Failure of geometric frustration to preserve a quasi-two-dimensional spin fluid

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Using spin-wave theory, we show that geometric frustration fails to preserve a two-dimensional spin fluid. Even though frustration can remove the interlayer coupling in the ground state of a classical antiferromagnet, spin layers inevitably develop a quantum-mechanical coupling via the mechanism of "order from disorder." We show how the order from disorder coupling mechanism can be viewed as a result of magnon pair tunneling, a process closely analogous to pair tunneling in the Josephson effect. In the spin system, the Josephson coupling manifests itself as a biquadratic spin coupling between layers, and for quantum spins, these coupling terms become comparable with the in-plane coupling terms. An alternative mechanism for decoupling spin layers occurs in classical *XY* models in which decoupled "sliding phases" of spin fluid can form in certain finely tuned conditions. Unfortunately, these finely tuned situations appear equally susceptible to the strong-coupling effects of quantum tunneling, forcing us to conclude that, in general, geometric frustration cannot preserve a two-dimensional spin fluid.

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I. INTRODUCTION

This study is motivated by recent theories of heavy electron systems[1,2] which propose that the formation of magnetically decoupled layers of spins plays a central role in the departures from Fermi liquid behavior observed near a magnetic quantum critical point. A wide variety of heavy electron materials develop logarithmically divergent specific heat coefficients and quasilinear resistivities in the vicinity of quantum critical points.[1–16] Several theories explaining these unusual properties have been proposed.[1–3,11–14,18–20] The standard model for these quantum phase transitions, proposed by Hertz and Moriya, involves a soft, antiferromagnetic mode coupled to a Fermi surface. The Hertz-Moriya spin-density wave (SDW) theory can account for the logarithmically divergent specific heat coefficients and quasilinear resistivities,[1,17] but only if the spin fluctuations are quasi-two-dimensional. An alternative local quantum critical description, based on the extended dynamical mean field theory, also requires a quasi-two-dimensional spin fluid.[2] Each of these theories can only account for the anomalies of quantum critical heavy electron materials if the spin fluctuations of these systems are quasi-two-dimensional.[1,3,14,18–20]

The hypothesis that heavy electrons involve decoupled layers of spins motivates a search for a mechanism that can generate a quasi-two-dimensional environment for the spin fluctuations out of a metal that is manifestly three dimensional. One such frequently cited mechanism is geometric frustration.[1,3] Here, the idea is that the frustration leads to a cancellation of the interlayer Weiss fields, so that spin layers decouple in the classical ground state (see Fig. 1).[1,3]

In this paper, we use the Heisenberg antiferromagnet as a simple example to explore this line of reasoning. The primary objective of our paper is not to examine whether interlayer coupling is relevant or irrelevant at the quantum critical point—but rather to examine whether frustration can set up an environment in which the interlayer coupling is sufficiently weak for us to consider the system to be quasi-two-dimensional. A key issue in this discussion is the effect of fluctuations and their potential to generate interlayer coupling via the mechanism of "order from disorder."[22,23] Using the spin-wave theory we show that, in general, zero-point fluctuations of the spin act as an extremely powerful force for coupling two-dimensional spin layers, making geometric frustration an unlikely candidate for the development of a quasi-two-dimensional spin fluid.

To illustrate the main points of our argument, consider two separate layers of Heisenberg spins. The Hamiltonian is

$$H_0 = H^{(B)} + H^{(T)}, \tag{1}$$

where $H^{(T)}$ and $H^{(B)}$ are the Hamiltonians for the top and bottom layers. Namely,

$$H_0 = J^{\parallel}\sum_{\mathbf{i},\mathbf{\Delta}} (\mathbf{S}_{\mathbf{i}}^{(B)}\mathbf{S}_{\mathbf{i}+\mathbf{\Delta}}^{(B)} + \mathbf{S}_{\mathbf{i}+\delta}^{(T)}\mathbf{S}_{\mathbf{i}+\delta+\mathbf{\Delta}}^{(T)}). \tag{2}$$

Here $\mathbf{S}_{\mathbf{i}}^{(B)}$ ($\mathbf{S}_{\mathbf{i}}^{(T)}$) is the spin variable defined at the site $\mathbf{i}$ in the bottom (top) layer. The vector $\mathbf{\Delta}$ denotes a displacement to the nearest neighbor sites within the plane, $\mathbf{\Delta}=(a,0)$ or $(0,a)$. $\delta=(a/2,a/2)$ defines a shift between layers.

