Role of quantum interference in ferromagnetic thin films

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The effect of quantum interference on the magnetoconductivity of a two-dimensional ferromagnet for arbitrary orientation of magnetization and external magnetic field has been investigated in the case when spin-orbit interaction plays an important role. By means of the diagrammatic perturbation technique, analytical results for the magnetoconductivity has been obtained as a function of the magnetization and characteristic relaxation times due to elastic, inelastic, and spin-orbit scattering. The result shows a strong dependence of the orientation of the magnetization with respect to the plane of the system on the conductivity. Depending on the orientation and strength of the magnetization and the coupling of the electronic spin with the magnetization both negative and positive magnetoresistance has been predicted. In addition, it is shown that, in order to explain the experimental variation of the conductivity in thin ferromagnetic films, electron-electron interaction and domain wall scattering must be considered.

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I. INTRODUCTION

The application of the effect of magnetoresistance in magnetoelectronics, field sensors, random access memory elements, and others[1] created a huge interest on the studies of the magnetic and transport properties of the magnetic materials realized in low dimensions. The interplay between magnetization and localization is central to the behavior of many artificially tailored materials and has given rise to phenomena such as giant magnetoresistance in metallic multilayers, spin polarons in high T_c superconductors, and skyrmion in two dimensional electron gases in semiconductors.[2] In magnetic materials the ferromagnetic *s*-*d* exchange yields spin splitting ΔE_s which is comparable or larger than the thermal energy or the Landau level splitting $\hbar\omega_c$ due to the external magnetic field.[3] Thus the conduction electron spins in these materials are polarized, which provides a flexible templet for the studies of spin polarized transport and tunneling.[4] At low temperatures, transport measurements (in ferromagnetic materials)[5,6] indicate remarkable modification of the quantum correction to the two dimensional (2D) magnetoconductivity in the weakly localized regime, and negative magnetoresistance (MR) at high field suggests suppression of weak electron localization (WEL) and spin disorder scattering. Although an extensive experimental[7–9] as well as theoretical[2,10–13] work related to the scattering of electrons from impurities in nonmagnetic metals and doped semiconductors have been performed to study the quantum correction to electrical conductivity, the problem of quantum effects on the ferromagnetic materials at low temperatures requires further investigations. Recently a few experimental[14–18] and theoretical[19–21] investigations have been devoted to explore the effect of quantum interference on the ferromagnetic systems.

At low temperatures the scattering of the electrons in metals is mainly governed by the elastic scattering from the impurities, which leads to WEL due to the phase coherent back scattering. It is well-known that in nonmagnetic metals electron-phonon and electron-electron interactions as well as spin-flip scattering processes can destroy the quantum interference effects and that the presence of spin-orbit scattering leads to weak antilocalization.[7,13] Moreover, the observation of anisotropic magnetoresistance (AMR)[22,23] and the anomalous Hall effect in ferromagnets[24–26] emphasizes the importance of spin-orbit scattering in the transport properties of the low-dimensional ferromagnetic materials. Thus, in this paper we will study the magnetoconductivity of a two-dimensional ferromagnet in presence of elastic scattering as well as spin-orbit scattering. Here we will provide a general description of the electronic transport in ferromagnets for arbitrary direction of magnetization and magnetic field. Thus our work provides the angular dependence of the magnetization and external magnetic field on the magnetoresistance of a ferromagnetic material. Our discussion also includes the theoretical prediction that, in order to achieve agreement with experiment, electron-electron interaction and domain wall scattering must be taken into account besides the usual quantum interference effects. In fact, it is shown that domain wall scattering destroys the WEL which confirms the experimental observation.[18]

II. CONDUCTIVITY AND MAGNETOCONDUCTIVITY

In order to investigate the transport properties of ferromagnetic metals we consider an idealized model where the electrons go through the elastic scattering as well as spin-orbit scattering from impurities. Here we consider a two-dimensional ferromagnetic system in the presence of magnetic field and magnetization in an arbitrary direction. The

Hamiltonian which represents our system (in the units $\hbar=1$ and electronic mass $m=1$) is

$$H = H_0 + H_I, \tag{1}$$

where

$$H_0 = \int \vec{d}r \Psi^\dagger(\vec{r})\Big\{\tfrac{1}{2}(\vec{p} - e\vec{A}/c)^2 - (J\vec{M} + g\mu_B\vec{B})\vec{s}\Big\}\Psi(\vec{r}),$$

$$H_I = \int \vec{d}r \Psi^\dagger(\vec{r})V(\vec{r})\Psi(\vec{r}). \tag{2}$$

Here, $\Psi(\vec{r})$ is a spinor field with components $\Psi(\vec{r})_\uparrow$ and $\Psi(\vec{r})_\downarrow$, $\vec{M}(M_x,M_z)$ is the magnetization of the system, J is the magnetic coupling strength between the magnetization $\vec{M}$ and the spin-angular momentum $\vec{s}$ of the electron. $B(=\vec{\nabla}\times\vec{A})$ is the external magnetic field with $\vec{A}$ being the vector potential. The random field $V(\vec{r})$ of impurities consists of two independent components: (i) The component independent of electron spins, described by the random potential $V_o(r)$ and (ii) the spin-orbit interaction $V_{so}(\vec{r})$. The matrix element of the potential has the form[27]

$$f_{\alpha\beta} = V_o\delta_{\alpha\beta} + iV_1(\vec{k}\times\vec{k}')\sigma_{\alpha\beta} \tag{3}$$

for the transition $(\vec{k},\alpha)\rightarrow(\vec{k},\beta)$ with V_o and V_1 being the constants representing the strength of the random nonmagnetic elastic interaction and the spin-orbit coupling. $\vec{k}$ and $\vec{k}'$ are the initial and the final wave vectors of an electron scattering from the spin state α to β and $\vec{\sigma}(\sigma_x,\sigma_y,\sigma_z)$ corresponds to the Pauli matrices.

