Polarization properties of a periodically-modulated metal film in regions of anomalous optical transparency

A. V. Kats,* M. L. Nesterov, and A. Yu. Nikitin

Usikov Institute for Radiophysics and Electronics, National Academy of Sciences of Ukraine, 61085 Kharkov, Ukraine Received 27 June 2005; revised manuscript received 8 September 2005; published 9 November 2005-

We present an analytical treatment of the resonance optical effects in which an arbitrarily polarized wave is incident at an arbitrary angle onto a periodically-modulated metal film in the conical mount. We show that under enhanced light transmittance conditions, not only intensity, but also the polarization of both reflected and transmitted waves undergoes significant changes. An in-depth investigation of the polarization transformation including the polarization conversion for both zeroth and nonzeroth diffraction orders is carried out. Besides, we derive novel polarization reciprocity relations.

DOI: [10.1103/PhysRevB.72.193405](http://dx.doi.org/10.1103/PhysRevB.72.193405)

Over the past few years, a number of important experiments have been made owing to major breakthroughs in manufacturing periodical nanostructures (nanoparticle ar- $+\sum$ *m*

rays, metal films with nanoholes, etc.). Specifically, in 1998, it was shown that metal films with subwavelength holes possess an enhanced light transmission (ELT).¹ The recent measurements have also indicated that the enhanced transmission through hole arrays in a zeroth diffraction order possesses a strong polarization dependence upon the symmetry of the array. $2-5$ The polarization analysis of the diffracted light plays a key role in understanding the true nature of ELT phenomena and elucidates the role of surface electromagnetic waves [known as surface plasmon-polariton (SPP) in optics] excitation.⁶

In the paper, we present a detailed analytical study of polarization properties of ELT phenomena for a metal film having one-dimensional (1D) periodic modulation. We consider briefly a two-dimensional (2D) case that has been basically investigated in the experiments. In fact, a number of results concerning the 2D case may well be described while considering appropriate 1D structures. This is due to the fact that 2D Fourier harmonics (which correspond to some 1D periodical substructures) responsible for SPP excitation by a one-step scattering process play a key role in the polarization effects under consideration. Other Fourier harmonics are far less important. An analytical approach is developed, allowing us to deal with the modulation of an arbitrary type and shape. We derive reflected and transmitted radiation polarization dependencies for periodicity formed by the permittivity modulation (for instance, holes and/or slits filled with other metal or a semiconductor, or nanowires and/or nanoparticle periodic arrays being introduced into the film). In addition, we discuss differences in polarization dependencies for permittivity and relief modulated films.

Consider an arbitrary polarized plane monochromatic wave with electric field amplitude \mathbf{E}^i and wave vector \mathbf{k}^i , which is incident onto a periodically-modulated metal film of thickness *d* surrounded by dielectric media having permittivities ε_r , $\tau = \pm$, from the medium corresponding to $\tau = -$. We imply that the periodicity is caused by the modulation of the surface impedance of the film, $\xi = 1/\sqrt{\varepsilon}$, where ε is the dielectric permittivity, the corresponding vector of the reciprocal grating is **g**, see Fig. 1.

The fields we seek in the form of the Fourier-Floquet expansion [we omit the time-dependent exp(-*iωt*) everywhere are as follows:

PACS number(s): 42.25. - p, 73.20.Mf, 78.20.Bh, 78.20.Ci

$$
\mathbf{E}^{\tau}(\mathbf{r}) = \delta_{\tau,-} \mathbf{E}^i \exp(i\mathbf{k}^i \mathbf{r}) \n+ \sum_m \mathbf{E}_m^{\tau} \exp[i\mathbf{k}_{mi}\mathbf{r} + ik_{\tau|mz}(z - \delta_{\tau,+}d)], \qquad (1)
$$

for $z \ge d(z \le 0)$ if $\tau = +(-)$; $\mathbf{r} = (x, y, z)$. Here subscript *t* indicates that it is tangential to the film faces components of vectors. The *m*th order wave vector is $\mathbf{k}_m^{\tau} = (\mathbf{k}_{mt}, k_{\tau|mz})$,

$$
\mathbf{k}_{mt} = \mathbf{k}_t^i + m\mathbf{g}, \quad k_{\tau/mz} = \tau \sqrt{k^2 \varepsilon_\tau - \mathbf{k}_{mt}^2}, \quad k = \omega/c. \tag{2}
$$

