). The irreducible current cross-correlator $C_{ud}(t_1, t_2) = \langle \delta\hat{I}_u(x, t_1)\delta\hat{I}_d(y, t_2)\rangle$ measured at the positions x and y in the leads "u" and "d" can be

splitted into two terms, one due to equilibrium fluctuations, $C_{ud}^{\mathrm{eq}}(t_1 - t_2) = \int (d\omega/2\pi) S^{\mathrm{eq}}(\omega) e^{i\omega(t_1 - t_2)}$ with

$$S^{\mathrm{eq}}(\omega) = -\frac{2e^2}{h} T_{ud} \cos(\omega\tau^+) \frac{\hbar\omega}{1 - e^{\hbar\omega/\theta}}, \quad (10)$$

and a second term describing the excess correlations at finite voltage

$$C_{ud}^{\mathrm{ex}}(t_1, t_2) = -\frac{4e^2}{h^2} T_u T_d \sin^2\frac{\phi(\xi_1) - \phi(\xi_2)}{2}\alpha(\tau - \tau^-, \theta),$$
$$(11)$$

with $\alpha(\tau, \theta) = \pi^2\theta^2/\sinh^2[\pi\theta\tau/\hbar]$ (θ is the temperature of electronic reservoirs), $\tau = t_1 - t_2$, $\tau^\pm = (x \pm y)/v_F$, $\xi_1 = t_1 - x/v_F$, and $\xi_2 = t_2 - y/v_F$. The coefficients $T_u = |t_{su}|^2$, $T_d = |t_{sd}|^2$, and $T_{ud} = |t_{ud}|^2 = |t_{du}|^2$ denote the transmission probabilities from the source to the "up", "down" leads, and from the "down" to the "up" lead.

The equilibrium part of the current cross-correlator $C_{ud}^{\mathrm{eq}}(t_1 - t_2)$ describes the correlations of the electrons in the Fermi sea propagating ballistically from lead "u" to lead "d" (or vice versa) with the retardation $\tau^+ = (x_1 + x_2)/v_F$. The corresponding equilibrium part of the particle-number cross correlator, $K_{ud}^{\mathrm{eq}} = \int_0^{t_{ac}} dt_1 dt_2 C_{ud}^{\mathrm{eq}}(t_1 - t_2)$ then takes the form

$$K_{ud}^{\mathrm{eq}} \approx \frac{e^2}{\pi^2} T_{ud}\ln\frac{t_{ac}}{\tau}, \quad \tau = \max\{\hbar/\epsilon_F, \tau^+\}, \quad (12)$$

where we have assumed the zero temperature limit and an accumulation time $t_{ac} \gg \tau$. The logarithmic divergence in t_{ac} reduces the violation of the Bell inequality Eq. (7) at large accumulation times and one has to suppress the equilibrium correlations between the upper and lower leads in the setup. This can be achieved via a reduction in the transmission probability T_{ud}, however, in the fork geometry of Fig. 1(a) the probability T_{ud} cannot be made to vanish. Alternatively, one may chose a setup with a reflectionless four-terminal beam splitter as sketched in Fig. 1(b) with no exchange amplitude between the upper and lower outgoing leads; using such a fork geometry, the equilibrium fluctuations K_{ud}^{eq} can be made to vanish.[24]

Next, we concentrate on the excess part K_{ud}^{ex} of the particle-number cross-correlator $\langle\hat{N}_u(t_{ac})\hat{N}_d(t_{ac})\rangle$. Note that the excess fluctuations are the same for both setups, Figs. 1(a) and 1(b), and we can carry out all the calculations for the fork geometry. We consider a sharp voltage pulse applied at time t_0, $0 < t_0 < t_{ac}$, with short duration δt. The total accumulated phase $\phi(t)$ then exhibits a steplike time dependence with the step height $\Delta\phi = \phi(t_0 + \delta t/2) - \phi(t_0 - \delta t/2) = -2\pi\Phi/\Phi_0$, where we have introduced the Faraday flux $\Phi = -c\int V(t)dt$ and $\Phi_0 = hc/e$ is the flux quantum. The excess part of the particle-number cross-correlator K_{ud} then takes the form (we consider again the zero temperature limit)

$$K_{ud}^{\mathrm{ex}} = -\frac{e^2}{\pi^2} T_u T_d \int_0^{t_{ac}} dt_1 dt_2 \frac{\sin^2\{[\phi(t_1) - \phi(t_2)]/2\}}{(t_1 - t_2)^2}. \quad (13)$$