Let us assume that the motion of the electrons is confined to the XY plane. From the form of the Hamiltonian it is evident that the component of the spin along $(g\mu_B\vec{B}+J\vec{M})$ commutes with the unperturbed Hamiltonian (H_0) and the quantized axis of the electronic spin should be in the direction of $(g\mu\vec{B}+J\vec{M})$. Therefore, we imagine a new system of axis, say, $X'Y'Z'$, where the axis Z' is along $(g\mu_B\vec{B}+J\vec{M})$ and express the scattering potential in this system of axis. However, the motion of the electrons is confined in the old XY plane and there would be no component of $\vec{k}$ or $\vec{k}'$ along the perpendicular direction of the plane. So we represent the momentum of the electron in the $X'Y'Z'$ axis such that the momentum k_z, which is along the perpendicular to the plane of the motion of the electron, is always zero. The momentum

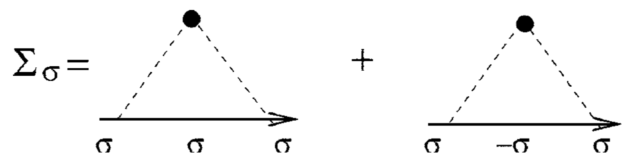

FIG. 1. Self-energies $\Sigma_\uparrow$ and $\Sigma_\downarrow$ corresponding to second-order perturbation theory.

$k_{x'}$, $k_{y'}$, $k_{z'}$ in the $X'Y'Z'$ coordinate system can be written in terms of k_x, k_y, k_z in the XYZ coordinate system as

$$k_{x'} = k_x \cos\theta, \quad k_{y'} = k_y, \quad k_{z'} = k_x \sin\theta, \tag{4}$$

with

$$\sin\theta = \left(\tfrac{1}{2}JM_x + \mu_B B_x\right)/\alpha_1, \quad \cos\theta = \left(\tfrac{1}{2}JM_z + \mu_B B_z\right)/\alpha_1, \tag{5}$$

$$\alpha_1 = \left[\left(\tfrac{1}{2}JM_x + \mu_B B_x\right)^2 + \left(\tfrac{1}{2}JM_z + \mu_B B_z\right)^2\right]^{1/2}. \tag{6}$$

In the case of weak scattering potential and upon averaging over the random impurity potential $V(\vec{r})$, we obtain the bare scattering amplitude

$$\Gamma^o_{\alpha\beta\gamma\delta} = x_1\delta_{\alpha\beta}\delta_{\gamma\delta} - y_1\sigma^z_{\alpha\beta}\sigma^z_{\gamma\delta} - z_1\sigma^x_{\alpha\beta}\sigma^x_{\gamma\delta} - z_2\sigma^y_{\alpha\beta}\sigma^y_{\gamma\delta}, \tag{7}$$

where

$$x_1 = n_i V_o^2 = \frac{1}{2\pi\nu_\uparrow\tau_{o\uparrow}} = \frac{1}{2\pi\nu_\downarrow\tau_{o\downarrow}},$$

$$y_1 = n_i V_1^2\overline{(\vec{k}\times\vec{k}')^2_{z'}} = n_i V_1^2\overline{(\vec{k}\times\vec{k}')^2_z}\cos^2\theta = \frac{\cos^2\theta}{2\pi\nu_\uparrow\tau^z_{so\uparrow}}$$

$$= \frac{\cos^2\theta}{2\pi\nu_\downarrow\tau^z_{so\downarrow}},$$

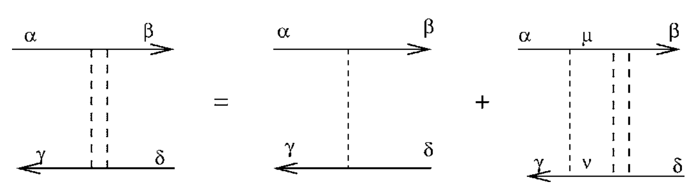

FIG. 2. The Dyson equation for particle-hole propagators. For the corresponding cooperon equation see (13).

$$z_1 = n_i V_1^2 \overline{(\vec{k}\times\vec{k}')^2_{x'}} = n_i V_1^2 \overline{(\vec{k}\times\vec{k}')^2_{x}} \sin^2\theta = \frac{\sin^2\theta}{2\pi\nu_\uparrow \tau^x_{so\uparrow}}$$
$$= \frac{\sin^2\theta}{2\pi\nu_\downarrow \tau^x_{so\downarrow}}, \quad z_2 = 0.$$

Here, n_i is the concentration of the impurities in the system, $\nu_\uparrow$ and $\nu_\downarrow$ are the densities of states at the Fermi-level for the up-spin and down-spin electrons. For a two-dimensional system $\nu_\uparrow = \nu_\downarrow = \nu$ is a constant and the relaxation times are $\tau_{0\uparrow} = \tau_{0\downarrow}$, $\tau^z_{so\uparrow} = \tau^z_{so\downarrow} = \tau^x_{so\uparrow} = \tau^x_{so\downarrow}$.

