

# Spin-dependent tunneling through a symmetric semiconductor barrier: The Dresselhaus effect

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Spin-dependent tunneling through a symmetric semiconductor barrier is studied including the  $k^3$  Dresselhaus effect. The spin-dependent transmission of an electron can be obtained analytically. By comparing with previous work [Phys. Rev. B **67**, 201304(R) (2003) and Phys. Rev. Lett. **93**, 056601 (2004)], it is shown that the spin polarization and interface current are changed significantly by including the off-diagonal elements in the current operator, and can be enhanced considerably by the Dresselhaus effect in the contact regions.

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Recently electron spin in semiconductors has attracted a rapidly growing interest due to its potential application in spintronics devices. To successfully incorporate spin into existing semiconductor technology, one has to overcome technical difficulties such as efficient spin-polarized injection, transport, control and manipulation, as well as measurement of spin polarization. The injection of spin-polarized electrons from ferromagnetic metals into semiconductors has low efficiency, less than 1%, because of a large resistivity mismatch between ferromagnetic and semiconductor materials.<sup>1</sup> Rashba proposed that this problem could be solved by inserting a tunneling barrier at the metal-semiconductor interface.<sup>2</sup> Asymmetric nonmagnetic semiconductor barriers are also used in the construction of spin filters.<sup>3</sup> This effect is caused by the interface-induced Rashba spin-orbit coupling<sup>4</sup> and can be quite significant for resonant tunneling through asymmetric double-barrier structure.<sup>5</sup> Very recently, a multichannel field-effect spin-barrier selector was investigated theoretically utilizing the Rashba and Dresselhaus effects.<sup>6</sup> A considerable spin polarization and an interesting “tunneling spin-galvanic” effect were found in the tunneling of electron through a single symmetric barrier utilizing the Dresselhaus effect in the barrier.<sup>7,8</sup> However, the off-diagonal elements in the current operator, the contribution from the Dresselhaus effect, are neglected in these works. These off-diagonal element in the currents operator could lead to the significant correction of the spin-dependent transmission, especially in a thin barrier case.

In this paper, we investigate theoretically the spin-dependent tunneling through a single symmetric barrier. The barrier and contacts consist of a zinc-blende-structure semiconductor lacking the inversion symmetry. It is shown that the spin-dependent tunneling and the electric current in the plane of the interfaces are different from the previous studies.<sup>7</sup> This difference is significant in thin barrier case and disappears gradually with increasing the thickness of the barrier. It is interesting to notice that the spin polarization and the interface current  $j_{\parallel}$  are enhanced by including the Dresselhaus effect in the contact regions.

We consider the transmission of an electron with initial wave vector  $\mathbf{k}=(k_{\parallel}, k_z)$  through a flat potential barrier of height  $V$  grown along the  $z$  [001] direction;  $k_{\parallel}$  is the in-plane

wave vector, and  $k_z$  is the wave vector along the growth direction, i.e.,  $z$  axis. Then the electron Hamiltonian including the spin-dependent  $k^3$  Dresselhaus term is

$$H = \frac{P^2}{2m^*} + V(z) + H_D,$$

$$H_D = \gamma_i [\sigma_x k_x (k_y^2 - k_z^2) + \sigma_y k_y (k_z^2 - k_x^2) + \sigma_z k_z (k_x^2 - k_y^2)], \quad (1)$$

where  $m^*$  is the effective mass of electron, and  $V(z)$  is the height of barrier.  $H_D$  describes the Dresselhaus spin-orbit coupling,  $\sigma_\alpha$  are the Pauli matrices, and  $\gamma_i (i=1,2)$  describe the strength of the Dresselhaus effect in the contact regions and the barrier. We assume that the kinetic energy of the electron is substantially smaller than the barrier height  $V$  in the barrier<sup>7,8</sup> (we take  $V=0.2$  eV,  $E_F=0.02$  eV in our paper). The Hamiltonian in the barrier is simplified to

$$H = \frac{p^2}{2m^*} + V(z) + \gamma_2 (k_x \sigma_x - k_y \sigma_y) \frac{\partial^2}{\partial z^2}. \quad (2)$$

