

Chirality in Coulomb-blockaded quantum dots

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We investigate the two-terminal nonlinear conductance of a Coulomb-blockaded quantum dot attached to chiral edge states. Reversal of the applied magnetic field inverts the system chirality and leads to a different polarization charge. As a result, the current-voltage characteristic is not an even function of the magnetic field. We show that the corresponding magnetic-field asymmetry arises from single-charge effects and vanishes in the limit of high temperature.

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The Onsager-Casimir symmetry relations^{1,2} establish that the linear-response transport is even under reversal of an external magnetic field. It is then of fundamental interest to investigate the conditions under which one can see deviations from the Onsager symmetries as one enters the nonlinear regime. Recently, it has been shown^{3,4} that in nonlinear mesoscopic transport there arise magnetic-field asymmetries entirely due to the effect of electron-electron interactions in the nonlinear regime.⁵ These works have been focused on quantum dots with large density of states and connected to leads via highly conducting openings; typically, quantum point contacts supporting more than one propagating mode.³ Recent experiments by Zumbühl *et al.*⁶ on large chaotic cavities are in good agreement with theory. In nonlinear transport magnetic-field asymmetries can occur under a wide variety of conditions.⁷⁻⁹ In particular, we considered³ a quasilocalized level separated from the leads with tunnel barriers but neglected single-charge effects. Therefore, it is natural to ask whether magnetic-field asymmetries are visible in the Coulomb-blockade regime.¹⁰⁻¹² Since Coulomb energies can be much larger than the energy scales for quantum interference, magnetic-field asymmetries induced by single electron effects should be visible at much higher temperatures.

The electrostatic approach used in the classical model of Coulomb blockade¹¹ predicts a potential ϕ_d in the quantum dot (QD) that depends on the QD charge Q_d ,

$$\phi_d = \frac{Q_d}{C_\Sigma} + \phi_{\text{ext}}, \quad (1)$$

and on an external potential ϕ_{ext} related to the polarization charge Q_{ext} externally induced by nearby reservoirs and gates,

$$\phi_{\text{ext}} = \frac{Q_{\text{ext}}}{C_\Sigma} = \frac{\sum_\alpha C_\alpha V_\alpha}{C_\Sigma}, \quad (2)$$

where the sum extends over all leads. This model assumes a uniform screening potential described by the QD (geometric) capacitance couplings C_α with the contacts. The total capacitance of the equivalent circuit is thus $C_\Sigma = \sum_\alpha C_\alpha$.

Consider now a two-terminal sample in the quantum Hall regime (see Fig. 1) with one edge state running along each side (top and bottom). With the help of gates it is possible to

create in the center a potential hill that behaves as a tunable quasilocalized state coupled to edge states acting as source and drain reservoirs. The resulting antidot¹³ connects the edge states in two ways:^{3,14} (i) *scattering coupling*, in which electrons tunnel from the edge states to the antidot, and (ii) *electrostatic coupling*, in which the antidot screening potential feels the repulsion through capacitive couplings: C_1 (C_2) between the dot and the upper (lower) edge state. The system has a definite chirality determined from the magnetic-field direction (upward or downward) since, e.g., the upper edge state originates from the left terminal for a given field $+B$ but carries current from the right terminal for the opposite field direction $-B$. Thus, the nonequilibrium polarization charge becomes $Q_{\text{ext}}(+B) = C_1 V_1 + C_2 V_2$ and $Q_{\text{ext}}(-B) = C_2 V_1 + C_1 V_2$, which is clearly magnetic-field asymmetric whenever the capacitance coupling is asymmetric. Thus, we expect that the current traversing the dot is not an even function of B .

The qualitative argument above can be traced back to the oddness of the Hall potential.³ We investigate now the effect in detail to give precise predictions for the dependence of the magnetic-field asymmetry on temperature, bias, and gate voltages.