In order to calculate the conductivity of the system, we follow the method introduced by Altshuler *et al.*[28] We first obtain the Green's function and the particle particle propagator with $\vec{A}=0$ and incorporate the contribution of the vector potential $\vec{A}$ due to the magnetic field perpendicular to the plain of motion of the electron in the quasiclassical approximation. However, this approximation is only applicable when the Landau orbit is much larger than the in-plane mean free path of the charge carrier. The conductivity of our system is calculated within the Kubo[29] formalism. In this method the dc-conductivity for $(\vec{A}=0)$ is expressed as

$$\sigma = \sigma_I^\uparrow + \sigma_I^\downarrow + \sigma_{II}^\uparrow + \sigma_{II}^\downarrow, \tag{8}$$

where, the Boltzmann contribution to the conductivity for up-spin ($\sigma=\uparrow$) and down-spin ($\sigma=\downarrow$) electrons is

$$\sigma_I^\sigma = \frac{e^2}{16\pi^3 m^2}\int d\epsilon \frac{df}{d\epsilon}\int d^2k\, k^2 [G^R_{\sigma\sigma}(\epsilon,\vec{k}) G^A_{\sigma\sigma}(\epsilon,\vec{k})], \tag{9}$$

and the weak localization correction to the conductivity of up-spin and down-spin electrons is given by

$$\sigma_{II}^\sigma = -\frac{e^2}{64\pi^5 m^2}\int d\epsilon \frac{df(\epsilon)}{d\epsilon}\int d^2k\, k^2 [G^R_{\sigma\sigma}(\epsilon,\vec{k}) G^A_{\sigma\sigma}(\epsilon,\vec{k}) G^R_{\sigma\sigma}(\epsilon,-\vec{k}) G^A_{\sigma\sigma}(\epsilon,-\vec{k})]$$
$$\times \int d^2q\, \Gamma_{\sigma\sigma\sigma\sigma}(0,q) - \frac{e^2}{64\pi^5 m^2}\int d\epsilon \frac{df(\epsilon)}{d\epsilon}\int d^2k\, k^2 [G^R_{\sigma\sigma}(\epsilon,\vec{k}) G^A_{\sigma\sigma}(\epsilon,\vec{k}) G^R_{-\sigma-\sigma}(\epsilon,-\vec{k}) G^A_{-\sigma-\sigma}(\epsilon,-\vec{k})]$$
$$\times \int d^2q\, \Gamma_{\sigma-\sigma-\sigma\sigma}(0,q). \tag{10}$$

In the above expression $f(\epsilon)$ is the usual Fermi-function, $G^{R(A)}_{\uparrow\uparrow}$, $G^{R(A)}_{\downarrow\downarrow}$ are the retarded (advanced) single-particle Green's-function for spin-up and spin-down electrons, and $\Gamma_{\uparrow\uparrow\uparrow\uparrow}(0,q)$, $\Gamma_{\uparrow\downarrow\downarrow\uparrow}(0,q)$, $\Gamma_{\downarrow\downarrow\downarrow\downarrow}(0,q)$, $\Gamma_{\downarrow\uparrow\uparrow\downarrow}(0,q)$ are the renormalized scattering amplitudes in the Cooper channel which represent the maximally crossed diagrams. These scattering processes are responsible for the quantum interferences or weak localization in the system.

In the presence of impurities the charge carriers acquire a relaxation time through the elastic and spin-orbit scattering from impurities. This is obtained from the imaginary part of the self-energies $\Sigma_\uparrow$ and $\Sigma_\downarrow$ sketched in Fig. 1.

After evaluating the self-energy diagram from the interaction expressed in Eq. (2) we obtain the scattering lifetime for up-spin and down-spin electrons,

$$\frac{1}{\tau_\sigma} = \frac{1}{\tau_{-\sigma}} = \frac{1}{\tau} = \pi\nu(x_1 + y_1 + z_1) = \frac{1}{\tau_o} + \frac{\cos^2\theta}{\tau^z_{so}} + \frac{\sin^2\theta}{\tau^x_{so}}, \tag{11}$$

and the corresponding single-particle Green's-function for up-spin and down-spin electrons is given by

$$G^R_{\sigma\sigma}(\epsilon,\vec{k}) = [\epsilon - k^2/(2m) + \mathrm{sgn}(\sigma)\alpha_1 - i/\tau]^{-1}. \tag{12}$$

