Macroscopic phase coherence of defective vortex lattices in two dimensions

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(Received 9 January 2005; published 7 December 2005)

The superfluid density is calculated theoretically for incompressible vortex lattices in two dimensions that have isolated dislocations quenched in by a random arrangement of pinned vortices. The latter are assumed to be sparse and to be fixed to material defects. It is shown that the pinned vortices act to confine a single dislocation of the vortex lattice along its glide plane. Plastic creep of the two-dimensional vortex lattice is thereby impeded, and macroscopic phase coherence results at low temperature in the limit of a dilute concentration of quenched-in dislocations.

DOI: 10.1103/PhysRevB.72.214503 PACS number(s): 74.25.Qt, 74.25.Op, 74.25.Dw, 61.72.Bb

I. INTRODUCTION

Consider either a bismuth-based, a mercury-based, or a thallium-based high-temperature superconductor in high enough external magnetic field so that magnetic flux lines appear, and so that these overlap considerably. Such materials are extreme type-II layered superconductors, with mass-anisotropy ratios between the layer direction and the perpendicular direction of order 10^{-3} or less.[1] To a first approximation then, it becomes valid to neglect the coupling of magnetic screening currents between layers, as well as the Josephson effect between them. The description of the initial physical situation is thereby reduced to a stack of isolated vortex lattices or vortex liquids within each layer.

High-temperature superconductors similar to those just mentioned typically also have crystalline defects and inhomogeneities that act as pinning centers for vortex lines in the mixed phase. Within the preceding approximation, theoretical and numerical studies indicate that *any* net concentration of randomly pinned vortices results in a net concentration of unbound dislocations quenched into the two-dimensional (2D) vortex lattices of each layer.[2–5] Defective vortex matter is then left as the only possible solid state of the mixed phase in two dimensions. It is useful to divide the last group into two classes: (i) configurations of vortices that contain no unbound disclinations and (ii) configurations of vortices that contain some concentration of unbound disclinations.[6] The amorphous vortex glass characterized by macroscopic phase coherence falls into the second class.[7] It is believed to exist only at zero temperature in two dimensions, however. Defective vortex lattices with no unbound disclinations, but with isolated dislocations,[8] or with dislocations arranged into grain boundaries,[9–11] are then perhaps left as the only solid states of the mixed phase that are possible in two dimensions above zero temperature.

In this paper, we demonstrate theoretically that defective vortex lattices in two dimensions show macroscopic phase coherence in the extreme type-II limit, in the regime of weak random pinning. We find, in particular, that the 2D vortex lattice exhibits a net superfluid density if it is void of disclinations, and if only a small number of isolated dislocations are quenched in in comparison to the total number of pinned vortices. This result is achieved in three steps. First, we demonstrate in Sec. II that a network of pinned vortices confines the motion of a single dislocation along its glide plane. This guarantees that the 2D vortex lattice remains elastic in the limit of a dilute concentration of such unbound dislocations. Next, the uniformly frustrated XY model for the 2D vortex lattice is introduced in Sec. III, through which a useful expression for the superfluid density is derived in terms of glide by unbound dislocations.[8] Interactions among the dislocations are notably ignored here. These results are then assembled in Sec. IV, where the final formula [Eq. (27)] for the superfluid density of the defective vortex lattice is obtained as a function of the ratio of the number of unbound dislocations to the number of pinned vortices.

II. COLLECTIVE PINNING OF ONE DISLOCATION

It is strongly believed that the vortex lattice in two-dimensions is unstable to the proliferation of dislocations in the presence of an arbitrarily weak field of random pinning centers.[4,5] Let us assume this to be the case. Let us also assume that the interaction between dislocations can be neglected due to the screening action by the random pins.[12] This requires a dilute concentration of dislocations in comparison to the concentration of pinned vortices. Consider then a *single* dislocation in the 2D vortex lattice at zero temperature in the extreme type-II limit. The latter implies that the vortex lattice is incompressible. The dislocation can therefore slide along its glide plane,[13] but it cannot climb across it. This would require the creation or the destruction of vortices, which is prohibited by the extreme type-II limit. Below, we shall demonstrate how randomly pinned vortices in the 2D vortex lattice act to pin the dislocation itself along its glide plane.

Consider a single dislocation that can move along its glide plane in the 2D vortex lattice with randomly located material defects present. Assume that a small fraction of the vortices are localized at some subset of the pinning centers. The former is guaranteed at zero-temperature for a sparse array of random pinning centers compared to the density of vortices. A vortex that lies at a point $\vec{R}$ in the case of the perfect triangular vortex lattice will in general be displaced to a position $\vec{R}+\vec{u}(\vec{R})$ by the action of thermal fluctuations and of the random pinning centers. We shall now make the approximation that the pinned vortices are *fixed*:

$$\vec{u}(\vec{R}_i) = \vec{v}_i \quad \text{for } i = 1,2, \ldots, N_{\text{pin}}, \tag{1}$$

where $\vec{R}_i$ is the home site of the vortex pinned down at $\vec{R}_i + \vec{v}_i$, and where N_{pin} denotes the total number of pinned vortices. This approximation is valid for physics at large length scales compared to the effective radius of a pinning center. It then requires low magnetic fields compared to the upper critical one if the radius of a pinning center is of order the coherence length. The energy of the pinned vortex lattice is then given by

$$E = \frac{1}{2}\mu_0 \int d^2R(\vec{\nabla} \times \vec{u})^2 + \int d^2R \vec{\lambda} \cdot [\vec{u} - \vec{v}] \tag{2}$$

in the continuum limit, where μ_0 denotes the shear modulus of the unpinned vortex lattice,[1,14] and where $\vec{\lambda}(\vec{R}) = \sum_{i=1}^{N_{\text{pin}}} \vec{\lambda}_i \delta^{(2)}(\vec{R} - \vec{R}_i)$ is the field of Lagrange multipliers that is introduced in order to enforce each of the N_{pin} constraints (1). Also, the vortex lattice is incompressible in the extreme type-II limit, and this requires that the displacement field satisfy the constraint