Electrons from lead α tunnel onto the dot via the edge states with a transmission probability characterized by a Breit-Wigner resonance with a width Γ_α . We assume that transport is governed by transitions between QD ground

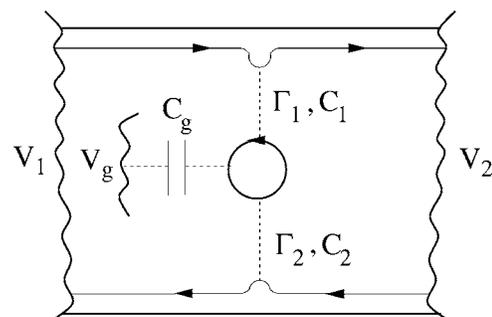


FIG. 1. Sketch of the system under consideration. The antidot is coupled to chiral edge states via tunnel barriers acting as leaky capacitors. A back-gate contact controls the dot occupation with a capacitive coupling C_g . When the magnetic field is reversed, both edge states invert their propagating direction.

states, which is a good approximation when both temperature and bias voltages are much smaller than any excitation energy. Then, the scattering matrix^{12,15} $S_{\alpha\beta}^N$ for the transition from $Q_d=N-1$ electrons to $Q_d=N$ electrons when an electron is transmitted from lead α to lead β ,

$$S_{\alpha\beta}^N(E) = \delta_{\alpha\beta} - i \frac{\sqrt{\Gamma_{\alpha}^N \Gamma_{\beta}^N}}{E - \mu_d(N) + i\Gamma^N/2}, \quad (3)$$

has a complex pole with a real part associated to the QD electrochemical potential $\mu_d(N)$. The total resonance width is proportional to $\Gamma^N = \sum_{\alpha} \Gamma_{\alpha}^N$. The widths fluctuate according to the Porter-Thomas distribution but in what follows we neglect intradot correlation effects in Γ and take it as energy independent, which works well provided bias variations are much smaller than the barrier height.

We emphasize that the scattering matrix is not only a function of the carrier's energy E but also depends on the full electrostatic configuration via $\mu(N) = E(N) - E(N-1)$, where $E(N)$ is the ground-state energy of an N -electron QD,

$$E(N) = \sum_{i=1}^N \varepsilon_i + \frac{(Ne)^2}{2C_{\Sigma}} - eN \sum_{\alpha} \frac{C_{\alpha} V_{\alpha}}{C_{\Sigma}}. \quad (4)$$

In Eq. (4), $E(N)$ consists of two terms. First, the kinetic energy is a sum over QD single-particle levels $E_k = \sum_{i=1}^N \varepsilon_i$ arising from confinement. These levels may be, in general, renormalized due to coupling to the leads: $\varepsilon_N \rightarrow \varepsilon_N + (\Gamma^N/\pi) \ln|(D-E)/(D+E)|$ with D the bandwidth assuming flat density of states in the leads. The renormalization term is a slowly increasing function of E and can be safely neglected. Therefore, the kinetic energy is invariant under reversal of B . The second contribution to $E(N)$ is the potential energy $U(N)$ that depends on the charge state of the dot and the set of applied voltages including nearby gates. We assume that the dot is in the presence of a back-gate potential V_g , which controls the number of electrons at equilibrium via the capacitance coupling C_g (see Fig. 1). Then, the QD charge, which is quantized to a value $Q_d = -Ne$ in the Coulomb valleys, determines the QD potential from the discretized Poisson equation,

$$C_1(\phi_d - V_1) + C_2(\phi_d - V_2) + C_g(\phi_d - V_g) = -Ne, \quad (5)$$

which amounts to the Hartree approximation, disregarding exchange and pairing effects. These effects might be important in certain situations¹² but we shall see below that this level of approximation already suffices to obtain a sizable magnetic-field asymmetry.

Equations (1) and (2) are readily derived from Eq. (5). Then, we find that the QD potential energy reads

$$U(N, +B) = \frac{N^2 \mathcal{U}}{2} - eN \left(\frac{C_1}{C_{\Sigma}} V_1 + \frac{C_2}{C_{\Sigma}} V_2 + \frac{C_g}{C_{\Sigma}} V_g \right), \quad (6)$$

where $C_{\Sigma} = C_1 + C_2 + C_g$ and $\mathcal{U} = e^2/C_{\Sigma}$. We now reverse the magnetic field,

$$U(N, -B) = \frac{N^2 \mathcal{U}}{2} - eN \left(\frac{C_2}{C_{\Sigma}} V_1 + \frac{C_1}{C_{\Sigma}} V_2 + \frac{C_g}{C_{\Sigma}} V_g \right). \quad (7)$$

From Eqs. (6) and (7) it is clear that the QD electrochemical potential shows a magnetic-field *asymmetry*, $\Phi_{\mu} = [\mu(N, +B) - \mu(N, -B)]/2$, given by

$$\Phi_{\mu} = \frac{C_2 - C_1}{2C_{\Sigma}} (V_1 - V_2). \quad (8)$$

Since $\mu(N)$ determines the position of the differential conductance resonance, it follows that the I - V characteristics of the antidot is asymmetric under B reversal. We remark that this model assumes full screening of the charges injected in the dot, i.e., the local potential neutralizes the excess charge: $C_{\alpha} \ll e^2 \nu_{\alpha}$ with ν_{α} the density of states of edge state α . Deviations from this limit would probably decrease the size of the asymmetry.³ Finally, we emphasize that magnetic field asymmetries develop only to the extent that capacitive interactions with surrounding contacts are considered.