For a sharp pulse with $\delta t \ll t_0, t_{ac}$ we can identify two distinct contributions arising from the integration domains $|t_1 - t_2|$

$\ll \delta t$ and $|t_1 - t_2| \gg \delta t$, cf. Refs. 25 and 26; we denote them with $K^<$ and $K^>$. Introducing the average and relative time coordinates $t = (t_1 + t_2)/2$ and $\tau = t_1 - t_2$ and expanding the phase difference $\phi(t_1) - \phi(t_2) = \phi(t + \tau/2) - \phi(t - \tau/2) \approx \dot{\phi}(t)\tau$, the first contribution $K^<$ reads

$$K^< = -\frac{e^2}{\pi^2}T_u T_d \int_0^{t_{ac}} dt \int d\tau \frac{\sin^2[\dot{\phi}(t)\tau/2]}{\tau^2}$$

$$= -\frac{e^2}{2\pi}T_u T_d \int_0^{t_{ac}} dt |\dot{\phi}(t)|. \tag{14}$$

Assuming that the phase $\phi(t)$ is a monotonic function of t [guaranteeing a unique sign for $\dot{\phi}(t)$] the last equation can be rewritten in terms of the Faraday flux Φ

$$K^< = -e^2 T_u T_d \frac{|\Phi|}{\Phi_0}; \tag{15}$$

this contribution to the particle-number cross-correlator K_{ud}^{ex} describes the correlations arising from the $n = |\Phi|/\Phi_0$ additional particles pushed through the fork by the voltage pulse $V(t)$, see Eq. (18) below.

The second contribution $K^>$ to K_{ud}^{ex} originates from the time domains $0 < t_{1(2)} < t_0 - \delta t/2$ and $t_0 + \delta t/2 < t_{2(1)} < t_{ac}$, where $|\phi(t_1) - \phi(t_2)| = 2\pi\Phi/\Phi_0$, hence

$$K^> \approx -\frac{2e^2}{\pi^2}T_u T_d \sin^2 \frac{\pi\Phi}{\Phi_0} \ln \frac{t_m}{\delta t}; \tag{16}$$

here we have kept the most divergent term in the measurement time $t_m = t_{ac} - t_0$, the time during which the pulse manifests itself in the detector. The above expression describes the response of the electron gas to the sudden perturbation $V(t)$; the logarithmic divergence in the measurement time t_m can be interpreted[25] along the lines of the orthogonality catastrophe,[27] with the isolated perturbation in space, the impurity, replaced by the sudden perturbation in time. The periodicity of the response in the Faraday flux Φ is due to the discrete nature of electron transport as expressed through the binomial character of the distribution function of transmitted particles.[25,26] Remarkably, the above logarithmically divergent contribution to K_{ud}^{ex} vanishes for voltage pulses carrying an integer number of electrons $n = |\Phi|/\Phi_0$, see Eq. (18) below. This follows quite naturally from the invariance of the scattering amplitudes t_{su} and t_{sd} in Eqs. (8) and (9) under the (adiabatic) voltage pulses carrying integer flux $\pm n\Phi$, $t_{sx} \rightarrow t_{sx}e^{\pm 2\pi n}$ with $x = u, d$; transmitting an integer number of particles at Faraday fluxes $\Phi = n\Phi_0$ avoids the system shakeup and the associated logarithmic divergence.

We proceed with the determination of the average number of transmitted (spinless) particles

$$\langle \hat{N}_{u(d)}(t_{ac}) \rangle = \int_0^{t_{ac}} dt \langle \hat{I}_{u(d)}(x,t) \rangle. $$

Within the scattering matrix approach the average currents in the upper and lower leads are given by the expression

$$\langle \hat{I}_{u(d)}(x,t) \rangle = \frac{e}{h}T_{u(d)}eV(t - x/v_F) = \frac{e}{2\pi}T_{u(d)}\dot{\phi}(t - x/v_F). \tag{17}$$

The time integration provides the average number of transmitted particles

$$\langle \hat{N}_{u(d)}(t_{ac}) \rangle = eT_{u(d)}\frac{\Phi}{\Phi_0}. \tag{18}$$

With $T_u + T_d = 1$, the result (18) tells that a voltage pulse corresponding to $n = |\Phi|/\Phi_0$ flux units pushes n spinless electrons through the fork, in the forward direction from the source lead "s" to the prongs "u" and "d" if $\Phi > 0$ and in the backward direction for $\Phi < 0$.