$$\vec{\nabla} \cdot \vec{u} = 0 \tag{3}$$

everywhere. By Eq. (2), the equilibrium configuration $\vec{u}_0$ of the dislocation confined to its glide plane then satisfies the field equation

$$\mu_0 \vec{\nabla} \times \vec{\nabla} \times \vec{u}_0 + \vec{\lambda}_0 = 0 \tag{4}$$

everywhere. It can be used to show that the elastic energy (2) for a fluctuation about equilibrium $\vec{u} = \vec{u}_0 + \delta\vec{u}$ takes the form

$$E = \frac{1}{2}\mu_0 \int d^2R(\vec{\nabla} \times \vec{u}_0)^2 + \frac{1}{2}\mu_0 \int d^2R(\vec{\nabla} \times \delta\vec{u})^2 + \int d^2R \, \delta\vec{\lambda} \cdot \delta\vec{u}, \tag{5}$$

where $\delta\vec{\lambda}(\vec{R}) = \sum_{i=1}^{N_{\text{pin}}} \delta\vec{\lambda}_i \delta^{(2)}(\vec{R} - \vec{R}_i)$ is the field of the fluctuation in the Lagrange multipliers $\delta\vec{\lambda}_i = \vec{\lambda}_i - \vec{\lambda}_i^{(0)}$.

To proceed further, it is convenient to decompose the displacement of vortices into *pure* wave and *pure* defect components: $\vec{u} = \vec{u}_{\text{wv}} + \vec{u}_{\text{df}}$. Suppose now that the dislocation is displaced by $\delta\vec{R}_{\text{df}}$ along its glide plane with respect to its equilibrium position. Notice then that the fluctuation in the defect component corresponds to a pair of dislocations with equal and opposite Burgers vectors oriented along the glide plane (see Fig. 1):

$$\delta\vec{u}_{\text{df}}(\vec{R}) = \vec{u}_{\text{df}}^{(0)}(\vec{R} - \delta\vec{R}_{\text{df}}) - \vec{u}_{\text{df}}^{(0)}(\vec{R}), \tag{6}$$

where $\vec{u}_{\text{df}}^{(0)}(\vec{R})$ denotes the displacement field of the pure dislocation at its home site. At this stage it becomes important to observe that the pure wave and the pure defect components do not interact elastically:[6] $\int d^2R(\vec{\nabla} \times \vec{u})^2 = \int d^2R(\vec{\nabla} \times \vec{u}_{\text{wv}})^2 + \int d^2R(\vec{\nabla} \times \vec{u}_{\text{df}})^2$. Application of this fact to Eq. (5) then ultimately yields the form

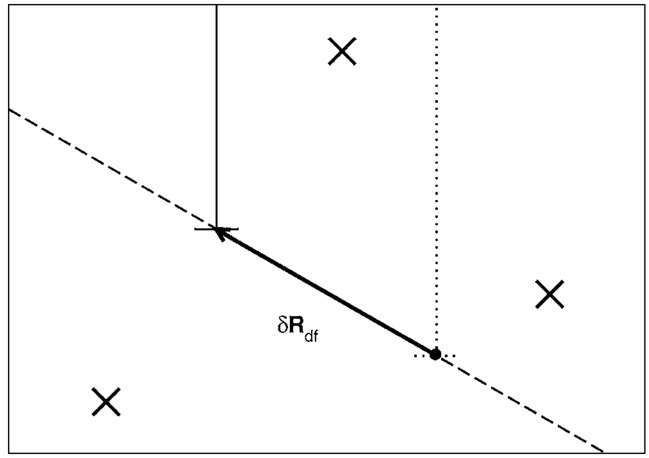

FIG. 1. Shown is a diagram for glide by a dislocation (upside-down "T") in the presence of randomly pinned vortices ("×").

$$E = E_{\text{df}} + E_{\text{wv}}^{(0)} + \frac{1}{2}\mu_0 \int d^2R(\vec{\nabla} \times \delta\vec{u}_{\text{wv}})^2 + \int d^2R \, \delta\vec{\lambda} \cdot (\delta\vec{u}_{\text{wv}} + \delta\vec{u}_{\text{df}}), \tag{7}$$

for the elastic energy, where $E_{\text{wv}}^{(0)} = (\mu_0/2) \int d^2R(\vec{\nabla} \times \vec{u}_{\text{wv}}^{(0)})^2$ is the wave contribution to the elastic energy at equilibrium. Also, $E_{\text{df}} = (\mu_0/2) \int d^2R(\vec{\nabla} \times \vec{u}_{\text{df}})^2$ is the elastic energy of the displaced pure dislocation, which is constant.

The energy (7) of the displaced dislocation is therefore optimized by minimization with respect to the pure wave component $\delta\vec{u}_{\text{wv}}$ along with the constraints

$$\delta\vec{u}_{\text{wv}}(\vec{R}_i) = -\,\delta\vec{u}_{\text{df}}(\vec{R}_i) \quad \text{for } i = 1,2, \ldots, N_{\text{pin}}. \tag{8}$$

