Statistics of charge transfer in a tunnel junction coupled to an oscillator

J. Wabnig, D. V. Khomitsky,* and J. Rammer

Department of Physics, Umeå University, SE-901 87, Umeå

A. L. Shelankov

Department of Physics, Umeå University, SE-901 87, Umeå and A. F. Ioffe Physico-Technical Institute, 194021 St. Petersburg, Russia

The charge transfer statistics of a tunnel junction coupled to a quantum object is studied using the charge projection technique. The joint dynamics of the quantum object and the number of charges transferred through the junction is described by the charge specific density matrix. The method allows evaluating the joint probability distribution of the state of the quantum object and the charge state of the junction. The statistical properties of the junction current are derived from the charge transfer statistics using the master equation for the charge specific density matrix. The theory is applied to a nanoelectromechanical system, and the influence on the average current and the current noise of the junction is obtained for coupling to a harmonic oscillator.

I. INTRODUCTION

In recent years, it has become possible to couple charge dynamics of electrons to vibrational modes of a nanostructure, and the new field of nanoelectromechanics has emerged.[1] Nanoelectromechanical devices are expected to lead to new technologies, such as ultrasmall mass detection techniques,[2,3] as well as stimulating fundamental studies of quantum phenomena in macroscopic systems.[4] For example, experiments have probed high-frequency nanomechanical resonators in order to reach the quantum uncertainty limit.[5–8] Charge transfer by mechanical motion has been studied in experiments shuttling single electrons.[9–11] Nanomechanical resonators are investigated for use in quantum information processing.[12,13]

The simplest possibility of detection and control of the vibrational degree of freedom in a nanomechanical system is at present via the coupling to a quantum point contact or a tunnel junction. Recent theoretical studies of nanoelectromechanical systems, therefore, have considered the coupling of a harmonic oscillator to a tunnel junction. The master equation for the oscillator was originally obtained by Mozyrsky and Martin,[14] using a method limiting considerations to the zero-temperature case, and the influence of the coupled oscillator on the average current was obtained. Smirnov, Mourokh, and Horing examined the nonequilibrium fluctuations of an oscillator coupled to a biased tunnel junction.[15] Clerk and Girvin derived the master equation for the oscillator and considered the current noise power spectrum in the shot noise regime, studying both the dc and ac cases.[16] Armour, Blencowe, and Zhang considered the dynamics of a classical oscillator coupled to a single electron transistor,[17] and Armour the induced current noise.[18] Related models were studied in molecular electronics.[19]

Tunnel junctions functioning as position detectors or being monitored by a vibrational mode put emphasis on a description of the current properties of a tunnel junction in terms of its charge dynamics. Recently, we considered a general many-body system coupled to a quantum object and considered their joint dynamics in the charge representation.[20] This approach, based on a previously introduced charge projection technique,[21] provides a quantum description of charge dynamics based directly on the density matrix for the system, and allows us to treat the number of particles in a given piece of material as a quantum degree of freedom, establishing, thereby, in proper quantum mechanical context, the charge representation. The evolution of the coupled systems is described in terms of the charge specific density matrix for the quantum object, $\hat{\rho}_n(t)$, i.e., the dynamics conditioned on the number n of charges in a specified spatial region of the environment. When a many-body environment is coupled to another quantum object, the method allows evaluating, at any moment in time, the joint probability distribution describing the quantum state of the object and the number of charges in the chosen region of the many-body system. The charge specific density matrix description of the dynamics of a quantum object, therefore, is an optimal tool to study transport in nanostructures, since in electrical measurements any information beyond the charge distribution is irrelevant. So far we have applied the method to charge counting in a tunnel junction coupled to a discrete quantum degree of freedom, viz. that of a two-level system, and shown that the charge state of the junction can function as a meter providing a projective measurement of the quantum state of the two-level system.[20]

In this paper, we shall apply the charge projection method to the case where the quantum object coupled to the junction is a continuous degree of freedom. In particular, we shall concentrate on the properties of the current through the junction due to the coupling to the quantum object. The statistical properties of the current through the junction and its correlations with the dynamics of the quantum object coupled to it, shall be expressed through the charge specific density matrix. We shall illustrate the results for the case of a harmonic oscillator coupled to the tunnel junction. It should be noted that current experimental setups studying nanoelectromechanical systems are operated under conditions where temperature, oscillator excitation energy, and voltage bias across

the junction are comparable.[3,5,6] Furthermore, nanomechanical oscillators, such as a suspended beam, are in addition to the charge dynamics of the electrons in the junction invariably coupled to a thermal environment, say the substrate upon which the oscillator is mounted. Thus, we are considering the situation where a quantum object in addition to interacting with a heat bath is interacting with an environment out of equilibrium. Having the additional parameter, the voltage, characterizing the environment in nonequilibrium, gives rise to features not present for an object coupled to a many-body system in equilibrium. The presented approach is applicable in a broad region of temperatures and voltages of the junction and arbitrary frequency of the oscillator and, thus, generalizes previous treatments.

The paper is organized as follows. In Sec. II, we introduce the model Hamiltonian for a generic electromechanical nanoresonator, a harmonic oscillator coupled to a tunnel junction, and derive the Markovian master equation for the charge specific density matrix. The master equation for the charge unconditional density matrix, i.e., the charge specific density matrix traced with respect to the charge degree of freedom of the junction, is discussed in detail for the case of a harmonic oscillator coupled to a tunnel junction. In Sec. III, we consider the influence of the oscillator on the current-voltage characteristic of the junction. In Sec. IV, we consider the properties of the stationary state of the oscillator. We calculate the heating of the oscillator due to the nonequilibrium state of the junction and calculate the steady-state *I-V* characteristic of the junction. In Sec. V, we consider the current noise in the junction using the charge representation and obtain the explicit expression for the current-current correlator in the Markovian approximation. In Sec. VI, the theory is then applied to the case of an oscillator influencing the current noise of the junction. Finally, in Sec. VII, we summarize and conclude. Details of calculations are presented in the appendices.

II. MASTER EQUATION

As a model of a nanoelectromechanical system, we consider a harmonic oscillator coupled to a tunnel junction. The transparency of the tunnel barrier is assumed perturbed by the displacement x of the oscillator. The resulting Hamiltonian is

$$\hat{H} = \hat{H}_0 + H_l + H_r + \hat{H}_T, \tag{2.1}$$

where $\hat{H}_0$ is the Hamiltonian for the isolated harmonic oscillator with bare frequency ω_B and mass m. A hat marks operators acting on the oscillator degree of freedom. The Hamiltonians $H_{l,r}$ specify the isolated left and right electrodes of the junction

$$H_l = \sum_{\mathbf{l}} \varepsilon_{\mathbf{l}} c_{\mathbf{l}}^\dagger c_{\mathbf{l}}, \quad H_r = \sum_{\mathbf{r}} \varepsilon_{\mathbf{r}} c_{\mathbf{r}}^\dagger c_{\mathbf{r}}, \tag{2.2}$$

where $\mathbf{l}, \mathbf{r}$ labels the quantum numbers of the single particle energy eigenstates in the left and right electrodes, respectively, with corresponding energies $\varepsilon_{\mathbf{l},\mathbf{r}}$. The operator $\hat{H}_T$ describes the tunneling,

$$\hat{H}_T = \hat{\mathcal{T}} + \hat{\mathcal{T}}^\dagger, \quad \hat{\mathcal{T}} = \sum_{\mathbf{l},\mathbf{r}} \hat{T}_{\mathbf{l}\mathbf{r}} c_{\mathbf{l}}^\dagger c_{\mathbf{r}} \tag{2.3}$$

with the tunneling amplitudes, $\hat{T}_{\mathbf{l}\mathbf{r}} = \hat{T}_{\mathbf{r}\mathbf{l}}^\dagger$, depending on the oscillator degree of freedom. Due to the coupling, the tunneling amplitudes and, thereby, the conductance of the tunnel junction depend on the state of the oscillator. In the following, we assume a linear coupling between the oscillator position and the tunnel junction

$$\hat{T}_{\mathbf{l}\mathbf{r}} = v_{\mathbf{l}\mathbf{r}} + w_{\mathbf{l}\mathbf{r}} \hat{x}, \tag{2.4}$$

where $v_{\mathbf{l}\mathbf{r}} = v_{\mathbf{r}\mathbf{l}}^*$ is the unperturbed tunneling amplitude and $w_{\mathbf{l}\mathbf{r}} = w_{\mathbf{r}\mathbf{l}}^*$ is its derivative with respect to the position of the oscillator. The derivation of the equation of motion for the charge specific density matrix presented in Appendix A shows that the following combinations of the model parameters $v_{\mathbf{l}\mathbf{r}}$ and $w_{\mathbf{l}\mathbf{r}}$ enter the master equation:

$$\begin{Bmatrix} G_0 \\ G_{xx} \\ G_x \\ g_x \end{Bmatrix} = \frac{2\pi}{\hbar} \sum_{\mathbf{l}\mathbf{r}} \begin{Bmatrix} |v_{\mathbf{l}\mathbf{r}}|^2 \\ |w_{\mathbf{l}\mathbf{r}}|^2 \\ \mathfrak{R}(v_{\mathbf{l}\mathbf{r}}^* w_{\mathbf{l}\mathbf{r}}) \\ \mathfrak{I}(v_{\mathbf{l}\mathbf{r}}^* w_{\mathbf{l}\mathbf{r}}) \end{Bmatrix} \left(-\frac{\partial f(\varepsilon_{\mathbf{l}})}{\partial \varepsilon_{\mathbf{l}}} \right) \delta(\varepsilon_{\mathbf{l}} - \varepsilon_{\mathbf{r}}). \tag{2.5}$$

These lumped parameters for the junction have the following physical meaning: Let $\mathsf{G}(x) = e^2 G(x)$, e being the electron charge, denote the conductance as a function of the oscillator coordinate x when it is treated as a classical variable. Then, G_0 gives the conductance of the junction in the absence of coupling to the oscillator, $G_0 = (1/e^2) \left. \mathsf{G} \right|_{x=0}$, and $G_x = (1/2e^2)(d\mathsf{G}/dx|_{x=0})$ and $G_{xx} = (1/2e^2)(d^2\mathsf{G}/dx^2|_{x=0})$. The coupling constant g_x cannot be expressed via $\mathsf{G}(x)$. Note that g_x changes its sign upon interchange of tunneling amplitudes between the states in the two electrodes, i.e., after the substitution $\mathbf{l} \leftrightarrow \mathbf{r}$. Therefore, it is only finite for an asymmetric junction and is a measure of the asymmetry. As shown in Sec. III, g_x generates effects similar to charge pumping, as well as nontrivial features in the electric current noise as discussed in Sec. VI. For later use, it is convenient to present the coupling constants in terms of *conductances* by introducing the characteristic length of the oscillator

$$\tilde{G}_{xx} = G_{xx} x_0^2, \quad \tilde{G}_x = G_x x_0, \quad \tilde{g}_x = g_x x_0, \tag{2.6}$$

where $x_0 = (\hbar/m\omega_0)^{1/2}$, and ω_0 is the frequency of the coupled oscillator as introduced in Appendix A.