IV. RESULTS

Substituting the above expressions for the particle-number cross correlators and for the average number of transmitted particles into Eq. (7) we arrive at the following general result for the Bell inequality:

$$E_{BI} = \left| \frac{n + (2/\pi^2)\sin^2(\pi n)\ln(t_m/\delta t)}{2n^2 - n - (2/\pi^2)\sin^2(\pi n)\ln(t_m/\delta t)} \right|. \tag{19}$$

A. Pulse with integer flux

For a voltage pulse with integer n the above expression simplifies dramatically as all logarithmic terms vanish, leaving us with the Bell inequality

$$E_{BI} = \left| \frac{1}{2n - 1} \right| \leq \frac{1}{\sqrt{2}}, \tag{20}$$

which we find maximally violated for $n = 1$ and never violated for larger integers $n > 1$ — any additional particle accumulated in the detector spoils the violation of the Bell inequality. Furthermore, this violation is independent of the transparencies T_u, T_d and hence universal; moreover, the Bell inequality (20) does not depend on the particular form or duration of the applied voltage pulse but involves only the number of electrons n carried by the voltage pulse.

A voltage pulse with $n = 1$ pushes two electrons with opposite spin polarization toward the beam splitter. Such a pair appears in a singlet state[8] and can be described by the wave function $\Psi_{in}^{12} = \phi_s^1 \phi_s^2 \chi_{sg}^{12}$ with the spin-singlet state $\chi_{sg}^{12} = [\chi_\uparrow^1 \chi_\downarrow^2 - \chi_\downarrow^1 \chi_\uparrow^2]/\sqrt{2}$; ϕ_s is the orbital part of the wave function describing a particle in the source lead "s" and the upper indices 1 and 2 denote the particle number. This local spin-singlet pair is scattered at the splitter and the wave function Ψ_{scat}^{12} transforms to $\Psi_{scat}^{12} = t_{su}^2 \phi_u^1 \phi_u^2 \chi_{sg}^{12} + t_{sd}^2 \phi_d^1 \phi_d^2 \chi_{sg}^{12} + t_{su}t_{sd}[\phi_u^1 \phi_d^2 + \phi_d^1 \phi_u^2]\chi_{sg}^{12}$, where the last term describes two particles in a singlet state shared between the upper and lower leads of the fork. The Bell inequality test is only sensitive to pairs of particles propagating in different arms, implying a projection of the scattered wave function Ψ_{scat}^{12} onto the spin-entangled component. Thus the origin of the entanglement is found in the post-selection during the cross-correlation measurement effectuated in the Bell inequality

test.[6,9] From an experimental point of view it may be difficult to produce voltage pulses driving exactly one (spinless) particle $n=1$. However, as follows from the full expression Eq. (19), for a sufficiently small deviation $|\delta n|=|n-1|\ll 1$ the logarithmic terms are small in the parameter $(\delta n)^2$ and thus can be neglected, provided the measurement time t_m satisfies the condition $(\delta n)^2 \ln(t_m/\delta t)\ll 1$.

B. Weak pumping regime

The weak pumping regime with $n<1$, corresponding to small voltage pulses carrying less than one electron per spin channel, deserves special attention. Inspection of Eq. (19) shows that the Bell inequality can be formally violated in this regime. We believe that this violation of the Bell inequality has no physical meaning. Below, we will show that for noninteger n many-particle effects generate backflow of particles in our setup. We argue that this backflow leads to an inconsistency in the derivation of the Bell inequality itself, as the assumption $0\leq|x/X|,|\bar{x}/X|,|y/Y|,|\bar{y}/Y|\leq 1$ may no longer hold true for $n<1$.

