Electromagnetic wave refraction at an interface of a double wire medium

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Plane-wave reflection and refraction at an interface with a double-wire medium is considered. The problem of additional boundary conditions (ABC) in application to wire media is discussed and an ABC-free approach, known in the solid state physics, is used. Expressions for the fields and Poynting vectors of the refracted waves are derived. Directions and values of the power density flow of the refracted waves are found and the conservation of the power flow through the interface is checked. The difference between the results given by the conventional model of wire media and the model, properly taking into account spatial dispersion, is discussed.

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I. INTRODUCTION

Wire medium (WM) is an artificial medium formed by a lattice of ideally conducting thin wires. Recently, interest in such artificial media which offer new physical phenomena and applications has been growing. This medium at low frequencies is usually described as a uniaxial crystal, whose axial permittivity tensor component is expressed by the plasma formula

$$\epsilon = \epsilon_h\left(1 - \frac{\omega_p^2}{\omega^2\epsilon_h}\right) = \epsilon_h\left(1 - \frac{k_p^2}{k^2}\right). \quad (1)$$

Here ϵ_h is the permittivity of the host medium, $k=\omega\sqrt{\epsilon_h}/c = k_o\sqrt{\epsilon_h}$, and c is the speed of light. The constant ω_p (or the corresponding k_p) is an equivalent "plasma frequency" that gives grounds to call the wire medium "artificial plasma." There exist different models for the plasma frequency, see Refs. 1–3. For thin wires all models give similar results, and we will use below in our calculations the following approximate formula:[2]

$$k_p^2 = \frac{2\pi/L^2}{\ln\dfrac{L}{2\pi r_0} + 0.5275}. \quad (2)$$

It has been shown,[4] that if the wave vector in a WM has a nonzero component along the wires, the plasma model should be corrected introducing spatial dispersion (SD). Similarly, spatial dispersion is inherent to the double WM (DWM) formed by two mutually orthogonal lattices of thin ideally conducting straight wires. See Fig. 1.

For consideration of waves in double wire media let us take a case where the wires are perpendicular to each other in the y and z directions. The waves in unbounded space filled with a DWM medium were studied in Refs. 5–7 numerically, and in Ref. 8, using a semianalytical approximation of the local field, and in Ref. 9 both numerically and using the effective medium (EM) approach. In a previous paper, a very good agreement between the results given by the EM and full-wave theories for all types of waves in DWM (if the wires are thin) has been demonstrated.

In this paper we consider the plane-wave reflection and refraction at an interface of DWM using the effective medium approach. We assume that the two orthogonal wire arrays are identical, the period of the lattice is equal to L in the x, y, and z directions, and the radius of the wires is equal to r_0. In this case the wire lattice is square in the plane of the wires, i.e., the (yz) plane. We assume also that the interface of DWM lies in the (xy) plane and the incident wave vector lies in the (yz) plane (see Fig. 2).

In the following sections we will give basic expressions, obtained in the framework of the EM approach, and formulate the wave refraction problem, demonstrating the neces-

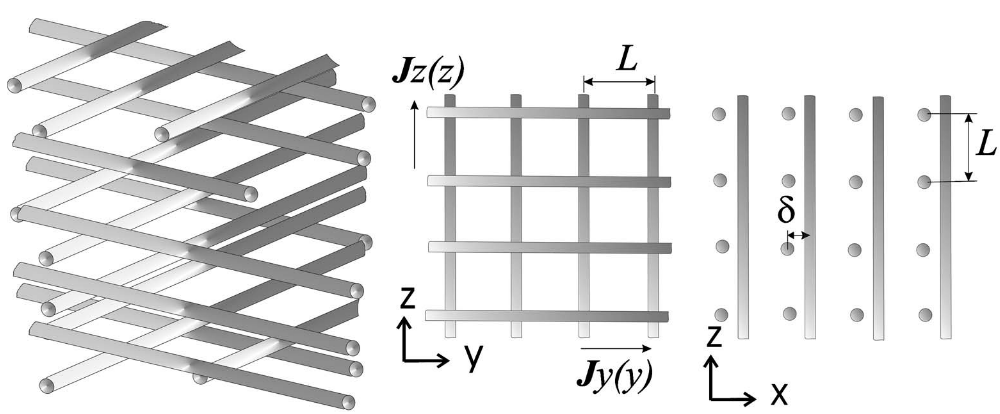

FIG. 1. Geometry of the unbounded DWM.

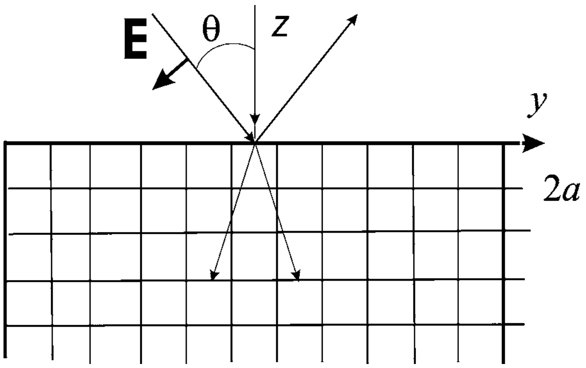

FIG. 2. Geometry of the wave reflection problem.

sity of additional boundary conditions (ABCs) both for interface problems with single and double wire media. Then we will discuss some approaches, applying in solid state physics in order to overcome the ABC problem and look what may be useful for us in application to WM. Using the ABC-free approach we will find the reflection coefficient, amplitudes and Poynting vectors for refracted waves. We will compare directions of the group velocity and the energy density flow found from the expression for the Poynting vector containing additional terms inherent for media with spatial dispersion. Conservation of the normal component of the power-flow vector at passing through the interface is checked.

II. FIELD EQUATIONS AND EIGENWAVES IN THE DOUBLE UNBOUNDED WIRE MEDIUM

Assuming space-time dependence of fields as $e^{j(\omega t - k_y y - k_z z)}$, there are nonzero wave vector components parallel to wires. Anisotropy appears in this electromagnetic crystal with square lattice and DWM behaves as a biaxial crystal with the relative permittivity dyadic

$$\overline{\overline{\epsilon}} = \epsilon_x \mathbf{u}_x \mathbf{u}_x + \epsilon_y \mathbf{u}_y \mathbf{u}_y + \epsilon_z \mathbf{u}_z \mathbf{u}_z, \tag{3}$$

with

$$\epsilon_x = \epsilon_h, \quad \epsilon_y = \epsilon_h\left(1 - \frac{k_p^2}{k^2 - k_y^2}\right), \quad \epsilon_z = \epsilon_h\left(1 - \frac{k_p^2}{k^2 - k_z^2}\right). \tag{4}$$

Note, that Eq. (4) works both for real and imaginary k_y (for propagating and evanescent waves, respectively), see Ref. 4.

