Fermi liquid parameters in two dimensions with spin-orbit interaction

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We derive analytical expressions for the quasiparticle lifetime τ, the effective mass m^*, and the Green's function renormalization factor Z for a two-dimensional Fermi liquid with electron-electron interaction in the presence of the Rashba spin-orbit interaction. We find that the modifications are independent of the Rashba band index ρ, and occur in second order of the spin-orbit coupling α. In the derivation of these results, we also discuss the screening of the Coulomb interaction, as well as the susceptibility and the self-energy in small α.

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I. INTRODUCTION

The lifetime of quasiparticle excitations determined by electron-electron collisions is a crucial quantity of the Fermi liquid theory[1] of interacting electron systems. In particular, the quasiparticle lifetime τ for a two-dimensional electron gas as found, e.g., in semiconductor heterostructures has been now studied in great detail.[2,3] While τ has been traditionally of importance for phenomena that are based on coherent transport such as, for example, conductance fluctuations, weak localization, or the Aharonov-Bohm effect,[4] this quantity is also important for the current strive toward quantum information processing in the solid state, which requires the coherent propagation of, e.g., entangled electrons. In this respect, as well as in the emerging field of spintronics, the spin degree of freedom is increasingly being investigated.[5]

The effect of spin-orbit (s-o) interaction in low-dimensional systems has consequently become an important issue, and has uncovered functionalities such as the spin-based transistor,[6] spin injection,[7] and the electric manipulation of spin in nonmagnetic semiconductors,[8] and has also led to new physics with the spin-Hall effect.[9–15] The consideration of s-o interaction in the framework of Fermi liquid theory is therefore desirable. Existing work has investigated electronic transport and plasmon excitations,[16,17] Friedel-like oscillations in the screened potential,[18] and the modification of the s-o coupling due to electron-electron interactions.[19] While the spin relaxation and decoherence rates have been widely studied in such systems,[20] the relaxation rate of the quasiparticle itself has not, to our knowledge, been studied so far.

An important contribution to the effective mass m^* comes from the renormalization of the electron band mass by electron-electron interactions. Simple expressions for m^* in two dimensions appear in early works addressing the g-factor[21] and the spin susceptibility,[22] and were followed by numerical studies.[23] Some recent work addressed nonanalytic corrections,[24] the temperature dependence,[25] and the effects of impurity scattering.[26] Another important parameter of Fermi liquid theory is the renormalization factor Z of the Green's function.[1] This quantity measures the quasiparticle spectral weight, and gives the size of discontinuity of the zero temperature Fermi occupation factor $n(\xi)$ at the Fermi surface. For a clean two-dimensional electron gas (2DEG) without impurities and s-o interaction, it has been studied for short-range potentials,[27] while the realistic case with Coulomb interaction has been studied numerically[28] and analytically.[29,30] Recent related work used Fermi liquid theory to study plasmons contributions to the effective mass in valley-degenerate systems,[41] spin resonance and the spin-Hall conductivity,[42] as well as screening and plasmon modes.[43]

This work presents an analytical study of the effect of s-o interaction on the quasiparticle lifetime, the Z factor, and the effective mass m^* in a two-dimensional Fermi liquid, taking the specific case of the Rashba interaction.[31] We consider the long-range Coulomb interaction, and work within the random phase approximation[1] (RPA) valid for small $r_s \ll 1$ (high densities). For the lifetime, we find that the spin-orbit contribution appears in second order of the s-o coupling α, and contains a logarithmic term similar to the standard lifetime,[2] where the excitation energy ξ is replaced by the Rashba splitting $2\alpha k_F/\hbar$. A similar result is found for the effective mass, with a modification of the form $\alpha^2 \ln \alpha$. For the Z factor, we find a quadratic term without logarithmic enhancement. In all these cases the modifications are independent of the Rashba band index ρ denoting the two directions of the eigenspinors of the Rashba Hamiltonian. We also discuss briefly the screening of the Coulomb interaction, and derive expressions both the real and imaginary parts of the susceptibility χ, complementing the expressions found in Refs. 16–18. We also give general arguments showing that the self-energy and, consequently, the Fermi liquid parameters, cannot have any modification linear in α. Throughout this work we consider a clean system at zero temperature.

II. 2D FERMI LIQUID WITH RASHBA S-O INTERACTION

A. Rashba eigenstates

We consider an electron in a 2D Fermi liquid in the presence of the Rashba spin-orbit interaction restricted to the $z=0$ plane, described by the Hamiltonian $H=p^2/2m+H_{\text{s-o}}$ with[31]

$$H_{\text{s-o}} = \frac{\alpha}{\hbar}(p_x\sigma^y - p_y\sigma^x). \tag{1}$$

Expressed in the σ_z-spin basis $|\pm\rangle_z$, the eigenstates are

$$|\mathbf{k},\rho\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\rho e^{i\phi(\mathbf{k})}\end{pmatrix}|\mathbf{k}\rangle, \tag{2}$$

with the polar angle $\phi(\mathbf{k})=\angle(\mathbf{k},\mathbf{Ox})$ and the momentum eigenstates $|\mathbf{k}\rangle$. The index $\rho=\pm$ defines the two Rashba bands. We define the s-o strength

$$\gamma = \frac{k_R}{k_F} = \frac{\alpha k_F/\hbar}{2E_F} = \sqrt{\frac{E_R}{2E_F}} \tag{3}$$

from the Rashba momentum

$$k_R = m\alpha/\hbar, \tag{4}$$

the Rashba energy $E_R=m\alpha^2/\hbar^2$, and the Fermi energy $E_F=k_F^2/2m$. We define the excitation energy by $\xi_{k\rho}=E_{k\rho}-E_F$, with the dispersion relations for the two branches $E_{k\rho}=(k^2+2\rho k_R k)/2m$. Setting $\xi_{k_\rho\rho}=0$ yields the two Fermi momenta

$$k_\rho = \kappa - \rho k_R \text{ with } \kappa = k_F\sqrt{1+\gamma^2}. \tag{5}$$

Note that both k_ρ and κ will replace k_F in a number of the expressions valid without s-o interaction. We define the unperturbed Matsubara[1] Green's function

$$G_\rho(k,ik_n) = \frac{1}{ik_n - \xi_{k\rho}} \tag{6}$$

corresponding to the Rashba eigenstates (2) without electron-electron interaction. We have introduced the fermionic Matsubara frequencies $k_n=(2n+1)\pi k_B T, n\in\mathbb{N}$.