In order to understand this weak pumping regime better, we analyze the sign of the full current cross-correlator $\mathcal{C}_{ud}(t_1,t_2)=\langle \hat{I}_u(t_1)\hat{I}_d(t_2)\rangle$ (for one spin component); expressing this quantity through the phase $\phi(t)$ we find the form

$$\mathcal{C}_{ud}(t_1,t_2) = \frac{e^2}{(2\pi)^2}T_uT_d$$
$$\times\left[\dot{\phi}(t_1)\dot{\phi}(t_2) - 4\frac{\sin^2[(\phi(t_1)-\phi(t_2)/2]}{(t_1-t_2)^2}\right]. \quad (21)$$

We consider again the case of a narrow voltage pulse applied at $t=t_0$ and assume a specific shape $\phi(t)=2n\arctan[(t-t_0)/\delta t]$. Choosing times $t_1<t_0<t_2$ before and after the application of the pulse at t_0, we find that the currents in the leads "u" and "d" predominantly flow in opposite directions: For a sharp pulse with $|t_{1,2}-t_0|\gg\delta t$ we can assume that $\phi(t_1)-\phi(t_2)=2\pi n$ and thus the correlator $\mathcal{C}_{ud}(t_1,t_2)$ takes the form

$$\mathcal{C}_{ud}(t_1,t_2) = \frac{e^2}{\pi^2(\delta t)^2}T_uT_d$$
$$\times\left[\frac{n^2}{(1+z_1^2)(1+z_2^2)} - \frac{\sin^2\pi n}{(z_1-z_2)^2}\right], \quad (22)$$

where we have introduced $z_{1,2}=(t_{1,2}-t_0)/\delta t$. For $|z_{1,2}|\gg 1$ the second (negative) term $\propto 1/(|z_1|+z_2)^2$ describing the irreducible correlations dominates over the first (positive) term $\propto 1/(z_1^2z_2^2)$ and hence the full current cross correlator is negative. This negative sign tells us that, despite application of a positive voltage pulse with $n>0$, the currents at times $t_1<t_0<t_2$ in leads "u" and "d" flow in opposite directions on average. Note that this unusual behavior is a specific feature of time dependent voltage pulses and does not appear for a constant dc voltage with $\phi(t)=eVt/\hbar$ — in this case the current cross correlator is always positive.

As a consequence, the time-integrated full particle-number cross correlator (per one spin component) may turn

out negative as well and it does so for voltage pulses carrying less then one electron per spin channel $n<1$,

$$\mathcal{K}_{ud}^{\rm ex} = e^2 T_u T_d\left[n^2 - n - \frac{2}{\pi^2}\sin^2(\pi n)\ln\frac{t_{\rm m}}{\delta t}\right]. \quad (23)$$

Hence, in the weak pumping regime the particles in the outgoing leads "u" and "d" are preferentially transmitted in opposite directions. Note that both (negative) terms in the correlator, the one $(-n)$ from short time differences as well as the contribution $[\propto -\sin^2(n\pi)\ln(t_{\rm m}/\delta t)]$ related to the "orthogonality catastrophe" dominate over the (positive) product term (n^2), with the second one becoming increasingly important at large measuring times. Furthermore, this second term also drives the particle-number cross-correlator negative at large noninteger $n>1$ and long measuring times, again signaling the presence of particle backflow in the device.

The derivation of the Bell inequality relies on the assumption that the quantities $|x/X|$, $|y/Y|$, etc. are bounded by unity. For our setup this implies that the particle number ratios of the type $|x/X|=|(N_1-N_3)/(N_1+N_3)|$ are bounded by unity, which is only guaranteed for particle numbers with equal sign, $N_1N_3>0$; hence, particles detected in the pair of spin filters with polarization $\pm\mathbf{a}$ in the upper arm have to be transmitted in the same direction. Next, we note that particles with opposite spin propagate independently and hence our finding that particles preferentially propagate in opposite directions of the outgoing leads "u" and "d" for $n<1$ also implies that the particle numbers N_1 and N_3 can be of opposite sign, hence the condition $x<1$ is not necessarily satisfied for $n<1$. On the contrary, for $n=1$ the full particle-number cross correlator (for one spin component) vanishes, $\mathcal{K}_{ud}=0$: in the simplest interpretation we may conclude that the single transmitted particle is propagating either through the upper or the lower lead, thus either $N_u=0$ and $N_d=1$ or vice versa and the quantity $|x/X|$ is properly bounded. The above arguments cannot exclude the relevance of additional many-particle effects, i.e., the appearance of additional particle-hole excitations in the system contributing to the particle count in the various detectors. A formal proof of the unidirectional propagation of particles confirming the applicability of the Bell inequality for the present nonstationary situation relies on the calculation of the full counting statistics of particles measured in the detectors 1 and 3, etc.; such a calculation has not been done yet.