The eigenvalues and eigenstates of the Hamiltonian are

$$E = \frac{\hbar^2 (k_{\parallel}^2 + q_{\pm}^2)}{2m^*} + V(z) \pm \gamma_2 k_{\parallel} q_{\pm}^2, \quad (3)$$

$$\chi_{\pm} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \mp e^{-i\varphi} \end{pmatrix}, \quad (4)$$

which correspond to the “+” and “-” electron states with opposite spin orientation. The wave functions of the electron in the left source, the barrier and right drain are

$$\Psi_L = \Psi_i + \exp(i\mathbf{k}_{\parallel} \cdot \boldsymbol{\rho}) \sum_{j=\pm} r_j \exp(-ik_j z) \chi_j,$$

$$\Psi_b = \exp(i\mathbf{k}_{\parallel} \cdot \boldsymbol{\rho}) \sum_{j=\pm} [A_j \exp(q_j z) + B_j \exp(-q_j z)] \chi_j,$$

$$\Psi_R = \exp(i\mathbf{k}_\parallel \cdot \boldsymbol{\rho}) \sum_{j=\pm} t_j \exp(ik_j z) \chi_j, \quad (5)$$

here  $\Psi_i = \exp(i\mathbf{k}_\parallel \cdot \boldsymbol{\rho}) \sum_{j=\pm} \exp(ik_j z) \chi_j$  corresponds to the injected spin state of electron,<sup>7,8</sup>  $t_\pm$ ,  $r_\pm$  are the transmission and reflection coefficients for the spin states  $\chi_\pm$ , respectively.  $\boldsymbol{\rho} = (x, y)$  is the coordinate in the barrier plane, and  $\varphi$  is the polar angle of the wave vector  $\mathbf{k}$  in the  $xy$  plane. The wave vectors  $q_\pm(k_\pm)$  inside and outside the barrier are given by

$$q_\pm = \sqrt{\frac{\frac{2m_2^*V}{\hbar^2} + k_\parallel^2 - \frac{2m_2^*E_F}{\hbar^2}}{1 \pm \frac{2m_2^*}{\hbar^2} \gamma_2 k_\parallel}}, \quad (6)$$

$$k_\pm = \sqrt{\frac{\frac{2m_1^*E_F}{\hbar^2} - k_\parallel^2}{1 \pm \frac{2m_1^*}{\hbar^2} \gamma_1 k_\parallel A}},$$

where  $A = \sqrt{1 + \tan^2 \theta + \tan^2 \theta \sin^2 2\varphi (\tan^2 \theta / 4 - 2)}$  and  $m_1^*$  ( $m_2^*$ ) is the effective masses outside (inside) the barrier. The boundary conditions including the Dresselhaus effect are

$$\Psi_{Lz=0} = \Psi_{bz=0},$$

$$\Psi_{bz=a} = \Psi_{Rz=a},$$

$$j_L \Psi_{Lz=0} = j_b \Psi_{bz=0},$$

$$j_b \Psi_{bz=a} = j_R \Psi_{Rz=a}, \quad (7)$$

where  $j_i$  ( $i=L, R, b$ ) are the current operators in the left side, the right side of the barrier, and inside the barrier,

$$j_{L,R} = \frac{1}{\hbar} \begin{pmatrix} -i \frac{\hbar^2}{m_1^*} \frac{\partial}{\partial z} + \gamma_1 k_\parallel^2 \cos(2\varphi) & -2\gamma_1 k_\parallel e^{i\varphi} \left(-i \frac{\partial}{\partial z}\right) \\ -2\gamma_1 k_\parallel e^{-i\varphi} \left(-i \frac{\partial}{\partial z}\right) & -i \frac{\hbar^2}{m_1^*} \frac{\partial}{\partial z} - \gamma_1 k_\parallel^2 \cos(2\varphi) \end{pmatrix},$$

$$j_b = \frac{1}{\hbar} \begin{pmatrix} \frac{\hbar^2}{m_2^*} & -2\gamma_2 k_\parallel e^{i\varphi} \\ -2\gamma_2 k_\parallel e^{-i\varphi} & \frac{\hbar^2}{m_2^*} \end{pmatrix} \left(-i \frac{\partial}{\partial z}\right). \quad (8)$$

Note that the off-diagonal elements of the current operators were neglected in Ref. 7. For the real case  $m_1^* \gamma_1 k_\parallel / \hbar^2 \ll 1$  and  $m_1^* \gamma_1 k_\parallel^2 \cos(2\varphi) / q_\pm / \hbar^2 \ll 1$ , we obtain the analytical expression of the spin-dependent transmission coefficient of the electron

$$t_\pm = - \frac{4i \delta_\pm \delta'_\pm k_\pm q_\pm e^{(-q_\pm a - ik_\pm a)}}{(i \delta'_\pm q_\pm - \delta_\pm k_\pm)^2 e^{-2q_\pm a} - (i \delta'_\pm q_\pm + \delta_\pm k_\pm)^2}, \quad (9)$$

where  $\delta_\pm = 1 \pm 2m_1^* \gamma_1 k_\parallel / \hbar^2$ , and  $\delta'_\pm = 1 \pm 2m_2^* \gamma_2 k_\parallel / \hbar^2$ .