Its solution can be obtained by a straightforward generalization of the solution for a pinned elastic string (see the Appendix). This yields

$$\delta\vec{u}_{\text{wv}}(\vec{R}) = -\sum_{i=1}^{N_{\text{pin}}} \sum_{j=1}^{N_{\text{pin}}} \overleftrightarrow{G}_{\perp}(\vec{R} - \vec{R}_i) \cdot \overleftrightarrow{G}_{i,j}^{-1} \cdot \delta\vec{u}_{\text{df}}(\vec{R}_j), \tag{9}$$

where

$$\overleftrightarrow{G}_{\perp}(\vec{R}) = \sum_{\vec{q}} e^{i\vec{q}\cdot\vec{R}} (\hat{z} \times \hat{q})(qL)^{-2}(\hat{z} \times \hat{q}) \tag{10}$$

is the transverse Greens function over an $L \times L$ square region with periodic boundary conditions, and where $\overleftrightarrow{G}_{i,j}^{-1}$ is the inverse of the $N_{\text{pin}} \times N_{\text{pin}}$ matrix $\overleftrightarrow{G}_{\perp}(\vec{R}_i - \vec{R}_j)$. Notice that Eq. (9) manifestly satisfies the constraints (8) and the incompressibility requirement $\vec{\nabla} \cdot \delta\vec{u}_{\text{wv}} = 0$. Also, direct substitution of the solution (9) into Eq. (7) yields a change in the elastic energy due to the displacement of the dislocation equal to

$$\delta E_{\rm pin} = \frac{1}{2}\mu_0 \sum_{i=1}^{N_{\rm pin}} \sum_{j=1}^{N_{\rm pin}} \delta\vec{u}_{\rm df}(\vec{R}_i) \cdot \overleftrightarrow{G}_{i,j}^{-1} \cdot \delta\vec{u}_{\rm df}(\vec{R}_j). \tag{11}$$

Yet $\overleftrightarrow{G}_{i,j}^{-1}$ is the inverse of the 2D Greens function, $G = -\boldsymbol{\nabla}^{-2}$, projected onto transverse displacements (3) and onto the sites of the pinned vortices $\{\vec{R}_i\}$. If these sites are *extensive and homogeneous*, then they resolve unity at long wavelength $\Sigma_{i=1}^{N_{\rm pin}} |i\rangle\langle i| \cong 1$. We therefore have that $\overleftrightarrow{G}_{i,j}^{-1} = \langle i | P_\perp^{-1}(-\boldsymbol{\nabla}^2) P_\perp | j\rangle$ at long wavelength, where $P_\perp$ denotes the projection operator for transverse displacements (3). Substitution into Eq. (11) then yields the expression

$$\delta E_{\rm pin} = \frac{1}{2}\mu_0 \int' d^2R (\vec{\nabla} \times \delta\vec{u}_{\rm df})^2 \tag{12}$$

for the change in the elastic energy due to the displacement of the dislocation, where the prime notation signals that the integral has an ultraviolet cutoff ($R_{\rm pin}$) of order the average spacing between pinned vortices [see Eq. (A7) and Ref. 15]. We conclude that the displacement of the dislocation along its glide plane generates shear stress on the vortex lattice via the array of pinned vortices (see Fig. 1). This then results in a restoring Peach-Kohler force on the displaced dislocation.[13]

Let us now compute the effective spring constant of the Peach-Kohler force experienced by the dislocation at small displacements from equilibrium due to the array of pinned vortices

$$\delta E_{\rm pin} = \frac{1}{2} k_{\rm pin} (\delta R_{\rm df})^2 \quad \text{for } n_{\rm pin}(\delta R_{\rm df})^2 \ll 1, \tag{13}$$

where $n_{\rm pin}$ denotes the density of pinned vortices per layer. The relative displacement field (6) then corresponds to that of a *pure* dislocation pair of extent $\delta R_{\rm df}$ that is oriented along its glide plane. Without loss of generality, it is sufficient to consider a pair of dislocations centered at the origin, with the glide plane located along the x axis. The displacement field is then given asymptotically by[13,16]

$$\delta\vec{u}_{\rm df}(\vec{R}) \cong (b/\pi)(\delta R_{\rm df})(XY/R^4)\vec{R}, \tag{14}$$

where b is one of the equal and opposite Burgers vectors oriented parallel to the glide vector $(\delta R_{\rm df})\hat{x}$. This expression is valid in the limit of small displacements relative to the spacing between pinned vortices $n_{\rm pin}(\delta R_{\rm df})^2 \ll 1$. Substitution into expression (12) for the change in the elastic energy yields the result

$$k_{\rm pin} = (2 n_{\rm pin} \pi R_{\rm pin}^2)^{-1} (\mu_0 b^2) n_{\rm pin} \tag{15}$$

for the effective spring constant of the Peach-Kohler force (13), where $R_{\rm pin}$ denotes the natural ultraviolet cutoff of the array of pinned vortices[15] $n_{\rm pin}\pi R_{\rm pin}^2 \sim 1$. Equations (13) and (15) represent the final result of this section. It indicates that the incompressible vortex lattice confined to two dimensions does *not* respond plastically to small shear stress[13] when a dilute enough concentration of unbound dislocations are quenched in. Instead, the response to small shear stress should remain elastic, like in the pristine case,[1,14] due to the pinning of the quenched-in dislocations.

III. UNIFORMLY FRUSTRATED *XY* MODEL

The minimal description of the mixed phase in a layered superconductor is given by a stack of isolated *XY* models with uniform frustration over the square lattice.[17] Both the effects of magnetic screening and of Josephson coupling between layers are neglected in this approximation. The thermodynamics of each layer is then determined by the superfluid kinetic energy

$$E_{XY}^{(2)} = -\sum_{\mu=x,y} \sum_r J_\mu \cos[\Delta_\mu \phi - A_\mu]|_r, \tag{16}$$

which is a functional of the superconducting phase $\phi(r)$ over the square lattice. The local phase rigidities within layers J_x and J_y are assumed to be constant over most of the nearest-neighbor links, with the exception of those links in the vicinity of a pinning site. The vector potential $A_\mu = (0, 2\pi f x/a)$ represents the magnetic induction $B_\perp = \Phi_0 f/a^2$ oriented perpendicular to each layer. Here a denotes the square lattice constant for each layer, which is of order the coherence length. Also, Φ_0 denotes the flux quantum, and f denotes the concentration of planar vortices per site. After taking the Villain approximation, which is generally valid at low temperature,[18] a series of standard manipulations then lead to a Coulomb gas ensemble with pinning centers that describes the vortex degrees of freedom on the dual square lattice.[19] The ensemble for each layer is weighted by the Boltzmann distribution set by the energy functional