The effect of quantum interferences can be taken into account by considering a particular group of electron hole propagators, the so-called maximally crossed diagrams.[7,28] The contribution of this group of electron-hole propagators, denoted by the renormalized scattering amplitudes $\Gamma_{\uparrow\uparrow\uparrow\uparrow}(0,q)$, $\Gamma_{\uparrow\downarrow\downarrow\uparrow}(0,q)$, $\Gamma_{\downarrow\downarrow\downarrow\downarrow}(0,q)$, $\Gamma_{\downarrow\uparrow\uparrow\downarrow}(0,q)$, are evaluated from the Dyson equation shown in Fig. 2. The diagrammatical expression is equivalent to the following equation:[27]

$$\Gamma_{\alpha\beta\gamma\delta}(0,q) = \Gamma^o_{\alpha\beta\gamma\delta} + \sum_{\mu\nu}\Gamma_{\alpha\mu\gamma\nu}\Pi_{\mu\nu}\Gamma_{\mu\beta\nu\delta}(0,q), \tag{13}$$

where

$$\Pi_{\mu\nu}(0,q) = \frac{1}{4\pi^2}\int d^2k\, G^R_\mu(\epsilon,\vec{k}) G^A_\nu(\epsilon,\vec{k}+\vec{q}). \tag{14}$$

Although the inelastic processes such as electron-electron scattering or electron-phonon scattering are not included in this calculation, one realizes that they cause an exponential decay proportional to $\exp(-t/\tau_\phi)$ with time t, where τ_ϕ is the relaxation time related to the inelastic processes mentioned above, in the current because the phase coherence between the interfering electronic wave functions decays with this characteristic time. When we introduce the destruction of the phase coherence due to the inelastic processes we finally obtain the conductivity of a two-dimensional ferromagnetic system when $B_z=0$,

$$
\begin{aligned}
\sigma_I^{\uparrow}+\sigma_I^{\downarrow}+\sigma_{II}^{\uparrow}+\sigma_{II}^{\downarrow} = e^2\nu(D_{\uparrow\uparrow}+D_{\downarrow\downarrow}) - \frac{e^2}{8\pi^2}
&\times\Bigg\{\left(1+\frac{P_1(D_{\uparrow\uparrow}-D_{\downarrow\downarrow})\tau}{\sqrt{S_1^2-4R_1T_1}}\right)\ln\left(\frac{\gamma_1+1/\tau}{\gamma_1}\right)+\left(1-\frac{P_1(D_{\uparrow\uparrow}-D_{\downarrow\downarrow})\tau}{\sqrt{S_1^2-4R_1T_1}}\right)\ln\left(\frac{\gamma_2+1/\tau}{\gamma_2}\right)\\
&+\left(1-\frac{P_1(D_{\uparrow\uparrow}-D_{\downarrow\downarrow})\tau}{\sqrt{S_1^2-4R_1T_1}}\right)\ln\left(\frac{\gamma_3+1/\tau}{\gamma_3}\right)+\left(1+\frac{P_1(D_{\uparrow\uparrow}-D_{\downarrow\downarrow})\tau}{\sqrt{S_1^2-4R_1T_1}}\right)\ln\left(\frac{\gamma_4+1/\tau}{\gamma_4}\right)\\
&-\frac{2\sin^2\theta(D_{\uparrow\uparrow}+D_{\downarrow\downarrow})}{(1+4\alpha_1^2\tau^2)\tau_{so}^x}\Re\Bigg[\frac{1}{\sqrt{Q_2^2-4R_2P_2}}\Bigg\{\ln\left(\frac{\gamma_5+2D_{\uparrow\downarrow}/[(D_{\uparrow\uparrow}+D_{\downarrow\downarrow})\tau]}{\gamma_5}\right)\\
&-\ln\left(\frac{\gamma_6+2D_{\uparrow\downarrow}/[(D_{\uparrow\uparrow}+D_{\downarrow\downarrow})\tau]}{\gamma_6}\right)\Bigg\}\Bigg]\Bigg\},
\end{aligned}
\tag{15}
$$

with

$$
\begin{aligned}
D_{\sigma\sigma} &= [\epsilon+\mathrm{sgn}(\sigma)\alpha_1]\tau/m,\\
D_{\sigma-\sigma} &= D_{-\sigma-\sigma}[1+2i\,\mathrm{sgn}(\sigma)\alpha_1\tau]^{-3},\\
\gamma_1 &= D_{\uparrow\uparrow}\frac{S_1-\sqrt{(S_1^2-4R_1T_1}}{2T_1}+\frac{1}{\tau_\phi},\\
\gamma_2 &= D_{\uparrow\uparrow}\frac{S_1+\sqrt{S_1^2-4R_1T_1}}{2T_1}+\frac{1}{\tau_\phi},\\
\gamma_3 &= D_{\downarrow\downarrow}\frac{S_1-\sqrt{S_1^2-4R_1T_1}}{2T_1}+\frac{1}{\tau_\phi},\\
\gamma_4 &= D_{\downarrow\downarrow}\frac{S_1+\sqrt{S_1^2-4R_1T_1}}{2T_1}+\frac{1}{\tau_\phi},\\
\gamma_5 &= D_{\uparrow\downarrow}\frac{Q_2-\sqrt{Q_2^2-4R_2P_2}}{2P_2}+\frac{1}{\tau_\phi},\\
\gamma_6 &= D_{\uparrow\downarrow}\frac{Q_2+\sqrt{Q_2^2-4R_2P_2}}{2P_2}+\frac{1}{\tau_\phi},
\end{aligned}
\tag{16}
$$