It is convenient to introduce the spin polarization  $P$  determined by the difference between the transmission of the spin states  $\chi_+$  and  $\chi_-$ ,

$$P = \frac{|t_+|^2 - |t_-|^2}{|t_+|^2 + |t_-|^2}. \quad (10)$$

The interface current due to spin-polarized electron transport through the tunneling structure can be obtained through the spin density matrix<sup>8</sup>

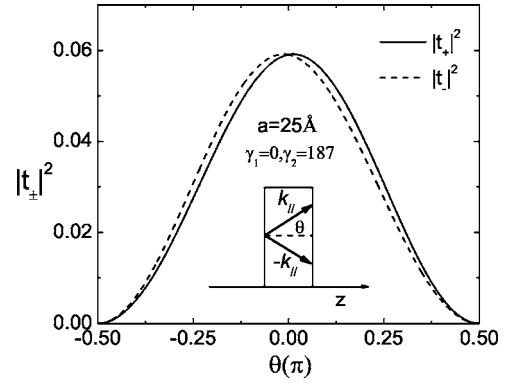


FIG. 1. The spin-dependent transmission of electron as a function of the angle  $\theta$  between the direction of  $\mathbf{k}$  and the  $z$  axis.

$$j_\parallel = e \sum_{k_\parallel, k_z > 0} \tau_p \text{Tr}[\mathcal{T} \rho_l \mathcal{T}^\dagger v_z] v_\parallel, \quad (11)$$

where  $\tau_p$  is the momentum relaxation time,  $\rho_l$  is the electron density matrix on the left side of the structure, and  $\mathcal{T}$  is the spin matrix of the tunneling transmission that links the incident spin wave function  $\Psi_L$  to the transmitted spin wave function  $\Psi_R$ ,  $\Psi_R = \mathcal{T} \Psi_L$ . The spin matrix of the electron transmission through the structure is given by

$$\mathcal{T} = \sum_{s=\pm} t_s \chi_s \chi_s^\dagger. \quad (12)$$

In the case of small degree of spin polarization, the density matrix has the form

$$\rho_l = f_0 I - \frac{df_0}{d\varepsilon} \frac{2p_s}{\langle 1/\varepsilon \rangle} (\mathbf{n}_s \cdot \hat{\sigma}), \quad (13)$$

where  $f_0$  is the equilibrium distribution function of nonpolarized carriers,  $p_s$  is the degree of the polarization, and  $\langle 1/\varepsilon \rangle$  is the average of the reciprocal kinetic energy of the carriers.  $\mathbf{n}_s$  is the unit vector directed along the spin orientation, and the orientations of spin  $s_\pm$  in the states “+” and “-” depend on the in-plane wave vector of the electron and are given by  $s_\pm = (\mp \cos \varphi, \pm \sin \varphi, 0)$ . Taking into account the spin matrix and the density matrix, the interface current is

$$j_\parallel = -C_0 \sum_{k_\parallel, k_z > 0} \frac{df_0}{d\varepsilon} [|t_+|^2 - |t_-|^2] v_z v_\parallel, \quad (14)$$

where  $C_0 = e \tau_p (p_s / \langle 1/\varepsilon \rangle)$ . It is interesting to notice that the direction of the interface current depends on the spin polarization of the injected electron  $p_s$ .

It is well known that normally the transmission through the barrier reaches maximum for carriers propagating along the normal to the barrier in the absence of spin-orbit interaction. But the spin-orbit coupling changes this rule as shown in Fig. 1 [see Eq. (9)]. The tunneling transmission for the spin-polarized electron with the finite in-plane wave vector  $\mathbf{k}_\parallel$  is larger than that for the electron with the opposite in-plane wave vector,  $-\mathbf{k}_\parallel$ . This asymmetry results in the in-plane flow of the transmitted electron near the barrier, i.e., an interface electric current.