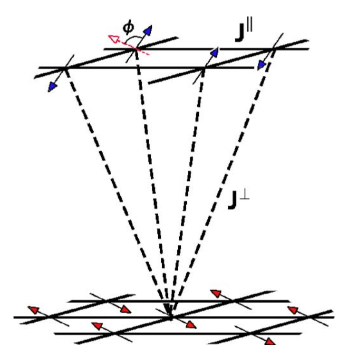

FIG. 1. (Color online) A lattice structure.

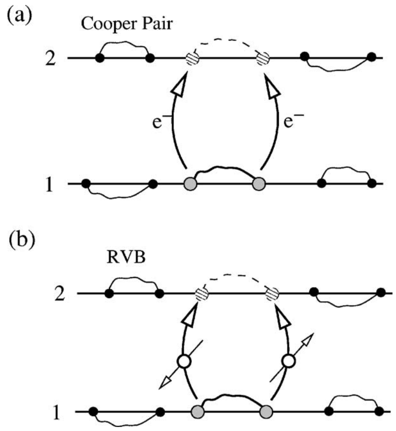

FIG. 2. The contrasting (a) Josephson tunneling between paired superconductors and (b) magnon tunneling between antiferromagnets, viewed within a resonating valence bond (RVB) picture.

At $T=0$ each layer is antiferromagnetically ordered and spin waves run along the layers. Now consider the effect of a small frustrated interlayer coupling. The Hamiltonian is then

$$H = H_0 + V, \tag{3}$$

with the interlayer coupling

$$V = J^{\perp} \sum_{\mathbf{i},\mathbf{\Delta}} (\mathbf{S}_{\mathbf{i}}^{(B)} \mathbf{S}_{\mathbf{i}\pm\delta}^{(T)} + \mathbf{S}_{\mathbf{i}}^{(B)} \mathbf{S}_{\mathbf{i}+\delta-\mathbf{\Delta}}^{(T)}). \tag{4}$$

Figure 1 illustrates the nature of the magnetic couplings between layers. Any given spin on a layer is coupled to four spins in the neighboring layer, two pointing in one direction, the other two pointing in the opposite direction, so that the Weiss fields of one layer on spins in the neighboring layer exactly cancel one another. The interlayer coupling is thus frustrated, and there is no preferred orientation of the spins on one layer with respect to the neighboring layer. Classically, the spin layers are thus decoupled, and there is no unique classical ground state. Let us now see how this picture changes when we take account of quantum zero-point fluctuations.

In the quantum-mechanical picture, even a small interlayer coupling enables magnons to virtually tunnel between layers. An antiferromagnet can be regarded as a long range resonating valence bond (RVB) state,[21] so individual magnon transfer is energetically unfavorable, and the transfer of magnons between the layers tends to occur in pairs, as in Josephson tunneling (see Fig. 2). Interlayer magnon pair tunneling is ubiquitous in three-dimensional spin systems, frustrated and unfrustrated alike. So unless the interlayer coupling constant is set exactly to zero, magnons travel between the layers, producing a coupling closely analogous to Josephson coupling of superconducting layers. Such a coupling is an alternative way of viewing the phenomenon of "order from disorder,"[22,23] whereby the free energy of zero-point or thermal fluctuations depends on the relative orientation of the classical magnetization.