The current around the $N-1 \rightarrow N$ resonance for spinless electrons reads

$$I_N(B) = -\frac{e}{h} \int dE (S_{12}^N)^{\dagger} S_{12}^N [f_1(E) - f_2(E)], \quad (9)$$

where the scattering matrix S from Eq. (3) depends on B because the QD potential response is asymmetric under B reversal, as shown above. $f(E)$ is the Fermi function and we take $V_1 = -V_2 = V/2$. Our goal is to calculate the asymmetry,

$$\Phi_G = \frac{G_N(+B) - G_N(-B)}{2}, \quad (10)$$

of the differential conductance $G_N = dI_N/dV$.

In the classical Coulomb-blockade regime, one neglects quantum fluctuations in Q_d . Since the coupling to the leads causes a finite lifetime of the QD charges, Q_d is quantized only when $k_B T \gg \Gamma^N$. Furthermore, one assumes that there is no overlap between the distinct resonances, thereby the mean level spacing in the dot $\Delta \varepsilon \gg \Gamma$. Hence, we expand Eq. (9) to leading order in Γ and obtain $G_N(V)$ for $B > 0$,

$$G_N(V, +B) = -\frac{e^2}{h} \frac{\Gamma_L \Gamma_R}{4C_{\Sigma} k_B T \Gamma^N} [y_2(V) + y_1(-V)], \quad (11)$$

with

$$y_{\alpha}(V) = (C_{\alpha} + C_g/2) \cosh^{-2} \left(\frac{\tilde{\varepsilon}_N + eV \frac{C_{\alpha} + C_g/2}{C_{\Sigma}}}{2k_B T} \right), \quad (12)$$

for $\alpha = 1, 2$ where $\tilde{\varepsilon}_N = \varepsilon_N - E_F + \mathcal{U}(N-1/2) - eC_g V_g / C_{\Sigma}$ with E_F the Fermi energy in the leads. For $B < 0$ one must make in Eq. (11) the replacement $1 \rightarrow 2$ and $V \rightarrow -V$. Then, our expression predicts a magnetic-field asymmetry that arises only in the nonlinear conductance (for voltages $V \neq 0$) and only due to electrostatic interactions with the leads. For $V = 0$ we reproduce the expression of the linear conductance $G_0 = G(V=0)$ as a function of V_g .¹¹ G_0 is independent on the sign of B , thus fulfilling the Onsager relation. Sharp Coulomb-blockade peaks are observed in the oscillating G_0 as a function of V_g when $k_B T \ll e^2/C_{\Sigma}$ (see inset of Fig. 2).

We illustrate the behavior of Φ_G in Figs. 2 and 3. We define a capacitance asymmetry factor,

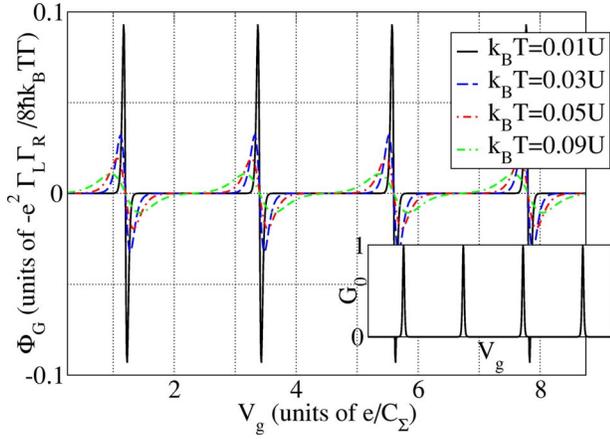


FIG. 2. (Color online) Magnetic-field asymmetry of the differential conductance vs gate voltage for different temperatures. We set $C_1 + C_2 = C_g = 0.5$ ($C_\Sigma = 1$), asymmetry factor $\eta = 0.5$, $\Delta = 0.1U$, $\Gamma = 0.002U$, and $V = 0.005U$ ($U = e^2/C_\Sigma$). Inset: Coulomb-blockade oscillations of the linear conductance ($V = 0$) for the same parameters and $k_B T = 0.01U$.