In the wire medium the Maxwell equations

$$\nabla \times \mathbf{E} = -j\omega\mu_o \mathbf{H} \tag{5}$$

$$\nabla \times \mathbf{H} = j\omega\epsilon_o \overline{\overline{\epsilon}} \cdot \mathbf{E} \tag{6}$$

split into two separate subsystems describing wave propagation of fields with TE and transverse magnetic (TM) polarizations:

$$-j(k_y\mathbf{u}_y + k_z\mathbf{u}_z) \times (E_x\mathbf{u}_x + E_y\mathbf{u}_y + E_z\mathbf{u}_z) = -j\omega\mu_o(H_x\mathbf{u}_x + H_y\mathbf{u}_y + H_z\mathbf{u}_z), \tag{7}$$

$$-j(k_y\mathbf{u}_y + k_z\mathbf{u}_z) \times (H_x\mathbf{u}_x + H_y\mathbf{u}_y + H_z\mathbf{u}_z) = j\omega\epsilon_o(\epsilon_x E_x\mathbf{u}_x + \epsilon_y E_y\mathbf{u}_y + \epsilon_z E_z\mathbf{u}_z). \tag{8}$$

For ordinary TE waves this leads to the wave equation

$$[k^2 - k_y^2 - k_z^2]E_x = 0. \tag{9}$$

The same equations hold for H_y and H_z. There are no effects due to wires, and ordinary waves propagate as in any isotropic dielectric medium.

Whereas for extraordinary TM waves the wires affect the propagation, and we obtain the wave equation

$$\left[k^2\epsilon_y - k_z^2 - k_y^2\frac{\epsilon_y}{\epsilon_z}\right]H_x = 0. \tag{10}$$

The same equation can be written for E_y and E_z.

In order to solve the wave reflection problem we need to evaluate the eigenwaves which are outgoing from the interface of DWM. It means that we have to find k_z under fixed k and k_y. Let $k_y = k\sin\theta$, where θ is the incidence angle.

It follows from (10) that the dispersion equation has the form

$$T(k_z,\omega) = k_z^2 - \left[k^2\epsilon_y(k_y) - k_y^2\frac{\epsilon_y(k_y)}{\epsilon_z(k_z)}\right] = 0, \tag{11}$$

and its solution is

$$k_{z\pm}^2 = \frac{2k^4 - 2k^2k_p^2 - 3k^2k_y^2 + 2k_p^2k_y^2 + k_y^4 \pm k_y\sqrt{(k_y^2 - k^2)[(2k_p^2 + k_y^2)^2 - k^2(4k_p^2 + k_y^2)]}}{2(k^2 - k_y^2)}. \tag{12}$$

Two waves propagating or attenuating in both directions follow from the effective medium theory (12).The conventional isotropic plasma model leads to only two waves for a certain direction, namely, $k_z = \pm\sqrt{k_0^2\epsilon - k_y^2}$, $\epsilon = \epsilon_h(1 - k_p^2/k^2)$.

For the following consideration of the refraction problem we will need to know properties of waves propagating in the z direction. Here we briefly revise these properties (see our previous paper[9] for more details). The real and imaginary parts of k_z versus the normalized frequency k/k_p are presented in Fig. 3 for $\theta = \pi/4$. Let us assume that $\epsilon_h = 1$. In the simple case of the conventional model (dotted curve) k_z is imaginary if $k < K_2 = k_p/\cos\theta$, and it is real if $k > K_2$. The

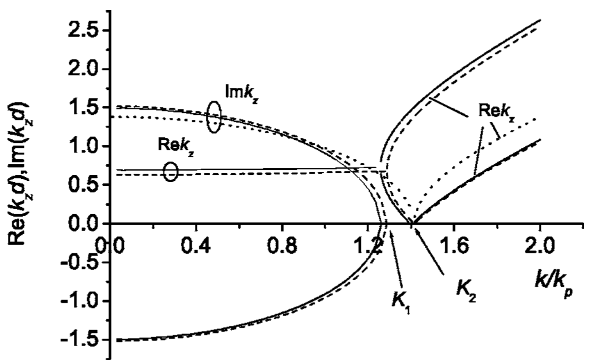

FIG. 3. Real and imaginary parts of k_z, calculated using the electrodynamical model (solid curves) and the EM theory (dashed curves). The dotted curve shows k_z given by the conventional plasma model.

solution of the conventional model is completely wrong between K_1 and K_2 because it gives an imaginary value of k_z instead of a real one which is obtained from the correct model. The correct, more complicated solutions, follow from Eq. (12). Analyzing Eq. (12), one can see that there exist three frequency regions, corresponding to different kinds of solution.

The first one is the low frequency band $k < K_1$, where

$$K_1 = k_p \frac{\sqrt{2}}{\sin\theta}\sqrt{\frac{1-\cos\theta}{\cos\theta}} \tag{13}$$

is the stop band edge: above this wave number the waves are propagating. Thus, the band edge shifts to smaller k compared with the point K_2 given by the conventional model. There the propagation constant k_z is complex despite the fact that we have assumed the medium to be lossless (see Fig. 3). Actually, there are two complex conjugate solutions for each $\mathrm{Re}(k_z) > 0$.

The second frequency area is $K_1 < k < K_2$. At point K_2 one of the solutions is zero and within the range $K_1 < k < K_2$ we have a forward wave and a backward wave with respect to the interface. It means that the energy density velocity (group velocity) for both waves points into the material, obeying causality, but for one of the waves the normal component of the wave vector (or phase velocity) is positive while for the other wave it is negative. The group velocity is equal to zero at point K_1. Note that it takes place under nonzero k_z and it is very similar to the vanishing of the group velocity in superconductors near the Josephson plasma resonance due to spatial dispersion.[10] A branch of the dispersion characteristics corresponding to the real part of k_z for the complex wave originates exactly at this point. Compare with the result obtained in the framework of the conventional model, where the point K_2, and not K_1, corresponds to the edge of the passband and $k_z = 0$ at the edge.

Finally, for $k > K_2$ both of the waves are propagating forward waves. Electrodynamical calculations[9] (using the three dimensional periodic Green's function) confirm the results of the effective medium theory with a high accuracy in a wide spectral range including the regions of evanescent and propagating waves (see solid and dashed curves in Fig. 3). Thus, the model taking into account spatial dispersion leads to a considerably more complicated structure of eigenwaves than the conventional model of an isotropic plasma, and it is in very good agreement with the results of the full-wave analysis.