If we use the analogy between superconductors and antiferromagnets, then spin rotations of an antiferromagnet map onto gauge transformations of the electron phase in a superconductor. In a superconducting tunnel junction, the Josephson energy is determined by the product of the order parameters in the two layers, i.e.,

$$\Delta E_J \sim -\frac{t_\perp^2}{\Delta} \mathrm{Re}[\langle \psi_{2\uparrow}^\dagger \psi_{2\downarrow}^\dagger \rangle \langle \psi_{1\uparrow} \psi_{1\downarrow} \rangle] \propto \cos(\phi_2 - \phi_1),$$

where $t_\perp$ is the tunneling matrix element, Δ the superconducting gap energy, and $\psi_{l\sigma}$ ($l=1,2$) represents an electron field in leads one and two. By analogy, in a corresponding "spin junction," the coupling energy is determined by the product of the spin-pair amplitudes. Suppose for simplicity that the system is an easy-plane XY magnet, then

$$\Delta E_S(\Delta\phi) \sim -\frac{J_\perp^2}{J_\parallel} \mathrm{Re}[\langle S_2^+(i) S_2^+(j) \rangle \langle S_1^-(i') S_1^-(j') \rangle]$$
$$\propto -\frac{J_\perp^2}{J_\parallel} S \cos(2\Delta\phi),$$

where $S_l^\pm(i)$ represents the spin raising, or lowering operator at site i in plane l, parallel to the local magnetization. The factor $2\Delta\phi$ arises because the spin pair carries a phase which is twice the angular displacement of the magnetization ($S_+ \equiv S_x + iS_y \sim Se^{i\phi}$). In other words, the effective Hamiltonian is

$$H_{eff} = H^{(B)} + H^{(T)} - \frac{\lambda J_\perp^2}{J_\parallel S^3} \sum_{\alpha,\mathbf{i},\mathbf{\Delta}} (\mathbf{S}_{\mathbf{i}}^{(\alpha)} \mathbf{S}_{\mathbf{i}+\mathbf{\Delta}}^{(\alpha)})^2, \tag{5}$$

where $\alpha = T$, B is a layer index, $\mathbf{i}$ is a site index, $\mathbf{\Delta}$ is a displacement to the nearest neighbor sites within a layer, and λ is a numerical constant. Obviously, the geometric frustration leads to a new ground state with

$$\Delta E_S(\Delta\phi) \sim -\frac{2J_\perp^2}{J_\parallel} S \cos^2(\Delta\phi) + \mathrm{const},$$

so the interlayer coupling induced by spin tunneling is expected to be biquadratic in the relative angle between the spins. Clearly, this is a much oversimplified argument. We need to take account of the $O(3)$, rather than the $U(1)$ symmetry of a Heisenberg system. Nevertheless, this simple argument captures the spirit of the coupling between spin layers, as we shall now see in a more detailed calculation.

II. SPIN-WAVE SPECTRUM FOR DECOUPLED LAYERS

Consider a Heisenberg model with nearest-neighbor antiferromagnetic interaction in its ground state defined on the body-centered tetragonal lattice. This choice of model is motivated by the structure of $CePd_2Si_2$, one of the compounds for which the idea of quasi-two-dimensionality was originally proposed.[3] In this lattice structure (Fig. 1), square lattices stack with a shift of $(a/2, a/2)$ between adjacent layers (a is the lattice constant within the layer). For simplicity, the distance between the layers is also a. The spins of the nearest neighbors in each layer are antiparallel. In the classical ground state the spins in different layers are decoupled and may assume any relative alignment.

For simplicity, let us consider just two adjacent layers, the argument being easily generalized to an infinite number of layers. The Hamiltonian is then given by Eqs. (2)–(4). Since the coupling between layers is small ($J^{\perp} \ll J^{\parallel}$), we may treat this model using perturbation theory where the ratio of coupling constants $J^{\perp}/J^{\parallel}$ is taken as a small parameter.

For our purposes, it is sufficient to consider a simple case with the spins lying in the planes of the two-dimensional lattice. At sites $\mathbf{i}=(la,ma)$ and $\mathbf{i}+\delta=\left(la+\frac{1}{2}a,ma+\frac{1}{2}a\right)$ the spins are

$$S_{\mathbf{i}}^{X(B)} = S(-1)^{l+m}, \quad S_{\mathbf{i}}^{Y(B)} = 0; \tag{6}$$

$$S_{\mathbf{i}+\delta}^{X(T)} = S(-1)^{l+m+1} \cos \phi, \quad S_{\mathbf{i}+\delta}^{Y(T)} = S(-1)^{l+m+1} \sin \phi; \tag{7}$$

where X and Y are mutually perpendicular directions in the plane and l and m are integers.