A. Charge specific master equation

To study the interaction of charge dynamics in a tunnel junction with the dynamics of a quantum object, we describe the combined system, the quantum degree of freedom coupled to a tunnel junction, using the charge specific density matrix method introduced in Ref. 20. The approach employs charge projectors to study the dynamics of the quantum object conditioned on the charge state of the junction. The charge projection operator, $\mathcal{P}_n$, projects the state of the conduction electrons in the junction onto its component for which exactly n electrons are in a given spatial region, say in

the left electrode. The charge specific density matrix is then specified by

$$\hat{\rho}_n(t) = \text{Tr}_{el}(\mathcal{P}_n \rho(t)), \tag{2.7}$$

where $\rho(t)$ is the full density matrix for the combined system, and Tr_{el} denotes the trace with respect to the conduction electrons in the junction. Provided the system at the initial time, $t=0$, is in a definite charge state, i.e., described by a charge specific density matrix of the form $\hat{\rho}_n(t) = \delta_{n0}\hat{\rho}_0$, where $\hat{\rho}_0$ is the initial state of the quantum object, the charge index n can be interpreted as the number of charges *transferred* through the junction. Thus, the charge projector method provides a basis for charge counting statistics in the cases where the distribution function for *transferred* charge is relevant as discussed in Ref. 21. The charge specific density matrix allows, therefore, the evaluation, at any moment in time, of the joint probability of the quantum state of the object and the number of charges transferred through the junction. For example, if the charge specific density matrix is traced over the quantum object degree of freedom, the probability $p_n(t)$ that n charges in time span t are *transferred* through the low transparency tunnel junction is the expectation value of the charge projector, or expressed in terms of the charge specific density matrix

$$p_n(t) = \text{Tr}(\rho_n(t)), \tag{2.8}$$

where the trace is with respect to the degree of freedom of the coupled quantum object.

The Markovian master equation for the charge specific density matrix, $\hat{\rho}_n(t)$, for the case of coupling of the junction to a quantum object is derived and discussed in Appendix A. The Markovian approximation is valid for describing slow time variations of the density matrix; the exact conditions of the applicability are specified later once the characteristic times of the problem have been identified. To the lowest order in the tunneling, the master equation for the charge specific density matrix can be generally written in terms of superoperators: a Lindblad-like term Λ, a diffusion term $\mathcal{D}$, and a drift term $\mathcal{J}$[20]

$$\dot{\hat{\rho}}_n = -\frac{i}{\hbar}[\hat{H}_0, \hat{\rho}_n] + \Lambda\{\hat{\rho}_n\} + \mathcal{D}\{\hat{\rho}_n''\} + \mathcal{J}\{\hat{\rho}_n'\}, \tag{2.9}$$

where $\hat{\rho}_n'$ and $\hat{\rho}_n''$ denote the discrete derivatives,

$$\hat{\rho}_n' = \frac{1}{2}(\hat{\rho}_{n+1} - \hat{\rho}_{n-1}), \tag{2.10}$$

$$\hat{\rho}_n'' = \hat{\rho}_{n+1} + \hat{\rho}_{n-1} - 2\hat{\rho}_n. \tag{2.11}$$

General expressions for the superoperators in Eq. (2.9) are presented in Appendix A as well as their specific form for the case of coupling to an oscillator. The equation shall, in Sec. IV, be used to study the current noise in the junction due to the coupling to a quantum object before we, in Sec. V, consider the explicit case of an oscillator coupled to the junction. However, first we analyze the master equation for the unconditional density matrix, i.e., the charge specific density matrix traced with respect to the charge degree of freedom of the junction.

B. Unconditional Master equation

Often interest is not in the detailed information of the charge evolution of the tunnel junction contained in the charge specific density matrix. If, for example, interest is solely in properties of the oscillator, this information is contained in the traced charge specific density matrix. Thus, we are led to study the master equation for the reduced or charge unconditional density matrix, the density matrix traced with respect to the charge degree of freedom, $\hat{\rho}(t) = \Sigma_n \hat{\rho}_n(t)$. Performing the charge trace on Eq. (2.9), the master equation for the reduced density matrix can be written in the form

$$\dot{\rho}(t) = \frac{1}{i\hbar}[\hat{H}_R, \hat{\rho}] + \frac{\gamma}{i\hbar}[\hat{x}, \{\hat{p}, \hat{\rho}\}] - \frac{D}{\hbar^2}[\hat{x}, [\hat{x}, \hat{\rho}]] + \frac{A}{\hbar^2}[\hat{x}, [\hat{p}, \hat{\rho}]]. \tag{2.12}$$

The form of the master equation is generic to any continuous quantum degree of freedom coupled linearly to the junction and has the well-known form for a particle coupled to a heat bath.[22,23] In the following, we consider the model Hamiltonian for a nanoelectromechanical system introduced in Sec. II and encounter the renormalized oscillator Hamiltonian

$$\hat{H}_R = \frac{\hat{p}^2}{2m} + \frac{m\omega_0^2 \hat{x}^2}{2}, \tag{2.13}$$

which in addition to having a renormalized oscillator frequency ω_0, suffers a voltage dependent linear shift in the equilibrium position of the oscillator, which in the following is assumed absorbed into the position of the oscillator (for details see Appendix A).

The second and third term on the right in Eq. (2.12) represent the physical influences of friction and fluctuations of the environment. For a nanomechanical object, the environment consists of several parts. The first one, which we have explicitly included in the model, is the tunnel junction. The other one is included phenomenologically by introducing γ_0 and D_0, the values of the friction and diffusion parameters in the absence of coupling to the junction. The physical mechanism generating the friction coefficient γ_0 and diffusion coefficient D_0 is, e.g., the heat exchange of the nanoscale oscillator and the bulk substrate it is mounted on. Thus the latter environment could also be modeled microscopically in the standard manner of coupling a quantum object to a heat bath.[22,24] Then, with the model Hamiltonian in Eq. (2.1) giving the electronic environment contribution due to the coupling to the junction, the total friction and diffusion coefficients become

$$\gamma = \gamma_0 + \gamma_e, \quad D = D_0 + D_e, \tag{2.14}$$

where γ_e is the electronic contribution to the damping coefficient

$$\gamma_e = \frac{\hbar^2 G_{xx}}{m} \tag{2.15}$$

proportional to the coupling strength G_{xx}, and the electronic contribution to the diffusion coefficient is

$$D_e = m \gamma_e \Omega \coth \frac{\Omega}{2T_e}, \qquad (2.16)$$

where $\Omega = \hbar \omega_0$ and the voltage dependent parameter T_e is given by the relation

$$\coth \frac{\Omega}{2T_e} = \frac{V+\Omega}{2\Omega} \coth \frac{V+\Omega}{2T} + \frac{V-\Omega}{2\Omega} \coth \frac{V-\Omega}{2T}. \qquad (2.17)$$

Here T is the temperature of the junction and we assume a dc voltage bias, $V = eU$, U being the applied voltage. We note that the right-hand side (r.h.s.) of Eq. (2.17) is proportional to the well-known value of the power spectrum of current noise of the isolated junction, taken at the frequency of the oscillator.[25] The fact that the junction as a part of the environment is in a nonequilibrium state is reflected in the voltage dependence of the electronic contribution to the diffusion coefficient. In Sec. IV, we show that T_e is the effective temperature of the junction seen by the oscillator.

The phenomenological parameters D_0 and γ_0 are related to each other by virtue of the fluctuation-dissipation theorem. Assuming that the junction and the part of the environment responsible for γ_0 and D_0 have the same temperature T, the diffusion coefficient can be generally presented in the form

$$D = m\Omega \left(\gamma_0 \coth \frac{\Omega}{2T} + \gamma_e \coth \frac{\Omega}{2T_e} \right). \qquad (2.18)$$

The master equation Eq. (2.12) contains the term proportional to the coupling constant A. A term with this structure has been obtained in previous discussions of quantum Brownian motion.[23,26] The derivation of the master equation for the oscillator (see Appendix A) shows that the main contribution to the coefficient A in Eq. (2.12) comes from virtual tunneling processes with an energy difference of initial and final states of the order of the Fermi energy, in contrast to the friction and diffusion coefficients which are controlled solely by the tunneling events in the vicinity of the Fermi surface. Besides, compared with the other terms in Eq. (2.12), the A term has different symmetry relative to time reversal, i.e., the transformation $\hat{\rho} \rightarrow (\hat{\rho})^*$ and $t \rightarrow -t$. The damping and diffusion terms, which are odd relative to time reversal, describe the irreversible dynamics of the oscillator, whereas the last term in Eq. (2.12), just like the Hamiltonian term, is time reversible. These observations give the hint that the A term is responsible for renormalization-like effects. This suggests that the A term should be treated on a different footing than the dissipative terms. Indirectly, the A term can be absorbed into the Hamiltonian dynamics at the price of having the time evolution of the oscillator described by a "renormalized" density matrix. Indeed, if we apply a (nonunitary) transformation to the density matrix, $\tilde{\hat{\rho}} = \mathcal{R}\{\hat{\rho}\}$, by acting on the density matrix $\hat{\rho}$ with the superoperator

$$\mathcal{R}\{\hat{\rho}\} = \hat{\rho} + \mu[\hat{p}, [\hat{p}, \hat{\rho}]], \qquad (2.19)$$

we can by proper choice of the parameter μ,

$$\mu = \frac{A}{2m\Omega^2}, \qquad (2.20)$$

cancel the A term in the equation for the transformed matrix $\tilde{\hat{\rho}}$. Leaving the γ and D terms intact, the counterterm is produced by the superoperator $\mathcal{R}$ acting on the Hamiltonian part of the master equation Eq. (2.12). Applied to the original A term, this procedure generates an additional contribution proportional to the product $\mu A \propto A^2$, and it can be neglected to the lowest order in the coupling, the limit which can be consistently studied.

The renormalized density matrix now obeys the master equation

$$\dot{\rho}(t) = \mathcal{K}\{\hat{\rho}\}, \qquad (2.21)$$

where

$$\mathcal{K}\{\hat{\rho}\} = \frac{1}{i\hbar}[\hat{H}_R, \rho] + \frac{\gamma}{i\hbar}[\hat{x}, \{\hat{p}, \hat{\rho}\}] - \frac{D}{\hbar^2}[\hat{x}, [\hat{x}, \hat{\rho}]], \quad (2.22)$$

up to a term quadratic in the coupling constant. For compact notation, we have introduced the superoperator $\mathcal{K}$ and dropped the tilde for marking the renormalized density matrix: thus, in the following, the renormalized density matrix will also be denoted by $\hat{\rho}$. The master equation being derived for the case of coupling to a tunnel junction is seen to be of the same *form* as for coupling linearly to a heat bath, i.e., an equilibrium state of a many-body system; the generic form of a damped quantum oscillator known from numerous investigations on quantum Brownian motion.[22] However, the diffusion term is qualitatively different from the usual case where the quantum object is coupled only to a heat bath. The nonequilibrium state of the junction, characterized by its voltage, gives rise to features not present when the coupling is simply to a many-body system in equilibrium.

We note that the superoperator $\mathcal{R}$ does not change the trace of the density matrix it operates on, and the renormalized density matrix is also normalized to unity. However, one has to keep in mind that the observables are to be calculated with the "unrenormalized" density matrix. Up to the first order in the coupling constant, the expectation value of an observable O are now given in terms of the renormalized density matrix according to

$$\langle O \rangle = \text{Tr}\left(\left(\hat{O} - \frac{A}{2m\Omega^2}[\hat{p}, [\hat{p}, \hat{O}]] \right) \hat{\rho} \right). \qquad (2.23)$$

This relation transfers renormalization from the density matrix to observables. In the language of the Feynman diagram technique, Eq. (2.23) corresponds to a vertex correction.

In this paper, we use the Markovian approximation to describe the time evolution of the density matrix. This approximation is valid in the low-frequency limit. For the dc-bias case, the characteristic frequency of time variation of the density matrix ω must be small enough to meet the condition

$$\omega \ll \omega_{max}, \quad \omega_{max} \equiv \max\left(\frac{T}{\hbar}, \frac{V}{\hbar}\right). \tag{2.24}$$

From the unconditional master equation, the characteristic frequency is seen to be determined by the friction coefficient, $\omega \sim \gamma$. This means that the coupling constant G_{xx} in Eq. (2.15) must be small enough to meet the condition $\gamma \ll \omega_{max}$.