III. WAVE REFLECTION FROM A WIRE MEDIUM INTERFACE AND THE PROBLEM OF ADDITIONAL BOUNDARY CONDITIONS

As was shown above, there exist two extraordinary waves with the wave vector and the electric field in the (yz) plane. Assuming the y component of the electric field of the incident wave to be equal to unity, and applying the continuity conditions of the tangential field components results in the reflection problem formulated as follows:

$$1 + R_E = E_+ + E_-,$$

$$(1 - R_E)/Z_0 = E_+/Z_+ + E_-/Z_-, \tag{14}$$

where R_E is the unknown reflection coefficient for electric field, E_+, E_- are the unknown amplitudes of refracted waves in the wire medium, Z_0 is the wave impedance (TM) in free space, and $Z_\pm$ are the wave impedances of the refracted waves. Thus the problem becomes similar to one appearing in crystallooptics, where excitons arise and spatial dispersion cannot be neglected.[11] The main difficulty here is the necessity to invoke additional boundary conditions in order to match solutions at the interface of media. It was pointed out by Pekar[12] (1956) that the well-known Maxwell's boundary conditions (14) are not sufficient to connect the amplitudes of the incident and transmitted waves in adjoining media, if more than one independent wave can propagate in any medium.

Probably, the first ABCs were proposed by Pekar,[12] and his ABCs stay among the more often used in the theory of media with SD. The most general ABC looks like[11]

$$P_z + \ell \frac{\partial P_z}{\partial z} = 0, \tag{15}$$

where ℓ is a phenomenological parameter to be determined from a microscopic model.

After pioneering Pekar's publication, different ABCs were proposed for problems of crystallooptics, as well as semiconductor and plasma electrodynamics. All of these works relate to specific media and take into account the properties of a subsurface layer at the media interface (see Refs. 11 and 13, and the bibliography presented there). Besides, phenomenological assumptions and experimental data are used very often in these theories. Since WM strongly differs from a solid-state crystal, only general approaches to ABCs which do not concern particular media may be interesting for us.

Many authors (see, for example, Refs. 14 and 15) derived ABCs from a given model of medium with an explicit specification of its surface. After simplifications these models give nothing more than Pekar's ABCs. Recently the microscopic approach was applied to study optical properties of layered superconductors near the Josephson plasma resonance.[10,16,17]

In these, media effects of spatial dispersion appear due to a strong delay of the group velocity in vicinity of the resonance. Despite that the wavelength is much larger than the lattice constant, spatial dispersion strongly influences transmission of light through a layered structure when the energy velocity approaches zero and the interaction time of light and matter remarkably increases. Actually, authors of Refs. 10, 16, and 17 derived parameter ℓ entering Eq. (15), solving the electrodynamical problem for the medium and neglecting some insignificant terms. Direct application of such an approach to WM is not possible due to the strongly different nature of superconductors near the Josephson plasma resonance and media consisting of infinitely long conductive wires. Among different theories of SD media, so-called "ABC-free" theories attract our attention. One of the ways for ABC derivation is a concept of "exciton dead layer," proposed by Hopfield and Thomas. Actually, there is a layer in which exciton wave function has an evanescent form. Cho[18] has pointed out that there is a certain limit of transition layer thickness, below which the ABC theory is unnecessary. However, the Cho theory is not yet free from some parameters determined by specific surfaces. Chen and Nelson declare that their work[19] solves the macroscopic ABC problem completely. Despite the fact that the authors of Ref. 19 used a complicated quantum mechanical model, their derivation found that the fully macroscopic solution is equivalent to the use of Pekar's ABC $P(z=0)=0$, whose application to the WM cannot be proved. Thus, such an approach also is not suitable for us. Vinogradov *et al.* considering in Ref. 20 the effects of SD in composite metamaterials, have obtained ABCs using an assumption of existence of additional waves in free space which are the same as in the metamaterial but have evanescent nature. This assumption leads to additional equations at the interface.

Another way to solve the problem was proposed recently by Henneberger.[21] It is based on the assumption of an abrupt transition from medium to vacuum. It is assumed that the incident wave excites a source $s(z,\omega)$, located within a subsurface layer $0<z<2a$, and its thickness is assumed to be negligible. This approach is appropriate for our problem of wire-media interface. Indeed, it is known, for problems of diffraction by single semiinfinite wire grids, that the induced currents deviate from the regular amplitudes far from the interface only in a very narrow region whose width is of the order of the grating period.[22] This conclusion holds for grids of wires that are both parallel and perpendicular to the edge. For this reason we assume that for the wire-medium interface the transition layer has the thickness of only a few periods of the lattice. This thickness is much smaller than the wavelength and negligible as compared with the length of the wires (wires are infinite in our model).

Applying this approach to our interface problem of free space and the wire medium, the wave equation for H_x in unbounded medium [Eq. (10) written for H_x] should be replaced by an inhomogeneous equation

$$\frac{\partial^2 H_x}{\partial z^2} + \left[k_0^2 \epsilon_y(k_y) - k_y^2 \frac{\epsilon_y(k_y)}{\epsilon_z(k_z)} \right] H_x = s(z,\omega), \qquad (16)$$

where H_x is the refracted field. It means that any propagating wave has to be created by a source. The proper source of the penetrating wave in the wire medium is the incident wave and the polarization induced by it in the medium. Such an externally controlled source can be identified with some polarization additionally induced to the one already described by $\overline{\overline{\epsilon}}$. Therefore it is located only on the surface and in the transition region, where the induced polarization deviates from that in the bulk medium. After the Fourier transform of Eq. (16) one obtains

$$H_x(z,\omega) = \int_{-\infty}^{\infty} \frac{dq}{2\pi} \frac{s(q,\omega)e^{jqz}}{T(q,\omega)}, \qquad (17)$$

where $s(q,\omega)$ is the Fourier transform of $s(z,\omega)$ and $T(q,\omega)$ is determined by Eq. (11). Assuming an abrupt transition from the medium to vacuum, we can present the source as a delta function $s(z,\omega)=s_0(\omega)\delta(z)$, then $s(q,\omega)=s_0(\omega)$. If $T(q,\omega)$ is an analytical function, the integration in Eq. (17) can be performed using the residue method. Residues can be found by presenting $1/T$ in the form

$$\frac{1}{T(k_z,\omega)} = \frac{1}{k_z^2 - \left[k_0^2 \epsilon_y(k_y) - k_y^2 \frac{\epsilon_y(k_y)}{\epsilon_z(k_z)} \right]} = \frac{\beta_+}{k_z^2 - k_{z+}^2} + \frac{\beta_-}{k_z^2 - k_{z-}^2}, \qquad (18)$$

where the coefficients are

$$\beta_+ = \frac{k^2 - k_p^2 - k_{z+}^2}{k_{z-}^2 - k_{z+}^2}, \qquad (19)$$

$$\beta_- = -\frac{k^2 - k_p^2 - k_{z-}^2}{k_{z-}^2 - k_{z+}^2}. \qquad (20)$$

The residues (the relative amplitudes of the transmitted field components) read

$$R_+ = \frac{\beta_+}{2k_{z+}} = \frac{k^2 - k_p^2 - k_{z+}^2}{2(k_{z-}^2 - k_{z+}^2)k_{z+}}, \qquad (21)$$

$$R_- = \frac{\beta_-}{2k_{z-}} = -\frac{k^2 - k_p^2 - k_{z-}^2}{2(k_{z-}^2 - k_{z+}^2)k_{z-}}. \qquad (22)$$