B. Renormalization due to the electron-electron interaction

Within Fermi liquid theory, the presence of electron-electron interaction modifies the retarded Green's function[1]

$$\bar{G}^R_\rho(k,\xi) = \bar{G}_\rho(k,ik_n\to\xi+i0^+) = \frac{1}{\xi-\xi_{k\rho}-\Sigma^R_\rho(k,\xi)}$$

$$\simeq \frac{Z_\rho}{\xi-\xi^*_{k\rho}+i(\hbar/2)\Gamma_\rho(k)} \tag{7}$$

describing a quasiparticle belonging to the Rashba band ρ with a momentum $\mathbf{k}$. To derive the expression above, one has expanded for small frequencies ξ and small excitation energies $\xi_{k\rho}$ above the Fermi surface, i.e., $\xi\ll E_F$, $0<\xi_{k\rho}\ll E_F \Leftrightarrow k-k_\rho\ll k_\rho$. In this procedure, one first shifts the Fermi momentum k_ρ via the requirement $\xi_{k_\rho\rho}+\mathrm{Re}\,\Sigma^R_\rho(k_\rho,0)=0$. The lifetime of the quasiparticle $\tau_\rho(k)=1/\Gamma_\rho(k)$ is given via

$$\Gamma_\rho(k) = -\frac{2}{\hbar}\,\mathrm{Im}\,\Sigma^R_\rho(k,\xi_{k\rho}), \tag{8}$$

where the self-energy Σ contains the effect of the Coulomb electron-electron interaction.

The Green's function acquires a renormalized weight

$$Z_\rho = \frac{1}{1-A}, \tag{9}$$

$$\text{with } A := \frac{\partial}{\partial\xi}\,\mathrm{Re}\,\Sigma^R_\rho(k_\rho,\xi=0) \tag{10}$$

which gives the size of the jump in the Fermi occupation factor $n(\xi)$ at the Fermi surface.

The effective mass enters in the renormalized excitation energy $\xi^*_{k\rho}=(k^2+2\rho k_R k-k_F^2)/2m^*$, and is defined by

$$\frac{m^*_\rho}{m} = \frac{1}{Z_\rho}\frac{1}{1+B}, \tag{11}$$

$$\text{with } B := \frac{m}{\kappa}\frac{\partial}{\partial k}\,\mathrm{Re}\,\Sigma^R_\rho(k=k_\rho,0). \tag{12}$$

As the excitation energy must vanish at the Fermi surface, one has $\xi^*_{k_\rho\rho}=0$ and thus k_F is also shifted with k_ρ. Note that it is κ, not k_ρ, that enters in the factor m/κ appearing in B.

In order to study the modifications introduced by the Rashba interaction, we first present here the results found without s-o interaction. The inverse lifetime reads[2,3,32]

$$\Gamma_0(k) = \frac{\xi_k^2}{E_F}\left[\ln\frac{\xi_k}{E_F} + O(r_s)\right]. \tag{13}$$

The r_s factor is defined here[33] as $r_s=k_{\mathrm{TF}}/2k_F=me_0^2/\hbar^2\sqrt{2\pi n}$, where n is the electronic sheet density (in the absence of s-o interaction), and k_{TF} is the Thomas-Fermi screening momentum. The two important characteristics of Eq. (13) are (i) $\Gamma_0\to 0$ when $\xi_k\to 0$, corresponding to long-lived quasiparticle excitations near the Fermi surface and (ii) the vanishing of Γ is slowed down by a logarithmic factor.

The effective mass contains a term $\sim r_s \ln r_s$, and is given by[21]

$$\frac{m_0^*}{m} - 1 = \frac{r_s}{\pi}[\ln r_s + 2 - \ln 2 + O(r_s)]. \tag{14}$$

The deviation of the renormalization weight Z from 1 is linear with r_s, and reads[29]

$$Z_0 - 1 = -\frac{r_s}{\pi}\left[1 + \frac{\pi}{2} + O(r_s)\right]. \tag{15}$$

Taking a GaAs 2DEG with[34] $n=4\times10^{15}\ \mathrm{m}^{-2}$, one has $r_s=0.614$, $Z_0=0.50$, and $m_0^*/m-1=0.16$. An InAs 2DEG with, e.g.,[35] $n=10\times10^{15}\ \mathrm{m}^{-2}$, $m=0.03m_e$ and $r_s=0.18$ has the parameters $Z_0=0.83$ and $m_0^*/m-1=0.019$.

C. Screening of the Coulomb interaction

In order to build a Fermi liquid theory including the s-o interaction, we must consider the matrix elements of the bare 2D Coulomb interaction $V_C(q)=2\pi e_0^2/q\hbar$ in the Rashba eigenstates basis. These matrix elements involve the overlap

$$\mathcal{F} = \frac{1}{4}[1+\rho_1\rho_1' e^{i(\phi_1'-\phi_1)}][1+\rho_2\rho_2' e^{i(\phi_2'-\phi_2)}] \tag{16}$$

of the eigenspinors (2), which depends on the directions $\{\phi_1,\phi_2,\phi_1',\phi_2'\}$ of the scattered states and on their band indices $\{\rho_1,\rho_2,\rho_1',\rho_2'\}$. In RPA,[1] we find the screened Coulomb potential

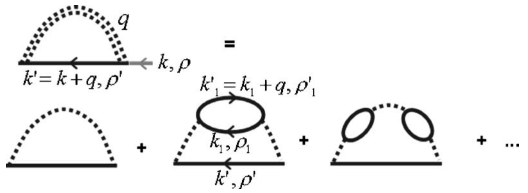

FIG. 1. The diagramatic representation of the self-energy $\Sigma_\rho(k)$ (23) in RPA. The full lines denote the electron Green's functions (6), the dashed lines the Coulomb interaction, the circles are the susceptibility bubble diagram (18), and the double dashed line is the screened Coulomb interaction (17). The Rashba bands are denoted by the ρ's, and yield the overlap factor $\mathcal{F}'$ (19).