$$E_{\rm vx} = (2\pi)^2 \sum_{(\vec{R},\vec{R}')} \delta Q J_0 G^{(2)} \delta Q' + \sum_{\vec{R}} V_{\rm pin} |Q|^2, \tag{17}$$

written in terms of the integer vorticity field $Q(\vec{R})$ over the sites $\vec{R}$ of the dual lattice in that layer, and of the fluctuation $\delta Q = Q - f$. A logarithmic interaction, $G^{(2)} = -\nabla^{-2}$, exists between the vortices, with a strength J_0 equal to the Gaussian phase rigidity. Last, $V_{\rm pin}(\vec{R})$ is the resulting pinning potential.[19]

The 2D Coulomb gas ensemble (17) can be used to test for the presence or the absence of superconductivity. In particular, the macroscopic phase rigidity parallel to the layers is given by one over its dielectric constant[20]

$$\rho_s^{(2D)}/J_0 = 1 - \lim_{k\to 0} (2\pi/\eta_{\rm sw}) \langle \delta Q_{\vec{k}} \delta Q_{-\vec{k}} \rangle / k^2 a^2 \mathcal{N}_\parallel. \tag{18}$$

Here $\delta Q_{\vec{k}} = Q_{\vec{k}} - \langle Q_{\vec{k}} \rangle$ is the fluctuation in the Fourier transform of the vorticity: $Q_{\vec{k}} = \Sigma_{\vec{R}} Q(\vec{R}) e^{i\vec{k}\cdot\vec{R}}$. Also, $\eta_{\rm sw} = k_B T / 2\pi J_0$ is the spin-wave component of the phase-correlation exponent, and $\mathcal{N}_\parallel$ denotes the number of points in the square-lattice grid. Now suppose that a given vortex is displaced by $\delta\vec{u}$ with respect to its equilibrium location at zero temperature $\vec{u}_0$. Conservation of vorticity dictates that the fluctuation in the vortex number is given by $\delta Q = -\vec{\nabla} \cdot \delta\vec{u}$. Substitution into Eq. (18) then yields the result[8]

$$\rho_s^{(2D)}/J_0 = 1 - (\eta'_{vx}/\eta_{sw}) \tag{19}$$

for the phase rigidity in terms of the vortex component of the phase-correlation exponent

$$\eta'_{vx} = \pi \left\langle \left[\sum_{\vec{R}}{}' \delta\vec{u} \right]^2 \right\rangle \Big/ N_{vx} a_{vx}^2. \tag{20}$$

The latter monitors fluctuations of the center-of-mass of the vortex lattice.[16] Above, N_{vx} denotes the number of vortices, while $a_{vx}=a/f^{1/2}$ is equal to the square root of the area per vortex. Also, the prime notation above signals that the summation is restricted to the vortex lattice. To proceed further, we again express the displacement field as a superposition of *pure* wave and of *pure* defect components of the triangular vortex lattice:[16] $\delta\vec{u}=\delta\vec{u}_{wv}+\delta\vec{u}_{df}$. Observe now that $\Sigma'_{\vec{R}}\delta\vec{u}_{wv}=0$ under periodic boundary conditions if rigid translations of the 2D vortex lattice are not possible. The latter is achieved by the array of pinned vortices (1) through the elastic forces (2). By Eq. (20), we therefore have the result

$$\eta'_{vx} = \pi \left\langle \left[\sum_{\vec{R}}{}' \delta\vec{u}_{df} \right]^2 \right\rangle \Big/ N_{vx} a_{vx}^2 \tag{21}$$

for the fluctuation in the center of mass of the 2D vortex lattice. The degree of phase coherence in the pinned vortex lattice is therefore insensitive to its *pure* wave contribution.

Consider now the hexatic vortex glass,[21,22] with a collection of N_{df} randomly located unbound dislocations that are quenched in by the random array of pinned vortices.[4,5] Suppose also that the temperature is low enough so that the thermal excitation of pairs of dislocations in the vortex lattice can be neglected. Within the elastic medium description (2), the pure defect component of the net displacement field is just a simple sum of the displacements due to each individual dislocation. And by analogy with the hexatic liquid phase of the pure 2D vortex lattice,[6] we shall assume that interactions in between the unbound dislocations can be neglected, be they direct or be they transmitted through the field of pinned vortices. Expression (21) for the fluctuation of the center-of-mass of the 2D vortex lattice then reduces to

$$\eta'_{vx} \cong \pi \overline{\left\langle \left[\sum_{\vec{R}}{}' \delta\vec{u}_{df}^{(1)} \right]^2 \right\rangle} n_{df}, \tag{22}$$

where $\delta\vec{u}_{df}^{(1)}(\vec{R})$ denotes the fluctuation field of a given dislocation displaced along its glide plane (6), where $n_{df}=N_{df}/N_{vx}a_{vx}^2$ is the density of unbound dislocations per layer, and where the overbar notation denotes a bulk average. A lone dislocation roams along its glide plane to an extent that is vanishingly small, however, in the zero-temperature limit: $\delta\vec{R}_{df}\to 0$ as $T\to 0$. Without loss of generality, we can then use the asymptotic expression (14) for the corresponding fluctuation in the displacement field, $\delta\vec{u}_{df}^{(1)}$. The fluctuation in the center-of-mass of the 2D vortex lattice (22) is dominated by the "diagonal" on-site contribution $\overline{\langle\Sigma'_{\vec{R}}|\delta\vec{u}_{df}^{(1)}|^2\rangle}$, which yields the estimate[8]

$$\eta'_{vx} \cong n_{df}\overline{\langle|\delta R_{df}|^2\rangle}(b/2a_{vx})^2 \ln R_0/a_{df}. \tag{23}$$