Within the film, we seek the electric field, $\overline{E}(\mathbf{r})$, in the form

$$
\overline{\mathbf{E}}(\mathbf{r}) = \sum_{m,\tau=\pm} \overline{\mathbf{E}}_m^{\tau} \exp[i\mathbf{k}_{mt}\mathbf{r} + \tau z/\delta],
$$

where δ is the decay length in the metal, $\delta = (k\sqrt{-\langle \varepsilon' \rangle})^{-1}$ oblique brackets and a prime or double prime denote spatial averaging and the real or imaginary part of a complex number, respectively). This representation is equivalent to neglecting the modulation in the film (cf. Ref. 7). Then from the Maxwell equations and the boundary conditions, it follows an infinite system of linear algebraic equations for the amplitudes \mathbf{E}_m^{τ} , $\mathbf{\bar{E}}_m^{\tau}$, which may be written in the matrix form as $\hat{D}\hat{E} = \hat{F}$. Here, \hat{D} is the matrix coupling the field harmonics through the periodical modulation, \hat{E} is the vector including all field harmonics, and \hat{F} is the source term proportional to the incident field. In spite of existence of the small parameter, $|\xi(\mathbf{r})| \ll 1$, resulting in smallness of the nondiagonal matrix elements, ordinary perturbation method fails due to SPP excitation. An appropriate method for solving the system is provided by the resonance perturbation theory cf. Refs. 7 and 8). It consists of subdividing the initial system into the resonance, $\hat{R}\hat{E}_r + \hat{U}\hat{E}_N = \hat{F}_r$, and nonresonance, $\hat{N}\hat{E}_N + \hat{L}\hat{E}_r$ $=\hat{F}_N$, subsystems where \hat{R} denotes the resonance part [it operates on transverse magnetic (TM) field components only], \hat{N} is the nonresonance part, and \hat{U} , \hat{L} are mixed parts of matrix \hat{D} . The subindex *r* corresponds to resonance diffraction orders here and below. The nonresonance subsystem can be solved analytically, expressing the nonresonance field amplitudes in terms of the resonance ones, $\hat{E}_N = \hat{N}^{-1}(\hat{F}_N - \hat{L}\hat{E}_r)$, using \hat{N}^{-1} series expansion. Excluding then \hat{E}_N from the resonance subsystem, we arrive at a closed finite system for the

FIG. 1. (Color online) Diffraction on the periodically-modulated metal film. The resonance occurs at the metal-superstrate interface in the *r*th diffraction order, $k_{rt} = K^-$.

resonance field amplitudes, $\hat{\tilde{D}}\hat{E}_r = \hat{\vec{F}}$, where $\hat{\tilde{D}} = \hat{R} - \hat{U}\hat{N}^{-1}\hat{L}$, $\hat{\vec{F}} = \hat{F}_r - \hat{U}\hat{N}^{-1}\hat{F}_N$. The reduced resonance system can be obtained in the vicinity of an arbitrary resonance with necessary accuracy and can be solved explicitly. Then, we obtain all field amplitudes. In a majority of cases of interest, it is sufficient to retain the quadratic-in-modulation amplitude terms in submatrix $\hat{\tilde{D}}$ and linear terms in renormalized righthand side term $\hat{\vec{F}}$, so that $\hat{\vec{D}} = \hat{R} + O(\tilde{\xi}^2)$, and $\hat{\vec{F}} \sim \tilde{\xi}$, where $\tilde{\xi}$ presents the modulated part of the surface impedance.