$$V(q,\omega) = \frac{V_C(q)}{\varepsilon(q,\omega)}\mathcal{F}, \tag{17}$$

where $\varepsilon = 1 - V_C\chi$ is the dielectric function. In Matsubara formalism,[1] the susceptibility is given by the bubble diagram

$$\chi(\mathbf{q},iq_n) = k_B T \sum_{\rho,\rho'} \sum_{\mathbf{k},ik_n} G_\rho(\mathbf{k},ik_n) G_{\rho'}(\mathbf{k}',ik_n')\mathcal{F}', \tag{18}$$

where $\mathbf{k}'=\mathbf{k}+\mathbf{q}$, $ik_n'=ik_n+iq_n$, $\theta=\angle(\mathbf{k},\mathbf{k}')$, $(2\pi)^2\Sigma_{\mathbf{k}} \to \int d\mathbf{k}$ and $q_n = 2n\pi k_B T$ are bosonic Matsubara frequencies. The corresponding diagrams are represented in Fig. 1. The last factor

$$\mathcal{F}' = \frac{1+\rho\rho'\cos\theta}{2} \tag{19}$$

is the overlap $\mathcal{F}$ for states with opposite momenta. After summing over ik_n and performing the analytical continuation $iq_n \to \omega + i0^+$, one finds

$$\mathrm{Re}\,\chi(\mathbf{q},\omega) = \sum_{\mathbf{k},\rho\rho'} \frac{n(\xi_{k\rho}) - n(\xi_{k'\rho'})}{\omega + \xi_{k\rho} - \xi_{k'\rho'}}\mathcal{F}', \tag{20}$$

$$\mathrm{Im}\,\chi(\mathbf{q},\omega) = -\pi \sum_{k,\rho\rho'} \delta(\omega + \xi_{k\rho} - \xi_{k'\rho'}) \times [n(\xi_{k\rho}) - n(\xi_{k'\rho'})]\mathcal{F}', \tag{21}$$

which is the standard form with the additional $\mathcal{F}'$ factors. As we consider zero temperature, the Fermi occupation factor reads $n(\xi) = \Theta(-\xi)$. Note that the effect of the spin-orbit interaction manifests itself in the energies $\xi_{\rho k}$, while the factors $\mathcal{F}'$ *alone* just describe a change of the spin basis. In particular, such a basis change could also be considered in the absence of s-o interaction. For instance, the scattering cross section for two electrons,[36] given in Born approximation by

$$\lambda = \frac{1}{2\pi k}\left|\frac{m}{\hbar^2}V(q)\right|^2, \tag{22}$$

vanishes for different Rashba bands in case of forward scattering (e.g., $\rho_1' = -\rho_1$, $\mathbf{k}_1' = \mathbf{k}_1$), while it vanishes for same bands in the case of backscattering ($\mathbf{k}_1' = -\mathbf{k}_1$). This only reflects the fact that the real spin is conserved by the Coulomb interaction. For forward scattering the spin basis does not change, so that the band index must be the same. The opposite happens for backscattering, where the spin basis is inverted and the band index must be changed in order to preserve the real spin. This is the same reason why the two conjugate states of the Kramers doublet belong to the same Rashba band.[37]

D. Self-energy

The self-energy is the central quantity that determines the other Fermi liquid parameters. In lowest order in the screened intraction (RPA), it is given by[1]

$$\Sigma_\rho(k,ik_n) = -k_B T \sum_{\rho'} \sum_{\mathbf{q},iq_n} G_{\rho'}(\mathbf{k}',ik_n')V'(q,iq_n), \tag{23}$$

and is represented in Fig. 1. Here the screened potential (17) $V' = \mathcal{F}'V_C/\varepsilon$ involves $\mathcal{F}'$ because of momentum conservation. At zero temperature the inverse lifetime $1/\tau_\rho = \Gamma_\rho$ is given by

$$\Gamma_\rho(k,\xi_{k\rho}) = \frac{2}{\hbar}\sum_{\mathbf{q}\rho'} \Theta(\xi_{k'\rho'})\Theta(\omega)\,\mathrm{Im}\,V'(q,\omega), \tag{24}$$

with $\omega = \xi_{k\rho} - \xi_{k'\rho'}$. We now introduce the parameters

$$x'' = \frac{q}{2k_\rho} \text{ and } y' = \frac{m\omega}{q\kappa}, \tag{25}$$

which are relevant for the susceptibility χ entering in V (see Sec. III). We consider small excitation energies above the Fermi surface

$$0 < \xi_{k\rho} \simeq \frac{\kappa\delta}{m} \ll E_F \Leftrightarrow \delta := k - k_\rho \ll k_\rho. \tag{26}$$

Using $\omega = [k^2 - k'^2 + 2k_R(\rho k - \rho' k')]/2m$, we see that the Θ functions in Eq. (24) yield

$$0 < y' < \bar{y}' \sim \frac{\max\{\delta, k_R\}}{q}. \tag{27}$$

Note that *a priori*, neither x'' nor y' have to be small; however, one can check numerically that the dominant contributions to Eq. (24) come from forward scattering, i.e., for

$$q \lesssim \delta \ll k_\rho \Leftrightarrow x'' \ll 1. \tag{28}$$

For the lifetime, one can also assume $y' \ll 1$. For these reasons we shall calculate the susceptibility in the limit $x'' \ll 1$, before taking $y' \ll 1$.

III. SUSCEPTIBILITY

The susceptibility χ (or, equivalently, the dielectric function $\epsilon = 1 - V_C\chi$) for a 2DEG with s-o interaction has been partially studied in Refs. 16–18 in the small $q \ll k_F$ limit. Expressions for the imaginary part of χ in the limit $q \ll k_R, k_F$ have been given in Refs. 16 and 17 in the context of transport. Ref. 18, which addressed nonanalytical contributions to the real part of ϵ, only gives expressions for $q \to 0$ for the interband case (different Rashba bands), while the intraband case is studied in the $q \to 2k_F$ case. Therefore, it is desirable to complement these studies by deriving expressions for both Re χ and Im χ in the $q\to 0$ limit.

We first write the 2D susceptibility χ_0 without Rashba interaction.[38] Introducing the parameters

$$x=\frac{q}{2k_F} \text{ and } y=\frac{m\omega}{qk_F}, \tag{29}$$

the susceptibility reads in the Matsubara formalism ($iy_n \to y+i0^+$)

$$\chi_0(x,iy_n)=-\frac{m}{\pi}\left\{1-\frac{1}{x}R[s(z)\sqrt{z^2-1}]\right\}, \tag{30}$$

where $z=x+iy_n$, $z^*=x-iy_n$, $R[f(z)]=[f(z)+f(z^*)]/2$, and $s(z)=\mathrm{sgn}[\mathrm{Re}(z)]$ arising from the choice of the $(-\infty,0]$ branch cut for $\sqrt{z}$.