$$\eta = \frac{C_1 - C_2}{C_1 + C_2}. \quad (13)$$

Clearly, Φ_G is nonzero only for asymmetric couplings. In Fig. 2, we show Φ_G as a function of the back-gate voltage V_g for a finite bias and different temperatures. For simplicity, we set $E_F = 0$ and take uniformly spaced levels: $\Delta = \varepsilon_N - \varepsilon_{N-1}$ independent of N (in reality, levels are Wigner-Dyson distributed). The curve is periodic since Φ_G reflects the periodicity of the conductance. The asymmetry vanishes exactly at the degeneracy points, i.e., at gate voltages $V_g = e(N - 1/2)/C_g$

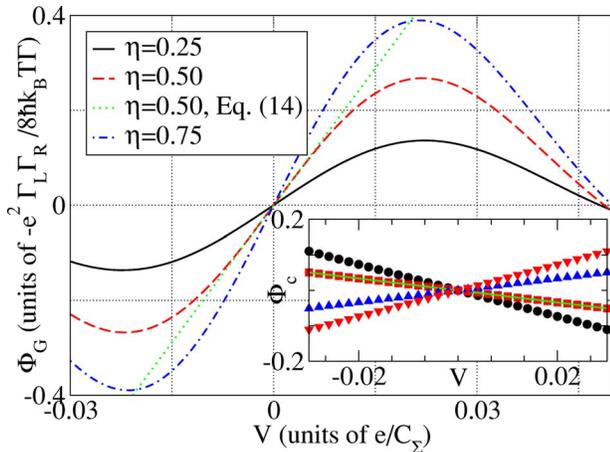


FIG. 3. (Color online) Magnetic-field asymmetry of the differential conductance vs bias voltage for different capacitance asymmetries. We set $k_B T = 0.01U$ ($U = e^2/C_\Sigma$), $C_1 + C_2 = C_g = 0.5$, $\Delta = 0.1U$, $\Gamma = 0.002U$, and $V_g = 1.173e/C_\Sigma$, which corresponds to a maximum in Fig. 2. The dotted line shows the low voltage result given by Eq. (14). Inset: Cotunneling magnetic-field asymmetry (in units of $\Gamma_L \Gamma_R / U^2$) vs bias for $eV_g/U = 1.8, 1.9, 2.1, 2.2$ (on the left, from top to bottom). The full line is obtained from Eq. (15) for $eV_g/U = 1.9$.

+ $\varepsilon_N C_\Sigma / eC_g$ (or simply $V_g = e(N - 1/2)/C_g$ for $\Delta \ll U$), where the conductance is maximum as $\tilde{\varepsilon}_N = 0$. Importantly, $|\Phi_G|$ reaches the maximum value on both sides of the degeneracy point and then decreases in the Coulomb-blockade valley, where the charge is fixed, because no transport is permitted. For very low voltages ($eV \ll k_B T$) a compact analytic expression can be found,

$$\Phi_G = -\frac{e}{\hbar} \frac{\Gamma_L^N \Gamma_R^N}{4\Gamma^N k_B T} \frac{\eta e V C_\Sigma - C_g}{C_\Sigma} \cosh^{-2} \frac{\tilde{\varepsilon}_N}{2k_B T} \tanh \frac{\tilde{\varepsilon}_N}{2k_B T}. \quad (14)$$

We find that the maxima of $|\Phi_G|$ take place approximately at $\tilde{\varepsilon}_N = k_B T$, i.e., for gate voltages of the order of $k_B T$ away from the degeneracy point. This explains as well why the maxima (minima) shift to lower (higher) values of V_g with increasing T . Moreover, it is worthwhile to note that the asymmetry effect vanishes overall in the high- T regime. This implies that when temperature is higher than the interaction e^2/C_Σ transport is mediated by thermal fluctuations only, which are B symmetric. We note in passing that our results are formally related to the voltage asymmetry that arises in a quantum dot that is more coupled to, say, the left lead than to the right lead.¹⁵ As a consequence, the conductance measured at forward bias differs from the backward bias case.¹⁶

Figure 3 presents the nonequilibrium conductance as a function of the bias voltage at a fixed V_g corresponding to one maximum in Fig. 2. The asymmetry increases rapidly with voltage and this increase is sharper for increasing capacitance asymmetry.