Here a_{df} is the core diameter of a dislocation, while R_0 is an infrared cutoff. The above logarithmic divergence associated with the latter scale justifies the neglect of the contribution to the fluctuation in the center-of-mass (22) by the autocorrelator $\overline{\langle \delta\vec{u}_{df}^{(1)} \cdot \delta\vec{u}_{df}^{(1)\prime}\rangle}$ at different points. This is due to the fact (i) that $-\overline{\langle \delta\vec{u}_{df}^{(1)}(a) \cdot \delta\vec{u}_{df}^{(1)}(b)\rangle}$ must decay faster than $[\overline{\langle|\delta\vec{u}_{df}^{(1)}|^2\rangle}/2\pi \ln(R_0/a_{df})](a_{vx}/R_{ab})^2$ by Eq. (22) because $\eta'_{vx}>0$ and to the fact (ii) that the former autocorrelator is short range as a result of disordering by the quenched-in dislocations. Last, given that expression (23) was obtained by neglecting interactions in between isolated dislocations, it is natural to assume that the infrared scale R_0 that appears there is set by their density n_{df}.

IV. LOW-TEMPERATURE PHASE COHERENCE

We shall now assemble the results of the previous two sections and compute the macroscopic phase rigidity of a defective vortex lattice in two dimensions. Recall that the energy cost for a small displacement of the dislocation along its glide plane takes the form $\delta E_{df}=(1/2)k_{pin}|\delta R_{df}|^2$, where k_{pin} is the effective spring constant (15) due to the randomly pinned vortices. This approximation for the elastic energy of the dislocation along its glide plane is valid in the zero-temperature limit $T\to 0$, where it amounts to a saddle-point approximation for the Boltzmann weight in the thermal average $\langle|\delta R_{df}|^2\rangle$. The periodic Peierls-Nabarro potential energy along the glide plane of the dislocation shall be neglected for the moment.[13] Application of the equipartition theorem to expression (23) for the fluctuation of the center of mass of the 2D vortex lattice then yields the result

$$\eta'_{vx} \cong (k_B T)(n_{df}/k_{pin})(b/2a_{vx})^2 \ln R_0/a_{df}, \tag{24}$$

which notably vanishes linearly with temperature. Substitution into expression (19) in turn yields the result

$$\rho_s^{(2D)}/J_0 \cong 1 - (2\pi J_0)(n_{df}/k_{pin})(b/2a_{vx})^2 \ln R_0/a_{df} \tag{25}$$

for the 2D phase rigidity in the zero-temperature limit. Substitution of the result (15) for the effective spring constant above yields the final formula for the macroscopic 2D phase rigidity near zero temperature

$$\frac{\rho_s^{(2D)}}{J_0} \cong 1 - (n_{pin}\cdot \pi R_{pin}^2)\frac{\pi J_0}{\mu_0 a_{vx}^2}\frac{N_{df}}{N_{pin}} \ln\left(\frac{R_0}{a_{df}}\right), \tag{26}$$

in terms of the number of unbound dislocations N_{df}, the number of vortices N_{vx}, and of the number of pinned vortices N_{pin} in an isolated 2D vortex lattice. Recall now the estimate for the shear modulus of the unpinned vortex lattice in the extreme type-II limit,[1,14] $\mu_0 a_{vx}^2=(\pi/4)J_0$. Substitution into expression (26) for the superfluid density then yields the yet simpler result

TABLE I. Listed are physical properties that characterize the low-temperature phases of vortex matter in two dimensions at the extreme type-II limit. The list is indexed by increasing levels of disorder.

Disorder index	Phase	$\rho_s^{(2D)}(0+)/J_0$	Unbound dislocations?	Unbound disclinations?
1	Bragg glass	unity	no	no
2	hexatic vortex glass	fraction	yes	no
3	hexatic vortex liquid	zero	yes	no
4	vortex liquid	zero	yes	yes

$$\frac{\rho_s^{(2D)}}{J_0} \cong 1 - (2n_{\rm pin} \cdot \pi R_{\rm pin}^2)\frac{N_{\rm df}}{N_{\rm pin}} \ln\left(\frac{R_0}{a_{\rm df}}\right)^2 \quad \text{near } T = 0. \tag{27}$$

The effect of the Peierls-Nabarro potential energy with period $\vec{b}$ that any dislocation experiences along its glide plane has been neglected above.[13] It becomes useful in this instance to define the temperature scale

$$k_B T_0 = k_{\rm pin} b^2 \sim (\mu_0 b^2)(n_{\rm pin} b^2), \tag{28}$$

at which point thermally induced excursions of the dislocation about its home site are typically of the size of a Burger's vector b. Notice first that $k_B T_0$ is an extremely small fraction of the elastic energy scale $\mu_0 b^2$ if the concentration of pinned vortices is dilute. Typical excursions of the dislocation will be large compared to the Burgers vector at temperatures above this extremely low scale: on average $|\delta R_{\rm df}| > b$ at $T > T_0$. In such case, the periodic Peierls-Nabarro potential can be neglected because the thermal motion of the dislocation becomes insensitive to its relatively short period b. The periodic potential will take effect at extremely low temperature $T \ll T_0$, on the other hand, in which case it will tend to localize the dislocation even further about its home site.