For an electron with wave vector  $\mathbf{k}$  [see  $\Psi_L$  in Eq. (5)] is

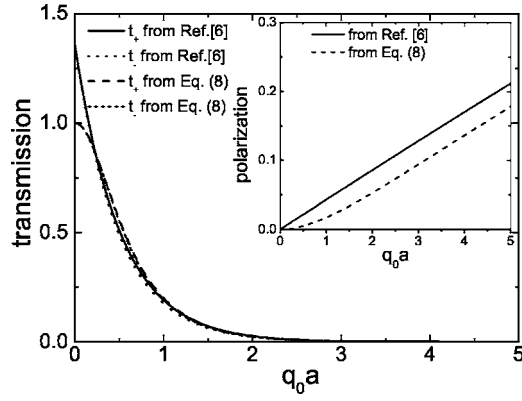


FIG. 2. The spin-dependent transmission of electron as a function of barrier width  $q_0 a$  for GaSb material ( $\gamma_1=0$ ,  $\gamma_2=187$ ,  $V=0.2$  eV). The inset shows the spin polarization  $P$  as a function of barrier width  $q_0 a$  for the same condition.

injected into the barrier, the spin-dependent transmission is determined by the width of the barrier and the Dresselhaus spin-orbit coupling strength [see Eq. (8)]. The spin-dependent transmission and the spin polarization  $P$  are plotted as a function of the barrier width  $q_0 a$  in Fig. 2. Here we consider the Dresselhaus effect only in the barrier material. The material parameter relevant to GaSb are used in our calculation  $\gamma_1=0$ ,  $\gamma_2=187$  and the effective masses are  $m^*=m_2^*=0.041m_0$ . It is shown that the spin-dependent transmission decreases rapidly with increasing barrier width  $q_0 a$ , while the spin polarization increases gradually. From this figure, we can see that the results of Ref. 7 agree with ours only in the thick barrier case, but there is a big difference in the thin barrier case. From the numerical results, we can find that the off-diagonal elements due to the Dresselhaus effect, in the current operator play an important role in the spin-dependent transport, especially in the thin barrier case.

Figure 3 describes the dependence of the magnitude of the interface current  $j_{||}$  on the width of the barrier in unit of  $C_0$  for incident electrons, which form spin-polarized degenerate gas as in GaAs [Fig. 3(a)] and in GaSb [Fig. 3(b)], respec-

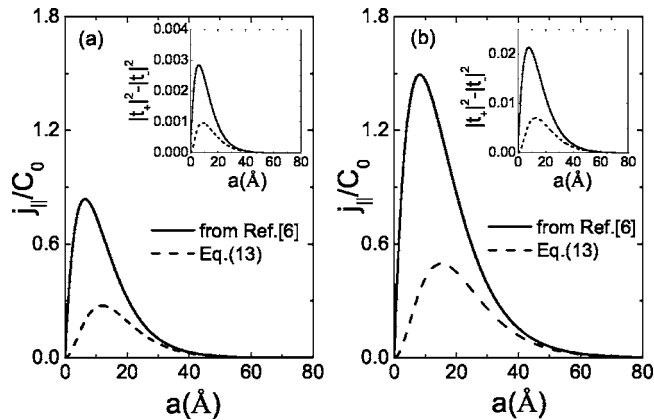


FIG. 3. The interface current  $j_{||}$  as a function of barrier width  $a$  in unit of  $C_0$  for (a) GaAs ( $\gamma_1=0, \gamma_2=24, V=0.2$  eV) and (b) GaSb ( $\gamma_1=0, \gamma_2=187, V=0.2$  eV). The insets show the difference in transmission between the spin states  $\chi_+$  and  $\chi_-$ .

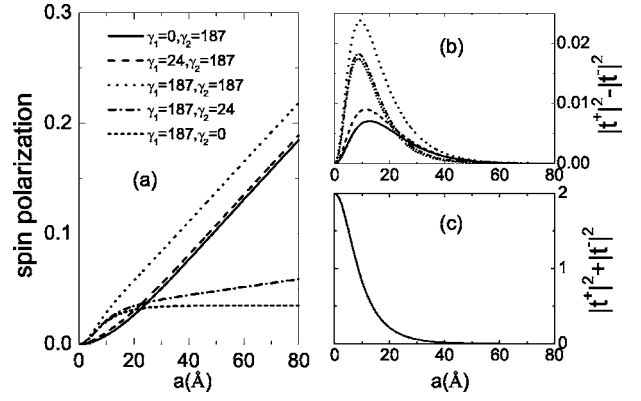


FIG. 4. (a) The spin polarization of electron as a function of barrier width  $a$  including the Dresselhaus effects inside and outside the barrier. (b) The difference between the transmission of the spin states  $\chi_+$  and  $\chi_-$ . (c) The total of the transmission of the spin states  $\chi_+$  and  $\chi_-$ .