Finally, the field component H_x in the wire medium is

$$H_x = s_o[R_+ e^{-jk_{z+}z} + R_- e^{-jk_{z-}z}] = s_o\left[\frac{\beta_+}{2k_{z+}} e^{-jk_{z+}z} + \frac{\beta_-}{2k_{z-}} e^{-jk_{z-}z} \right]. \qquad (23)$$

The expression for E_y and E_z can be obtained from the Maxwell equations as

$$k_z H_x = -\omega\epsilon_0\epsilon_y E_y, \quad -k_y H_x = -\omega\epsilon_0\epsilon_z E_z, \qquad (24)$$

which gives us the electric field components in the wire medium

$$E_y = -\frac{s_o}{2\omega\epsilon_0\epsilon_y}[\beta_+ e^{-jk_{z+}z} + \beta_- e^{-jk_{z-}z}], \qquad (25)$$

$$E_z = \frac{k_y s_o}{\omega \epsilon_o}\left[\frac{\beta_+}{2k_{z+}\epsilon_{z+}}e^{-jk_{z+}z} + \frac{\beta_-}{2k_{z-}\epsilon_{z-}}e^{-jk_{z-}z}\right]. \tag{26}$$

Now we have expressions for all field components induced in the wire medium for the TM polarization.

In free space there exist incident and reflected TM waves. Magnetic field components are

$$H_x^i = H_0 e^{-jk_y y}e^{-j\beta_0 z}, \quad H_x^r = H_r e^{-jk_y y}e^{j\beta_0 z} = R_H H_0 e^{-jk_y y}e^{j\beta_0 z}, \tag{27}$$

where $\beta_0=\sqrt{k_0^2-k_y^2}$ and the electric field components are

$$E_y^i = -\frac{\beta_0}{k_0}\eta H_0 e^{-jk_y y}e^{-j\beta_0 z}, \quad E_y^r = R_H\frac{\beta_0}{k_0}\eta H_0 e^{-jk_y y}e^{j\beta_0 z}, \tag{28}$$

$$E_z^i = \frac{k_y}{k_0}\eta H_0 e^{-jk_y y}e^{-j\beta_0 z}, \quad E_z^r = R_H\frac{k_y}{k_0}\eta H_0 e^{-jk_y y}e^{j\beta_0 z}. \tag{29}$$

At the interface $z=0$ the continuity of the tangential field components leads to relations

$$H_0 + R_H H_0 = \frac{s_0}{2}\left[\frac{\beta_+}{k_{z+}} + \frac{\beta_-}{k_{z-}}\right], \tag{30}$$

$$-\frac{\beta_0}{k_0}H_0 + \frac{\beta_0}{k_0}R_H H_0 = -\frac{s_0}{2k_0\epsilon_y}[\beta_+ + \beta_-], \tag{31}$$

from which the reflection coefficient for the magnetic field is obtained

$$R_H = \frac{\left(\frac{\beta_+}{k_{z+}} + \frac{\beta_-}{k_{z-}}\right) - \frac{1}{\epsilon_y}\left(\frac{\beta_+}{\beta_0} + \frac{\beta_-}{\beta_0}\right)}{\left(\frac{\beta_+}{k_{z+}} + \frac{\beta_-}{k_{z-}}\right) + \frac{1}{\epsilon_y}\left(\frac{\beta_+}{\beta_0} + \frac{\beta_-}{\beta_0}\right)}. \tag{32}$$

Finally, the explicit expression for the coefficient s_0 (the transmission source) is obtained,

$$s_0 = \frac{1+R_H}{\frac{1}{2}\left(\frac{\beta_+}{k_{z+}} + \frac{\beta_-}{k_{z-}}\right)}H_0 = \frac{4H_0}{\left(\frac{\beta_+}{k_{z+}} + \frac{\beta_-}{k_{z-}}\right) + \frac{1}{\epsilon_y}\left(\frac{\beta_+}{\beta_0} + \frac{\beta_-}{\beta_0}\right)}. \tag{33}$$

In the region of complex waves $k/k_p < K_1$ we have to choose the branches of the square roots for $k_{z\pm}$ having positive imaginary parts. In the region $K_1 < k/k_p < K_2$ it is necessary to take for the backward wave ("–" wave) the root branch with the opposite sign. In the above derivation we evaluated the reflection coefficient for the magnetic field. The reflection coefficient for the electric field is $R_E=-R_H$.

Reflection from a single wire medium interface

The plane-wave reflection coefficient from an interface of a single wire medium where wires are along the z axis is easily obtained as a special case of the previously considered double-wire medium reflection problem. In a single wire medium the permittivity components are $\epsilon_x=\epsilon_y=\epsilon_h$ and $\epsilon_z=\epsilon_h\{1-[k_p^2/(k^2-k_z^2)]\}$. Evaluating the dispersion equation (11) we have as solutions $k_{z+}=k$, which is the propagation factor for the TEM mode and $k_{z-}=\sqrt{k^2-k_y^2-k_p^2}$ for the TM mode. Thus, in the single wire medium the two extraordinary eigenwaves are TEM and TM polarized.