We now derive the susceptibility (20) and (21) in the limit of small q, $k_R \ll k_\rho$. We first write $\chi(q,\omega)=\Sigma_{\rho\rho'}\chi_{\rho,\rho'}$. We define $\varphi=\angle(\mathbf{k},\mathbf{q})$, and expand $k'=k\sqrt{1+2\cos\varphi(q/k)+(q/k)^2}$ in small q to get the energy difference

$$\xi_{k\rho}-\xi_{k'\rho'}\simeq\frac{k}{m}k_R(\rho-\rho')-\frac{q}{m}(k+\rho' k_R)\cos\varphi. \tag{31}$$

We also expand the Fermi function

$$n(\xi_{k'\rho'})-n(\xi_{k\rho})\simeq(\xi_{k\rho}-\xi_{k'\rho'})\delta(\xi_{k\rho}), \tag{32}$$

which selects $k=k_\rho$, and the spinor overlap

$$\frac{1\pm\cos\theta}{2}=\frac{1}{2}\left(1\pm\frac{k+q\cos\varphi}{k'}\right)\simeq\frac{1}{2}\pm\left[\frac{1}{2}-\left(\frac{q}{2k}\sin\varphi\right)^2\right]. \tag{33}$$

These expansions are valid for q, $k_R\ll k_\rho$.

A. Intraband contributions ($\rho'=\rho$)

We first consider transitions within a given Rashba band. We can neglect $(q/k)^2$ in the spinor overlap, and integrate over φ and k. We find

$$\mathrm{Re}\,\chi_{\rho,\rho}=-\frac{m}{2\pi}\left[1-\frac{|y'|}{\sqrt{y'^2-1}}\Theta(|y'|-1)\right]\left(1-\rho\frac{k_R}{\kappa}\right). \tag{34}$$

For the imaginary part, $\delta(\omega+\xi_{k\rho}-\xi_{k'\rho'})$ selects $\varphi=\arccos(y')$ if $|y'|<1$, and we get

$$\mathrm{Im}\,\chi_{\rho,\rho}=-\frac{m}{2\pi}\frac{y'}{\sqrt{1-y'^2}}\Theta(1-|y'|)\left(1-\rho\frac{k_R}{\kappa}\right), \tag{35}$$

which agrees with Eq. (35) of Ref. 16. Summing over ρ, we see that the intraband contributions to χ are independent of the band index ρ.

B. Interband contributions ($\rho'=-\rho$)

For transitions between two different Rashba bands, it is necessary to distinguish between two cases.

(a) $k_R\ll q\ll k_\rho$. We find

$$\mathrm{Re}\,\chi_{\rho,-\rho}=-\frac{m}{2\pi}x^2\left[\frac{1}{2}+(|y|\sqrt{y^2-1}-y^2)\Theta(|y|-1)\right], \tag{36}$$

$$\mathrm{Im}\,\chi_{\rho,-\rho}=-\frac{m}{2\pi}x^2y\sqrt{1-y^2}\Theta(1-|y|), \tag{37}$$

where we have also expanded in $y'\simeq y$.

(b) $q\ll k_R$, k_ρ. We get

$$\mathrm{Re}\,\chi_{\rho,-\rho}=\frac{1}{16\pi}\frac{q^2}{\omega}\ln\left(\frac{\rho\kappa/k_R+4E_F/\omega-1}{\rho\kappa/k_R+4E_F/\omega+1}\right)$$
$$\simeq-\frac{m}{4\pi}x^2\frac{1}{1+m\omega/2\rho k_Rk_F}, \tag{38}$$

where we have expanded in small $k_R\ll\kappa$ in the second equality. Note the unusual term $m\omega/k_Rk_F$. Setting $\omega=0$ and summing over ρ yields the static result (24) of Ref. 18 in the limit $k_BT\to 0$. For the imaginary part we find

$$\sum_\rho \mathrm{Im}\,\chi_{\rho,-\rho}=-\frac{m}{8}\frac{x}{y}\Theta(\omega_-<|\omega|<\omega_+), \tag{39}$$

with $\omega_\pm=2(\kappa\pm k_R)k_R/m$. This expression (39) agrees with Eq. (37) of Ref. 16 and Eq. (10) of Ref. 17, which are relevant for the optical conductivity. One can neglect this contribution when calculating the lifetime, as in this case $q\sim\delta$, $\omega\sim k_Rk_F/m\Rightarrow x/y\ll y$. The other interband contributions are negligible compared to the intrabands ones, as they are smaller by the factor $x^2\ll 1$.

C. Total susceptibility χ

Adding the two intrabands branches, we find for the susceptibility

$$\mathrm{Re}\,\chi(q\to 0,\omega)=-\frac{m}{\pi}\left[1-\frac{|y'|}{\sqrt{y'^2-1}}\Theta(|y'|-1)\right], \tag{40}$$

$$\mathrm{Im}\,\chi(q\to 0,\omega)=-\frac{m}{\pi}\frac{y'}{\sqrt{1-y'^2}}\Theta(1-|y'|), \tag{41}$$

which corresponds to the case without Rashba interaction (30) in the limit $x\to 0$, where one replaces y by the new parameter y'. We can now take the limit of small energy, $y'\ll 1$, and we finally find the susceptibility in the presence of Rashba s-o interaction

$$\chi(q\to 0,\omega\to 0)=-\frac{m}{\pi}(1+iy'), \tag{42}$$

which we shall use in the calculations of the Fermi-liquid parameters below. Note that in general Eqs. (20) and (21) yields that $\chi(q,-\omega)=\chi^*(-q,\omega)$. In particular, Re χ and Im χ are, respectively, even and odd in y in the limit $q=0$, as seen in our expressions above.

IV. SUSCEPTIBILITY AND SELF-ENERGY FOR SMALL α

In this section we show on general grounds that the expansion of χ and Σ in small α have no term linear in α.

A. Susceptibility

The susceptibility has only a second-order contribution from the s-o interaction,

$$\chi = \chi_0 + O(\alpha^2), \tag{43}$$

because χ is an even function of $k_R = m\alpha/\hbar$. This can be seen by expanding χ around $\alpha=0$ via the function $h(\xi_{k\rho};\xi_{k'\rho'}) := [n(\xi_{k\rho}) - n(\xi_{k'\rho'})]/(i\omega_n + \xi_{k\rho} - \xi_{k'\rho'})$. We use $d\xi_{k\rho}/dk_R = \rho k/m$ and $\xi_k = k^2/2m - E_F$, and find

$$\chi(q,i\omega_n) = \frac{1}{2}\sum_{\rho\rho'}\sum_{\mathbf{k}}(1+\rho\rho'\cos\theta) \times \sum_{j\geq 0}\frac{1}{j!}\left(\frac{k_R}{m}\right)^j\left(\rho k\frac{\partial}{\partial\xi_k} + \rho' k'\frac{\partial}{\partial\xi_{k'}}\right)^j h(\xi_k;\xi_{k'}), \tag{44}$$

and notice that the sums over ρ, ρ' cancel for odd powers j. This result is consistent with Eqs. (34)–(39), where the terms linear in k_R vanish after summing over ρ, ρ'.