Following a standard procedure,[24,25] we use the Holstein-Primakoff approximation for the spin operators to determine the spin-wave spectrum:

$$S_{\mathbf{i}}^{+(\alpha)} \approx \sqrt{2S}a_{\mathbf{i}}^{(\alpha)}, \quad S_{\mathbf{i}}^{-(\alpha)} \approx \sqrt{2S}a_{\mathbf{i}}^{+(\alpha)}, \quad S_{\mathbf{i}}^{z(\alpha)} = S - a_{\mathbf{i}}^{+(\alpha)}a_{\mathbf{i}}^{(\alpha)}, \tag{8}$$

where

$$[a_{\mathbf{i}}^{(\alpha)}, a_{\mathbf{j}}^{+(\alpha)}] = \delta_{\mathbf{ij}}, \quad \alpha = B,T. \tag{9}$$

The Fourier transforms of $a_{\mathbf{i}}^{(B)}$, $a_{\mathbf{i}}^{+(B)}$, $a_{\mathbf{i}}^{(T)}$, and $a_{\mathbf{i}}^{+(T)}$ are

$$a_{\mathbf{q}}^{(B)} = \frac{1}{\sqrt{N}}\sum_{\mathbf{i}} a_{\mathbf{i}}^{(B)}e^{i\mathbf{qi}}, \quad a_{\mathbf{q}}^{+(B)} = \frac{1}{\sqrt{N}}\sum_{\mathbf{i}} a_{\mathbf{i}}^{+(B)}e^{-i\mathbf{qi}}, \tag{10}$$

$$a_{\mathbf{q}}^{(T)} = \frac{1}{\sqrt{N}}\sum_{\mathbf{i}} a_{\mathbf{i}+\delta}^{(T)}e^{i\mathbf{q}(\mathbf{i}+\delta)}, \quad a_{\mathbf{q}}^{+(T)} = \frac{1}{\sqrt{N}}\sum_{\mathbf{i}} a_{\mathbf{i}+\delta}^{+(T)}e^{-i\mathbf{q}(\mathbf{i}+\delta)}, \tag{11}$$

where N is the number of spin sites in the layer.

$a_{\mathbf{q}}^{+(\alpha)}$ and $a_{\mathbf{q}}^{(\alpha)}$ are the spin-wave creation and annihilation operators.

$$[a_{\mathbf{q}}^{(\alpha)}, a_{\mathbf{q}'}^{+(\alpha)}] = \delta_{\mathbf{qq}'}. \tag{12}$$

The single-layer Hamiltonian $H^{(\alpha)}$ becomes

$$H^{(\alpha)} = -4NS^2J^{\parallel} + \sum_{\mathbf{q}} \{8SJ^{\parallel}a_{\mathbf{q}}^{+(\alpha)}a_{\mathbf{q}}^{(\alpha)} + SJ^{\parallel}(\mathbf{q})[a_{\mathbf{q}}^{(\alpha)}a_{-\mathbf{q}}^{(\alpha)} + \text{H.c.}]\}, \tag{13}$$

and on diagonalization the Hamiltonian H_0 for the decoupled layers can be written as

$$H_0 = E_0 + \sum_{\alpha=T,B}\sum_{\mathbf{q}} \omega_{\mathbf{q}}^{\parallel} b_{\mathbf{q}}^{+(\alpha)}b_{\mathbf{q}}^{(\alpha)}. \tag{14}$$

The ground state energy of the decoupled two-layer system is then

$$E_0 = -8NS(S+1)J^{\parallel} + \sum_{\mathbf{q}} \omega_{\mathbf{q}}^{\parallel}. \tag{15}$$

$\omega_{\mathbf{q}}^{\parallel}$ defines the spectrum of spin waves propagating in each of the layers

$$\omega_{\mathbf{q}}^{\parallel} = 4SJ^{\parallel}\sqrt{4 - [\cos q_x a + \cos q_y a]^2}. \tag{16}$$