We can use exactly the same expressions for the reflection coefficient as in the case of the double wire medium by simply substituting $k_{z+}=k$ and $k_{z-}=\sqrt{k^2-k_y^2-k_p^2}$. This leads to expressions for the coefficients

$$\beta_+ = \frac{k_p^2}{k_y^2+k_p^2}, \tag{34}$$

$$\beta_- = \frac{k_y^2}{k_y^2+k_p^2}, \tag{35}$$

and residues

$$R_+ = \frac{k_p^2}{2k(k_y^2+k_p^2)}, \tag{36}$$

$$R_- = \frac{k_y^2}{2\sqrt{k^2-k_y^2-k_p^2}(k_y^2+k_p^2)}. \tag{37}$$

IV. GROUP VELOCITY, POYNTING VECTOR AND REFRACTED WAVES IN DWM

In this section we will discuss the group velocity and Poynting vectors of waves excited in a double wire medium and check the power conservation at the interface. It is well known that the group velocity is defined as

$$v_{gr} = \text{grad}_{\mathbf{k}}\omega. \tag{38}$$

The Poynting vector that determines the energy density flow for media with spatial dispersion has the form[23]

$$\mathbf{S} = \frac{1}{2}\text{Re}\{\mathbf{E}\times\mathbf{H}^*\} - \frac{\omega}{4}\frac{\partial\epsilon_{ik}}{\partial\mathbf{k}}E_i^*E_k, \tag{39}$$

where the permittivity dyadic components are expressed by Eq. (4) for DWM. Their partial derivatives read

$$\frac{\partial\epsilon_x}{\partial k_x}=0, \quad \frac{\partial\epsilon_y}{\partial k_y} = -\frac{2k_p^2k_y}{(k^2-k_y^2)^2}, \quad \frac{\partial\epsilon_z}{\partial k_z} = -\frac{2k_p^2k_z}{(k^2-k_z^2)^2}. \tag{40}$$

Let us derive expressions for the Poynting vector in free space and in the wire medium. In free space the field expressions are

$$\mathbf{H}_1 = H_0[e^{-j\beta_0 z} + R_H e^{j\beta_0 z}]e^{-jk_y y}\mathbf{u}_x, \tag{41}$$

$$\mathbf{E}_1 = \frac{\beta_0\eta H_0}{k_0}[-e^{-j\beta_0 z} + R_H e^{j\beta_0 z}]e^{-jk_y y}\mathbf{u}_y + \frac{k_y\eta H_0}{k_0}[e^{-j\beta_0 z} + R_H e^{j\beta_0 z}]e^{-jk_y y}\mathbf{u}_z. \tag{42}$$

The fields of the waves, marked by + and –, in the wire medium are

$$\mathbf{H}_{2\pm} = \frac{s_0}{2}\frac{\beta_\pm}{k_{z\pm}} e^{-jk_{z\pm}z} e^{-jk_y y}\mathbf{u}_x, \tag{43}$$

$$\mathbf{E}_{2\pm} = -\frac{s_0\eta}{2k_0\epsilon_y}\beta_\pm e^{-jk_{z\pm}z}e^{-jk_y y}\mathbf{u}_y + \frac{k_y s_0\eta}{2k_0}\frac{\beta_\pm}{k_{z\pm}\epsilon_{z\pm}}e^{-jk_{z\pm}z}e^{-jk_y y}\mathbf{u}_z. \tag{44}$$

Now we can write the Poynting vector in free space and in the wire medium (at the interface)

$$\mathbf{S}_1(0) = \frac{1}{2}\mathrm{Re}\{\mathbf{E}_1 \times \mathbf{H}_1^*\} = \frac{|H_0|^2\eta}{2k_0}[\beta_0(1-|R_H|^2)\mathbf{u}_z + k_y(1+2|R_H|\cos\phi + |R_H|^2)\mathbf{u}_y] \tag{45}$$

with the notation $R_H = |R_H| e^{j\phi}$. In the wire medium, the cross product term is

$$\mathbf{S}_{2\pm}^0(0) = \frac{1}{2}\mathrm{Re}\{\mathbf{E}_{2\pm}\times\mathbf{H}_{2\pm}^*\} = \frac{|s_0|^2\eta}{2k_0}\frac{1}{4}\,\mathrm{Re}\left\{\frac{1}{\epsilon_y}\frac{\beta_\pm\beta_\pm^*}{k_{z\pm}^*}\mathbf{u}_z + k_y\frac{\beta_\pm\beta_\pm^*}{k_{z\pm}k_{z\pm}^*\epsilon_{z\pm}}\mathbf{u}_y\right\}, \tag{46}$$

and the spatial dispersive term is

$$\mathbf{S}_{2\pm}^d(0) = \frac{|s_0|^2\eta}{2k_0}\frac{1}{4}\left[\frac{k_p^2k_y^2k_{z\pm}\beta_\pm\beta_\pm^*}{(k^2-k_{z\pm}^2)^2k_{z\pm}k_{z\pm}^*\epsilon_{z\pm}\epsilon_{z\pm}^*}\mathbf{u}_z + \frac{k_p^2k_y}{(k^2-k_y^2)^2\epsilon_y^2}\beta_\pm\beta_\pm^*\mathbf{u}_y\right]. \tag{47}$$

The total Poynting vector in wire medium is

$$\mathbf{S}_{2\pm}(0) = \mathbf{S}_{2\pm}^0(0) + \mathbf{S}_{2\pm}^d(0). \tag{48}$$

As an example, we consider excitation of modes, located above the plasma frequency.[5] For the taken parameters of DWM, the wire radius $r_0 = 0.01$ cm, and the period $L = 1$ cm, the plasma wave number is $k_p \approx 1.38\ \mathrm{cm}^{-1}$. The dispersion diagram in the form of isofrequencies and directions of the Poynting vector (the energy velocity) for these modes with respect to the normal to the interface (z axis) are shown in Fig. 4. Isofrequencies, the lines of constant frequencies in the plane of wave vectors, are presented in Fig. 4 in coordinates k_y, k_z. If one draws a vector from the origin to a point at an isofrequency, it will show the direction of the phase velocity corresponding to this frequency and certain k_y, k_z. Normal to the isofrequency at the same point shows the direction of the group velocity. Isofrequencies are very convenient for construction of refraction and reflection laws using the following rules: projection of the wave vector on the interface is the same for the incident, refracted and reflected waves, and the group velocity that is outgoing from the interface.

The calculations were performed at $k/k_p = 1.35$. Here we observe two elliptic-type isofrequencies for TM modes, that is, appearance of SD, because without SD we would have one circle for a square-wire lattice in the (yz) plane. The value of the tangential component k_y is determined by the incidence angle θ, namely, $k_y = k\sin\theta$. The direction of the group velocity, found by numerical differentiation of the dispersion characteristics (which are the isofrequencies in Fig. 4), exactly coincides with the direction of $\mathbf{S}_{2\pm}$, calculated using formulas (46)–(48). The solid curves in Fig. 4 show the angle ψ between the Poynting vector (or the group velocity) and the normal to interface versus k_y. Disregarding the term that takes into account spatial dispersion leads to a strongly incorrect result, illustrated by dashed curves.