C. Many integer-flux pulses

Above, we have concentrated on the situation where only a single voltage pulse has been applied. Let us consider another situation where a sequence of voltage pulses driving an integer number of electrons is applied to the source lead "s." In contrast to the previous analysis, we study the total transmitted charge from $t=-\infty$ to $t=\infty$, $\hat{N}_i(\infty)=\int_{-\infty}^{\infty}dt\hat{I}_i(t)$; the excess part of the irreducible particle-number cross correlator takes the form [we remind that the equilibrium part can be quenched in going to the four-terminal beam splitter of Fig. 1(b)]

$$K_{ud}^{\text{ex}} = -\frac{e^2}{\pi^2} T_u T_d \int_{-\infty}^{\infty} dt_1 dt_2 \frac{\sin^2\{[\phi(t_1) - \phi(t_2)]/2\}}{(t_1 - t_2)^2}. \quad (24)$$

In our further analysis, we closely follow the technique developed in Ref. 28. The double integral in the above expression is logarithmically divergent at large times t_1, t_2, producing the logarithmic dependence on the measurement time t_m noted above for the finite accumulation time. However, for pulses with an integer number of electrons, this problematic term disappears; in this case we are allowed to regularize the integral in Eq. (24) with the help of

$$\frac{1}{(t_1 - t_2)^2} \to \frac{1}{2}\left[\frac{1}{(t_1 - t_2 + i\delta)^2} + \frac{1}{(t_1 - t_2 - i\delta)^2}\right] \quad (25)$$

and $\delta \to 0$ a small cutoff. Expressing the factor $\sin^2(\dots)$ in Eq. (24) in terms of exponential functions, we arrive at the form

$$K_{ud}^{\text{ex}} = \frac{e^2 T_u T_d}{(2\pi)^2} \int dt_1 dt_2 \left[\frac{e^{i\phi(t_1) - i\phi(t_2)}}{(t_1 - t_2 + i\delta)^2} + \frac{e^{i\phi(t_1) - i\phi(t_2)}}{(t_1 - t_2 - i\delta)^2}\right]. \quad (26)$$

In order to proceed further, we split the exponential into two terms $e^{i\phi(t)} = f_+(t) + f_-(t)$ with $f_+(t)$ and $f_-(t)$ two bounded analytic functions in the upper and lower complex-t plain. Substitution into the above expression and using Cauchy's formula for the derivative

$$\dot{f}_\pm(t) = \pm\frac{i}{2\pi}\int dt' \frac{f_\pm(t')}{(t - t' \pm i\delta)^2},$$

allows us to write the particle-number correlator in the form

$$K_{ud}^{\text{ex}} = -\frac{e^2}{2\pi i} T_u T_d \int dt[\dot{f}_+(t)f_+^*(t) - \dot{f}_-(t)f_-^*(t)]. \quad (27)$$

In Eq. (27) we have made use of the analytical properties of $f_\pm(t)$; in particular, with the complex conjugate functions $f_+^*(t)$ and $f_-^*(t)$ bounded and analytic in the lower and upper half-planes, respectively, we easily find that $\int dt \dot{f}_+(t) f_-^*(t) = \int dt \dot{f}_-(t) f_+^*(t) = 0$. In addition, we can also express the average number of transmitted particles in terms of the functions $f_\pm(t)$ introduced above

$$\langle \hat{N}_{u(d)} \rangle = \frac{e}{\pi} T_{u(d)} \int dt \dot{\phi}(t) = \frac{e}{\pi i} T_{u(d)} \int dt e^{-i\phi(t)} \frac{d}{dt} e^{i\phi(t)}$$

$$= \frac{e}{\pi i} T_{u(d)} \int dt[\dot{f}_+(t)f_+^*(t) + \dot{f}_-(t)f_-^*(t)]. \quad (28)$$