III. CURRENT-VOLTAGE CHARACTERISTIC

The average value of the current through the junction is given in terms of the probability distribution for charge transfers, i.e.,[20]

$$I(t) = -e\frac{d}{dt}\sum_n n \,\mathrm{Tr}\,\hat{\rho}_n(t), \tag{3.1}$$

where Tr denotes the trace with respect to the degree of freedom of the coupled quantum object. However, to the lowest order in the tunneling, the average current turns out to be expressible through the charge unconditional density matrix, the reduced density matrix for the coupled quantum object. Indeed, the master equation for the charge specific density matrix then enables one to express the time derivative in Eq. (3.1) in terms of the reduced density matrix for the quantum object, the charge unconditional density matrix

$$I(t) = e\,\mathrm{Tr}\,\mathcal{J}\{\hat{\rho}(t)\}, \tag{3.2}$$

where the drift superoperator $\mathcal{J}$ is specified in Eq. (A11).

A. Contributions to the current under dc bias

For a dc bias $V = eU$, U being the applied voltage, the drift operator $\mathcal{J}$ is given by Eq. (A11), and the current, Eq. (3.2), is specified by

$$\frac{1}{e}I(t) = V\langle G\rangle_t + I_q(V) + I_p(t) \tag{3.3}$$

and is, in general, time dependent due to the coupling to the oscillator. The current consists of three physically distinct contributions. The first term is the Ohmic-like part of the current proportional to the conductance

$$\langle G\rangle_t = G_0 + 2G_x\langle x\rangle_t + G_{xx}\langle x^2\rangle_t, \tag{3.4}$$

the instantaneous value of the conductance operator, Eq. (A12), where $\langle x^n\rangle_t \equiv \mathrm{Tr}(\hat{x}^n\hat{\rho}(t))$. We note that besides the pure Ohmic term of the isolated junction, the additional terms due to the coupling to the oscillator will in general contribute to the nonlinear part of the current-voltage characteristic since the state of the oscillator will depend on the voltage. A case in question is discussed in Sec. III B, where the stationary state of the oscillator is considered.

The second term, I_q, originates from the commutator of position and momentum operators, and for this reason, we refer to it as the quantum correction to the current

$$I_q(V) = -\frac{1}{2}\widetilde{G}_{xx}\Delta_V, \tag{3.5}$$

where Δ_V is specified in Eq. (A13) or equivalently

$$\Delta_V = V + (\Omega + V)N_{\Omega+V} - (\Omega - V)N_{\Omega-V} \tag{3.6}$$

and $N_{\Omega\pm V} = 1/(e^{(\Omega\pm V)/T} - 1)$.

The last term in Eq. (3.3),

$$I_p(t) = e\hbar g_x\langle \dot{x}\rangle_t, \quad \dot{x} = \frac{\hat{p}}{m}, \tag{3.7}$$

is proportional to the average velocity of the oscillator and is present only for an asymmetric junction, $g_x \neq 0$.

The Ohmic part of the current is calculated in Sec. IV B for the stationary case. Next, we discuss the quantum correction and the dissipationless contribution to the current.

B. Quantum correction to the current

The quantum dynamics of the oscillator leads to a suppresion of the dc current as expressed by the quantum correction to the current, $I_q(V)$. Unlike the other terms in the expression for the current, Eq. (3.3), the quantum correction, $I_q(V)$, does not depend on the state of the oscillator, but only on its characteristic energy and the temperature of the junction. At low voltages, $V \ll T$, the quantum correction is linear in the voltage

$$I_q = -V\widetilde{G}_{xx}\left(\frac{1}{2} + N_\Omega - \frac{\Omega}{T}N_\Omega(N_\Omega + 1)\right), \tag{3.8}$$

where $N_\Omega = 1/(e^{\Omega/T} - 1)$. At large voltages, it reaches a constant value

$$I_q \approx -\frac{1}{2}\widetilde{G}_{xx}\Omega, \quad V \gg T, \Omega, \tag{3.9}$$

in agreement with an earlier result obtained by a technique valid at zero temperature.[14] Our approach generalizes the expression for the current to arbitrary relations between junction voltage and temperature and the frequency of the oscillator. The voltage dependence of the quantum correction to the conductance, $G_q = I_q/V$, is shown in Fig. 1 for different temperatures.

C. Dissipationless current

The last contribution in Eq. (3.3) to the current I_p is qualitatively different from the other terms. From Eq. (3.7), one sees that I_p is proportional to the velocity of the oscillator. Therefore, the corresponding transferred charge through the junction, $\delta Q_p = I_p\delta t$, is controlled by the coordinate of the oscillator: $\delta Q_p = e\hbar g_x\delta\langle x\rangle$. Being proportional to a velocity, the current contribution I_p is odd with respect to time reversal and, therefore, a dissipationless current.

The presence of the term I_p in the current, which does not depend explicitly on the voltage, means that a current through the junction can be induced just by the motion of the oscillator alone, i.e., by the time variation of a system parameter which in our case is the junction transparency. This effect is closely related to the well-known physics of quantum pumping,[27,28] but has not, to our knowledge, been discussed in the present context. The dissipationless "pumping"-like current I_p is proportional to the coupling
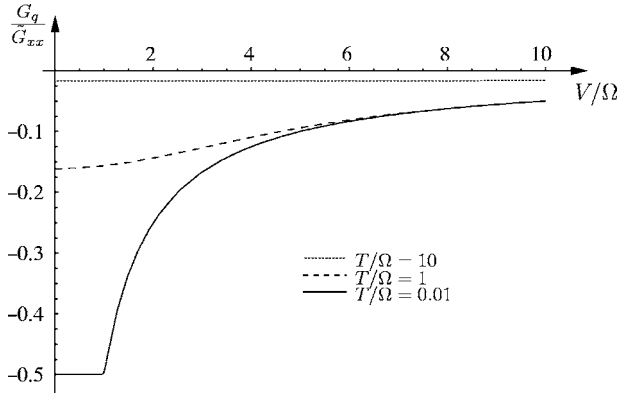
FIG. 1. Voltage dependence of the quantum correction to the junction conductance G_q. The three curves correspond to the following relations between the junction temperature T and the oscillator frequency: $T = 0.01\ \Omega$, $T = \Omega$, and $T = 10\ \Omega$. At high temperatures, $T \gg \Omega$, the quantum correction is small and shows no particular voltage dependence (dotted line). At low temperatures, $T \ll \Omega$, two distinct regions can be distinguished. At voltages lower than the oscillator frequency the quantum correction stays constant, and at voltages higher than the oscillator frequency, it is inversely proportional to the voltage.

constant g_x. This contribution to the current, thus, is only present if the tunnel junction is asymmetric. This is in concordance with the quantum pumping effect.[29] The global symmetry properties of the system, thus, crucially determine the existence and magnitude of the induced current.

A single mode oscillator driven by an external periodic force at frequency ω induces an ac current $I_\sim$ with the same frequency and a phase of the ac current rigidly following the phase of the external force. For a given amplitude of the oscillations, x_{max}, the magnitude of the ac current can be estimated as

$$I_\sim \sim \alpha_{as} e \omega, \qquad (3.10)$$

where the dimensionless parameter $\alpha_{as} = \hbar g_x x_{max}$ characterizes the effective asymmetry of the junction. In principle, α_{as} may be comparable to unity so that $I_\sim \sim e\omega$, provided that the amplitude of the oscillations x_{max} is large enough and the conductance of the junction is not too small.

One can show that in the case of an oscillator with two or more modes interacting with the junction, the corresponding term generates directed pumping of charge.[30]

IV. STATIONARY STATE PROPERTIES

In this section, we shall study the stationary state of the reduced density matrix for the oscillator in the Markovian approximation. The question arises whether the stationary state of the oscillator is a thermal equilibrium state even though the environment is in a nonequilibrium state as the junction is biased. According to Eq. (2.21), the stationary renormalized density matrix of the oscillator $\hat{\rho}_s$ is determined by the equation

$$\mathcal{K}\{\hat{\rho}_s\} = 0, \qquad (4.1)$$

and the solution is indeed of the form of a thermal density matrix, $\hat{\rho}_s \propto \exp(-\hat{H}_R/T^*)$, where the temperature of the oscillator T^* is specified by the relation

$$\coth \frac{\Omega}{2T^*} = \frac{D}{\gamma m \Omega}.$$

Using Eq. (2.18), the temperature of the oscillator is related to the environment temperature and the voltage bias according to

$$\coth \frac{\Omega}{2T^*} = \frac{\gamma_0}{\gamma_0 + \gamma_e} \coth \frac{\Omega}{2T} + \frac{\gamma_e}{\gamma_0 + \gamma_e} \coth \frac{\Omega}{2T_e}. \qquad (4.2)$$

The average occupation number for the oscillator N^* is given by the Bose function, $N^* = 1/(e^{\Omega/T^*} - 1)$, and seen to be populated separately by the interaction with the two environments

$$N^* = \frac{\gamma_0}{\gamma} N_\Omega + \frac{\gamma_e}{\gamma} N_e, \qquad (4.3)$$

where $N_e = 1/(e^{\Omega/T_e} - 1)$.[31]

We observe that the oscillator acquires the temperature of the bath, $T^* \approx T$, if the interaction with the junction is weak and γ_0 is the dominant contribution to the friction, $\gamma_0 \gg \gamma_e$. The general case and the opposite limit, where the dynamics of the junction is dominating, $\gamma_e \gg \gamma_0$, we proceed to consider.

A. Oscillator heating

When the oscillator is well isolated and the interaction with the junction dominates, $\gamma_e \gg \gamma_0$, the oscillator attains, according to Eq. (4.2), the effective temperature of the junction T_e, as given by Eq. (2.17). As expected, in the absence of a bias voltage across the junction, $V = 0$, the temperature of the oscillator equals that of the junction irrespective of its temperature. When the junction is biased, the oscillator is generally heated except at zero temperature and low voltages, and we first discuss the case of a junction at zero temperature.

At zero junction temperature, $T = 0$, we must distinguish two voltage regions. If the voltage is smaller than the frequency of the oscillator, $V < \Omega$, the temperature of the oscillator is also zero, $T^* = 0$, independent of the voltage as it follows from Eq. (4.2). In this regime, the interaction with the tunneling electrons is unable to excite the oscillator from its ground state. Heating can only take place beyond the voltage threshold given by the oscillator frequency. If instead the voltage is larger than the frequency of the oscillator, $V > \Omega$, the temperature of the oscillator T^* is determined by the following relation to the voltage

$$\tanh \frac{\Omega}{2T^*} = \frac{\Omega}{|V|}. \qquad (4.4)$$

At high voltages, $V \gg \Omega$, the temperature of the oscillator approaches half the bias voltage, $T^* = V/2$, in agreement with the result obtained in a previous study where the temperature of the junction was assumed to vanish.[14]
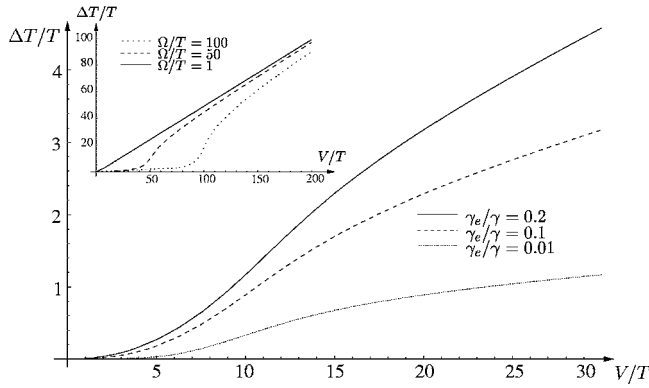
FIG. 2. Heating of the oscillator, its excess temperature, $\Delta T = T^* - T$, as a function of the bias voltage for the temperature $T = 0.1\,\Omega$, and different ratios of γ_e/γ, covering from moderate to strong external coupling. The influence of the external damping reduces the heating effect. The inset shows the heating with negligible external damping ($\gamma_e/\gamma = 1$) as a function of the bias voltage for different junction temperatures. At junction temperatures that are low compared to the oscillator frequency, $T \ll \Omega$, two regions of voltage dependence can be distinguished with a rapid switch as the voltage passes the value of the frequency of the oscillator. If the bias voltage is smaller than the oscillator frequency, $V < \Omega$, the oscillator temperature T^* depends only weakly on the voltage. In the region where the voltage exceeds the oscillator frequency, $V > \Omega$, the oscillator temperature approaches $V/2$.