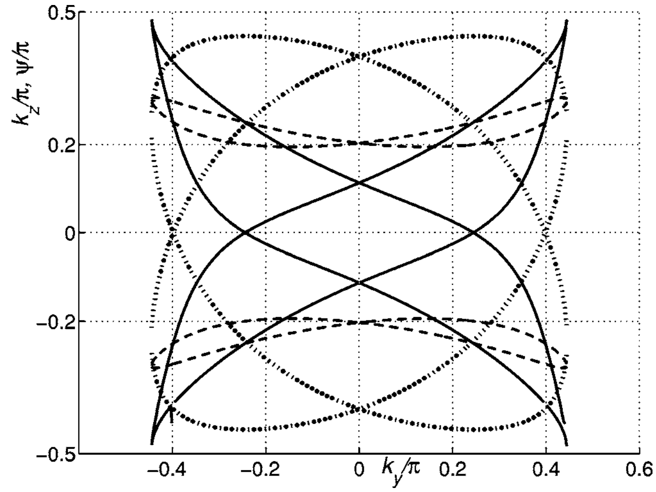

FIG. 4. Dotted curves indicate isofrequencies. Solid curves show the angle ψ between the energy velocity and the z axis versus k_y. Dashed curves show the angle between $\mathbf{S}_{2\pm}^0(0)$ and the z axis versus k_y.

Next, let us consider the power conservation at the interface of free space and the wire medium. It is important to check this properly because in the wire-medium region there exist two waves, and the wire medium is spatially dispersive. In the frequency range considered here, the parameter values are assumed to be real. The normal components of the Poynting vector on both sides of the interface are (using the continuity condition of the tangential field components)

$$S_{z1}(0) = \frac{|s_o|^2}{8\omega\epsilon_o\epsilon_y}\left(\frac{\beta_+}{k_{z+}} + \frac{\beta_-}{k_{z-}}\right), \tag{49}$$

and

$$S_{2z}(0) = \frac{|s_o|^2}{8\omega\epsilon_o}\left[\frac{1}{\epsilon_y}\left(\frac{\beta_+^2}{k_{z+}} + \frac{\beta_-^2}{k_{z-}}\right) + \frac{k_p^2k_y^2\beta_+^2}{(k^2-k_{z+}^2)^2\epsilon_{z+}^2k_{z+}} + \frac{k_p^2k_y^2\beta_-^2}{(k^2-k_{z-}^2)^2\epsilon_{z-}^2k_{z-}}\right]. \tag{50}$$

Subtracting these power density components leads to the expression

$$S_{z1}(0) - S_{z2}(0) = \frac{|s_o|^2}{8\omega\epsilon_o}\left(\frac{1}{k_{z_+}} + \frac{1}{k_{z_-}}\right)\left[\frac{\beta_+\beta_-}{\epsilon_y} - \frac{k_p^2k_y^2}{(k_{z-}^2 - k_{z+}^2)^2}\right]. \tag{51}$$

Using the expressions for $\beta_\pm$, $k_{z\pm}^2$, and ϵ_y, we find that the term inside the square brackets vanishes. The normal component of the Poynting vector is continuous across the interface, which means that the power conservation law is satisfied.

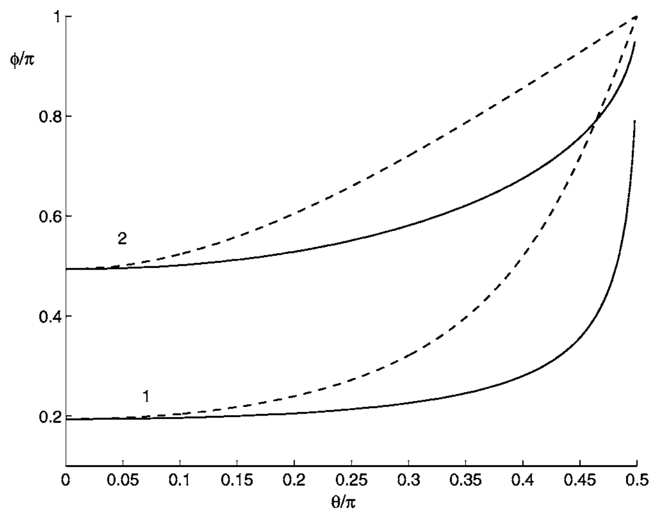

FIG. 5. Normalized phase of the reflected wave ϕ/π versus the incidence angle θ/π, calculated at different k/k_p: curves 1 correspond to $k/k_p=0.3$ and curves 2 correspond to $k/k_p=0.7$. Solid and dashed curves show the reflection phase given by the new and conventional models, respectively.

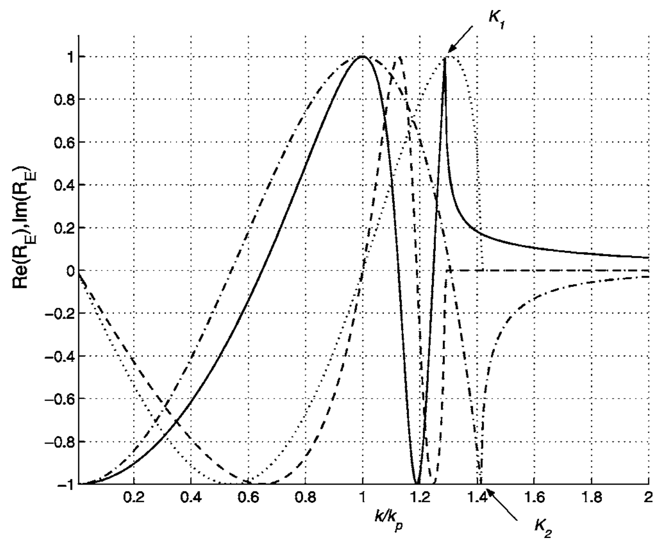

FIG. 6. Real and imaginary parts of the reflection coefficient versus the normalized wave vector k/k_p. The solid line shows $\mathrm{Re}(R_E)$, the dashed line shows $\mathrm{Im}(R_E)$ (the new model), the dash-dot and dot lines correspond to $\mathrm{Re}(R_E)$ and $\mathrm{Im}(R_E)$, respectively, obtained in the framework of the old model.