and

$$P_1=(2\pi\nu)^2[(x_1-y_1)(2y_1+z_1)+z_1^2],$$

$$S_1=P_1(D_{\uparrow\uparrow}+D_{\downarrow\downarrow})\tau,$$

$$R_1=(4\pi\nu)^2[y_1^2+y_1z_1],$$

$$T_1=(2\pi\nu)^2[(x_1-y_1)^2-z_1^2]D_{\downarrow\downarrow}D_{\uparrow\downarrow}\tau^2,$$

$$P_2=2\pi\nu\tau(x_1+y_1-z_1)D_{\downarrow\uparrow}D_{\uparrow\downarrow},$$

$$
\begin{aligned}
Q_2=2\pi\nu\Bigg[&\frac{z_1-2i\alpha_1\tau(x_1+y_1)}{1-2i\alpha_1\tau}D_{\uparrow\downarrow}\\
&+\frac{z_1+2i\alpha_1\tau(x_1+y_1)}{1+2i\alpha_1\tau}D_{\downarrow\uparrow}\Bigg],
\end{aligned}
$$

$$R_2=\frac{4\alpha_1^2}{1+4\alpha_1^2\tau^2}.$$

An external magnetic field perpendicular to the two-dimensional plane (B_z) has a strong influence on quantum interference. The vector potential due to the perpendicular magnetic field modifies the phase of the wave functions and leads to a partial destruction of quantum interference. Here we consider a two-dimensional system in which the mean free path is much smaller than the cyclotron radius. In this case the major effect of the vector potential on the electronic wave function or the Green's function is the change of the phase between two different points.[28] Then, in presence of the breakdown of the time reversal symmetry of the system due to the magnetic field B_z, the weak localization correction to the conductivity ($\sigma_{II}^{\uparrow}+\sigma_{II}^{\downarrow}$) reduces to

$$\sigma_{II}^{\uparrow}+\sigma_{II}^{\downarrow}=-\frac{e^2}{8\pi^2}\Bigg\{\left(1+\frac{P_1(D_{\uparrow\uparrow}-D_{\downarrow\downarrow})\tau}{\sqrt{S_1^2-4R_1T_1}}\right)\left[\Psi\left(\frac{1}{2}+\frac{\gamma_1+1/\tau}{D_{\uparrow\uparrow}a}\right)-\Psi\left(\frac{1}{2}+\frac{\gamma_1}{D_{\uparrow\uparrow}a}\right)\right]+\left(1-\frac{P_1(D_{\uparrow\uparrow}-D_{\downarrow\downarrow})\tau}{\sqrt{S_1^2-4R_1T_1}}\right)\left[\Psi\left(\frac{1}{2}+\frac{\gamma_2+1/\tau}{D_{\uparrow\uparrow}a}\right)\right.$$
$$\left.-\Psi\left(\frac{1}{2}+\frac{\gamma_2}{D_{\uparrow\uparrow}a}\right)\right]+\left(1-\frac{P_1(D_{\uparrow\uparrow}-D_{\downarrow\downarrow})\tau}{\sqrt{S_1^2-4R_1T_1}}\right)\left[\Psi\left(\frac{1}{2}+\frac{\gamma_3+1/\tau}{D_{\downarrow\downarrow}a}\right)-\Psi\left(\frac{1}{2}+\frac{\gamma_3}{D_{\downarrow\downarrow}a}\right)\right]+\left(1+\frac{P_1(D_{\uparrow\uparrow}-D_{\downarrow\downarrow})\tau}{\sqrt{S_1^2-4R_1T_1}}\right)$$
$$\times\left[\Psi\left(\frac{1}{2}+\frac{\gamma_4+1/\tau}{D_{\downarrow\downarrow}a}\right)-\Psi\left(\frac{1}{2}+\frac{\gamma_4}{D_{\downarrow\downarrow}a}\right)\right]-\frac{2\sin^2\theta(D_{\uparrow\uparrow}+D_{\downarrow\downarrow})}{(1+4\alpha_1^2\tau^2)\tau_{so}^x}\Re\left[\frac{1}{\sqrt{Q_2^2-4R_2P_2}}\left\{\Psi\left(\frac{1}{2}+\frac{\gamma_5}{D_{\uparrow\downarrow}a}+\frac{2/\tau}{(D_{\uparrow\uparrow}+D_{\downarrow\downarrow})a}\right)\right.\right.$$
$$\left.\left.-\Psi\left(\frac{1}{2}+\frac{\gamma_5}{D_{\uparrow\downarrow}a}\right)-\Psi\left(\frac{1}{2}+\frac{\gamma_6}{D_{\uparrow\downarrow}a}+\frac{2/\tau}{(D_{\uparrow\uparrow}+D_{\downarrow\downarrow})a}\right)+\Psi\left(\frac{1}{2}+\frac{\gamma_6}{D_{\uparrow\downarrow}a}\right)\right\}\right]\Bigg\}, \tag{17}$$

where $a=4eB_z/c$ and $\Psi(x)$ is the digamma function.