Rewriting Eqs. (27) and (28) in terms of the real numbers $n_\pm$

$$n_\pm = \pm\frac{1}{2\pi i}\int dt \dot{f}_\pm(t)f_\pm^*(t), \quad (29)$$

we obtain the particle-number cross correlator and the average number of transmitted particles in the form

$$K_{ud}^{\text{ex}} = -e^2 T_u T_d (n_+ + n_-), \quad (30)$$

$$\langle N_{u(d)} \rangle = 2e T_{u(d)}(n_+ - n_-). \quad (31)$$

Substituting these expressions into the Bell inequality Eq. (7), we arrive at the result

$$E_{\text{BI}} = \left|\frac{n_+ + n_-}{2(n_+ - n_-)^2 - (n_+ + n_-)}\right| \leq \frac{1}{\sqrt{2}}. \quad (32)$$

The physical meaning of the numbers $n_\pm$ is easily identified for the specific form of Lorentzian voltage pulses

$$V(t) = \sum_i n_i \frac{2\hbar \gamma_i/e}{1 + (t - t_i)^2 \gamma_i^2}, \quad (33)$$

where the index i denotes the number of the pulse in the sequence, t_i is the moment of its appearance, γ_i^{-1} is the pulse width, and n_i the number of spinless electrons carried by the ith pulse with the sign of n_i defining the sign of the applied voltage. Such a sequence of pulses produces the phase

$$e^{i\phi(t)} = \prod_i \left(\frac{t - t_i - i/\gamma_i}{t - t_i + i/\gamma_i}\right)^{n_i}, \quad (34)$$

from which the decomposition into the terms $f_\pm(t)$ can be found. The further analysis is straightforward for unidirectional pulse sequences with all $n_i > 0$, in which case $\exp[i\phi(t)] = f_+(t)$ and $n_+ = \Sigma_i n_i$, $n_- = 0$, or all $n_i < 0$ when $\exp[i\phi(t)] = f_-(t)$ and $n_+ = 0$, $n_- = -\Sigma_i n_i$. It then turns out[26,28] that all results for the irreducible particle-number cross-correlator (30), the average currents (31), and the Bell inequality (32) do neither depend on the separation $t_{i+1} - t_i$ between the pulses nor on their widths γ_i^{-1}. Furthermore, the result (32) for the Bell inequality agrees with the previous expression (20) where a single pulse is carrying $n = n_+$ (or n_-) electrons in one go and we confirm our finding that the violation of the Bell inequality is restricted to pulses containing only one pair of electrons with opposite spin. Also, we note that for the case of well-separated pulses we can restrict the accumulation time over the duration of the individual pulses, in which case the Bell inequality is violated for all pulses with $|n_i| = 1$.

Another remark concerns the case of an alternating voltage signal with no net charge transport and hence zero accumulated particle numbers $\langle N_{u(d)} \rangle = 0$. Equation (31) then tells us that $n_+ = n_-$ and the Bell inequality (32) is formally violated. However, we argue that this violation is again unphysical and due to the same improper normalization of the basic quantities $|x/X|$, $|y/Y|$, etc. as encountered previously for the case of small Faraday flux $n < 1$: Concentrating on the expression $x/X = (N_1 - N_3)/(N_1 + N_3)$, we note that two pulses with opposite signs allow for processes where the charge driven through the two spin detectors satisfies $N_1 N_3 < 0$ and hence $|x/X| > 1$, in contradiction to the requirements of the Lemma. Note that the manner of violating the Bell inequality is quite different for the physical cases involving pulses with a single particle (see the discussion of single integer-flux pulses in Sec. IV A with $n = 1$, or the discussion of many integer-flux pulses in Sec. IV C with $n_+ = 1$, $n_- = 0$, and $n_+ = 0$, $n_- = 1$) and for the unphysical situation of an alternating signal with $n_+ = n_-$ discussed above: In the first case the small

denominator results from a cancellation between the product term $2n$ and the negative number correlator -1, hence $E_{BI} = |1/(2-1)| = 1$, while in the second case, the product term vanishes and there is no compensation, although the final result is the same, $E_{BI} = |1/-1| = 1$. The same apparent violation appears at large noninteger values of $n > 1$ and long measuring times, where the term $\propto \sin^2(\pi n) \ln(t_m/\delta t)$ becomes dominant, cf. Eq. (19).