The heating of the oscillator, its excess temperature, $\Delta T = T^* - T$, as a function of the bias, is shown in Fig. 2, both for the case where the coupling to the junction dominates and the opposite case of dominating external damping. The effect of the external damping is shown for moderate to strong external coupling, $\gamma/\gamma_e = 5, 10, 100$. Increasing the coupling to the external heat bath leads to the suppression of the heating of the oscillator, the additional environment acting as a heat sink. In the case where the coupling to the junction dominates, the inset shows that at low temperatures, the oscillator is not excited at voltages below the frequency of the oscillator.

At high voltages, $V \gg \Omega, T$, in the shot noise regime, the temperature of the oscillator T^* can be found from Eq. (4.4). Just as in the case of vanishing junction temperature, the oscillator temperature approaches half the bias voltage, $T^* = V/2$, at large bias, and these results generalize previous studies which were limited to zero temperature and high voltages, $V \gg \Omega$.[14,16]

In the quest for using tunnel junctions to measure the position of a coupled object with the ultimate precision set by the uncertainty principle,[5–8] it is important to take into account that the measuring, involving a finite voltage, will invariably heat the oscillator. In this respect, the presence of the additional heat bath, described by the coupling γ_0, is important. For example, envisioning the oscillator has been cooled to a temperature T much lower than Ω and a voltage has been turned on. In order to obtain an appreciable signal, the voltage must be larger than Ω. The oscillator will then in a time span of the order of γ^{-1} be heated and attain a temperature for which the average number of quanta in the oscillator is

$$N^* = \frac{\gamma_e}{2\gamma} \frac{|V| - \Omega}{\Omega}. \tag{4.5}$$

Estimating the oscillator temperature, we have $T^* \sim \Omega/\ln(\gamma/\gamma_e)$. A strong environmental coupling can thus be beneficial for retaining the oscillator in the ground state.

B. *I-V* characteristic

In the stationary state, the dc current $I(V)$, Eq. (3.3), is conveniently written in the form

$$\frac{1}{e}I(V) = VG_0 + \frac{1}{2}\widetilde{G}_{xx}(2VN^* + (\Omega - V)N_{\Omega-V} - (\Omega + V)N_{\Omega+V}). \tag{4.6}$$

As before, N^* is the occupation number of the oscillator at the temperature T^*. In the stationary state, only the coupling constant G_{xx} is effective.

The first term in Eq. (4.6) is the current through the isolated junction, and the remaining term describes the influence of the oscillator on the current. It is interesting to consider the latter in the limit of vanishing environment temperature, $T = 0$. At zero temperature, we must distinguish two voltage regimes. If the voltage is smaller than the frequency of the oscillator, $V < \Omega$, we observed in Sec. IV A that the oscillator remains in the ground state, $N^* = 0$, and the average current turns out to be equal to that of an isolated junction, $I = eG_0V$. One observes that the effect of zero point fluctuations present in the conductance, Eq. (3.4), is exactly canceled by the quantum correction, Eq. (3.5). This lack of influence of zero point fluctuations is expected since the oscillator in its ground state is inert to the tunneling electrons for such low bias.

If the voltage is larger than the oscillator frequency, $V > \Omega$, the slope of the *I-V* characteristic, at $T = 0$, abruptly increases, $I = VG_0 + (V - \Omega)\widetilde{G}_{xx}/2$.

At arbitrary temperatures, the linear conductance of the junction, $G = I/V$, $V \to 0$, is given by

$$\frac{1}{e^2}G = G_0 + \widetilde{G}_{xx}\frac{\Omega}{T}N_\Omega(N_\Omega + 1). \tag{4.7}$$

To derive this formula, we recall that in the limit of vanishing bias, $V \to 0$, the oscillator attains the temperature of the junction, $T^* = T$.

At high temperatures, $T \gg V, \Omega$, the quantum correction to the current, and, thereby, the nonlinear quantum corrections, vanish. We obtain the result

$$\frac{1}{e}I(V) = V(G_0 + G_{xx}\langle x^2\rangle_*), \quad \langle x^2\rangle_* = \frac{T^*}{m\omega_0^2}, \tag{4.8}$$

where $\langle x^2\rangle_*$ is the mean square of the oscillator coordinate at temperature T^*. This result is to be expected from a classical oscillator in thermal equilibrium influencing the conductance of a tunnel junction. The *I-V* characteristic is in this regime nonlinear due to the voltage dependence of T^*, Eq. (4.2).

V. CURRENT NOISE

In this section, we use the charge projection technique to develop the description of the statistical properties of the current of a tunnel junction coupled to a quantum object. The discussion will be kept quite general before we, in Sec. V A, specialize to the case of a harmonic oscillator, the nanoelectromechanical model of Sec. II. We shall show how the charge dynamics, described by the master equation for the charge specific density matrix, can be used to obtain the statistical properties of the junction current, such as the noise power spectrum. The prerequisite for the success of this endeavor is that for the considered low transparency tunnel junction, the charge representation in fact provides the probability distribution for the charges *transferred* through the junction.[20,21]

A. Current noise in the charge representation

The probability, $p_n(t)$, for n charge transfers in time span t is according to Eq. (2.8) given by $p_n(t) = \text{Tr } \hat{\rho}_n(t)$, where $\hat{\rho}_n(t)$ is the charge specific density matrix, Eq. (2.7). The charge-transfer probability distribution specifies the stochastic process of charge transfers, $n(t)$. The variance of the charge fluctuations,

$$\langle\langle n^2(t)\rangle\rangle = \langle n^2(t)\rangle - \langle n(t)\rangle^2 \tag{5.1}$$

is defined in terms of the moments of the probability distribution of charge transfers

$$\langle n^r(t)\rangle = \sum_n n^r p_n(t), \quad p_n(t) = \text{Tr}(\hat{\rho}_n(t)). \tag{5.2}$$

To express the statistical properties of the current in terms of the probabilities for charge transfers, we inherit the stochastic current process, $i(t)$, through its relation to the charge transfer process

$$n(t) = \int_0^t dt'\, i(t'). \tag{5.3}$$

The average current, given by $\langle i(t) = d\langle n(t)\rangle/dt$, is in accordance with Eq. (3.1). The variance of the charge fluctuations are expressed via the current fluctuations according to

$$\langle\langle n^2(t)\rangle\rangle = \int_0^t dt_1 \int_0^t dt_2 \langle \delta i(t_1)\, \delta i(t_2)\rangle, \tag{5.4}$$

where $\delta i(t) = i(t) - \langle i\rangle$, and $\langle i\rangle$ is the average dc current. In the following, we shall consider the stationary state. Stationary current noise is characterized by the current-current correlator

$$S(\tau) = \langle \delta i(t + \tau)\, \delta i(t)\rangle. \tag{5.5}$$

Inserting this expression into Eq. (5.4), one obtains

$$\langle\langle n^2(t)\rangle\rangle = 2 \int_0^t d\tau (t - \tau) S(\tau). \tag{5.6}$$

This expression allows one to relate the charge and current fluctuations.

Taking time derivatives of Eq. (5.6) gives

$$\frac{d\langle\langle n^2(t)\rangle\rangle}{dt} = 2 \int_0^t d\tau S(\tau), \tag{5.7}$$

and

$$S(t) = \frac{1}{2} \frac{d^2\langle\langle n^2(t)\rangle\rangle}{dt^2}, \tag{5.8}$$

i.e., the current-current correlator equals the second derivative of the variance of charge transfers. This relation allows one to calculate the current-current correlator, $S(t)$, by evaluating the charge fluctuations using the master equation for the charge specific density matrix.

Eventually, interest is in the current noise power spectrum, S_ω, given by

$$S_\omega = 4 \int_0^\infty dt \cos(\omega t) S(t), \tag{5.9}$$

where ω is the frequency at which the noise is measured.[32]

We observe that the zero frequency noise power, according to Eqs. (5.7) and (5.9), can be calculated from the general relation

$$S_{\omega=0} = 2 \left. \frac{d\langle\langle n^2(t)\rangle\rangle}{dt} \right|_{t\to\infty}, \tag{5.10}$$

i.e., as the rate of change of the charge variance at large times.

In the present approach, it is convenient to calculate directly the current-current correlator, as is done in Sec. V B and Appendix B. Only at the end, we then transform to obtain the noise power spectrum. However, we note that the approach is equivalent to employing the widely used Mac-Donald formula.[33]

B. Current-current correlator

In this section, we show how the master equation for the charge specific density matrix can be used to obtain the noise power spectrum of the current. A convenient feature of the method is that it allows one directly to obtain the time dependence of the current noise.

The probability distribution of charge transfers, $p_n(t)$, is obtained from the master equation for the charge specific density matrix given the initial condition corresponding to a state of definite initial charge

$$\hat{\rho}_n(t = 0) = \delta_{n,0}\hat{\rho}_s \tag{5.11}$$

at the time when the charge counting starts, the initial time $t=0$. We are interested in the noise properties of the stationary state and, therefore, the stationary density matrix of the oscillator, the thermal state $\hat{\rho}_s$, enters the initial condition.

In the following, we shall treat the charge specific dynamics in the Markovian approximation. The charge specific density matrix, $\hat{\rho}_n(t)$, is obtained as the solution of the master equation, Eq. (2.9). For notational convenience, we write the charge specific master equation in the form

$$\frac{d\hat{\rho}_n}{dt} = \mathcal{K}\{\hat{\rho}_n\} + \mathcal{D}\{\hat{\rho}_n''\} + \mathcal{J}\{\hat{\rho}_n'\}, \qquad (5.12)$$

where $\mathcal{K}$ is the superoperator introduced in Eq. (2.21). Although the $\hat{\rho}_n$'s are time dependent, the unconditional density matrix, $\hat{\rho} = \Sigma_n \hat{\rho}_n(t)$, remains equal to the thermal state, $\hat{\rho}_s$, by virtue of its stationarity property, $\mathcal{K}\{\hat{\rho}_s\} = 0$. Our goal is now to evaluate the variance of the charge transfers and, thereby, the current-current correlator with the help of Eq. (5.8).

The rate of change of the first charge moment, i.e., the dc current according to Eq. (3.1), becomes in the stationary state

$$\frac{1}{e}I = -\operatorname{Tr}(\mathcal{J}\{\hat{\rho}_s\}), \qquad (5.13)$$

following from Eq. (3.2) and the stationarity property of the density matrix for the coupled quantum object, $\hat{\rho}(t) = \hat{\rho}_s$. The dc current was calculated in Sec. IV B.