Let us finally close the chain of calculations by estimating the ratio of the number of unbound dislocations that are quenched into the vortex lattice to the number of pinned vortices. Notice that it determines the phase rigidity (27) of the defective vortex lattice at $T > T_0$. A variational calculation by Mullock and Evetts finds that this ratio is given by

$$\frac{N_{\rm df}}{N_{\rm pin}} = \left[\frac{\pi}{\ln(n_{\rm df} a_{\rm df}'^2)^{-1}}\right]^2 \left(\frac{f_{\rm pin}}{\mu_0 b}\right)^2, \tag{29}$$

where $f_{\rm pin}$ denotes the maximum pinning force, and where $a'_{\rm df}$ is of order the core diameter of a dislocation in the vortex lattice.[2] This result is valid only in the collective-pinning regime at $N_{\rm df} \leq N_{\rm pin}$. Equation (29) therefore implies a small ratio of topological defects to pinned vortices $N_{\rm df} \ll N_{\rm pin}$ for weak pinning forces compared to the elastic forces $f_{\rm pin} \ll \mu_0 b$. Substitution of the estimate for the shear modulus quoted earlier[1,14] yields the field dependence $N_{\rm df}/N_{\rm pin} = B_{\rm cp}/B$ for this ratio (29), where $B_{\rm cp} = (\sqrt{3}/2)[\pi/\ln(n_{\rm df} a_{\rm df}'^2)^{-1}]^2 (4f_p/\pi J_0)^2 \Phi_0$ is the threshold magnetic field above which collective pinning holds. Last, the effect of substrate pinning by the XY model grid (16), which tends to suppress the number of unbound dislocations even further, is neglected here. This is valid in the regime of dilute vortex lattices compared to the model grid[17] $f < 1/36$.

Three important conclusions can be reached from the estimate for the degree of macroscopic phase coherence encoded by Eqs. (27) and (29) above. First, observe that $N_{\rm vx}$, $N_{\rm pin}$, and $N_{\rm df}$ are all extensive thermodynamic variables that scale with the area of each layer L^2. The macroscopic phase rigidity (27) hence attains its maximum value J_0 near zero temperature when the total number of unbound dislocations is subthermodynamic: e.g., if $N_{\rm df} \propto L$, or if $N_{\rm df}$ remains finite as $L \to \infty$. This state is then a Bragg glass.[4,23] The variational result (29) for the number of unbound dislocations obtained by Mullock and Evetts[2] indicates that it exists only in the absence of bulk point pins. Other types of pinning, such as surface barriers or planar defects, must therefore be present in order to impede flux flow by the 2D Bragg glass.[16] Second, recall that $n_{\rm pin} \cdot \pi R_{\rm pin}^2 \sim 1$. Expression (27) therefore implies that weaker macroscopic phase coherence exists near zero temperature at dilute concentrations of unbound dislocations: $\rho_s^{(2D)}(0+) > 0$ for $N_{\rm df} \ll N_{\rm pin}$. Such a state is then a hexatic vortex glass.[21,22] The variational result (29) for the number of unbound dislocations indicates that this state exists at weak pinning $f_{\rm pin} \ll \mu_0 b$, which occurs at large magnetic fields $B \gg B_{\rm cp}$. Third, expression (27) also implies that a pinned vortex liquid that shows *no* macroscopic phase coherence is possible in the zero-temperature limit at sufficiently high concentrations of unbound dislocations: $\rho_s^{(2D)}(0+) = 0$ if $N_{\rm df} \sim N_{\rm pin}$. This phase is then a (pinned) hexatic vortex liquid.[6,16] The variational result (29) for the number of dislocations indicates that such a phase-incoherent state can occur in the regime of strong pinning forces, at low fields compared to the collective pinning threshold $B_{\rm cp}$. We remind the reader that expression (27) was derived by neglecting the interactions in between dislocations. This approximation, and Eq. (27) as a result, may not necessarily be valid in the regime of relatively dense dislocations last discussed.

V. DISCUSSION

The results of the previous sections are summarized by Table I. Here the low-temperature phases are listed by increasing order of the random pinning force $f_{\rm pin}$. Below, we confront Table I with previous theoretical work on 2D vortex matter.

Order parameters. It is natural to ask what order parameters characterize the phases listed in Table I. Let us begin by defining the autocorrelation function

$$G_{\rm BrG}(\vec{r}) = \overline{\left\langle \exp i\left[\phi(\vec{r}') - \phi(\vec{r}+\vec{r}') + \int_{\vec{r}'}^{\vec{r}+\vec{r}'} d\vec{l} \cdot \vec{A}/a\right]\right\rangle} \tag{30}$$

for the Bragg-glass order parameter, where the overbar denotes a bulk average over $\vec{r}'$. Drawing the analogy with the thermal degradation of phase coherence in the pristine 2D vortex lattice[16] implies that $G_{\mathrm{BrG}}(\vec{r})$ is *not* short range in the zero-temperature limit if no unbound dislocations are quenched in. The latter is consistent with with the properties of the Bragg glass listed in Table I (see Ref. 23). We conclude that the Bragg-glass phase studied here displays conventional phase coherence at long range. Again in analogy with the case of thermal disordering of the pristine vortex lattice,[16] the unbound dislocations that are quenched inside of the hexatic vortex glass, on the other hand, will result in short-range order in $G_{\mathrm{BrG}}(\vec{r})$ over a scale set by the density of such defects n_{df}. The absence of Bragg-glass order in the hexatic vortex glass, as defined above by Eq. (30), is then consistent with the presence of unbound dislocations in the vortex lattice.[23]

Following Fisher, Fisher, and Huse,[7] we can next define the vortex glass autocorrelation function

$$G_{\mathrm{VG}}(\vec{r}) = \overline{|\langle \exp i[\phi(\vec{r}\,') - \phi(\vec{r}+\vec{r}\,')]\rangle|^2}. \tag{31}$$

In the zero-temperature limit, this function is notably identical to unity if the ground-state configuration $e^{i\phi_0}$ is unique. Recall now that the present treatment of the hexatic vortex glass has been restricted to the limit of vanishing dislocation density, in which case a unique ground state is to be expected. The autocorrelation function $G_{\mathrm{VG}}(\vec{r})$ then is *not* short range for the case of the hexatic vortex glass in the limit of weak disorder pinning. The zero-temperature configuration may not be unique, on the other hand, for the case of the (pinned) hexatic vortex liquid. The number of unbound dislocations is comparable to the number of pinned vortices in such case. Interactions in between dislocations may become important, and these may frustrate the confining action on the dislocations by the pinned vortices. The end result could be multiple ground states that lead to a vortex glass autocorrelation function $G_{\mathrm{VG}}(\vec{r})$ that shows only short-range order.