V. DISCUSSION OF THE REFLECTION AND REFRACTION AT THE INTERFACE OF WM

Figure 5 shows the phase of the reflected wave $\phi = \arctan\{\mathrm{Im}(R_H)/\mathrm{Re}(R_H)\}$ versus the incidence angle θ, calculated for different k/k_p, corresponding to the region of complex waves (see Fig. 3). Here $\mathrm{Re}(R_H)$ and $\mathrm{Im}(R_H)$ are the real and imaginary parts of the reflection coefficient R_H. The calculations have been performed using the model (4), taking into account spatial dispersion, and the conventional one, $\epsilon_y=\epsilon_z=\epsilon$, ϵ is expressed by formula (1). As it is expected, both models give the modulus of the reflection coefficient equal to unity due to the absence of losses and propagating waves in wire medium. It is remarkable that the spatial dispersion results in a weak dependence of the phase of the reflected wave on the incidence angle. The parameters of the wire medium are taken as above in the eigenvalues calculations.

The real and imaginary parts of the reflection coefficient in a wide spectral range cover areas with complex waves, forward and backward waves, and forward waves only, presented in Fig. 6. The incidence angle is taken to be $\pi/4$. To understand these characteristics, it is useful to compare them with the eigenvalue dispersion (Fig. 3). Distinctive points K_1, where propagating waves appear in the framework of a new model, and K_2, where both of the waves become forward ones. Results, given by the old model, are also shown here. The most important feature is that the area of propagating waves shifts by $\Delta k/k_p = K_2 - K_1$ in comparison with that given by the old model.

The normal to the interface components of the Poynting vectors of the refracted waves as well as the reflection coefficient (which is purely real beyond K_1) are shown in Fig. 7. The values of S_{2z+} and S_{2z-} are normalized to the power density of the incident plane wave $|S_i| = (1/2)\eta H_0^2 \cos\theta$. These results agree with those shown in Fig. 3, namely, S_{2z-} becomes zero at point $k/k_p = K_2$. The point K_2 is a transition point of the "–" wave because it is a backward wave with respect to the interface if $k/k_p < K_2$, and a forward wave if $k/k_p > K_2$. It is important, that birefringence takes place for the waves having the same TM polarization as found in Ref. 10 for superconductors.

Angular dependence of the normal components of the Poynting vector is presented in Fig. 8. Both refracted modes are TM modes in this case. At low frequencies ($k/k_p < 1$), a plane wave cannot excite propagating TM modes at any incidence angle θ, so we present here the results for $k/k_p = 1.5$. There exists a critical angle $\theta_c \simeq 0.3\pi$, such that $S_{z1} = S_{z2} = 0$ if $\theta > \theta_c$. As in the previous case, $R_H = +1$ if $\theta = \theta_c$, and $R_H \to 1$ if $\theta \to \pi/2$, so DWM is an electric wall for grazing incidence.

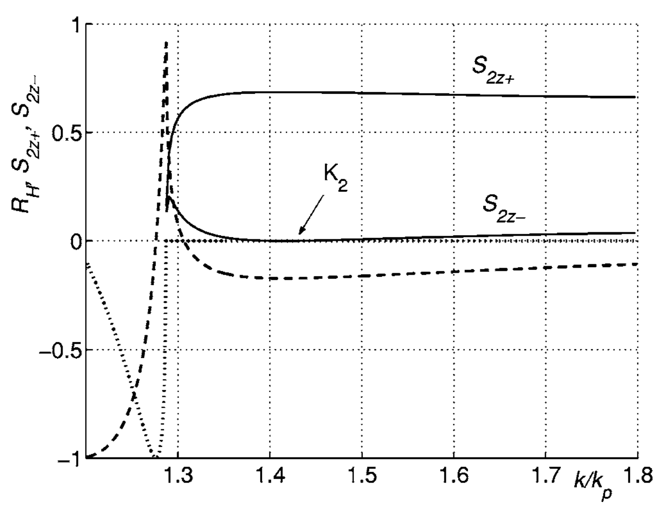

FIG. 7. Reflection coefficient R_H [$\mathrm{Re}(R_H)$ dotted curve, $\mathrm{Im}(R_H)$ dashed curve], S_{2z+} and S_{2z-} (solid curves) versus the normalized wave vector.

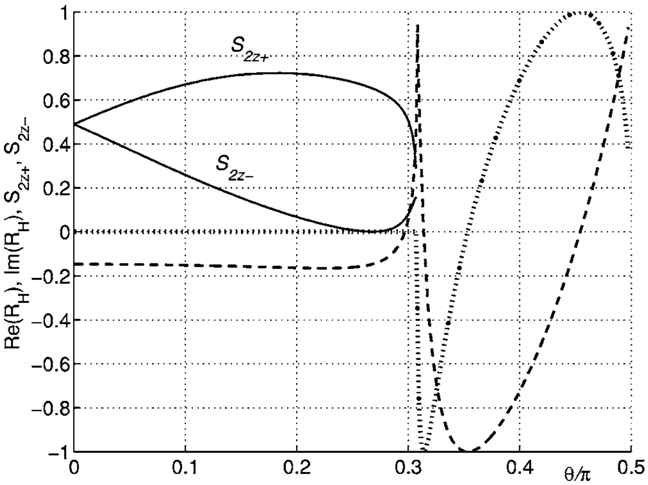

FIG. 8. The real and imaginary parts of R_H (dashed and dotted curves, respectively), S_{z+} and S_{z-} (solid curves) versus the incidence angle, calculated at $k/k_p=1.5$.

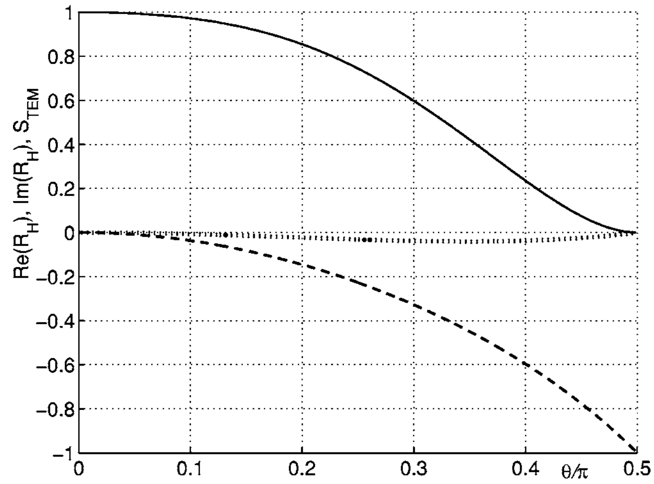

FIG. 9. The real and imaginary parts of R_H (dashed and dotted curves, respectively) and S_{TEM} (solid curves) versus the incidence angle, calculated at $k/k_p=0.5$.