In Ref. 3 we distinguished between capacitive asymmetry and scattering asymmetry, the latter arising from asymmetric tunnel couplings $\Gamma_L^N \neq \Gamma_R^N$. Both asymmetries can be varied independently by changing the height and width of the tunnel barrier separating the dot and the edge states. This distinction was possible because the problem could be solved exactly at all orders in the coupling Γ^N (coherent tunneling). When the dot is Coulomb blocked, tunneling is sequential and tunnel couplings are treated to first order (Γ^N is the lowest energy scale). Thus, the effect of a tunnel asymmetry is trivially incorporated in our equations since $\Gamma_L^N \Gamma_R^N / \Gamma^N = (1 - \xi^2) / 4\Gamma^N$ with the scattering asymmetry factor $\xi = (\Gamma_L^N - \Gamma_R^N) / \Gamma^N$. However, in the classical treatment of Coulomb blockade given here, the asymmetry Φ_G vanishes when $\eta = 0$ independently of ξ . To include quantum fluctuations is a difficult task since the charge Q_d is not simply Ne and the self-consistent procedure to find the dot potential becomes involved. In the absence of Coulomb blockade effects, but in the presence of a Hartree potential, the task can be solved¹⁴ to all orders in Γ .

Cotunneling processes contribute to the conductance to order Γ^2 . Thereby a residual asymmetry is expected around the conductance minima.¹⁷ We consider elastic cotunneling, which is the dominant off-resonance mechanism at low bias when $k_B T \ll \Gamma$, as experimentally demonstrated.¹⁸ Elastic cotunneling consists of the virtual tunneling of an electron in a coherent fashion without leaving the dot in an excited state. Hence, our theory for transport between ground states is applicable. For definiteness, we investigate the minimum between the $N = 1$ and $N = 2$ resonances. Due to large denomi-

nators in Eq. (3) we can use $T=0$ Fermi functions in Eq. (9) and expand in powers of Γ . The resulting conductance goes as $(\Gamma/\mathcal{U})^2$. In the inset of Fig. 3 we plot the numerical result of the asymmetry of the cotunneling conductance, Φ_c , as a function of V for gates voltages around the conductance minimum, which represents the electron-hole (e - h) symmetry point. Interestingly enough, Φ_c changes sign about the minimum and exactly vanishes (not shown) at the e - h symmetry point since charge fluctuations are quenched there (the mean charge is $1/2$ per channel). For $E_F = \varepsilon_1 + \Delta/2$ the G_0 minimum takes place at $Q_g = C_g V_g = +e$. Then, to leading order in $(Q_g/e - 1)$ we find

$$\Phi_c = -\frac{e^2}{h} \Gamma_L \Gamma_R \frac{192\eta(C_\Sigma - C_g)\mathcal{U}eV}{C_\Sigma(\Delta + \mathcal{U})^4} \left(\frac{Q_g}{e} - 1 \right), \quad (15)$$

valid in the limit $eV \ll \mathcal{U}$ and $k_B T \ll \Gamma \ll \Delta < \mathcal{U}$. This expression reproduces the effects discussed above and is in remarkable agreement with the numerical results (see the inset of Fig. 3).

Thus far we have neglected the spin degeneracy. When T is further lowered, spin-flip cotunneling processes lead to Kondo effects and the corrections of the conductance become of the order of e^2/h . Notably, a dependence on the bias

polarity¹⁹ due to asymmetric couplings²⁰ has been observed. Therefore, one might expect a large magnetic-field asymmetry. However, recent works^{21,22} have emphasized the robustness of the e - h symmetry point in the Kondo regime against external disturbances, which would suggest that also the magnetic-field asymmetry vanishes at this point.

We have demonstrated that careful consideration of the interaction between a quantum dot and the edge states to which it is coupled leads to an out-of-equilibrium charging that is asymmetric under magnetic-field reversal. Crucial to this result is the chirality of the polarization charge. Obviously, any model generating an uneven polarization charge would similarly and quite generally predict an asymmetry. Importantly, the temperature scale of the magnetic field asymmetry we find is determined by the Coulomb charging energy. Consequently, the effect reported here should be readily observable in a wide range of systems.

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