B. Self-energy

We first perform the sum over the Rashba band index ρ' in the self-energy (23)

$$\Sigma_\rho(k,ik_n) = -k_B T\sum_{\mathbf{q},iq_n} H(\mathbf{q},ik_n')\frac{V_C(q)}{\varepsilon(q,iq_n)}, \tag{45}$$

where

$$H(\mathbf{q},ik_n') = \frac{\zeta + \rho\alpha(k+q\cos\phi)}{\zeta^2 - (\alpha k')^2}, \tag{46}$$

and $\zeta = ik_n' - \xi_{k'}$. We expand in small α and find

$$H(\mathbf{q},ik_n') \simeq \frac{1}{\zeta_0} + O(\alpha^2)\left(\frac{1}{\zeta_0^2} + \frac{1}{\zeta_0^3}\right). \tag{47}$$

where $\zeta_0 = \zeta(\alpha\to 0)$. (We recall that k, being close to k_ρ, depends on α.)

The integrations of the first and third terms do not yield any logarithmic term in α because their divergence at $\zeta_0=0$ is odd with respect to q. On the contrary, the term $O(\alpha^2)/\zeta_0^2$ brings logarithmic contributions, as will be seen in the lifetime and the effective mass below. Because $\varepsilon = 1 - V_C\chi$ has also no term linear in α, we find that the modification of the self-energy due to the s-o interaction can only appear in second order.

V. LIFETIME

In this section we calculate the lifetime as given by Eq. (8). We first define the Thomas-Fermi momentum

$$k_{\mathrm{TF}} = \frac{2me_0^2}{\hbar} = 2r_s k_F, \tag{48}$$

and assume that the small q contributions dominate such that $mV(q)/\pi = k_{\mathrm{TF}}/q \gg 1$ (this is justified in GaAs where $k_s \simeq 1.2k_F$). We find

$$\mathrm{Im}\,V'(q,\omega) = \frac{V_C^2(q)}{(1+k_{\mathrm{TF}}/q)^2 + (k_{\mathrm{TF}}/q)^2 y'^2}\left(-\frac{m}{\pi}y'\right)\mathcal{F}' \simeq -\frac{\pi}{m}y'\mathcal{F}'. \tag{49}$$

Note that it is $\mathcal{F}'$—and not $\mathcal{F}'^2$—that appears here with V^2, because the screening involves only χV, without $\mathcal{F}'$. Writing $\Gamma_\rho(k) = \Sigma_{\rho'}\Gamma_{\rho,\rho'}(k)$ and changing variables $\Sigma_{\mathbf{q}} \to \Sigma_{\mathbf{k}'}$, we have

$$\Gamma_{\rho,\rho'}(k) = \frac{1}{8\pi\hbar m\kappa}\int_{k_{\rho'}}^{\bar{k}} dk' k'(\xi_{k'\rho'} - \xi_{k\rho})I_{\rho'}, \tag{50}$$

where

$$I_{\rho'} = \int_0^{2\pi} d\theta\frac{1+\rho\rho'\cos\theta}{q(k',\theta)}. \tag{51}$$

Here $\bar{k} = k + k_R(\rho - \rho')$, $\theta = \angle(\mathbf{k},\mathbf{k}')$, and $q(k',\theta) = \sqrt{k^2 + k'^2 - 2kk'\cos\theta}$. We distinguish intra- and interband contributions.

A. Intraband case ($\rho' = \rho$)

We find

$$I_\rho = \frac{2}{kk'|k-k'|}[(k+k')^2 K(-z) - (k-k')^2 E(-z)], \tag{52}$$

where $z = 4kk'/(k-k')^2$, and K and E are the complete Elliptic integrals of the first and second kind, respectively. We use their asymptotics[39] $E(-z) \sim \sqrt{z}$ and $K(-z) \sim \log(4\sqrt{z})/\sqrt{z}$ for large $z \gg 1$, as $k - k' \sim \delta \ll k \simeq k_\rho$. After performing the k integration and expanding in small $\delta \ll k_\rho$ up to second order, we finally get

$$\Gamma_{\rho,\rho}(k) = -\frac{\delta^2}{2\pi\hbar m}\left\{\ln\left(\frac{\delta}{8k_\rho}\right) + \frac{1}{2}\right\} \simeq -\frac{\delta^2}{2\pi\hbar m}\left\{\ln\left(\frac{\delta}{8k_F}\right) + \frac{1}{2} + \rho\gamma\right\}. \tag{53}$$

We also expanded in small $k_R \ll k_F$ in the second line.

B. Interband case ($\rho' = -\rho$)

We find

$$I_{-\rho} = -\frac{2|k-k'|}{kk'}[K(-z) - E(-z)]. \tag{54}$$

We repeat the same procedure and expand in δ, $k_R \ll k_F$. We get

$$\Gamma_{\rho,-\rho}(k) = \frac{\delta^2}{2\pi\hbar m}\left\{1 + \rho\gamma + \gamma^2\ln\frac{\gamma}{4}\right\}. \tag{55}$$

We now add the two Rashba branches. The term linear in k_R vanishes and we finally get for the lifetime including the Rashba s-o interaction

$$\Gamma_\rho(k) = -\frac{\delta^2}{2\pi\hbar m}\left\{\ln\left(\frac{\delta}{8k_F}\right) - \frac{1}{2} - \gamma^2 \ln\frac{\gamma}{4}\right\}$$
$$= -\frac{\xi_{k\rho}^2}{4\pi\hbar E_F}\left\{\ln\left(\frac{\xi_{k\rho}}{16E_F}\right) - \frac{1}{2} - \frac{E_R}{E_F}\ln\frac{E_R}{8E_F}\right\}, \quad (56)$$

valid up to $\delta/(\hbar m)\times O(\delta,\gamma^2)$. We recognize in the first term the standard lifetime for a 2D Fermi liquid without Rashba interaction,[2,3] with the logarithmic enhancement $\log(\delta/k_F) \sim \ln(\xi_k/E_F)$.

The modification to the lifetime due to the spin-orbit interaction also contains a logarithmic factor $\ln(k_R/k_F) \sim \ln(\Delta_R/E_F)$ involving the Rashba splitting at the Fermi surface, $\Delta_R=2\alpha k_F/\hbar$. We note that for typical GaAs 2DEGs this modification is rather weak, because of the factor $\gamma=k_R/k_F \ll 1$, and therefore does not modify significantly the usual term valid without s-o interaction.