It readily follows from Eq. (5.12) that the time derivative of the variance of charge transfers, $\langle\langle n^2(t)\rangle\rangle$, can be presented in the form

$$\frac{d}{dt}\langle\langle n^2(t)\rangle\rangle = 2\operatorname{Tr}(\mathcal{D}\{\hat{\rho}_s\}) - 2\operatorname{Tr}(\mathcal{J}\{\delta\hat{N}(t)\}), \quad (5.14)$$

where $\delta\hat{N}(t)$ denotes the traceless matrix

$$\delta\hat{N}(t) = \sum_n (n - \langle n(t)\rangle)\hat{\rho}_n(t). \qquad (5.15)$$

We observe that only the truncated density matrix, $\delta\hat{N}(t)$, is needed to calculate the noise.

Comparing Eqs. (5.14) and (5.7), one concludes that the current-current correlator has a δ function like singularity at the initial time, $t=0$, where the charge counting starts. Indeed, the r.h.s. of Eq. (5.14) has a finite limit as $t \rightarrow 0$ given by the first term, since the second term initially vanishes, $\delta\hat{N}(t=0) = 0$. For this result to be compatible with Eqs. (5.7) and (5.8), the current-current correlator, $S(t)$, must have the following structure:

$$S(t) = S_1(t) + S_2(t), \qquad (5.16)$$

the sum of a singular contribution,

$$S_1(t) = 2\operatorname{Tr}(\mathcal{D}\{\hat{\rho}_s\})\delta(t), \qquad (5.17)$$

where $\delta(t)$ denotes a function peaked at $t=0$ and normalized according to the condition $\int_0^\infty dt\,\delta(t) = \frac{1}{2}$, and a regular part given by

$$S_2(t) = -\operatorname{Tr}(\mathcal{J}\{\delta\hat{I}(t)\}), \qquad (5.18)$$

where $\delta\hat{I}$ denotes the matrix, $\delta\hat{I} = (d/dt)\delta\hat{N}$. The finite time correlation of the current described by the regular part $S_2(t)$ is solely due to the interaction with the quantum object, as follows from $\delta\hat{I}(t)$ being traceless. We note here that the δ-function singularity, which would provide noise at arbitrary high frequencies, is an artifact of the Markovian approximation.

The task of calculating the time-dependent current noise is thus reduced to obtaining the time derivative of the charge-averaged density matrix, $\delta\hat{N}(t)$, given in Eq. (5.15). From the master equation for the charge specific density matrix, one obtains the following equation for $\delta\hat{I}(t)$:

$$\frac{d}{dt}\delta\hat{I} = \mathcal{K}\{\delta\hat{I}\}, \qquad (5.19)$$

and the initial condition

$$\delta\hat{I}|_{t=0} = -\delta\mathcal{J}\{\hat{\rho}_s\}. \qquad (5.20)$$

Here, the superoperator $\delta\mathcal{J}$ acts on its argument matrix according to

$$\delta\mathcal{J}\{X\} = \mathcal{J}\{X\} - X(\operatorname{Tr}\mathcal{J}\{X\}). \qquad (5.21)$$

We note, that acting on a matrix X with unit trace, $\operatorname{Tr} X = 1$, the superoperator $\delta\mathcal{J}$ returns a traceless matrix. The dynamics of the charge averaged quantity $\delta\hat{I}$ is thus identical to that of the charge unconditional density matrix of the oscillator.

The formal solution to Eq. (5.19) can be written in terms of the time evolution superoperator for the charge unconditional density matrix of the oscillator

$$\mathcal{U}_t = e^{\mathcal{K}t} \qquad (5.22)$$

as

$$\delta\hat{I}(t) = -\mathcal{U}_t\{\delta\mathcal{J}\{\hat{\rho}_s\}\} \qquad (5.23)$$

and the regular part of the current-current correlator can be written on the form

$$S_2(t) = \operatorname{Tr}(\delta\mathcal{J}\{\mathcal{U}_t\{\delta\mathcal{J}\{\hat{\rho}_s\}\}\}). \qquad (5.24)$$

Here, $\mathcal{J}$ in Eq. (5.18) has been replaced for $\delta\mathcal{J}$ in Eq. (5.18); the replacement is valid under the trace operation since $\operatorname{Tr}\delta\hat{I}(t) = 0$. Combined with the singular part in Eq. (5.16), this gives the general expression in the Markovian approximation for the current-current correlator of a tunnel junction interacting with a quantum system in its stationary state. The current noise correlator has thus conveniently been written with the help of the Markovian superoperators $\mathcal{K}$, $\mathcal{J}$, and $\mathcal{D}$.

In Sec. VI, we shall turn to calculating the noise properties for the case of the nanoelectromechanical device described in Sec. II.

VI. NOISE POWER SPECTRUM

We now turn to calculate the current-current correlator of the tunnel junction coupled to the harmonic oscillator as described by the model of Sec. II. Taking advantage of the general analysis in the Markovian approximation presented above in Sec. V B, the current-current correlator can be written in the following form:

$$S(t) = S_1(t) + S_x(t) + S_{x^2}(t). \qquad (6.1)$$

Here S_1 is the singular part defined in Eq. (5.17) and specified in Eq. (B11). The regular contribution is given by sec-
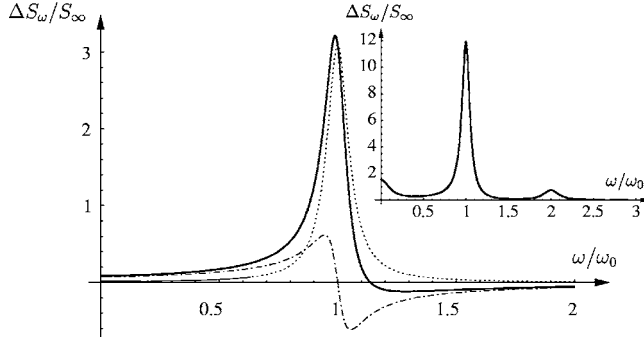
FIG. 3. The excess noise power spectrum, $\Delta S_\omega = S_\omega - S_\infty$, due to an oscillator coupled to an asymmetric junction is shown by the full line for the parameter values $V/\Omega = 10$, $T/\Omega = 0.01$, $\gamma_0/\omega_0 = 0.05$, and conductances $G_0 = 1$, $\widetilde{G}_x = 0.07$, $\widetilde{g}_x = 0.05$, and $\widetilde{G}_{xx} = 0.005$ giving for the electronic coupling constant $\gamma_e/\omega_0 = 0.005$. The dotted line shows the contribution symmetric in the voltage, S_ω^+, and the dashed-dotted line shows the contribution asymmetric in the voltage, S_ω^-. The inset shows the noise power spectrum for the high temperature $T = 100\,\Omega$, but otherwise the same set of parameters, a regime where the peaks at zero and twice the oscillator frequency are also visible.

ond and third terms on the right in Eq. (6.1), as obtained by inserting the expression for the drift superoperator $\mathcal{J}$, Eq. (A11), into the expression for the regular contribution, Eq. (5.24), giving

$$S_x(t) = 2VG_x x_J(t) + g_x \frac{\hbar}{m} p_J(t) \tag{6.2}$$

and

$$S_{x^2}(t) = VG_{xx} x_J^2(t), \tag{6.3}$$

where the time dependent quantities are given by

$$X_J(t) = \text{Tr}(\hat{X}\mathcal{U}_t\{\delta\mathcal{J}\{\hat{\rho}_s\}\}), \tag{6.4}$$

with $\hat{X} = \hat{x}, \hat{p}, \hat{x}^2$, respectively. It is readily checked that the latter quantities evolve in time in accordance with their corresponding classical equations of motion for a damped oscillator with initial conditions given by

$$X_J(0) = \text{Tr}(\hat{X}\delta\mathcal{J}\{\hat{\rho}_s\}), \quad \hat{X} = \hat{x}, \hat{p}, \hat{x}^2, \hat{p}^2. \tag{6.5}$$

The calculation of these quantities are presented in Appendix B, giving, according to Eqs. (6.2) and (6.3), an explicit expression for the current-current correlator $S(t)$ and, thereby, according to Eq. (5.9), for the noise power S_ω. Fourier transforming the current-current correlator gives, according to Eqs. (B9)–(B11), peaks in the noise power as well as a constant upshift in the noise floor, the noise pedestal. The noise power spectrum is displayed in the inset in Fig. 3. As expected, there is a pronounced peak at the frequency of the oscillator, and in addition, two side peaks each shifted by frequency of the oscillator, one at zero frequency and one at twice the oscillator frequency.

Below, we analyze the noise in two frequency regions: (i) low frequency noise at frequencies $\omega \lesssim \gamma$; and (ii) noise in the vicinity of the oscillator resonance frequency $\omega \approx \omega_0$ and $\omega \approx 2\omega_0$. We examine the voltage and temperature features of the noise power.

A. Low-frequency noise

Let us consider low-frequency noise, at frequencies of the order of the damping rate and lower, $\omega \lesssim \gamma$. Then the noise power spectrum is given by the Fourier transform of the correlation functions $S_1(t)$ and $S_{x^2}(t)$ in Eqs. (B10) and (B11), respectively, giving

$$S_\omega = S^{(0)} + S^{(1)} + S_\omega^{(2)}, \tag{6.6}$$

where

$$S^{(0)} = 2G_0 V \coth \frac{V}{2T} \tag{6.7}$$

is the low frequency, $\omega \ll V$, white Nyquist or Schottky noise of the isolated junction, and $S^{(1)}$ is the correction to the white noise due to the interaction with the oscillator

$$S^{(1)} = 2\widetilde{G}_{xx}\Omega(N^*(N_e + 1) + N_e(N^* + 1)). \tag{6.8}$$

Together, these two contributions form the noise pedestal, $S_\infty = S^{(0)} + S^{(1)}$. The frequency-dependent part, $S_\omega^{(2)}$, becomes at low frequencies

$$S_\omega^{(2)} = \widetilde{G}_{xx}\frac{4\gamma\gamma_e}{\omega^2 + 4\gamma^2}\frac{V}{\Omega}(2VN^{*2} + (V - \Delta_V)(2N^* + 1)). \tag{6.9}$$

The low-frequency noise is displayed explicitly proportional to the coupling to the electronic tunnel junction environment as we have taken advantage of the relation $\gamma_e = \widetilde{G}_{xx}\Omega$. The width of the low-frequency peak is twice the damping rate 2γ. At zero bias, $V = 0$, where $S_\omega^{(2)} = 0$ and the oscillator and effective junction temperatures equal the environment temperature, leaving $N^* = N_e = N_\Omega$, one recovers the fluctuation-dissipation relation for the noise power, $S_{\omega=0} = 4TG$, where G is the linear conductance of the junction in the presence of the interaction with the oscillator, i.e., given by Eq. (4.7).

In the following, we discuss the features of the low-frequency excess noise, the noise due to the coupling to the oscillator, and in particular the noise peak height at zero frequency, in the limits of temperatures high and low compared to the oscillator frequency.

1. Low-temperature noise

First, we consider the low-frequency noise at low temperatures, $T \ll \Omega$. As expected, no excess noise is according to Eq. (6.6) generated by the oscillator at zero temperature and voltages below the oscillator frequency, $|V| < \Omega$, where the oscillator cannot be excited from its ground state. Indeed, in the region of low temperatures, $T \ll \Omega$, and low voltages, $V < \Omega$, the oscillator is nonresponsive and the excess noise, $S^{(1)}$ and $S_\omega^{(2)}$, vanishes exponentially in Ω/T below the activation energy Ω.

Close to the noise onset threshold, $|V| \sim \Omega$, in the narrow region, $||V| - \Omega| \ll T$, the peak height relative to the pedestal rises linearly with temperature

$$S^{(2)}_{\omega=0} = \frac{\gamma_e}{\gamma} T \widetilde{G}_{xx}.$$ (6.10)

At voltages much higher than threshold, $|V| \gg \Omega$, the peak height relative to the pedestal becomes

$$S^{(2)}_{\omega=0} = 2\frac{\widetilde{G}^2_{xx}}{\gamma} V^2 \left(\frac{1}{2} + \frac{\gamma_e}{\gamma} \frac{|V|}{2\Omega} + \left(\frac{\gamma_e}{\gamma} \frac{V}{2\Omega} \right)^2 \right).$$ (6.11)

The zero frequency noise is proportional to V^4 if the effective coupling to the electronic environment is appreciable, i.e., the ratio γ_e/γ is not too small. In the high voltage limit, the oscillator is in the classical regime, but with the oscillator temperature given by $T^* = |V|/2$ as discussed in Sec. IV.