Poynting vector in single wire media

The Poynting vector in a single wire medium can be simply deduced from the expressions for the double wire media. In this case, the partial derivatives of the permittivity components read

$$\frac{\partial \epsilon_x}{\partial k_x}=0, \quad \frac{\partial \epsilon_y}{\partial k_y}=0, \quad \frac{\partial \epsilon_z}{\partial k_z}=-\frac{2k_p^2 k_z}{(k^2-k_z^2)^2}. \tag{52}$$

The additional term arises now only in the z component of the Poynting vector, and it is equal to

$$\mathbf{S}_{2\pm}^d=\frac{|s_o|^2\eta}{2k_o}\frac{1}{4}\frac{k_p^2k_y^2|\beta_\pm|^2}{k_{z\pm}(k^2-k_{z\pm}^2-k_p^2)^2}\mathbf{u}_z. \tag{53}$$

Thus, the total Poynting vector reads for real parameter values

$$\mathbf{S}_{2\pm}=\frac{|s_o|^2\eta}{2k_o}\frac{\beta_\pm^2}{4k_{z\pm}^2}\left[\frac{k_y}{\epsilon_{z\pm}}\mathbf{u}_y+k_{z\pm}\left(\frac{1}{\epsilon_y}+\frac{k_p^2k_y^2}{(k^2-k_{z\pm}^2-k_p^2)^2}\right)\mathbf{u}_z\right]. \tag{54}$$

This gives us after substituting $k_{z\pm}$ and $\beta_\pm$

$$\mathbf{S}_{2+}=\frac{|s_o|^2\eta}{2k_o}\frac{kk_p^2}{4(k_y^2+k_p^2)}\mathbf{u}_z \tag{55}$$

and

$$\mathbf{S}_{2-}=\frac{|s_o|^2\eta}{2k_o}\frac{k_y^2+k_p^2}{4k_y^2}(k_y\mathbf{u}_y+\sqrt{k^2-k_y^2-k_p^2}\mathbf{u}_z). \tag{56}$$

One can see from (56) that for the TM mode, the Poynting vector is also in the same direction as the phase velocity. The continuity condition of the power flow across the interface (continuity of the normal component of the Poynting vector) can be easily checked similarly as for double wire media. For the TEM mode directions of the phase velocity and the Poynting vector are different. The Poynting vector and energy flow are always directed along the wires, see Eq. (55). The angle φ between the phase velocity and the interface is equal to

$$\varphi=\arctan\sin\theta. \tag{57}$$

Figure 9 illustrates the angular dependence of the reflection coefficient R_H and the normal to the interface component of the Poynting vector S_z for the TEM mode (S_{TEM}). Parameters of the WM are taken the same as in the case of DWM. The calculated Poynting vector is normalized to the incident wave power flow density. Wave number k/k_p corresponds to the frequency below the plasma resonance and the TM mode cannot be excited at any θ. For grazing incidence $R_H\to-1$. It is obvious that in this case the surface must behave as a wall, because no traveling waves in the medium can be excited. Actually, it behaves as a magnetic wall, because in this geometry the wires are orthogonal to the interface, and the normal component of the electric field is zero.

More complicated is the case when the wave number is taken beyond the plasma resonance, i.e., $k/k_p=1.5$, see Fig. 10. However, it follows from Eq. (4) that for oblique wave incidence, the "effective" plasma wave number becomes $k_p'=k_p/\cos\theta$ because $k_y=k\sin\theta$. The grazing incidence for $R_H\to-1$, is similar to the previous case.

For small incidence angles, both TEM and TM modes are excited. There exists an incidence angle θ', which is the angle of total reflection. It takes place when $k=k_p'$, so $\cos\theta'=2/3$. Neither TEM nor TM modes are excited, and $R_H\to1$ which corresponds to the case of an electric wall. For larger θ, the TM mode disappears.

VI. CONCLUSIONS

Considering the problem of a plane-wave refraction at the interface of a double wire medium exhibiting strong spatial dispersion, we have analyzed the literature discussion on the ABC problem and have come to the point of view that Hennenberger's approach[21] for SD media can be applied for some kinds of metamaterials in the microwave range including single and double wire media.

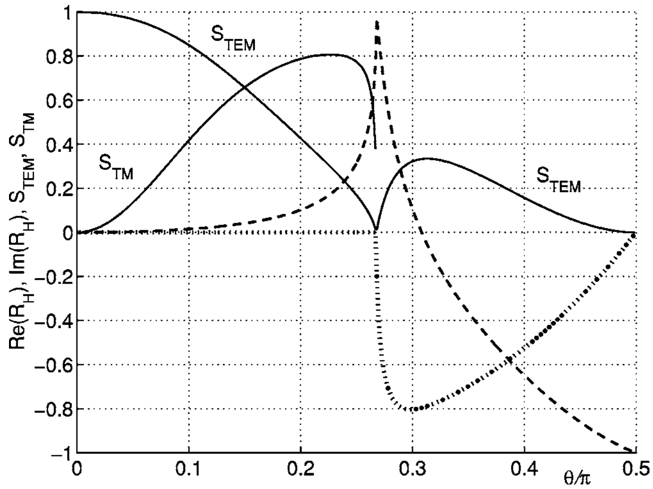

FIG. 10. The real and imaginary parts of R_H (dashed and dotted curves, respectively) and S_{TEM} and S_{zTM} (solid curves) versus the incidence angle, calculated at $k/k_p=1.5$.

Despite criticism of the used ABC-free method (see Refs. 24–26), we suppose that application of this approach has allowed us to overcome the problem of additional waves appearing in media with spatial dispersion and to obtain reasonable results for the reflection coefficient and for the power densities and group velocities of the refracted waves. Fulfilment of the conservation of the power, passing through the interface, can be considered as evidence of the correctness of this approach. A series of phenomena, similar to those found in superconductors near the Josephson plasma resonance,[10,16,17] can be observed in WM. There are excitation of multiple propagating or evanescent eigenmodes with the same polarization, nonapplicability of the conventional Fresnels reflection formulas, and stopping of a wave in the vicinity of some spectral point under a nonzero wave vector ("stop light"). We have shown that such an extremal point appears when two modes in DWM with positive and negative dispersion are mixed, and this effect is almost the same as described in Ref. 10 for superconductors. All those effects cannot be obtained in the single-mode approach without dispersion. However, there is an essential difference in the reason of appearance of spatial dispersion in the medium studied in Refs. 10, 16, and 17 and in artificial WM. Namely, in superconductors near the Josephson plasma resonance, spatial dispersion is caused by a strong delay of the group velocity and in WM a strong SD appears in the model of infinitely long conductive wires due to the nonlocal response even at very low frequencies. Thus SD in WM has a nonresonant nature and exists in a very wide frequency range.

Possible applications of DWM include antenna structures, low-frequency filters, frequency selective radomes, and double-negative metamaterials.

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