VI. RENORMALIZATION FACTOR Z

We now derive the expression for the renormalization factor Z (9). We give some details of the calculation, in order to show the cancellation of the $\sim\ln r_s$ term, as well as to introduce integrations that will also be useful for the calculation of the effective mass. Our starting point is the real part of the self-energy entering in Eq. (10). At $k_BT=0$, one can replace[1] the sum over the Matsubara frequencies appearing in Eq. (23) by an integral along the imaginary axis, $k_BT\Sigma_{iq_n}f(iq_n) \to (1/2\pi)\int du f(iu)$. Thus we need to evaluate

$$A = -\frac{\partial}{\partial\xi}\,\mathrm{Re}\sum_{\mathbf{q}\rho'}\frac{1}{2\pi}\int_{-\infty}^{\infty} du G_{\rho'}(k', ik_n + iu)V'(q,iu), \quad (57)$$

where $\mathbf{k}'=\mathbf{k}+\mathbf{q}$, $k=k_\rho$ and one sets $\xi=0$ after taking the derivative. While the analytical continuation $ik_n\to\xi+i0_+$ must be taken after the integration, one can reverse this order (i.e., make the analytical continuation first)[1]

$$A = A^{\mathrm{res}} - \sum_{\mathbf{q}\rho'}\frac{\mathrm{Re}}{\pi}\int_0^{\infty} du\ \frac{\partial}{\partial\xi}G_{\rho'}(k', \xi + iu)V'(q,iu)\bigg|_{\xi=0}, \quad (58)$$

provided that one compensates for the contributions of the poles of G by adding the "residue" term

$$A^{\mathrm{res}} = -\,\mathrm{Re}\sum_{\mathbf{q}\rho'}\frac{\partial}{\partial\xi}[\Theta(-\xi_{k'\rho'}) - \Theta(\xi - \xi_{k'\rho'})]$$
$$\times\ V'(q,\xi_{k'\rho'} - \xi)|_{\xi=0} = \sum_{\mathbf{q}\rho'}\delta(\xi_{k'\rho'})V'(q,0). \quad (59)$$

We have used in Eq. (58) the fact that the integrated function is even in u. For the remaining term, we notice that $-\partial_\xi G_{\rho'}(k',\xi+iu)=i\partial_u G_{\rho'}(k',iu)$ when $\xi=0$ and integrate by parts over u. The boundary term with $u\to\infty$ vanishes, while the term with $u\to 0_+$ gives

$$A_{\mathrm{boundary}}^{(u\to 0_+)} = \mathrm{Re}\ i\frac{1}{\pi}\sum_{\mathbf{q}\rho'}G_{\rho'}(k', i0_+)V'(q,i0_+)$$
$$= -\sum_{\mathbf{q}\rho'}\delta(\xi_{k'\rho'})V'(q,0), \quad (60)$$

where we have used $-\mathrm{Im}\,G_{\rho'}(k',i0_+)=-\pi\delta(\xi_{k'\rho'})$. We see that this boundary term cancels with the residue term A_ρ^{res}. This is important, as these terms actually contain a term that is logarithmic in r_s [see Eq. (70) in the calculation of m^* below]. Thus we have

$$A = \mathrm{Im}\,\frac{1}{\pi}\sum_{\mathbf{q}\rho'}\int_0^{\infty} du G_{\rho'}(k', iu)\frac{\partial}{\partial u}V'(q,iu) \quad (61)$$

$$= -\frac{r''}{2\pi^2}\sum_{\rho'}\ \mathrm{Im}\int_0^{\infty} dy''\int_0^{2\pi} d\phi\int_0^{\infty} dx'' f(x'',y'',\phi). \quad (62)$$

We have defined $r''=me_0^2/k_\rho\hbar=k_{\mathrm{TF}}/2k_\rho$, $y''=mu/qk_\rho$, and $x''=q/2k_\rho$. The integrand is

$$f(x'',y'',\phi) = \frac{\mathcal{F}'}{iy'' - \mu}\frac{1}{x''\varepsilon^2}\frac{\partial\varepsilon}{\partial y''}, \quad (63)$$

where $\mathcal{F}'(x'',\phi)=1/2+\rho\rho'(1+2x''\cos\phi)/2\ell$ is the overlap of the eigenspinors, $\ell(x'',\phi)=\sqrt{1+4x''\cos\phi+4x''^2}=k'/k$, $\mu(x'',\phi)=\cos\phi+x''+(\rho\rho'\ell-1)\gamma'/2x''$ is the dimensionless energy $\xi_{k'\rho'}$, and $\gamma'=\rho k_R/k_\rho$ is a modified s-o strength.

We now consider the RPA limit of high density, which corresponds to small $r''\ll 1$. In this case, the dominant contribution comes from the intraband case $(\rho'=\rho)$ with $x''\ll 1$, where we can use the approximations $\mathcal{F}'\simeq 1+O(x''^2)$, $x''\varepsilon\simeq x''+r''a(y')+O(x''^2)$, $\mu\simeq\cos\phi(1+\gamma')+O(x'')$, and we have defined $a(y')=1-y'/\sqrt{y'^2+1}$ and $y'=mu/q\kappa = y''/(1+\gamma'')$. Defining $r'=me_0^2/\kappa\hbar=r''/(1+\gamma'')$, we have

$$A \simeq -\frac{r'}{2\pi^2}\,\mathrm{Im}\int_0^{\infty} dy'\frac{\partial}{\partial y'}a(y')\int_0^{2\pi} d\phi\frac{1}{iy' - \cos\phi}$$
$$\times\int_0^{\infty} dx''\frac{\partial}{\partial x''}\frac{-r''}{x'' + ra(y')} = -\frac{r'}{\pi}\int_0^{\infty} dy'\frac{1}{(y^2+1)^2}\frac{1}{a(y')}$$
$$= -\frac{r'}{\pi}\left(1 + \frac{\pi}{2}\right) + O(r'^2). \quad (64)$$

The remaining terms (in particular, the contribution from the interband transition with $\rho'=-\rho$) are neglected as they are $O(r'^2)$. We now use

$$r' = r_s(1 + \gamma^2)^{-1/2} \simeq r_s\left(1 - \frac{1}{2}\gamma^2\right), \quad (65)$$

and the renormalization factor reads

$$Z_\rho = 1 - \frac{r_s}{\pi}\left(1 + \frac{\pi}{2}\right)\left(1 - \frac{1}{2}\gamma^2\right) = 1 - \frac{r_s}{\pi}\left(1 + \frac{\pi}{2}\right)\left(1 - \frac{E_R}{E_F}\right). \quad (66)$$

This result is valid up to $O(r_s^2, r_s\gamma^3)$, so that the modification from the result (15) without s-o disappears in the case γ

$\ll\sqrt{r_s}\ll 1$. Similar to the inverse lifetime, we see that the Z factor is independent of the Rashba band index ρ, and that its modification appears only in second order in the strength of the s-o interaction. This modification can be traced back to the small shift of the Fermi surface due to the s-o interaction. Without s-o inteaction ($\gamma=0$), Eq. (66) corresponds to the result presented in Refs. 29 and 30.