Consider again the (pinned) hexatic vortex liquid state that is possible at relatively dense concentrations of unbound dislocations $N_{\mathrm{df}} \sim N_{\mathrm{pin}}$, by expression (27) for the superfluid density. Some fraction of the pairs of fivefold and sevenfold coordinated disclinations that make up the unbound dislocations present in this state will unbind in the strong pinning limit $f_{\mathrm{pin}} \gg \mu_0 b$. This is confirmed by direct Monte Carlo simulations[24] of the Coulomb gas ensemble (17). Such a state then shows only short-range translational and orientational order. It also cannot have a net superfluid density $\rho_s^{(2D)} > 0$, since it is yet more disordered than the hexatic vortex liquid from whence it originates. For the same reason, it can neither show long-range Bragg glass nor vortex glass order in the respective autocorrelation functions (30) and (31). This strongly pinned state is then a "conventional" vortex liquid (see Table I).

Vortex Glass in 2D? Fisher, Fisher, and Huse argue in Ref. 7 that the vortex-glass state is not possible in two dimensions above zero temperature. This statement conflicts at first sight with the phase-coherent vortex lattice state with a dilute concentration of quenched-in dislocations that we have discovered in the uniformly frustrated *XY* model (16) in the limit of weak random pinning. This state also shows vortex-glass order (31). Study of Ref. 7 reveals that the authors presume the strong-pinning limit, however. The vortex glass is necessarily amorphous under such conditions, where it possesses a net concentration of unbound disclinations. It hence lies in the same topological class as the "conventional" vortex liquid state discussed above and listed in Table I. No conflict then truly exists between Ref. 7 and the present results concerning the impossibility of observing an amorphous vortex glass in two dimensions. The latter is topologically distinct from the hexatic vortex glass discovered here.

The hexatic vortex glass will also have some concentration of bound pairs of dislocations quenched into the vortex lattice in the zero-temperature limit. Vinokur and co-workers have argued that the absence of infinite potential barriers along the corresponding glide planes will result in thermally activated plastic creep of magnetic flux due to the diffusion of such pairs of dislocations, and in thermally activated electrical resistance as a result.[25,1] Hence, although a stack of uncoupled sheets of hexatic vortex glass shows magnetic screening in direct proportion to the superfluid density $\rho_s^{(2D)}$, this system may not in fact be a perfect conductor. Flux creep immediately becomes neutralized, however, once macroscopically big layers (compared to the Josephson penetration depth) are coupled through the Josephson effect. This is due to the binding of pairs of dislocations into "quartets" that carry no net magnetic flux.[12,1] The possibility just raised of three-dimensional vortex matter that is ohmic, but that nevertheless shows macroscopic phase coherence, therefore remains unrealistic.

VI. CONCLUSIONS

We have demonstrated that incompressible vortex lattices that are confined to two dimensions and that are void of unbound disclinations show macroscopic phase coherence near zero temperature in the limit of weak random pinning. The latter ensures that the total number of unbound dislocations quenched into the vortex lattice by the randomly pinned vortices is small in comparison to the total number of such pins. This in turn results in a net superfluid density. The hexatic vortex glass predicted here is consistent with the observation of isolated dislocations that are quenched into the vortex lattice of extremely layered high-temperature superconductors at low temperature,[21] and with the observation of superconductivity in 2D Josephson junction arrays in external magnetic field.[26] How exactly such a superconducting vortex lattice transits into a vortex liquid with increasing temperature remains unclear. Equations (19) and (21) indicate that the superfluid density vanishes either once the quenched-in dislocations delocalize and begin to cross the length of the vortex lattice, or once thermally activated pairs of dislocations unbind and begin to cross the length of the vortex lattice.[16] Both mechanisms cause plastic creep of the 2D vortex lattice,[13] which destroys macroscopic phase coherence. Continuity implies that the melting temperature $T_g^{(2D)}$ of the defective vortex lattice lies near that of the pristine vortex lattice[17] $k_B T_m^{(2D)} \cong J_0/20$, in the limit of weak random pinning.

ACKNOWLEDGMENTS

The author thanks M. Maley and L. Civale for useful discussions. He is also grateful for the hospitality of the Superconductivity Technology Center at Los Alamos National Laboratory during the last stages of this work.

APPENDIX: PINNED ELASTIC STRING

Consider a tense elastic string of length L that lies along the x axis under periodic boundary conditions, and suppose that only transverse displacements $u(x)$ along the y axis are allowed. Suppose further that the string is pinned down at N_{pin} sites:

$$u(x_i) = v_i \quad \text{for } i = 1,2,\ \ldots\ ,N_{\mathrm{pin}}. \tag{A1}$$

The shape of the string with the lowest energy can then be determined by minimizing the elastic energy along with appropriate terms that enforce the constraints

$$E = \frac{1}{2}\mu_0 \int_0^L dx \left(\frac{du}{dx}\right)^2 + \int_0^L dx\, \lambda \cdot [u - v]. \tag{A2}$$