III. MAGNON PAIR TUNNELING BETWEEN THE LAYERS

Now we express the perturbation V in (4) in terms of $a_{\mathbf{q}}^{+(\alpha)}$, $a_{\mathbf{q}}^{(\alpha)}$ as

$$V = S\sum_{\mathbf{q}} A_{\mathbf{q}}^{\perp}(a_{\mathbf{q}}^{+(T)}a_{\mathbf{q}}^{(B)} + \text{H.c.}) + S\sum_{\mathbf{q}} B_{\mathbf{q}}^{\perp}(a_{\mathbf{q}}^{(T)}a_{-\mathbf{q}}^{(B)} + \text{H.c.}), \tag{17}$$

where $A_{\mathbf{q}}^{\perp}$ and $B_{\mathbf{q}}^{\perp}$ are defined as

$$A_{\mathbf{q}}^{\perp} = 2J^{\perp}\left[\cos\left(\frac{q_x a}{2}\right)\cos\left(\frac{q_y a}{2}\right) + \sin\left(\frac{q_x a}{2}\right)\sin\left(\frac{q_y a}{2}\right)\cos\phi\right], \tag{18}$$

$$B_{\mathbf{q}}^{\perp} = 2J^{\perp}\left[\cos\left(\frac{q_x a}{2}\right)\cos\left(\frac{q_y a}{2}\right) - \sin\left(\frac{q_x a}{2}\right)\sin\left(\frac{q_y a}{2}\right)\cos\phi\right]. \tag{19}$$

In terms of $b_{\mathbf{q}}^{+(\alpha)}$, $b_{\mathbf{q}}^{(\alpha)}$

$$V = S\sum_{\mathbf{q}} \alpha_q(b_{\mathbf{q}}^{(T)}b_{-\mathbf{q}}^{(B)} + b_{\mathbf{q}}^{+(T)}b_{-\mathbf{q}}^{+(B)}) + V_{ph}, \tag{20}$$

where $\alpha_q=(A_{\mathbf{q}}^{\perp}\sinh 2u_{\mathbf{q}}+B_{\mathbf{q}}^{\perp}\cosh 2u_{\mathbf{q}})$ describes the amplitude for magnon pair tunneling, and $V_{ph}=S\Sigma_{\mathbf{q}}(A_{\mathbf{q}}^{\perp}\cosh 2u_{\mathbf{q}} + B_{\mathbf{q}}^{\perp}\sinh 2u_{\mathbf{q}})(b_{\mathbf{q}}^{+(T)}b_{\mathbf{q}}^{(B)}+b_{\mathbf{q}}^{+(B)}b_{\mathbf{q}}^{(T)})$ describes single magnon tunneling between layers.

It is straightforward to see that the particle-hole terms do not affect the ground-state energy, for $V_{ph}|GS\rangle=0$, where $|GS\rangle$ denotes the ground state wave function of the system. The second order correction to the ground state energy E_0 is then

$$\Delta E_0^{(2)} = \sum_{\lambda} \frac{|\langle\lambda|\hat{V}|GS\rangle|^2}{E_{\lambda} - E_{GS}}, \tag{21}$$

where $|\lambda\rangle$ denotes a state with two magnons being transferred between layers. Thus,

$$\Delta E_0^{(2)} = -\sum_{\mathbf{q}} S^2\alpha_q^2/(2\omega_{\mathbf{q}}^{\parallel}). \tag{22}$$

To understand the nature of coupling between the layers (dipolar or quadrupolar), let us retrieve the dependence of $\Delta E_0^{(2)}$ on the angle ϕ.

$$\Delta E_0^{(2)} = -\frac{S}{2}\frac{(J^{\perp})^2}{J^{\parallel}}[C_0 + C_2\cos^2\phi] = -\frac{S}{2}\frac{(J^{\perp})^2}{J^{\parallel}}\left[\left(C_0 + \frac{C_2}{2}\right) + \frac{C_2}{2}\cos 2\phi\right]. \tag{23}$$

The particular form of the coefficients is

$$C_0 = \int_{-\pi}^{\pi} \frac{dx\,dy}{(2\pi)^2} \sqrt{1 - \tfrac{1}{4}[\cos x + \cos y]^2} \times \cos^2\frac{x}{2}\cos^2\frac{y}{2}\left(1 - \frac{1}{2}[\cos x + \cos y]\right)^2, \qquad (24)$$

$$C_2 = \int_{-\pi}^{\pi} \frac{dx\,dy}{(2\pi)^2} \sqrt{1 - \tfrac{1}{4}[\cos x + \cos y]^2} \times \cos^2\phi \sin^2\frac{x}{2}\sin^2\frac{y}{2}\left(1 + \frac{1}{2}[\cos x + \cos y]\right)^2. \qquad (25)$$

The interlayer coupling is indeed quadrupolar in nature, as foreseen earlier.