III. DISCUSSION AND CONCLUSION

In this work we have investigated the effect of quantum interference on the conductivity of a two-dimensional ferromagnet in presence of spin-orbit scattering. By means of the diagrammatic techniques in perturbation theory, we have calculated the magnetoresistance of a ferromagnetic metal when magnetization of the system and external magnetic field are along any arbitrary direction, which generalizes the results obtained by Dugaev *et al.*[20,21] and by Kirkpatrick *et al.*[12,25]

The analytical results for the magnetoconductivity have been obtained as a function of magnetization, coupling of the electronic spin with the magnetization of the system and the characteristic lifetimes due to elastic, inelastic, and spin-orbit scattering. We show that the effect of spin-orbit (SO) scattering on conductivity depends strongly on the orientation of the magnetization with respect to the plane of the motion of the charge carriers.

In the case of a two-dimensional ferromagnet with in-plane magnetization, spin-flip process due to the SO interaction introduces a weak antilocalization in the system. However, the spin polarization of the conducting electrons due to its coupling with the magnetization reduces the spin-flip scattering in the system and leads to a suppression of the antilocalization effect. As a result, the quantum correction to the conductivity is always negative for a strong ferromagnet ($JM\tau \gg 1$), which manifests negative magnetoresistance. However, in the absence of magnetization the system shows weak antilocalization instead of weak localization.

In Fig. 3 we show the variation of weak localization or weak antilocalization contributions to the magnetoconductivity as a function of the applied field (perpendicular to the film) of a nonmagnetic as well as a ferromagnetic system in presence of spin-orbit scattering. When the magnetization is vanishingly small we observe weak antilocalization which is in good agreement with the well-known results of Hikami *et al.*[27] (solid and dashed curve). The destruction of weak antilocalization in the presence of magnetization, both, in plane (dotted curve) and out of plane (dash-dotted curve), are shown in this figure to understand the role of ferromagnetic correlation in the conductivity. When the magnetization is perpendicular to the plane of the motion of the electrons, the spin-flip scattering due to the SO interaction is absent and only the spin-conserving scattering process contributes to the conductivity. This spin-conserving scattering also suppresses the localization correction to the conductivity, but the WEL correction to the conductivity is always negative and the conductivity increases with the increase of the external magnetic field. When the orientation of magnetization is along the plane only spin-flip scattering is present in the system. Since the presence of magnetization destroys spin-flip scattering, a stronger localization is observed in Fig. 3.

However, for an arbitrary orientation of magnetization both scattering processes, namely the spin-conserved scattering and the spin-flip scattering, are present and since the two scattering processes affect the response of the external field differently, the magnetoresistance of a 2D ferromagnet depends significantly on the orientation of the magnetization. In Fig. 4 we have demonstrated the dependence of the ori-

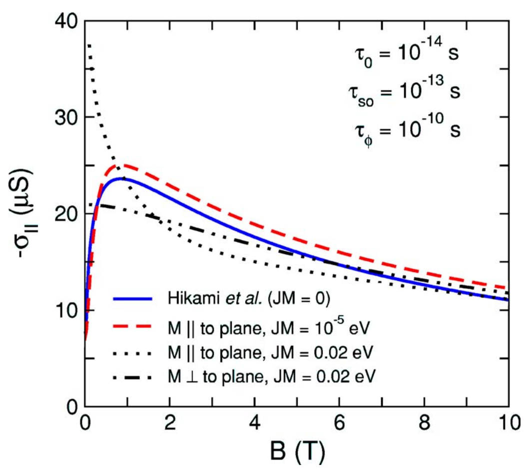

FIG. 3. (Color online) The variation of weak-localization or weak-antilocalization contributions to the conductivity (σ_{II}) with respect to the external magnetic field $\vec{B}$ (perpendicular to the film) is shown for nearly nonmagnetic systems (solid and dashed curves) as well as ferromagnetic systems with out of plane magnetization (dash-dotted curve) and in-plane magnetization (dotted curve). The results of a nearly nonmagnetic system (dashed curve) are compared with the well-known results of Hikami *et al.* (Ref. 27) (solid curve).

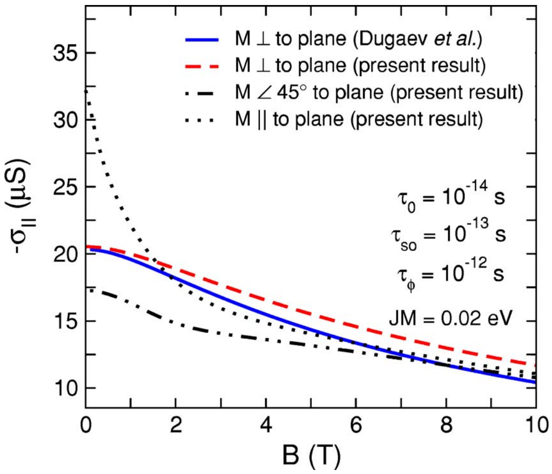

FIG. 4. (Color online) The weak localization contribution to the magnetoconductivity (σ_{II}) for a ferromagnetic system with magnetization (M) perpendicular to the thin film (dashed curve) is compared with the results of Dugaev *et al.* (solid curve) (Ref. 21). To show the dependence of magnetoconductivity with the orientation of magnetization, the magnetic field dependence of σ_{II} is presented for three different orientations of magnetization, (i) M perpendicular to the plane of the thin film, (ii) M along the 45° direction to the plane of the thin film, and (iii) M parallel to the plane of the thin film. In all cases the external field $\vec{B}$ is applied along the perpendicular to the thin film.