To deal with an arbitrary polarized incident radiation, let us introduce a polarization decomposition for waves in dielectric media. Using polarization unit vectors $\mathbf{e}_m^{\dagger\sigma}$ in π th media,

$$
\mathbf{e}_m^{\tau|+} = \frac{\mathbf{e}_z \times \mathbf{k}_{mt}}{k_{mt}}, \quad \mathbf{e}_m^{\tau|+} = \frac{\mathbf{e}_m^{\tau|+} \times \mathbf{k}_m^{\tau}}{k \sqrt{\varepsilon_\tau}},
$$

where $\sigma = -$ and $\sigma = +$ correspond to *TM*(*p*) and *TE*(*s*) polarization and analogous unit vectors for the incident field, **e** (changing \mathbf{k}_m^{τ} and \mathbf{k}_{m}^{τ} to \mathbf{k}^i and \mathbf{k}_t^i , and setting $\tau = -$), the above vector amplitudes are

$$
\mathbf{E}^i = \sum_{\sigma} E^{\sigma} \mathbf{e}^{\sigma}, \quad \mathbf{E}_m^{\tau} = \sum_{\sigma} E_m^{\tau | \sigma} \mathbf{e}_m^{\tau | \sigma}.
$$

Due to linearity of the problem, the polarization amplitudes of the diffracted waves are related to those of the incident wave by *transformation matrices* \hat{T}_m^{τ} , formed by the polarization transformation coefficients (TCs), $T_m^{\pi/\sigma\sigma'}$,

$$
E_m^{\tau|\sigma} = \sum_{\sigma'} T_m^{\tau|\sigma\sigma'} E^{\sigma'}.
$$
 (3)

We will concentrate mainly on the case of a singlediffraction-order resonance in the limit of a thick film, exp($-\Phi'$) ≤ 1 , where $\Phi = k\sqrt{-\langle \varepsilon \rangle}d$ is the dimensionless film thickness (its real part defines the thickness in the decay lengths). This means that the condition $k_{rt} \simeq K^{\tau}$ (where K^{τ}

 $=\sqrt{\varepsilon_{\tau}+\varepsilon_{\tau}^2 \zeta_0^2} \simeq \sqrt{\varepsilon_{\tau}}$ denotes the wave number of an unmodulated-film SPP) holds for a single diffraction order *r*. Then the *r*th-order polarization matrix¹² for $r \neq 0$ reads as

$$
\begin{pmatrix} T_r^{\eta|++} & T_r^{\eta|+-} \\ T_r^{\eta|+-} & T_r^{\eta|--} \end{pmatrix} = \begin{pmatrix} O(\widetilde{\xi}_r) \cos \theta \cos \psi_{r0} & O(\widetilde{\xi}_r) \sin \psi_{r0} \\ L_r^{\tau} \cos \theta \sin \psi_{r0} & -L_r^{\tau} \cos \psi_{r0} \end{pmatrix},\qquad(4)
$$

where the resonance factor L_r^{τ} is

$$
L_r^{\tau} = -\frac{2\tau \tilde{\xi}_r}{\Delta_r} (\tilde{\beta}_{r|\bar{\tau}} - \delta_{\tau,+} Y_r)(\cosh \Phi)^{-(1+\tau)/2},\tag{5}
$$

$$
\tilde{\beta}_{r|\tau} = \beta_{r|\tau} \tanh \Phi + \xi_0 + G_r^{\tau}, \tag{6}
$$

$$
\Delta_r = \widetilde{\beta}_{r+} \widetilde{\beta}_{r-} - \Upsilon_r^2 \cosh^{-2} \Phi, \quad \Upsilon_r = \xi_0 + G_r^+ + G_r^-, \quad (7)
$$

$$
G_r^{\tau} = -\sum_N \frac{\tilde{\xi}_{r-N}\tilde{\xi}_{N-r}}{\beta_{N|\tau}} (\cos^2 \psi_{rN} + \varepsilon_\tau \sin^2 \psi_{rN} \beta_{N|\tau}^2), \qquad (8)
$$