VII. EFFECTIVE MASS

The calculation for the effective mass (11) is similar in spirit to the Z factor calculation, but is more involved. We start with

$$B=-\frac{m}{\kappa}\frac{\partial}{\partial k}\mathrm{Re}\sum_{\mathbf{q}\rho'}\frac{1}{2\pi}\int_{-\infty}^{\infty}duG_{\rho'}(k',ik_n+iu)V'(q,iu), \tag{67}$$

where $k\to k_\rho$ after taking the derivative. Again, we first perform the analytical continuation $ik_n\to\xi+i0_+$ by adding a residue term $\sim\Theta(-\xi_{k'\rho'})-\Theta(\xi-\xi_{k'\rho'})$. Contrary to the case for Z, this residue term identically vanishes once we take $\xi=0$. Hence we have

$$B=-\mathrm{Re}\,\frac{m}{\kappa}\sum_{\mathbf{q}\rho'}\frac{1}{\pi}\int_0^{\infty}du\;\;\frac{\partial}{\partial k}G_{\rho'}(k',iu)V'(q,iu)\bigg|_{k=k_\rho}, \tag{68}$$

which we integrate by parts. With the change of variables $\mathbf{q}\to\mathbf{k}'$, the boundary term reads

$$\begin{aligned}B_{\text{boundary}}^{(u\to 0_+)}&=\frac{m}{\kappa}\sum_{\mathbf{k}'\rho'}\delta(\xi_{k'\rho'})V'(q,0)\frac{\partial\xi_{k'\rho'}}{\partial k}\\&=\frac{r'}{8\pi}\sum_{\rho'}\frac{k_{\rho'}}{k_\rho}\int_0^{2\pi}d\theta\frac{\cos\theta}{x''_{\rho\rho'}(\theta)+r''}(1+\rho\rho'\cos\theta),\end{aligned} \tag{69}$$

where $x''_{\rho\rho'}(\theta)=(1/2k_\rho)\sqrt{k_{\rho'}^2-2k_{\rho'}k_\rho\cos\theta+k_\rho^2}$ and we recall that $r'=me_0^2/\kappa\hbar$. We consider the case r_s, $\gamma\ll 1$ and finally get

$$B_{\text{boundary}}^{(u\to 0_+)}=-\frac{r_s}{\pi}\left[\ln\left(\frac{r_s}{2}\right)+2+\frac{4\rho\gamma}{3}-\frac{\gamma^2}{2}\ln\gamma+O(r_s,\gamma^2)\right]. \tag{70}$$

The remaining integrated term of the integration by parts in Eq. (68) contains two terms. The first one reads

$$B_{\text{int}}^{(1)}=-\mathrm{Im}\,\frac{m}{\kappa\pi}\sum_{\mathbf{q}\rho'}\int_0^{\infty}duG_{\rho'}(k',iu)\frac{\partial}{\partial u}V'(q,iu)\frac{\partial\xi_{k'\rho'}}{\partial k}. \tag{71}$$

We see that the integrand is identical to the expression (61) appearing in the calculation made for Z, apart from the factor $\partial\xi_{k'\rho'}/\partial k\simeq(k/m)(1+\rho'\gamma'+2x\cos\phi)$ for $x\ll 1$. We proceed as before, and get in lowest order

$$B_{\text{int}}^{(1)}=\frac{r'}{\pi}\left(1+\frac{\pi}{2}\right). \tag{72}$$

The second term reads

$$B_{\text{int}}^{(2)}=-\mathrm{Re}\,\frac{m}{\kappa\pi}\sum_{\mathbf{q}\rho'}\int_0^{\infty}duG_{\rho'}(k',iu)\frac{V_C(q)}{\varepsilon(q,iu)}\frac{\partial\mathcal{F}'}{\partial k}, \tag{73}$$

where $\partial\mathcal{F}'/\partial k=\rho\rho' q^2\sin^2\phi/2k'^3$. The analysis of this integral is rather demanding, as no approximation is accurate and only a numerical solution seems possible. However, a careful examination of the different terms shows that there is no logarithmic contribution—in particular, the small x contributions are suppressed by the overall $\sim x^3$ dependence. We can now use the general argument about the self-energy (see Sec. IV B), which states that no term linear in γ can be present in the effective mass. This implies that this integral $B_{\text{int}}^{(2)}$ compensates for the linear term appearing in Eq. (70), which is confirmed by numerical integrations.

Finally, we obtain the effective mass (11)

$$\frac{m_\rho^*}{m}-1=\frac{r_s}{\pi}\left(\ln r_s+2-\ln 2-\frac{1}{2}\gamma^2\ln\gamma\right), \tag{74}$$

where we used the expression (66) for Z. We recognize in the first three terms the unperturbed result (14). The modification induced by the s-o interaction has the form $\gamma^2\ln\gamma$, similar to the lifetime, Eq. (56) We see that particles in different Rashba spin eigenstates have, to lowest order, the same effective mass.

VIII. CONCLUSION

We have calculated the main quasiparticle parameters (the inverse quasiparticle lifetime $1/\tau$, the renormalization factor Z, and the effective mass m^*) due to the Coulomb electron-electron interaction in a 2D Fermi liquid with Rashba interaction. The modifications due to s-o interaction are found to be independent of the Rashba band index ρ, and to appear only in second order in the s-o strength $\gamma\sim\sqrt{E_R/E_F}$ with some logarithmic enhancement for the lifetime and the effective mass.

The spin-orbit constant being rather small in typical semiconductor 2DEGs, these modification will be very small, around 10^{-3}. For instance, a GaAs 2DEG with[40] $\alpha=0.5\times10^{-12}$ eV m, $n=4\times10^{15}$ m^{-2} yields $k_R=0.43\ \mu$m^{-1}. This gives a rather small $\gamma=2.7\times10^{-3}$, so that one gets only very small modifications. Even for an InGaAs 2DEGs with larger s-o coupling and with[35] $\alpha=30\times10^{-12}$ eV m for $n=10\times10^{15}$ m^{-2}, $m=0.03m_e$ and $r_s=0.18$, one has a modest $\gamma=0.051$.