Here μ_0 denotes the shear modulus, while the field $\lambda(x) = \Sigma_{i=1}^{N_{\mathrm{pin}}} \lambda_i \delta(x - x_i)$ is weighted by the Lagrange multipliers λ_i that correspond to each of the constraints (A1). The configuration that minimizes the elastic energy (A2) satisfies the field equation

$$-\mu_0 \frac{d^2 u}{dx^2} + \lambda = 0 \tag{A3}$$

everywhere. Equation (A2) can be easily minimized in the wave representation

$$u(x) = \sum_q u_q e^{iqx} \quad \text{and} \quad \lambda(x) = \sum_q \lambda_q e^{iqx}, \tag{A4}$$

expressed as a sum over allowed wave numbers q that are multiples of $\pm 2\pi/L$. One then obtains the solution

$$u(x) = \sum_{i=1}^{N_{\mathrm{pin}}} \sum_{j=1}^{N_{\mathrm{pin}}} G(x - x_i) G_{i,j}^{-1} v_j, \tag{A5}$$

where $G(x) = \Sigma_q e^{iqx}/Lq^2$ is the Greens functions in one dimension, and where $G_{i,j}^{-1}$ is the inverse of the $N_{\mathrm{pin}} \times N_{\mathrm{pin}}$ matrix $G(x_i - x_j)$. Notice that the above solution satisfies the constraints (A1). It also satisfies the field equation (A3), with Lagrange multipliers $\lambda_i = -\mu_0 \Sigma_{j=1}^{N_{\mathrm{pin}}} G_{i,j}^{-1} v_j$ that weight the field $\lambda(x)$ at each of the pins located at x_i. Direct substitution of the solution (A5) into the elastic energy (A2) then yields the result

$$E = (\mu_0/2) \sum_{i=1}^{N_{\mathrm{pin}}} \sum_{j=1}^{N_{\mathrm{pin}}} v_i G_{i,j}^{-1} v_j. \tag{A6}$$

It reduces to the expression

$$E_{1,2} = (\mu_0/2)(v_1 - v_2)^2/|x_1 - x_2| \tag{A7}$$

for the elastic energy in the special case of two pins at the thermodynamic limit $L \to \infty$. Here we used the result $G(x) = G(0) - (|x|/2)$ for the Greens function in one dimension, with a limiting value $G(0) \to \infty$ for the constant.

[1] G. Blatter, M. V. Feigel'man, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, Rev. Mod. Phys. **66**, 1125 (1994).
[2] S. J. Mullock and J. E. Evetts, J. Appl. Phys. **57**, 2588 (1985).
[3] A. C. Shi and A. J. Berlinsky, Phys. Rev. Lett. **67**, 1926 (1991).
[4] T. Nattermann and S. Scheidl, Adv. Phys. **49**, 607 (2000).
[5] C. Zeng, P. L. Leath, and D. S. Fisher, Phys. Rev. Lett. **82**, 1935 (1999).
[6] D. R. Nelson and B. I. Halperin, Phys. Rev. B **19**, 2457 (1979).
[7] D. S. Fisher, M. P. A. Fisher, and D. A. Huse, Phys. Rev. B **43**, 130 (1991).
[8] J. P. Rodriguez, Phys. Rev. B **69**, 100503(R) (2004).
[9] C. Dasgupta and O. T. Valls, Phys. Rev. Lett. **91**, 127002 (2003); M. Chandran, R. T. Scalettar, and G. T. Zimanyi, Phys. Rev. B **69**, 024526 (2004).
[10] M. Menghini, Y. Fasano, F. de la Cruz, S. S. Banerjee, Y. Myasoedov, E. Zeldov, C. J. van der Beek, M. Konczykowski, and T. Tamegai, Phys. Rev. Lett. **90**, 147001 (2003).
[11] P. Moretti, M. Carmen Miguel, M. Zaiser, and S. Zapperi, Phys. Rev. Lett. **92**, 257004 (2004).
[12] M. Feigel'man, V. B. Geshkenbein, and A. I. Larkin, Physica C **167**, 177 (1990).
[13] D. Hull and D. J. Bacon, *Introduction to Dislocations*, 3rd ed. (Pergamon, Oxford, 1984).
[14] E. H. Brandt, J. Low Temp. Phys. **26**, 735 (1977).
[15] Arrangements of pinned vortices that show clusters and voids do not possess a well defined ultraviolet scale R_{pin}. They can occur near the lower-critical magnetic field, in soft vortex lattices with a purely random arrangement of columnar pinning centers, at relatively low matching fields (see Ref. 10).
[16] J. P. Rodriguez, Phys. Rev. Lett. **87**, 207001 (2001).
[17] Søren A. Hattel and J. M. Wheatley, Phys. Rev. B **51**, 11 951 (1995).
[18] J. V. José, L. P. Kadanoff, S. Kirkpatrick, and D. R. Nelson, Phys. Rev. B **16**, 1217 (1977).
[19] C. E. Creffield and J. P. Rodriguez, Phys. Rev. B **67**, 144510 (2003).
[20] P. Minnhagen and G. G. Warren, Phys. Rev. B **24**, 2526 (1981).
[21] C. A. Murray, P. L. Gammel, D. J. Bishop, D. B. Mitzi, and A. Kapitulnik, Phys. Rev. Lett. **64**, 2312 (1990); P. Kim, Z. Yao, and C. M. Lieber, Phys. Rev. Lett. **77**, 5118 (1996).
[22] E. M. Chudnovsky, Phys. Rev. B **43**, 7831 (1991).
[23] T. Giamarchi and P. Le Doussal, Phys. Rev. B **52**, 1242 (1995).
[24] J. P. Rodriguez and C. E. Creffield (unpublished).
[25] V. M. Vinokur, P. H. Kes, and A. E. Koshelev, Physica C **168**, 29 (1990).
[26] Y.-J. Yun, I.-C. Baek, and M.-Y. Choi, Phys. Rev. Lett. **89**, 037004 (2002).