and $\bar{\tau}$ =− τ . In these formulas, θ is the angle of incidence, $\beta_{m|\tau}$ is the normalized *z* component of the wave vector corresponding to *m*th field harmonic, $\beta_{m|\tau} = \tau k_{\tau | m z} / k \varepsilon_{\tau}$, ψ_{nm} is the angle between the tangential components of wave vectors and $\bar{\tau} = -\tau$. In these formulas, θ is the angle $\beta_{m|\tau}$ is the normalized z component of the wave
responding to *m*th field harmonic, $\beta_{m|\tau} = \tau k_{\tau | m z}$
the angle between the tangential components of of *n*th an $\overline{\mathbf{R}_{mt}}$). Here $\tilde{\xi}_n$ is *nth* complex amplitude of $\xi(\mathbf{r})$ Fourier expansion, ξ_0 is the mean impedance value, $\xi_0 = \langle \xi \rangle$, $\tilde{\xi}_0 = 0$. The summation in (8) is performed over all diffraction orders except for those corresponding to the resonance ones (with diffraction order r). We neglect the small value $|\xi_0|$ as compared with $|\beta_{N|\tau}| \gtrsim 1$, which is valid well away from grazing incidence. In the resonance vicinity, coefficients $T_r^{\bar{T}+\sigma}$ are small in comparison with $T_r^{\pi|\sigma}$, because SPP is *p* polarized. Poles of the resonance coefficient $L_r^{\tau}(\Delta_r=0)$ give us the dispersion relation for SPP modes existing in the modulated film. Depending on the dielectric surrounding of the film, the SPP excited in *r*th order may be either single-boundary (SB) localized at the interface between the metal and τ th media (when $\varepsilon_+ \neq \varepsilon_-$) or double-boundary (DB) localized at both interfaces of the film (when $\varepsilon_{+} = \varepsilon_{-}$).¹³ DB SPPs, in turn, are subdivided into the short-range (SR) and long-range (LR) waves (cf. Ref. 6).

We would like to stress that the TCs $T_r^{\pi_{l+1}}$, $T_r^{\tau|-} (T_r^{\tau|+}, T_r^{\tau|-+})$ are proportional to cosine (sine) of the angle ψ_{r0} between the plane of incidence and the propagation direction of the resonance diffraction wave. This property is easy to understand. Namely, since the SPP magnetic field \mathbf{H}_r^{τ} is perpendicular to the propagation direction and parallel to the boundary, SPP excitation results from the projection of the incident wave magnetic field H^i onto the direction of the vector \mathbf{H}_r^{τ} . When the incident wave is p polarized, the vector H^i is perpendicular to the plane of incidence so that the projection of \mathbf{H}^i (= \mathbf{H}^i_t) onto \mathbf{H}^{τ}_t is proportional to cos ψ_{r0} . For *s* polarization, \mathbf{H}^i lies in the incident plane, $\mathbf{H}^i_t \|\mathbf{k}_{0t}$, so that the projection \mathbf{H}^i onto \mathbf{H}^{τ}_r is proportional to cos θ sin ψ_{r0} [see Fig. 1(b)]. These arguments clear up a simple angular dependence of coefficients $T_r^{\pi/-}$, $T_r^{\pi/+}$ (and $T_r^{\pi/+}$, $T_r^{\pi/+}$ as well) [see Fig. $2(b)$].

Consider first the case of a SB SPP excitation at the in-

FIG. 2. (Color online) Polarization dependence upon the SPP tilting angle for the single-order resonance, $r=1$, in the harmonically modulated film. The film thickness and the dimensionless grating vector are $\Phi' = 3$, $g/k = 1.2K_-, \xi_0 = 0.133i + 0.00071$ (Ag at 1.06 μ m). (a) corresponds to zeroth-order transmittance and/or reflectance under DB resonance for $\varepsilon_=\varepsilon_+ = 1$, $\tilde{\xi}_{\pm 1} = 2.65i\sqrt{\xi'_0}$. Circular (triangular) symbols correspond to $|T_0^{+|+|}$ ² ($|T_0^{-|+|}$ ²) numerically simulated dependency. Small symbols are for a LR mode, big ones are for a SR mode. Curves are the <u>normalized $\sin^2(2\psi_{10})$.</u> (b) corresponds to nonzeroth-order transmittance under SB resonance for $\varepsilon_0 = 1$, $\varepsilon_+ = 2.31$ (quartz), $\tilde{\xi}_{\pm 1}$ $= 1.25i\sqrt{\xi'_0}$. Circular (triangular) symbols correspond to $|T_1^{+|-}|^2$ ($|T_1^{+|-}|^2$) numerically simulated dependency. Solid [dashed] curve is the normalized squared $\cos \theta \sin(\psi_{10})[\cos(\psi_{10})]$.