2. High-temperature noise

At temperatures higher than the oscillator frequency, $T \gg \Omega$, we can distinguish two voltage regimes. At low voltages, $V \ll T$, the peak height scales quadratically in both the temperature and voltage

$$S^{(2)}_{\omega=0} = \frac{2}{\gamma} \widetilde{G}^2_{xx} V^2 \left(\frac{T}{\Omega} \right)^2,$$ (6.12)

and we recall that the oscillator temperature equals the junction temperature, $T^* = T$.

At high voltages, $V \gg T$, the peak height becomes

$$S^{(2)}_{\omega=0} = \frac{2}{\gamma} V^2 \widetilde{G}^2_{xx} \left(\frac{\gamma_0}{\gamma} \frac{T}{\Omega} + \frac{\gamma_e}{\gamma} \frac{|V|}{2\Omega} \right)^2.$$ (6.13)

At high temperatures, the oscillator is in the classical regime and the average occupation number depends linearly on the oscillator temperature. The peak height, proportional to the fluctuations in the oscillator position squared, is proportional to the square of the average occupation number and is, therefore, proportional to the square of the oscillator temperature.

B. High-frequency noise

Next, we investigate the properties of the peaks in the noise power spectrum occurring at finite frequencies, at the oscillator frequency, $\omega \approx \omega_0$, and its harmonic, $\omega \approx 2\omega_0$. The Markovian approximation allows us to consider the high-frequency noise only under the condition, Eq. (2.24), that the frequency is much smaller than the maximum value of the voltage or the temperature, and for frequencies in question, this means that $\max(T, V) \gg \Omega$ for consistency. The inset in Fig. 3 shows the frequency dependence of the noise power spectrum, Eq. (5.9), in the case of high temperatures, $T \gg \Omega$. The noise power displays three peaks. The noise power spectrum consists at $\omega \approx \omega_0$ of a Lorentzian part, as given by the Fourier transform of Eq. (B9), with a width given by the damping rate γ and an asymmetric part specified by Eq. (B9) and present only for an asymmetric junction. At $\omega \approx 2\omega_0$, the noise power spectrum is according to Eq. (B10), a Lorentz-

ian with a width given by twice the damping rate 2γ. We now discuss these peak heights at high and low temperatures.

1. High-frequency noise at high temperatures

At high temperatures, $T \gg V \gg \Omega$, the height of the peak at the oscillator frequency relative to the pedestal depends linearly on temperature

$$S^{(2)}_{\omega=\omega_0} = 2\frac{T}{\gamma} \left[\widetilde{G}^2_x \frac{4V^2}{\Omega} - \widetilde{g}^2_x \Omega \right]$$ (6.14)

and is determined by the conductances $\widetilde{G}_x$ and $\widetilde{g}_x$.

At double the oscillator frequency, the peak height depends quadratically on temperature

$$S^{(2)}_{\omega=2\omega_0} = \frac{1}{\gamma} \widetilde{G}^2_{xx} \frac{V^2 T^2}{\Omega^2}$$ (6.15)

and just as the peak at zero frequency determined by the conductance $\widetilde{G}_{xx}$. We note that its height is half that of the peak at zero frequency, Eq. (6.12).

The expressions for the excess noise, Eqs. (6.14) and (6.15), are in fact also valid at low voltage. Contrary to the peaks at $\omega = 0$ and $\omega = 2\omega_0$, which vanish in the absence of voltage, the excess noise power at $\omega \approx \omega_0$ is, therefore, finite for an asymmetric junction even at zero voltage and, according to Eq. (6.14), in fact negative. An asymmetric junction with a Q factor much larger than T/Ω can thus at zero voltage lead to a suppression of the noise power below that of an isolated junction.

2. High-frequency noise at low temperatures

At high voltages and low temperatures, $V \gg \Omega \gg T$, the peak height at the oscillator frequency becomes

$$S^{(2)}_{\omega=\omega_0} = \frac{2V}{\gamma} \left[2\widetilde{G}^2_x V \left(1 + \frac{\gamma_e}{\gamma} \frac{|V|}{\Omega} \right) - \widetilde{g}^2_x \Omega \left(1 - \frac{\gamma_e}{2\gamma} \right) \right],$$ (6.16)

and the peak height at twice the oscillator frequency becomes

$$S^{(2)}_{\omega=2\omega_0} = \frac{1}{\gamma} \widetilde{G}^2_{xx} V^2 \frac{\gamma_e}{\gamma} \frac{|V|}{2\Omega} \left(\frac{\gamma_e}{\gamma} \frac{|V|}{2\Omega} + 1 \right).$$ (6.17)

The noise can be large due to the high oscillator temperature.

3. Noise asymmetry

A striking feature of the finite frequency noise is the contribution proportional to $g_x G_x$. It is *odd* relative to the sign of the voltage and does *not* depend on the state of the oscillator [see Eq. (B10)]. This term, which is only present for an asymmetric junction, $g_x \neq 0$, does not contribute to the peak height at the oscillator frequency, but provides the asymmetry of the peak in the frequency region around the oscillator frequency, $\omega = \omega_0$. Separating the even and odd voltage contributions in the noise power, $S^{\pm}_{\omega} = \frac{1}{2}(S_{\omega}(V) \pm S_{\omega}(-V))$, the odd contribution becomes

$$S_\omega^- = \widetilde{g}_x \widetilde{G}_x (F_{\omega_0}(\omega) - F_{-\omega_0}(\omega))$$

$$\times \left(V^2 \coth \frac{V}{2T} + \Omega V(2N_e + 1) - \frac{\Omega}{2}\Delta_V \right) \quad (6.18)$$

with the frequency dependence given by the function

$$F_{\omega_0}(\omega) = \frac{2(\omega_0 - \omega)}{\gamma^2 + (\omega - \omega_0)^2}.$$

The noise power spectrum is displayed by the full line in Fig. 3 for the temperature $T=0.01\Omega$, where only the peak at the oscillator frequency is appreciable. The even part in voltage of the noise power, S_ω^+, displayed by the dotted line, is a symmetric function of the frequency relative to the frequency of the oscillator, and the odd part in the voltage, S_ω^-, displayed by the dashed-dotted line, is an antisymmetric function of the frequency relative to the oscillator frequency. If the voltage is reversed, the frequency dependence of the asymmetric part is mirrored around the frequency $\omega=\omega_0$.

In contrast to an isolated junction, the noise power of the coupled junction-oscillator system shows asymmetry in the voltage. This behavior is a novel feature that arises when an asymmetric junction is coupled to an additional degree of freedom.

VII. CONCLUSIONS

We have applied the charge projection technique to obtain the charge specific dynamics of a continuous quantum degree of freedom coupled to a tunnel junction. The master equation for the charge specific density matrix has been derived, describing the charge conditioned dynamics of the coupled object as well as the charge transfer statistics of the junction. The method allows evaluating at any moment in time the joint probability distribution describing the quantum state of the object and the number of charges transferred through the junction.

The approach, generally valid for any quantum object coupled to the junction, has been applied to the generic case of a nanoelectromechanical system, a harmonic oscillator coupled to the charge dynamics of a tunnel junction. In this regard, it is important that the method allows inclusion of a thermal environment in addition to the electronic environment of the tunnel junction since nanoresonators are invariably coupled to a substrate. The oscillator dynamics, described by the reduced density matrix for the harmonic oscillator, the charge specific density matrix traced with respect to the charge index, has upon a renormalization been shown to satisfy a master equation of the generic form valid for coupling to a heat bath. Even though the electronic environment is in a nonequilibrium state, the master equation is of the Caldeira-Leggett type, consisting of a damping and a fluctuation term. Though the coefficients of the terms are not related by the equilibrium fluctuation-dissipation relation, the fluctuation term originating from the coupling to the junction is of the steady-state fluctuation-dissipation type, containing the current noise power spectral function of the isolated junction taken at the frequency of the oscillator. The diffusion parameter is thus determined by all energy scales of the

problem including temperature, voltage, and oscillator frequency. The presence of an environment in a nonequilibrium state thus leads to features which are absent when the oscillator is only coupled to a heat bath.

The Markovian master equation for the charge specific density matrix has been used to calculate the current. In general, the average junction current consists of an Ohmic term, however, with a conductance modified due to the coupling to the oscillator dynamics, a quantum correction, and a dissipationless ac current only present for an asymmetric junction and proportional to the instantaneous velocity of the oscillator. The latter term does not depend on voltage explicitly and is an example of an effect similar to quantum pumping.

The stationary state of the oscillator has been shown to be a thermal state even though the environment is in a nonequilibrium state. Therefore, the only effect of the bias is heating of the junction. Thus, the stationary oscillator state is a thermal equilibrium state, though in equilibrium at a higher temperature than that of the environment if the junction is in a nonequilibrium state of finite voltage. This is a back-action effect of the measuring device, the tunnel junction, on the oscillator. At zero temperature and voltages below the oscillator frequency, the oscillator remains, to lowest order in the tunneling, in its ground state, and the dc current equals that of an isolated junction. The coupling of the oscillator to the additional heat bath, described by the coupling constant γ_0, is shown to be beneficial for avoiding heating of the oscillator due to a finite voltage. This is of importance for application of quantum point contacts and tunnel junctions to position measurements aiming at a precision reaching the quantum limit.

The charge projection method has been used to infer the statistical properties of the junction current from the charge probability distribution. For example, the noise power spectrum is specified in terms of the variance of the charge distribution. The master equation for the charge specific density matrix, therefore, can be used to obtain the current-current correlator directly, and this has been done explicitly in the Markovian approximation. The excess noise power spectrum due to the coupling to the oscillator consists of a main peak located at the oscillator frequency and two smaller peaks located at zero frequency and twice the oscillator frequency, respectively. The peaks at zero frequency and at twice the oscillator frequency have heights proportional to the coupling constant G_{xx} squared, whereas the height of the peak at the oscillator frequency is proportional to the coupling constants G_x and g_x squared. The voltage and temperature dependencies of the peaks has been examined in detail.

For an asymmetric junction, the noise power spectrum contains a term with the striking feature of being an odd function of the voltage and independent of the state of the oscillator. Contrary to the case of a symmetric junction, the coupling of an oscillator to an asymmetric junction with temperature higher than the oscillator frequency results, even at zero voltage, in a suppression of the noise power at the oscillator frequency, the excess noise power being negative. For an asymmetric junction, the noise power at $\omega \approx \omega_0$ can thus be suppressed below the Nyquist level of the isolated junction.

The Markovian approximation employed to calculate the noise power cannot be validated at arbitrary frequencies

compared to temperature or voltage. Not surprisingly, naive attempts to extend expressions beyond the Markovian applicability range, Eq. (2.24), leads to unphysical results for the noise. For example, at zero temperature and voltages below the oscillator frequency, a spurious noise power arises even for the oscillator in the ground state.