We note that replacing the Rashba interaction by the Dresselhaus interaction $H_D=\beta(-p_x\sigma_x+p_y\sigma_y)/\hbar$ yields exactly the same results. Indeed, the only difference lies in the eigenspinors (the phase is decreased by $\pi/2$), so that their overlap $\mathcal{F}'$, Eq. (19), is unchanged and the energies have the same form. Natural extensions of this work are the studies of the effect of the s-o interaction on the renormalized g factor,[21] the consideration of short-range potential instead of the Coulomb interaction, finite temperatures, and the presence of a perpendicular or parallel magnetic field as used to measure the mobility and the effective mass or to manipulate electron spins.

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[1] G. D. Mahan, *Many-Particle Physics* (Plenum, New York, 1990); H. Bruus and K. Flensberg, *Many-Body Quantum Theory in Condensed Matter Physics: An Introduction* (Oxford University Press, New York, 2004).

[2] A. V. Chaplik, Sov. Phys. JETP **33**, 997 (1971); G. F. Giuliani and J. J. Quinn, Phys. Rev. B **26**, 4421 (1982).

[3] L. Zheng and S. Das Sarma, Phys. Rev. B **53**, 9964 (1996); D. Menashe and B. Laikhtman, *ibid.* **54**, 11561 (1996); T. Jungwirth and A. H. MacDonald, *ibid.* **53**, 7403 (1996); M. Reizer and J. W. Wilkins, *ibid.* **55**, R7363 (1997).

[4] See, e.g., S. Washburn and R. A. Webb, Rep. Prog. Phys. **55**, 1311 (1992).

[5] *Semiconductor Spintronics and Quantum Computation*, edited by D. D. Awschalom, D. Loss, and N. Samarth (Springer-Verlag, Berlin, 2002).

[6] S. Datta and B. Das, Appl. Phys. Lett. **56**, 665 (1990).

[7] A. Voskoboynikov, S. S. Lin, and C. P. Lee, J. Appl. Phys. **87**, 387 (2000); T. Koga, J. Nitta, H. Takayanagi, and S. Datta, Phys. Rev. Lett. **88**, 126601 (2002); A. A. Kiselev and K. W. Kim, Appl. Phys. Lett. **78**, 775 (2001).

[8] Y. Kato, R. C. Myers, A. C. Gossard, and D. D. Awschalom, Nature (London) **427**, 50 (2004).

[9] S. Murakami, N. Nagaosa, and S. C. Zhang, Science **301**, 1348 (2003).

[10] J. Sinova, D. Culcer, Q. Niu, N. A. Sinitsyn, T. Jungwirth, and A. H. MacDonald, Phys. Rev. Lett. **92**, 126603 (2004).

[11] Y. K. Kato, R. C. Myers, A. C. Gossard, and D. D. Awschalom, Science **306**, 1910 (2004).

[12] J. Wunderlich, B. Kaestner, J. Sinova, and T. Jungwirth, Phys. Rev. Lett. **94**, 047204 (2005).

[13] J. Schliemann and D. Loss, Phys. Rev. B **69**, 165315 (2004).

[14] O. Chalaev and D. Loss, Phys. Rev. B **71**, 245318 (2005).

[15] S. I. Erlingsson, J. Schliemann, and D. Loss, Phys. Rev. B **71**, 035319 (2005).

[16] E. G. Mishchenko and B. I. Halperin, Phys. Rev. B **68**, 045317 (2003).

[17] L. I. Magaril, A. V. Chaplik, and M. V. Éntin, JETP **92**, 153 (2001).

[18] G.-H. Chen and M. E. Raikh, Phys. Rev. B **59**, 5090 (1999).

[19] G.-H. Chen and M. E. Raikh, Phys. Rev. B **60**, 4826 (1999).

[20] R. J. Elliott, Phys. Rev. **96**, 266 (1954); M. I. D' yakonov and V. I. Perel, Fiz. Tverd. Tela (Leningrad) **13**, 3581 (1971) [Sov. Phys. Solid State **13**, 3023 (1971)]; P. Boguslawski, Solid State Commun. **33**, 389 (1980); M. M. Glazov and E. L. Ivchenko, JETP Lett. **75**, 403 (2002).

[21] J. F. Janak, Phys. Rev. **178**, 1416 (1969).

[22] S. Yarlagadda and G. F. Giuliani, Phys. Rev. B **40**, 5432 (1989).

[23] C. S. Ting, T. K. Lee, and J. J. Quinn, Phys. Rev. Lett. **34**, 870 (1975).

[24] A. V. Chubukov and D. L. Maslov, Phys. Rev. B **68**, 155113 (2003).

[25] Y. Zhang and S. Das Sarma, Phys. Rev. B **70**, 035104 (2004).

[26] K. Morawetz, Europhys. Lett. **67**, 77 (2004).

[27] P. Bloom, Phys. Rev. B **12**, 125 (1975).

[28] R. Jalabert and S. Das Sarma, Phys. Rev. B **40**, 9723 (1989).

[29] G. Burkard, D. Loss, and E. V. Sukhorukov, Phys. Rev. B **61**, R16303 (2000).

[30] V. M. Galitski and S. Das Sarma, Phys. Rev. B **70**, 035111 (2004).

[31] Y. A. Bychkov and E. I. Rashba, J. Phys. C **17**, 6039 (1984).

[32] Note that the expressions given in Refs. 2 and 3 only agree for the dominant logarithm $\log_{10}(\xi/E_F)$, while the rest of $O(1)$ differ.

[33] The paramater $\bar{r}_s = r_s\sqrt{2}$ is also used in the literature.

[34] T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982).

[35] Y. Sato, T. Kita, S. Gozu, and S. Yamada, J. Appl. Phys. **89**, 8017 (2001).

[36] For a detailed study of the Coulomb scattering of two electrons in a 2DEG, see D. S. Saraga, B. L. Altshuler, D. Loss, and R. M. Westervelt, Phys. Rev. Lett. **92**, 246803 (2004); Phys. Rev. B **71**, 045338 (2005).

[37] E. I. Rashba, Phys. Rev. B **68**, 241315(R) (2003).

[38] F. Stern, Phys. Rev. Lett. **18**, 546 (1967).

[39] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (Dover Publications, New York, 1965).

[40] J. B. Miller, D. M. Zumbuhl, C. M. Marcus, Y. B. Lyanda-Geller, D. Goldhaber-Gordon, K. Campman, and A. C. Gossard, Phys. Rev. Lett. **90**, 076807 (2003).

[41] S. Gangadharaiah and D. L. Maslov, cond-mat/0505534 (unpublished).

[42] A. Shekhter, M. Khodas, and A. M. Finkel'stein, Phys. Rev. B **71**, 165329 (2005).

[43] M. Pletyukhov and V. Gritsev, cond-mat/0506227 (unpublished).