terface between τ th media and a metal, which is of interest due to the possibility of a nonzeroth-order ELT.⁷ Then $\beta_{r|\tau}$ $\approx -i\xi_0''$ and $\beta_{r|\bar{\tau}} \approx 1$. Equality $\Im m(\tilde{\beta}_{r|\tau}) = 0$ for given *r* defines the resonance curve in θ , ψ_{r0} plane. This curve corresponds to the extreme value of diffracted wave amplitudes. For other parameters being fixed, this equation may be solved for θ as $\theta = \theta(\psi_{r0})$ (see Fig. 2). The maxima of the resonance amplitudes correspond to the equality between the radiation losses which exist even for a harmonic grating, being of order $(\vec{\xi}_r|^2)^{14}$ and the dissipation losses in the metal (it is determined by ξ_0'). This best matching condition defines the optimal value of $\tilde{\xi}_r$, $|\tilde{\xi}_r|_{opt} \sim \sqrt{\xi'_0}$. For the optimal grating amplitude, we have $\tilde{\beta}_{r|\tau} \sim \xi_0'$, $\tilde{\beta}_{r|\tau} \sim 1$ (see Ref. 7), thus obtaining the maximal value of the resonance factor $|L_r^+|_{max}$ $\sim e^{-\Phi'}/\sqrt{\xi'_0}$.

If the diffraction order r corresponds both to the homogeneous outgoing wave in the τ th media ($k_{\tau | rz}$ is real) and to the SB SPP on the $-\pi h$ media boundary, then SPP excitation results not only in zeroth-order ELT (and a specular reflection decrease), but in a far greater leakage effect. Specifically, for metal-superstrate SPP excitation, an *r*th order propagating wave provides the nonzeroth-order ELT. Due to *p* polarization of the SPP, the corresponding homogeneous outgoing wave is likewise *p* polarized. Therefore, in the nonzeroth-order channel, we obtain a plane-polarized wave having a transparent vector form,

$$
\mathbf{E}_r^{\tau} = -(k_{rt})^{-1} L_r^{\tau} (\mathbf{k}_{rt} \times \mathbf{H}^i)_z \mathbf{e}_r^{\tau -},
$$

for an arbitrary polarization of the incident wave.

The zeroth-order polarization matrix is

$$
\begin{pmatrix}\nT_0^{\dagger++} & T_0^{\dagger+-} \\
T_0^{\dagger-+} & T_0^{\dagger--}\n\end{pmatrix} = \begin{pmatrix}\nT_F^{\dagger+} & 0 \\
0 & T_F^{\dagger-}\n\end{pmatrix} \\
+ \begin{pmatrix}\n2\cos\theta_{\tau}\cos\theta\sin^2\psi_{r0} & -\cos\theta_{\tau}\sin 2\psi_{r0} \\
-\tau\cos\theta\sin 2\psi_{r0} & 2\tau\cos^2\psi_{r0}\n\end{pmatrix} L_{0|r}^{\tau}, (9)
$$

where

$$
L_{0|r}^{\tau} = -\frac{\sqrt{\varepsilon_r \tilde{\xi}_r \tilde{\xi}_{-r}}}{\cos \theta_r \Delta_r} [\tilde{\beta}_r |_{\tilde{\tau}} + (\tilde{\beta}_r |_{\tau} - \Upsilon_r) (\cosh \Phi)^{\tau-1}] \times (\cosh \Phi)^{-(1+\tau)/2}.
$$
 (10)

 θ_{τ} is the angle of propagation of zeroth-order wave in τ th dielectric media relative to the *z* axis (in the superstrate θ $\equiv \theta$). $T_F^{\dagger \sigma}$ corresponds to the transmission (reflection) coefficient for $\tau = +(-)$ of $s(p)$ polarized wave for $\sigma = +(-)$ in case of a nonmodulated film. It is well known that $|T_F^{|\sigma}| \approx 1$ and $|T_F^{+|\sigma|} \propto |\xi_0| e^{-\Phi'} \ll 1$ for a thick film. Polarization TCs $T_0^{\tau_1^+ +}(p \to s)$ and $T_0^{\tau_1^+ +}(s \to p)$ vanish both for sin $\psi_{r0} = 0$ and $\cos \psi_{r0} = 0$ [see Fig. 2(a)]. Therefore, there is no polarization conversion of a purely *p*-polarized or *s*-polarized wave if the propagation direction of the excited SPP is parallel or perpendicular to the plane of incidence.