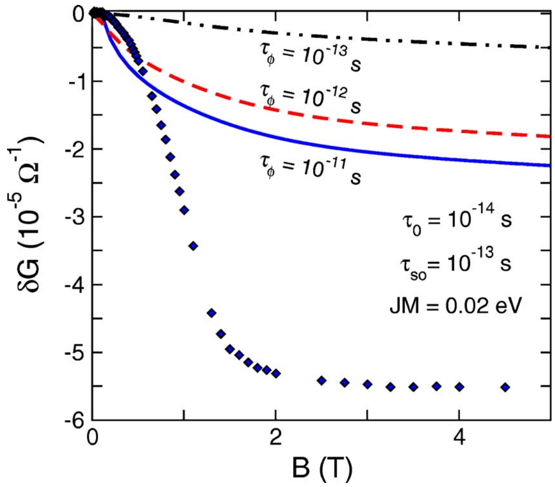

FIG. 5. (Color online) The weak localization contribution to the magnetoresistance $\delta G = \sigma(0) - \sigma(B)$ $(=[R_s(B) - R_s(0)]/R_s(0)^2$, $R_s(0)$ being the zero field resistance of the unit area of the material) of a 2D ferromagnetic system with magnetization parallel to the plane is compared with the experimental data from Ref. 18 (♦) for the external magnetic field $\vec{B}$ applied in the out of plane direction for $\tau_\phi = 10^{-11}$ s (solid curve), 10^{-12} s (dashed curve), and 10^{-13} s (dash-dotted curve).

entation of magnetization on the magnetoconductivity of a two-dimensional ferromagnet. For this purpose we considered three cases: (i) The magnetization is perpendicular to the 2D plane (dashed curve), (ii) the magnetization is along the 45° direction to the 2D plane, and (iii) the magnetization is parallel to the 2D plane. A comparison of our result for the perpendicular magnetization configuration (dashed curve) with the previous calculation of Dugaev *et al.* (solid curve)[21] shows a good agreement of our result with the corresponding results of Dugaev *et al.*

In view of our observation from Fig. 3 it is clear that the magnetoresistance can be positive or negative depending on the orientation and the strength of the magnetization. Moreover, as shown in Fig. 4, the orientation of the magnetization is determined by the strength of the anisotropic field and the strength of the magnetic field. Thus, the direction of the system magnetization changes with the variation of the magnetic field. Therefore, one should take into account the proper orientation of the magnetization for the comparison of the experimental results with theoretical predictions. We believe that our results will provide a realistic estimation for the WEL correction to the magnetoresistance of a ferromagnetic thin film.

In Figs. 5–7 we made an effort to understand whether the WEL is the only major effect which introduces the magnetoresistance in the two-dimensional ferromagnetic system at low temperatures. Here we compare the response of the variation of magnetic field and temperature to the WEL correction to the transport properties with the corresponding experimental observation of magnetoconductivity of a cobalt film of thickness 10 nm.[18] As the magnetization of this system is along the 2D plane we restrict ourself to the in-plane magnetization configuration in order to compare our result with experimental findings. In Fig. 5 we have compared the variation of the localization contribution to the magnetoresistance with the experimental observations of Brands *et al.*[18] We tentatively associate the discrepancy of the theoretical prediction and the experimental observation (Fig. 5) to the strong anisotropic magnetoresistance (AMR) effects in the ferromagnetic system. It is to be noted that in our calculation we did not take into account the AMR effect which significantly contributes to the magnetoresistance of a ferromagnetic system.

At low temperatures the variation of resistance with respect to temperature mainly originates from the quantum interference and from the electron-electron interaction (EEI). The electron-electron interaction modifies the transport properties in two ways, (i) it modifies the self-energy, hence producing the inelastic lifetime for the electronic states and leads to a phase decoherence, characterized by τ_ϕ, in the system, (ii) it modifies the current-current correlation giving an additional temperature dependent contribution to the conductivity. For a two-dimensional electronic system the correction due to the EEI effects to the conductivity is given by[30] (assuming the screening length to be large),

$$\delta\sigma(\mathrm{EEI}) = \frac{e^2}{2\pi^2\hbar}\ln(k_B T \tau_0). \tag{18}$$

In Figs. 6 and 7 we have compared the theoretical and experimental results of the temperature variation of resistance (R_s) of a thin Co film for different magnetic fields. Although the weak localization contribution to the resistance