V. CONCLUSION

The application of voltage pulses to a mesoscopic fork allows generation of spin-entangled pairs of electrons through post-selection; the presence of these entangled pairs can be observed in a Bell inequality measurement based on particle-number cross correlators. A number of items have to be observed in producing these entangled objects: (i) Equilibrium fluctuations competing with the pulse signal have to be eliminated. This can be achieved with the help of a four-channel beam splitter as sketched in Fig. 1(b) where the channel mixing is tuned such that the transmission T_{ud} between the upper and lower channel is blocked. (ii) Pulses $V(t)$ with integer Faraday flux $\Phi = -c \int dt V(t) = n\Phi_0$ injecting an integer number of particles shall be used. Otherwise, the "fractional injection" of a particle induces a long-time perturbation in the system producing a logarithmically divergent contribution to the excess number correlator. The flux $\Phi = (n + \delta n)\Phi_0$, $n =$ an integer, associated with the voltage pulse has to be precise within the limit $(\delta n)^2 \ll 1/\ln(t_m/\delta t)$, with t_m the measurement time of the pulse and δt the pulse width. (iii) The Bell inequality is violated for pulses injecting a single pair of electrons with opposite spin, i.e., pulses with one Faraday flux and hence $n = 1$. The maximal violation of the inequality points to the full entanglement of the pair—the question, "What type of pulses produce only partially entangled states?" (as quantified in terms of concurrence or negativity of the partially transposed density matrix[29]) has not been addressed here. (iv) Although weak pumping with pulses carrying less than one Faraday flux, i.e., $n < 1$, formally violate the Bell inequality [note the proviso (ii), however], we associate this spurious violation with an improper normalization of the particle-number ratios $(N_i - N_j)/(N_i + N_j)$ entering the Bell inequality. (v) The same argument also applies to the case of pumping with an alternating signal—we find the Bell inequality always violated when the average injected current vanishes (i.e., when the number of carriers transmitted in the forward and backward directions are equal). Again, the origin of this spurious violation is located in the improper normalization of the particle-number ratios $(N_i - N_j)/(N_i + N_j)$ for this situation.

The above points suggest the following physical interpretation: An integer-flux pulse with Faraday flux $n\Phi_0$ extracts exactly n electron pairs from the reservoir that then are tested in the Bell measurement setup. For $n = 1$ we find the Bell inequality maximally violated, implying that the electrons within the pair are maximally entangled and not entangled with the remaining electrons in the Fermi sea. On the other

hand, the application of a fractional-flux pulse with noninteger n produces a superposition of states with a different number of excess electron pairs in the fork. The electrons injected into the fork then remain entangled with those in the Fermi sea and their analysis in the Bell measurement setup makes no sense.

In our analysis of the spurious violations of Bell inequalities for weak pumping and for alternating drives we have identified the presence of reverse particle flow as the problematic element. In the weak pumping limit this conclusion has been conjectured from the appearance of negative values in the current cross correlator, implying negative values of the particle-number correlator for $n < 1$. Although we believe that these are strong arguments supporting our interpretation, we are not aware of a formal analysis of the backflow appearing in systems of this type. The question to be addressed then is: "Given a bias signal driving particles through the device in the forward direction, what are the circumstances and what is the probability of finding particles moving in the opposite direction (backflow)?" A related problem has been addressed by Levitov[30] (see also Ref. 31), who has derived the full counting statistics for the charge transport across a quantum point contact in the weak ac pumping regime and has identified parameters producing a strictly unidirectional flow. The corresponding analysis for our system remains to be done.

A similar scheme for producing spin-entangled pairs of electrons has been discussed in Ref. 8, where a constant voltage V has been applied to the source lead "s." In this case, the source reservoir injects a regular sequence of spin-singlet pairs of electrons separated by the voltage time $\tau_V = h/eV$; the Bell inequality then is violated at short times only. The main difference of the present proposal is the generation of well-separated spin-entangled electron pairs in response to distinguished voltage pulses, thus avoiding the short time correlation measurement at time scales of order τ_V.

Our mesoscopic fork device produces entangled pairs of electrons with a probability of 50%, i.e., half of the single-flux pulses will produce a useful pair with one spin propagating in the upper and the other in the lower channel. The competing events with both particles moving in one channel produce no useful outcome. This is similar to the finding of Beenakker et al.[13], who derive a concurrence corresponding to the production of one entangled pair per two pumping cycles. In how far this represents an upper limit in the performance of this type of device or what type of entanglement generators are able to reach (at least ideally) 100% efficiency is an interesting problem.

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