From (9), we derive the *polarization reciprocity relations* for the zeroth-order channel,

$$
T_0^{\tau|-*} = \tau \frac{\cos \theta}{\cos \theta_\tau} T_0^{\tau|+}, \quad \tau = \pm . \tag{11}
$$

Under the conditions of a single-diffraction-order resonance, these relations are valid for an arbitrary grating shape (cf. Ref. 8 for the half-space problem), the generalization for multiple resonance case will be discussed elsewhere. The reflectance cross-polarization coefficients differ only by the sign both for symmetric $(\varepsilon_-\varepsilon_+)$ and nonsymmetric $(\varepsilon_ \neq \varepsilon_{+}$) surrounding. The transmittance cross-polarization coefficients are of the same sign, but differ by the multiplier $\cos \theta / \cos \theta_+$ for the nonsymmetric case. The relations (11) generalize those of Ref. 9 in two aspects: they are proved for (i) the nonsymmetric dielectric surroundings and (ii) arbitrary modulation shape.

It is remarkable that the transmitted zeroth-order wave is always linear-polarized independently of the film parameters and of the incident wave polarization. Indeed, the conditions of linear polarization of the zeroth-order wave in τ th media,

which are the zero values of corresponding polarization matrix determinant, det $\hat{T}_0^{\tau} = 0$, and $\Im m(T_0^{\tau_{1}^{+}} + T_0^{\tau_{1}^{+}}) = 0$, are fulfilled identically for $\tau = +$ [if one neglects a small term $T_F^{+|\sigma}$ in comparison with the resonance one in (9)]. To be more precise, the polarization of the outgoing zeroth-order wave is elliptical, but with a very high ratio of the ellipse semiaxes, which is of order ξ_0^{-1} . It is convenient to write the zerothorder transmitted wave in a vector form,

$$
\mathbf{E}_{0}^{+} = - 2 L_{0|r}^{+} \frac{\left[\mathbf{k}_{rt} \times \mathbf{H}^{i}\right]_{z}}{k_{0r} k_{rt}^{2}} \left[\left([\mathbf{k}_{rt} \times \mathbf{k}_{0r}]\hat{\mathbf{e}}_{\mathbf{k}_{0}^{+}} \right) \mathbf{e}_{0}^{+} \right]^{+} - (\mathbf{k}_{rt} \cdot \mathbf{k}_{0r}) \mathbf{e}_{0}^{+} \left| - \right].
$$

In the case of SB SPP excitation, the zeroth-order transmittance is small as compared to the nonzeroth-order one, since their resonance factors ratio is $L_{0|r}^+ / L_r^+ \propto \tilde{\xi}_r$. Therefore, only the excitation of DB SPP (in the symmetrically surrounded film) corresponds to the most pronounced zeroth-order ELT effect, when the resonance conditions are $\tilde{\beta}_r|_{\tau} = \tilde{\beta}_r|_{\tilde{\tau}} \approx \xi_0'$ (see Ref. 7). Then for the maximal value of the resonance factor, we have $|L_{0|r}^{+}|_{max} \sim |\xi_{0}'| e^{-\Phi'}/\xi_{0}'$ for the optimal modulation amplitude.

In contrast to the transmitted zeroth-order wave, the reflected one is elliptically polarized with the semiaxes being dependent upon the parameters (modulation harmonic amplitudes, the angle of incidence, and the wavelength). It becomes linear polarized only for the specific parameters. These parameters may be found from the above formulas, and are close to those for the corresponding half-space problem,⁸ with accuracy $O(e^{-2\Phi'})$.

For the corrugated films, the structure of the solution is similar, but the polarization dependence is different. As the analysis indicates, it is necessary to replace angle ψ_{r0} in the above expressions with the angle $(\mathbf{g}, \mathbf{k}_{0t})$. Nevertheless, the analysis indicates, it is necessary to replace angle ψ_{r0} in the above expressions with the angle $(\mathbf{g}, \mathbf{k}_{0t})$. Nevertheless, the statement concerning the independence of the polarization of the transmitted radiation (it is always linear for both zeroth and nonzeroth diffraction channels) upon the polarization of the incident light remains valid for this type of modulation as well.