IV. DISCUSSION

In the above calculation, we considered an ordered Heisenberg antiferromagnet at zero temperature. In practice, provided the spin-spin correlation length ξ is large compared with the lattice constant a, $\xi \gg a$, a biquadratic interlayer coupling will still develop. Moreover, at finite temperatures, thermal fluctuations will produce further interlayer coupling. Both thermal and quantum interlayer coupling processes are manifestations of "order from disorder." The main difference between the thermal and quantum coupling processes lies in the replacement of the magnon occupation numbers with a Bose-Einstein distribution function, and, in general, both the sign and the angular dependences of the two couplings are expected to be the same.[22] In general, geometrical frustration is an extremely fragile mechanism for decoupling spin layers and will always be overcome by quantum and thermal fluctuations. Our work was motivated by heavy electron systems. These are much more complex systems than insulating antiferromagnets, but if our mechanism for the formation of two-dimensional spin fluid is to be frustration, it is difficult to see how similar interlayer coupling effects might be avoided. We are led to conclude that for the hypothesis of the reduced dimensionality of the spin fluid in heavy fermion materials to hold, a completely different decoupling mechanism must be at work.

In the special case of *XY* magnetism there is, in fact, one such alternative mechanism, related to "sliding phases." Here the idea is to create an environment in which interlayer coupling, though present, becomes irrelevant, ultimately scaling to zero at long distances. Some heavy fermion systems, such as $YbRh_2Si_2$, are *XY*-like; most others, such as $CeCu_6$, are Ising-like. It is, therefore, instructive to consider whether the sliding phase mechanism might be generalized to Heisenberg or Ising spin systems to provide an escape from the fluctuation coupling that we have discussed.

The existence of a "sliding phase" in weakly coupled stacks of two-dimensional (2D) *XY* models was predicted by O'Hern, Lubensky, and Toner.[26] Sliding phases are of particular current interest in the context of Josephson junction arrays.[27] In addition to Josephson interlayer couplings, O'Hern *et al.* included higher-order gradient couplings between the layers. In the absence of Josephson couplings, these gradient couplings preserve the decoupled nature of the spin layers, only modifying the power-law exponents of the 2D correlation functions, $\langle S_i S_j \rangle \sim r^{-\eta}$. As the temperature is raised, Josephson interlayer couplings become irrelevant above a particular "decoupling temperature" T_d. One can always select interlayer gradient couplings to satisfy $T_d < T_{KT}$ and produce a stable sliding phase in the temperature window $T_d < T < T_{KT}$.

To see this in a bit more detail, consider the continuous version of the Hamiltonian of two layers of *XY* models, $H = H_0 + V$, where H_0 is a sum of independent layer Hamiltonians and V is the usual Josephson-type interlayer coupling

$$H_0 = \frac{J^{\parallel}}{2}\int d^2r[\nabla_{\perp}\phi_T(\mathbf{r})]^2 + \frac{J^{\parallel}}{2}\int d^2r[\nabla_{\perp}\phi_B(\mathbf{r})]^2, \qquad (26)$$

$$V = J^{\perp}\int d^2r \cos[\phi_T(\mathbf{r}) - \phi_B(\mathbf{r})]. \qquad (27)$$

At low temperature, when the interlayer coupling $J^{\perp}$ is zero, the average of the intralayer spin-spin correlation function with respect to H_0 is

$$\langle \phi^2(\mathbf{r}) \rangle_0 = \eta \ln(L/b), \qquad (28)$$

and

$$\langle \cos[\phi(\mathbf{r}) - \phi(\mathbf{0})] \rangle_0 \sim (L/b)^{-\eta}, \qquad (29)$$

where $\eta = T/2\pi J^{\parallel}$, L is the sample width, and b is a short-distance cutoff in the *XY* plane.