APPENDIX A: CHARGE SPECIFIC MASTER EQUATION

In a previous paper, we introduced the charge representation for a general many-body system.[20] The approach is based on the use of charge projectors previously introduced in the context of counting statistics.[21] In the charge representation, the dynamics of a quantum object coupled to a many-body system is described by the charge specific density matrix

$$\hat{\rho}_n(t) = \mathrm{Tr}_{el}(\mathcal{P}_n \rho(t)), \tag{A1}$$

where $\rho(t)$ is the full density matrix for a many-body system and a quantum object coupled to it. Tr_{el} denotes the trace with respect to the degrees of freedom of the many-body system, which, in the following, is assumed to be the conduction electrons of a tunnel junction. The charge projection operators $\mathcal{P}_n$, which project the state of the system onto its component for which exactly n electrons are in a specified region of space, have been discussed in detail earlier.[20,21] There, we discussed the circumstances under which the charge index n can be interpreted as the number of charges *transferred* through the junction, and the charge projector method, thus, provides a basis for charge counting statistics in the cases where the distribution function for transferred charge is a relevant concept. In this case, the charge specific density matrix allows the evaluation, at any moment in time, of the joint probability of the quantum state of the object and the number of charges transferred through the junction. In the previous paper, the non-Markovian master equation for the charge specific density matrix for an arbitrary quantum object coupled to a low transparency tunnel junction was derived.[20] A non-Markovian master equation is less tractable for calculational purposes and the Markovian approximation is employed in the present paper. This is quite sufficient for calculations of average properties, such as the average current through the tunnel junction, where only the long time behavior needs to be addressed. However, when calculating the current noise of the junction, the Markovian approximation limits the description to the low-frequency noise as discussed in Sec. VI.

The Markovian charge specific master equation for a quantum object coupled to the junction was in general shown to have the form[20]

$$\dot{\hat{\rho}}_n(t) = \frac{1}{i\hbar}[\hat{H}_0, \hat{\rho}_n(t)] + \Lambda\{\hat{\rho}_n(t)\} + \mathcal{D}\{\hat{\rho}_n''(t)\} + \mathcal{J}\{\hat{\rho}_n'(t)\}, \tag{A2}$$

where $\hat{\rho}_n'$ and $\hat{\rho}_n''$ denote the "discrete derivatives" introduced in Eqs. (2.10) and (2.11), and the Lindblad-like superoperator, $\Lambda\{\hat{\rho}_n\}$, has the form

$$\Lambda\{\hat{\rho}\} = \frac{1}{\hbar}\left[\sum_{\mathbf{lr}} f_{\mathbf{l}}(1 - f_{\mathbf{r}})([\hat{T}_{\mathbf{lr}}^\dagger]\hat{\rho}\hat{T}_{\mathbf{lr}} - \hat{T}_{\mathbf{lr}}[\hat{T}_{\mathbf{lr}}^\dagger]\hat{\rho}) \right.$$
$$\left. + \sum_{\mathbf{lr}} f_{\mathbf{r}}(1 - f_{\mathbf{l}})(\hat{T}_{\mathbf{lr}}\hat{\rho}[\hat{T}_{\mathbf{lr}}^\dagger] - \hat{\rho}[\hat{T}_{\mathbf{lr}}^\dagger]\hat{T}_{\mathbf{lr}}) \right] + \mathrm{H.c.}, \tag{A3}$$

where here, and in the following, H.c. represents the Hermitian conjugate term with respect to the variable of the quantum object. The bracket denotes the operation

$$[\hat{T}_{\mathbf{lr}}] = \frac{1}{\hbar}\int_0^\infty d\tau e^{i(\varepsilon + eV)\tau/\hbar} e^{-i\hat{H}_0\tau/\hbar} \hat{T}_{\mathbf{lr}} e^{i\hat{H}_0\tau/\hbar}, \tag{A4}$$

where $\hat{T}_{\mathbf{lr}}$ is the oscillator perturbed tunneling amplitude, Eq. (2.4), and $\varepsilon = \varepsilon_{\mathbf{l}} - \varepsilon_{\mathbf{r}}$, and $f_{\mathbf{l}}$ and $f_{\mathbf{r}}$ are the single particle energy distribution functions for the electrodes, which in the following are assumed in equilibrium, described by the junction temperature T. In this paper, we restrict ourselves to the case where the junction is biased by a constant voltage U, denoting $V = eU$, where e is the electron charge. The dagger indicates Hermitian conjugation of operators of the coupled quantum object.

The drift superoperator is

$$\mathcal{J}\{\hat{R}\} = \frac{1}{\hbar}\sum_{\mathbf{lr}} \frac{F_{\mathbf{lr}}^a}{2}([\hat{T}_{\mathbf{lr}}^\dagger]\hat{R}\hat{T}_{\mathbf{lr}} + \hat{T}_{\mathbf{lr}}\hat{R}[\hat{T}_{\mathbf{lr}}^\dagger] + \mathrm{H.c.})$$
$$+ \frac{F_{\mathbf{lr}}^s}{2}([\hat{T}_{\mathbf{lr}}^\dagger]\hat{R}\hat{T}_{\mathbf{lr}} - \hat{T}_{\mathbf{lr}}\hat{R}[\hat{T}_{\mathbf{lr}}^\dagger] + \mathrm{H.c.}), \tag{A5}$$

and it has been written in terms of the symmetric and antisymmetric combinations of the distribution functions

$$F_{\mathbf{lr}}^s = f_{\mathbf{l}} + f_{\mathbf{r}} - 2f_{\mathbf{l}}f_{\mathbf{r}}, \quad F_{\mathbf{lr}}^a = f_{\mathbf{l}} - f_{\mathbf{r}}.$$

The diffusion superoperator can be obtained from the drift superoperator according to

$$\mathcal{D}\{\hat{R}\} = \frac{1}{2}\mathcal{J}_{s\leftrightarrow a}\{\hat{R}\}, \tag{A6}$$

where the subscript indicates that symmetric and antisymmetric combinations of the distribution functions should be interchanged, $F_{\mathbf{lr}}^s \leftrightarrow F_{\mathbf{lr}}^a$.

The bracket notation is specified in Eq. (A4), where, in general, H_0 denotes the Hamiltonian for the isolated arbitrary quantum object. In the following, we consider an oscillator coupled to the junction, and H_0 represents the isolated harmonic oscillator.

As expected, the coupling of the oscillator to the tunnel junction leads to a renormalization of its frequency, $\omega_B^2 \rightarrow \omega_0^2$. The renormalization originates technically in the term present in the Lindblad-like operator, $\Lambda\{\hat{\rho}\}$, which is qua-

dratic in the oscillator coordinate and of commutator form with the charge specific density matrix, and gives for the renormalized frequency

$$\omega_0^2 = \omega_B^2 - \frac{2}{m} \sum_{\mathbf{lr}} F_{\mathbf{lr}}^a (P_+ + P_-) |w_{\mathbf{lr}}|^2,$$

where

$$P_\pm = \Re \frac{1}{\varepsilon - V \mp \hbar\omega_0 + i0}. \qquad (A7)$$

For the considered interaction, the renormalization can be simply handled by changing in Eq. (A4) from the evolution by the bare oscillator Hamiltonian to the oscillator Hamiltonian with the renormalized frequency, the shift being compensated by subtracting an identical counter term. The above frequency shift is then identified by the counter term having to cancel the quadratic oscillator term of commutator form generated by the $\Lambda\{\hat{\rho}\}$ part in Eq. (A2). Substituting, in (A4), the renormalized Hamiltonian, Eq. (2.13), for the bare oscillator Hamiltonian, $\hat{H}_0$, all quantities are expressed in terms of the physically observed oscillator frequency ω_0. In particular, the bracket becomes

$$[\hat{T}_{\mathbf{lr}}] = \pi\left(\delta_0 v_{\mathbf{lr}} + (\delta_+ + \delta_-)\frac{w_{\mathbf{lr}}}{2}\hat{x} - i(\delta_+ - \delta_-)\frac{w_{\mathbf{lr}}}{2m\omega_0}\hat{p} \right)$$
$$+ i\left(P_0 v_{\mathbf{lr}} + (P_+ + P_-)\frac{w_{\mathbf{lr}}}{2}\hat{x} - i(P_+ - P_-)\frac{w_{\mathbf{lr}}}{2m\omega_0}\hat{p} \right),$$
$$(A8)$$

where

$$\delta_0 = \delta(\varepsilon - V), \quad \delta_\pm = \delta(\varepsilon - V \mp \hbar\omega_0), \qquad (A9)$$

and

$$P_0 = \Re \frac{1}{\varepsilon - V + i0}. \qquad (A10)$$

In the following, the notation $\Omega = \hbar\omega_0$ for the characteristic oscillator energy is introduced.

Evaluating the diffusion and drift operators for the case of position coupling of the oscillator to the junction, Eq. (2.4), we obtain for the drift superoperator

$$\mathcal{J}\{\hat{R}\} = V\mathcal{G}\{\hat{R}\} + \frac{\hbar}{m\Omega}G_{xx}\Delta_V\mathfrak{I}(\hat{x}\hat{R}\hat{p}) + G_x\frac{i\hbar}{2m\Omega}\Delta_V[\hat{p},\hat{R}]$$
$$+ g_x\frac{\hbar}{2m}\{\hat{p},\hat{R}\} + g_x\frac{1}{2i}\left(V \coth \frac{V}{2T} + S_V \right)[\hat{x},\hat{R}],$$
$$(A11)$$

where the conductance superoperator is defined as

$$\mathcal{G}\{\hat{R}\} \equiv G_0\hat{R} + G_x\{\hat{x},\hat{R}\} + G_{xx}\hat{x}\hat{R}\hat{x} \qquad (A12)$$

and

$$\Delta_V = \frac{V+\Omega}{2} \coth \frac{V+\Omega}{2T} - \frac{V-\Omega}{2} \coth \frac{V-\Omega}{2T} \quad (A13)$$

and

$$S_V = \frac{V+\Omega}{2} \coth \frac{V+\Omega}{2T} + \frac{V-\Omega}{2} \coth \frac{V-\Omega}{2T}, \quad (A14)$$

the latter being proportional to the current noise power spectrum at the frequency of the oscillator. For the diffusion superoperator, we obtain

$$\mathcal{D}\{\hat{R}\} = \frac{V}{2} \coth \frac{V}{2T}\mathcal{G}\{\hat{R}\} + \frac{G_{xx}}{2}\left(B_V\hat{x}\hat{R}\hat{x} + \frac{\hbar}{m}\mathfrak{I}(\hat{x}\hat{R}\hat{p}) \right)$$
$$+ \frac{\hbar A}{m\Omega}\Re(\hat{p}\hat{R}\hat{x}) + \frac{G_x}{4}\left(B_V\{\hat{x},\hat{R}\} + \frac{i\hbar}{m}[\hat{p},\hat{R}] \right)$$
$$+ g_x\left(\frac{V}{2i}[\hat{x},\hat{R}] + \frac{\hbar\Delta_V}{4m\Omega}\{\hat{p},\hat{R}\} \right), \qquad (A15)$$

where

$$B_V = S_V - V \coth \frac{V}{2T}, \qquad (A16)$$

and we have introduced the notation

$$\Re(\hat{p}\hat{R}\hat{x}) = \frac{1}{2}((\hat{p}\hat{R}\hat{x}) + (\hat{p}\hat{R}\hat{x})^\dagger) \qquad (A17)$$

and

$$\mathfrak{I}(\hat{p}\hat{R}\hat{x}) = \frac{1}{2i}((\hat{p}\hat{R}\hat{x}) - (\hat{p}\hat{R}\hat{x})^\dagger) \qquad (A18)$$

in (A11) and (A15).