As for 2D structures, the specific solution presented in the paper is equally applicable to them if SPP is excited in a single-diffraction order (r_1, r_2) (this means that at each interface there is no more than one excited SPP, but not a set of them, as it occurs in case of a high symmetry, for instance, for the normal incidence onto the lattice possessing C_4 or C_6 symmetry). The modification of our formulas for the 2D case consists of all the diffraction indexes m [corresponding to the tangential components of wave vectors \mathbf{k}_{tm} of the form (2)] being replaced by the multi-indexes (m_1, m_2) (corresponding to those of the form $\mathbf{k}_{t(m_1,m_2)} = \mathbf{k}_t^i + m_1 \mathbf{g}_1 + m_2 \mathbf{g}_2$, where \mathbf{g}_1 , \mathbf{g}_2 are reciprocal lattice vectors). Accordingly, the polarization dependencies are defined by the angle between \mathbf{k}_t^i and $\mathbf{k}_{t(r_1, r_2)}$. In case of a multiple SPP excitation at both interfaces or at one of them, the contribution to the polarization of the transmitted waves is defined by the linear combination of the excited SPP magnetic fields. For instance, square symmetry of the modulation corresponds to the excitation of one or two standing SPP at normal incidence. In this instance, the polarization of the transmitted radiation does not depend upon orientation of the incident plane, coinciding with the polarization of the incident wave. This is in accordance with the experiment.¹⁰

We have provided an insight into the energy and polarization properties of reflectance and the enhanced light transmittance through modulated metal films. We have derived the novel reciprocity relation, which generalizes the one discovered recently for the polarization transformation matrix, and found that the light transmitted through the film in the ELT case is approximately linear polarized for the arbitrary polarization of the incident plane wave both in zeroth and nonzeroth diffraction orders finiteness of the illuminated region and the beam divergence can be easily taken into account and results in the effects being partly observed in the experiments (cf. Refs. 4, 5, and 11)]. The results thus obtained highlight both the basic properties of the ELT effect from the stand point of theoretical physics, allowing one to design necessary experiments specifically for incline incidence, and the optic devices offering unique prescribed properties. We would like to emphasize the analytical approach, as opposed to the numeric one, that enabled us to consider the effects for modulation of an arbitrary type and shape, thereby expressing the solution in terms of the Fourier amplitudes of the periodical structures, and to find the structures responsible for the maximal effects of the polarization transformation.

This work was partially supported by Ukrainian President Grant No. F8/323-2004.

- *Corresponding author. Electronic address: avkats @online.kharkiv.com
- ¹ T. W. Ebbesen *et al.*, Nature (London) **391**, 667 (1998).
- ²R. Gordon *et al.*, Phys. Rev. Lett. **92**, 037401 (2004).
- ³ J. Elliott *et al.*, Opt. Lett. **29**, 1414 (2004).
- ⁴E. Altewischer *et al.*, Opt. Lett. **30**, 90 (2005).
- 5C . Genet *et al.*, Phys. Rev. B **71**, 033409 (2005).
- ⁶H. Raether, Surface Plasmons (Springer-Verlag, Berlin, 1988).
- ⁷ A. V. Kats and A. Y. Nikitin, Phys. Rev. B **70**, 235412 (2004); JETP Lett. **79**, 625 (2004).
- ⁸ A. V. Kats and I. S. Spevak, Phys. Rev. B **65**, 195406 (2002).
- ⁹L. Li, J. Opt. Soc. Am. A 17, 881 (2000).
- ¹⁰W. L. Barnes *et al.*, Phys. Rev. Lett. **92**, 107401 (2004).
- 11 F. Miyamaru and M. Hangyo, Appl. Opt. 43, 1412 (2004).
- ¹²The zeroth diffraction order can become resonant for incline incidence if a SPP is excited on the far interface, that is $\varepsilon \geq \varepsilon_+$. This case is of specific interest (cf. Ref. 7)
- 13 Under specific conditions (see Ref. 7), DB SPP can be excited for $\varepsilon_{+} \neq \varepsilon$
- ¹⁴The radiation losses are defined by those Fourier amplitudes of the grating, $\tilde{\xi}_n$ resulting in transformation of the resonance wave into the propagating ones via one-step scattering processes. Each harmonic of this type contributes a term $\sim |\tilde{\xi}|^2$ to the radiation losses.