The average of Josephson interlayer coupling V scales as $\langle V \rangle_0 \sim L^{2-\eta}$, so Josephson couplings become irrelevant at $T_d = 4\pi J^{\parallel}$. At temperatures above the Kosterlitz-Thouless transition temperature $T_{KT} = \pi J^{\parallel}/2$, thermally excited vortices destroy the quasi-long-range order and drive the system to disorder. In this simple example, it happens that $T_d > T_{KT}$, which does not permit a sliding phase. However, higher-order gradient interlayer couplings between the layers, when added to this model, suppress T_d below T_{KT}, producing a stable sliding phase for $T_d < T < T_{KT}$.

So can the sliding phase concept be generalized to Heisenberg spin systems? A sliding phase develops in the *XY* model because power-law spin correlations introduce an anomalous scaling dimension, but unfortunately, a finite temperature Heisenberg model has no phases with power-law correlations.[28] In general, biquadratic interlayer couplings will always remain relevant in Heisenberg models. In the quantum-mechanical picture, as soon as a frustrated interlayer coupling is introduced, the order-from-disorder phenomenon[22,23] generates a coupling $\lambda \sim S J^{\perp 2}/J^{\parallel}$ between the layers.

$$H = \int \frac{\rho}{2}\sum_i (\nabla \hat{n}_i)^2 + \frac{\lambda}{2}\sum_i (\hat{n}_i - \hat{n}_{i+1})^2, \qquad (30)$$

where $\rho = S^2 a^2 J^{\parallel}$. This coupling gives us a length scale l_0 determined from $(l_0)^{-2} \sim \lambda/\rho$ or $l_0 \sim a\sqrt{S}J^{\parallel}/J^{\perp}$. Once the spin correlation length $\xi \sim a \exp(2\pi J^{\parallel} S^2/T)$ within a layer grows to become larger than l_0, i.e., $l_0 < a \exp(2\pi J^{\parallel} S^2/T)$, a 3D-ordering phase transition occurs. An estimate of the 3D-ordering transition temperature is then $T_c \sim 2\pi J^{\parallel} S^2/$

$\ln(\sqrt{S}J^{\parallel}/J^{\perp})$. The answer is essentially identical in the classical picture, for here, thermal fluctuations generate an entropic interlayer coupling $\lambda \sim \max(SJ^{\perp 2}/J^{\parallel}, TS^2)$, so at high enough temperatures, for large S, $\lambda \sim TJ^{\perp 2}/J^{\parallel 2}$, $l_0 \sim SaJ^{\parallel\, 3/2}/J^{\perp}\sqrt{T}$. A classical estimate of the 3D-ordering temperature is $T_c \sim 2\pi J^{\parallel} S^2/\ln(J^{\parallel}/J^{\perp})$.

Another interesting question is whether XY models permit sliding phases at $T=0$. The decoupling temperature, as found by O'Hern, Lubensky, and Toner,[26] is

$$T_d(p) = \frac{4\pi\rho}{f_o^{-1} - f_p^{-1}}. \tag{31}$$

One sees no obvious mechanism of suppressing T_d to zero. A 2D sliding phase is equivalent to a 3D finite temperature sliding phase, so the existence of a sliding phase in the XY model at zero temperature would mean a power-law phase in the 3D XY model. Since no power-law phase exists in 3D XY-like systems, sliding phases at $T=0$ are extremely unlikely. In conclusion, the sliding phase scenario also fails to provide a valid general mechanism for decoupling layers in Ising-like and Heisenberg-like systems.

Let us return momentarily to consider the implications of these conclusions for the more complex case of heavy electron materials. It is clear from our discussion that simple models of frustration do not provide a viable mechanism for decoupling spin layers. One of the obvious distinctions between an insulating and a metallic antiferromagnet is the presence of dissipation which acts on the spin fluctuations. The interlayer coupling we considered here relies on short-wavelength spin fluctuations, and these are the ones that are most heavily damped in a metal. Our exclusion of such effects does hold open a small possibility that order-from-disorder effects might be substantially weaker in a metallic antiferromagnet. However, if we are to take this route, then we can certainly no longer appeal to the analogy of the insulating antiferromagnet while discussing a possible mechanism for decoupling spin layers.

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