The parameter A in Eq. (A15) is given by

$$A_V = \frac{\hbar^2}{2m\Omega} \sum_{\mathbf{lr}} |w_{\mathbf{lr}}|^2 F_{\mathbf{lr}}^s (P_+ - P_-) \qquad (A19)$$

and was encountered and discussed in connection with the unconditional master equation, Eq. (2.12). Technically, it originates in our model from the principal value of integrals, i.e., from virtual processes where electronic states far from the Fermi surface are involved. Estimating its magnitude under the assumption that the couplings $|w_{\mathbf{lr}}|^2$ are constants, one obtains with logarithmic accuracy

$$A \approx \frac{2\hbar^2 G_{xx}}{\pi m} \ln \frac{E_F}{\max(V,T,\Omega)}. \qquad (A20)$$

In the course of evaluating the diffusion and drift operators, combinations like $\mathfrak{I}/\Re(v_{\mathbf{lr}}^* w_{\mathbf{lr}})$ appear together with principal value terms. The phase of $v_{\mathbf{lr}}^* w_{\mathbf{lr}}$ will in general be a random function of the electron reservoir quantum numbers **l** and **r**. Summing over these quantum numbers, where the principal value term does not provide any restriction of the energy interval, as it happens in the case of terms proportional to delta functions, they will tend to average to zero. Therefore, in the following, we shall neglect such terms.[34]

APPENDIX B: NOISE IN MARKOVIAN APPROXIMATION

In this appendix, we evaluate in the Markovian approximation the current-current correlator of the tunnel junction

for the case of a harmonic oscillator coupled to the junction. The task has been reduced to evaluating the expressions in Eqs. (6.2) and (6.3), i.e., quantities of the form Eq. (6.4) where the involved superoperator is the evolution operator for the charge unconditional density matrix given in Eq. (2.21).

It immediately follows from the master equation, Eq. (2.21), that quantities like $X(t) = \text{Tr}(\hat{X}\hat{\rho}(t))$, where $\hat{X}$ can denote $\hat{x}$, $\hat{p}$, $\hat{x}^2$, and $\hat{p}^2$, and $\hat{\rho}$ is an arbitrarily normalized solution to the master equation, satisfy the corresponding classical equations of motion for a damped oscillator. The variables entering Eq. (6.4), therefore, can be expressed in terms of their initial values at time $t=0$. Restricting ourselves, for simplicity, to the case of weak damping, $\gamma \ll \omega_0$, they have the form corresponding to that of an underdamped classical oscillator

$$x_J(t) = x_J(0)e^{-\gamma t}\cos \omega_0 t + \frac{p_J(0)}{m\omega_0}e^{-\gamma t}\sin \omega_0 t \quad \text{(B1)}$$

and

$$p_J(t) = -m\omega_0 x_J(0)e^{-\gamma t}\sin \omega_0 t + p_J(0)e^{-\gamma t}\cos \omega_0 t \quad \text{(B2)}$$

and

$$x_J^2(t) = e^{-2\gamma t}x_J^2(0)\cos^2 \omega_0 t + \frac{p_J^2(0)}{m^2\omega_0^2}e^{-2\gamma t}\sin^2 \omega_0 t$$

$$+ e^{-2\gamma t}\frac{\{x,p\}_J(0)}{2m\omega_0}\sin 2\omega_0 t. \quad \text{(B3)}$$

The initial values in these equations are found from Eq. (6.5) to be

$$x_J(0) = G_x\left(\frac{\hbar^2}{m\Omega}\right)\left(V(2N^* + 1) - \frac{1}{2}\Delta_V\right), \quad \text{(B4)}$$

$$\frac{p_J(0)}{m} = g_x\frac{\hbar}{2}\left(\Omega(2N^* + 1) - \left(V\coth\frac{V}{2T} + S_V\right)\right), \quad \text{(B5)}$$

$$\frac{p_J^2(0)}{2m} + \frac{m\omega_0^2 x_J^2(0)}{2} = \widetilde{G}_{xx}\left(\Omega VN^{*2} + \Omega(V - \Delta_V)\left(N^* + \frac{1}{2}\right)\right), \quad \text{(B6)}$$

$$\frac{m\omega_0^2 x_J^2(0)}{2} - \frac{p_J^2(0)}{2m} = \widetilde{G}_{xx}\Omega VN^*(N^* + 1), \quad \text{(B7)}$$

$$\{x,p\}_J(0) = 0. \quad \text{(B8)}$$

Substituting these initial values into equations Eqs. (B1)–(B3), we obtain, using Eqs. (6.2) and (6.3), the expressions in Eq. (6.1) for the regular part of the current-current correlator

$$S_x(t) = \widetilde{G}_x^2 e^{-\gamma t}\cos \omega_0 t V(2V(2N^* + 1) - \Delta_V)$$

$$- \tilde{g}_x^2 e^{-\gamma t}\cos \omega_0 t\left(\frac{1}{2}\Omega V\coth\frac{V}{2T} + \Omega^2(N_e - N^*)\right)$$

$$- \tilde{g}_x\widetilde{G}_x e^{-\gamma t}\sin \omega_0 t$$

$$\times\left(V^2\coth\frac{V}{2T} + \Omega V(2N_e + 1) - \frac{1}{2}\Omega\Delta_V\right) \quad \text{(B9)}$$

and

$$S_{x^2}(t) = \frac{1}{2}\widetilde{G}_{xx}^2 e^{-2\gamma t}V(2VN^{*2} + (V - \Delta_V)(2N^* + 1))$$

$$+ \widetilde{G}_{xx}^2 e^{-2\gamma t}V^2 N^*(N^* + 1)\cos 2\omega_0 t. \quad \text{(B10)}$$

Evaluating in Eq. (5.17), the trace of the diffusion superoperator in the stationary state of the oscillator, $\mathcal{D}\{\hat{\rho}_s\}$, we obtain, according to Eq. (A15), for the singular part of the noise correlator,

$$S_1(t) = 2\delta(t)\left(G_0\frac{V}{2}\coth\frac{V}{2T} + \widetilde{G}_{xx}\frac{\Omega}{2}(N^*(N_e + 1) + N_e\right.$$

$$\left.\times(N^* + 1))\right). \quad \text{(B11)}$$

We observe that as to be expected, the Markovian approximation for the dynamics of the charge specific density matrix only captures the low-frequency noise, $\omega < \max(T/\hbar, V/\hbar)$. Therefore, in Sec. VI, we shall only discuss this limit.[35]

---

*Present address: Nizhny Novgorod State University, 603950 Nizhny Novgorod, Russia.

[1] M. Blencowe, Phys. Rep. **395**, 159 (2004).

[2] B. Ilic, H. G. Craighead, S. Krylov, W. Senaratne, C. Ober, and P. Neuzil, J. Appl. Phys. **95**, 3694 (2004).

[3] K. L. Ekinci, X. M. H. Huang, and M. L. Roukes, Appl. Phys. Lett. **84**, 4469 (2004).

[4] W. Marshall, C. Simon, R. Penrose, and D. Bouwmeester, Phys. Rev. Lett. **91**, 130401 (2003).

[5] R. G. Knobel and A. N. Cleland, Nature (London) **424**, 291 (2003).

[6] M. D. LaHaye, O. Buu, B. Camarota, and K. C. Schwab, Science **304**, 74 (2004).

[7] A. Gaidarzhy, G. Zolfagharkhani, R. L. Badzey, and P. Mohanty, Phys. Rev. Lett. **94**, 030402 (2005).

[8] A. N. Cleland, J. S. Aldridge, D. C. Driscoll, A. C. Gossard, Appl. Phys. Lett. **81**, 1699 (2002).

[9] R. I. Shekhter, L. Y. Gorelik, M. Jonson, Y. M. Galperin, and V. M. Vinokur, *Shuttling Transport in Nanostructures*, in Handbook of Theoretical and Computational Nanotechnology, edited by M. Rieth and W. Schommers (American Scientific Publishers, Stevenson Ranch, CA, 2005).

[10] A. Erbe, C. Weiss, W. Zwerger, and R. H. Blick, Phys. Rev. Lett. **87**, 096106 (2001).

[11] D. V. Scheible and R. H. Blick, Appl. Phys. Lett. **84**, 4632 (2004).

[12] A. D. Armour, M. P. Blencowe, and K. C. Schwab, Phys. Rev. Lett. **88**, 148301 (2002).

[13] M. R. Geller and A. N. Cleland, Phys. Rev. A **71**, 032311 (2005).

[14] D. Mozyrsky and I. Martin, Phys. Rev. Lett. **89**, 018301 (2002).

[15] A. Yu. Smirnov, L. G. Mourokh, and N. J. M. Horing, Phys. Rev. B **67**, 115312 (2003).

[16] A. A. Clerk and S. M. Girvin, Phys. Rev. B **70**, 121303 (2004).

[17] A. D. Armour, M. P. Blencowe, and Y. Zhang, Phys. Rev. B **69**, 125313 (2004).

[18] A. D. Armour, Phys. Rev. B **70**, 165315 (2004).

[19] A. Mitra, I. Aleiner, and A. J. Millis, Phys. Rev. B **69**, 245302 (2004).

[20] J. Rammer, A. L. Shelankov, and J. Wabnig, Phys. Rev. B **70**, 115327 (2004).

[21] A. L. Shelankov and J. Rammer, Europhys. Lett. **63**, 485 (2003).

[22] A. O. Caldeira and A. J. Leggett, Physica A **121A**, 587 (1983).

[23] H. P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, Oxford, 2002).

[24] R. P. Feynman and F. L. Vernon, Ann. Phys. (N.Y.) **24**, 118 (1963).

[25] Sh. Kogan, *Electronic Noise and Fluctuations in Solids* (Cambridge University Press, Cambridge, U.K.,1996).

[26] F. Haake and R. Reibold, Phys. Rev. A **32**, 2462 (1985).

[27] P. W. Brouwer, Phys. Rev. B **58**, R10135 (1998).

[28] B. L. Altshuler and L. I. Glazman, Science **283**, 1864 (1999).

[29] I. L. Aleiner, B. L. Altshuler, and A. Kamenev, Phys. Rev. B **62**, 10373 (2000).

[30] D. V. Khomitsky (unpublished).

[31] It is interesting to note that the virial theorem turns out to be violated due to the coupling. Indeed, the renormalization in Eq. (2.23) modifies the potential energy, $V_{pot}=(m\omega^2 x^2)/2$, but does not influence the kinetic energy $K=p^2/2m$. As a result, $\langle V \rangle -\langle K \rangle =\frac{1}{2}A$, in disaccord with the virial theorem for a harmonic oscillator, $\langle V \rangle =\langle K \rangle$. In fact, this is an exact result in the sense that it can be obtained, without reference to the renormalization procedure, directly from Eq. (2.12) by the standard Ehrenfest construction of the equation of motion for expectation values.

[32] S. O. Rice, in *Selected Papers on Noise and Stochastic Processes*, edited by N. Wax (Dover, New York, 1954).

[33] The current-current correlator, $S(t)$, tends to zero at large times and a convergence factor, $\exp(-\delta t)$, can be inserted into the integrand in the expression for the power spectrum, Eq. (5.9). Upon inserting the expression for the current-current correlator, Eq. (5.8), into the expression for the power spectrum, a partial integration gives MacDonald's formula $S_\omega =2\omega\int_0^\infty dt e^{-\delta t}\sin(\omega t)d\langle\langle n^2(t)\rangle\rangle/dt$, since the vanishing of the boundary term at $t=0$ is ensured by virtue of Eq. (5.7).

[34] A similar renormalization was encountered and discussed in connection with a two-level system coupled to a tunnel junction. See Ref. 20.

[35] We recall the result for an isolated junction (Ref. 20): using the non-Markovian master equation for the charge specific density matrix, one obtains for the power spectrum of current noise $S_\omega =I(V+\omega)\coth[(V+\omega)/2T]+I(V-\omega)\coth[(V-\omega)/2T]$, the general result for a biased junction, valid for arbitrary frequency and nonlinear I-V characteristic, $I=I(V)$. The Markovian approximation only captures the low-frequency limit of the noise power.