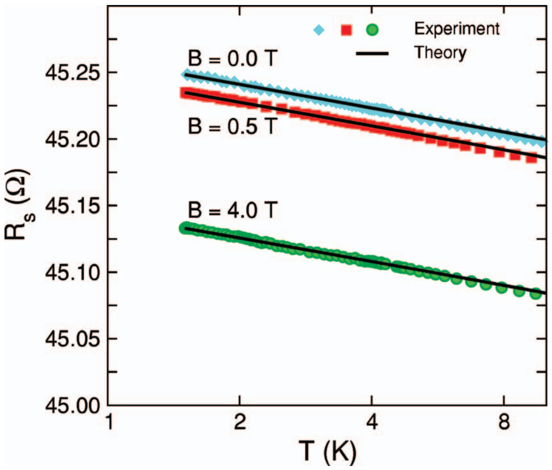

FIG. 6. (Color) The temperature variance of resistance R_s of a ferromagnetic system with magnetization parallel to the plane of the thin film has been shown for different perpendicular magnetic fields B_z of strength 0.0 T, 0.5 T, and 4.0 T (solid curves) for $JM=0.02$ eV, $\tau_0=10^{-14}$ s, and $\tau_{so}=10^{-13}$ s. The result of our calculation is compared with the experimental observation of Brands *et al.* (Ref. 18) for the magnetic field $B_z=0.0$ T (blue diamonds), 0.5 T (red squares), and 4.0 T (green circles).

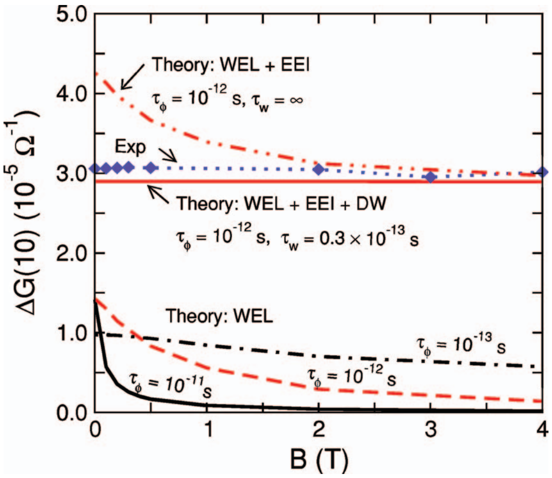

FIG. 7. (Color) The lower three curves show the theoretical WEL contribution of the change of the conductivity due to the change of the temperature defined by $\Delta G(10) = \sigma(10\text{ K}) - \sigma(1\text{ K})\{=[R_s(1\text{ K}) - R_s(10\text{ K})]/R_s(10\text{ K})^2\}$, which has been plotted as a function of magnetic field (perpendicular to the film) for $\tau_\phi=10^{-11}$ s (solid curve), 10^{-12} s (dashed curve), and 10^{-13} s (dash-dotted curve) at 1 K when $JM=0.02$ eV, $\tau_0=10^{-14}$ s, $\tau_{so}=10^{-13}$ s. When EEI is included the theoretical curve for $\tau_\phi=10^{-12}$ is shifted upward (double-dotted dashed curve). If in addition domain wall scattering is taken into account (WEL+EEI+DW, solid red curve), the results of our theoretical calculations compare very well with the experimental observation of Brands *et al.* (Ref. 18) (blue diamonds).

predicts a strong variation of slopes in R_s vs T curves with respect to the magnetic field (curves with $\tau_w=\infty$ in Fig. 7) this feature is absent in the experimental observation. We tentatively associate this discrepancy to the existence of conductance fluctuations originating from inhomogeneous magnetization and magnetic fields due to the polycrystalline morphology of the samples.[31] The interfaces of the differently oriented polycrystallines produce changes in the orientation of magnetization, which introduces phase decoherence between the interfering electrons and results in a suppression of the quantum interference effect. In order to determine the influence of misorientation of the local magnetization on WEL, we estimate the modification of the conductivity due to scattering of the electrons from a domain wall following Tatara *et al.*[32] In this work[32] it has been shown that the contribution due to domain wall scattering (scattering life time being τ_w) to the conductivity can be taken into account by changing the phase coherence lifetime τ_ϕ by

$$\frac{1}{\tau_\phi} \to \frac{1}{\tau_\phi} + \frac{1}{\tau_w}, \tag{19}$$

with

$$\frac{1}{\tau_w} = \frac{n_w}{6\lambda k_F^2 \tau_0}\left(\frac{\epsilon_F}{JM}\right)^2, \tag{20}$$

where λ is the width of the domain wall, n_w is the concentration of domain walls, and k_F and ϵ_F are the Fermi momentum and Fermi energy of the system. For $\lambda=500$ Å, $JM=0.02$ eV, and $\epsilon=3.0$ eV, τ_w is estimated as $3.03\tau_0=0.3\times10^{-13}$ s.[32] When we introduce the EEI effect and the domain wall scattering due to the polycrystalline morphology of the sample we observe a good agreement with the theoretical and experimental observation of the temperature dependence of the resistivity in the low temperature regime (Fig. 6). In Fig. 7 also we found a good agreement between the theoretical and experimental results when we plot the logarithmic slope $\Delta G(10)$ as a function of the magnetic field.

From our studies we find that the complete features of magnetoresistance of a realistic ferromagnetic system at low temperatures cannot be explained by taking into account the quantum-interference phenomena only. For a more complete understanding of the transport properties of a 2D ferromagnet in the presence of a magnetic field, the effect of AMR and nonhomogeneity of the magnetization in the system[33] should be considered rigorously.

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