tively. From this figure, we can see that the two results are very different. Our numerical results show that the maximum interface current is overestimated in Refs. 7 and 8, the two curves will merge gradually together with increasing the barrier width. The numerical accuracy spin-dependent tunneling and interface current could be improved significantly by including the effect of the off-diagonal elements of the current operator due to the Dresselhaus effect. The difference between the present results and Ref. 8 will increase with increasing  $m_2^* k_{||} \gamma_2 / \hbar^2$  comparing Figs. 3(a) and 3(b).

The spin polarization  $P$  is plotted as a function of the barrier width  $a$  for different strengths of the Dresselhaus effect in Fig. 4. From this figure, we see that the spin polarization depends on the Dresselhaus effect  $\gamma_i (i=1, 2)$  and the barrier width  $a$ . It is interesting to notice that the spin polarization is enhanced by including the Dresselhaus effect  $\gamma_1$  in the contact regions, and increases with increasing the barrier width  $a$ . On the other hand, the spin polarization decreases with diminishing the strength of the Dresselhaus effect  $\gamma_2$  in the barrier for a fixed  $\gamma_1$ . The spin polarization saturates gradually with increasing barrier width  $a$  for  $\gamma_2=0$ , and reaches maximum when  $\gamma_1=\gamma_2=187$ . These features can be understood from Figs. 4(b) and 4(c). The total transmission decreases with increasing barrier width and almost the same for different strengths of the Dresselhaus effect [see Fig. 4(c)], but the differences between the spin states  $\chi_+$  and  $\chi_-$  reach the maxima for different strengths  $\gamma_1, \gamma_2$  [see Fig. 4(b)]. Therefore the spin polarizations exhibit different behavior for different strengths of Dresselhaus effect in the contact and barrier regions. The spin polarization saturates gradually with increasing barrier width for  $\gamma_1=187, \gamma_2=0$  since the spin polarization is mainly caused by the Dresselhaus effect  $\gamma_1$  at both sides of the barrier, and consequently almost independent of the barrier width. The spin polarization approaches the maximum when  $\gamma_1=\gamma_2$ .

Figure 5 shows the interface current  $j_{||}$  as a function of the barrier width  $a$  for different strengths of the Dresselhaus effect. From this figure, we can see that the interface current  $j_{||}$  is enhanced by the strength of the Dresselhaus effect  $\gamma_1$  in the contact regions, and show maxima at certain values of

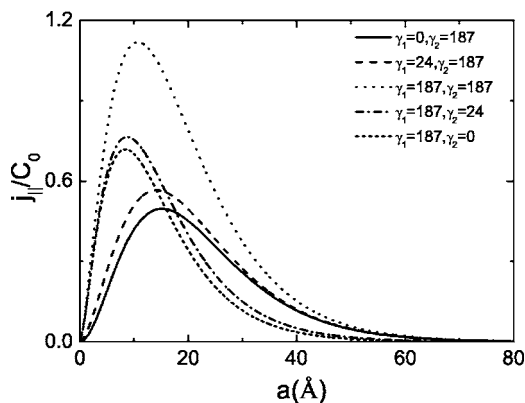


FIG. 5. The interface current  $j_{\parallel}$  as a function of barrier width  $a$  in unit of  $C_0$  including the Dresselhaus effects inside and outside the barrier.

the barrier width  $a$ . It is interesting to notice that the interface current  $j_{\parallel}$  reaches maximum when  $\gamma_1 = \gamma_2$ , and decreases as the strength of the Dresselhaus effect either in the barrier or in the contact regions decreases. Considering the kinetic

energy of the electron is substantially smaller than the barrier height  $V$  and  $e^{-q_{\pm}a} \ll 1$ , we can obtain

$$j_{\parallel} \propto |t_+|^2 - |t_-|^2 \propto (\delta'_+ \delta_+ e^{-2q_+a} - \delta'_- \delta_- e^{-2q_-a}). \quad (15)$$

It is obvious that the interface current  $j_{\parallel}$  increases with increasing  $\gamma_2$  if we fix  $\gamma_1$ . Similarly, the interface current  $j_{\parallel}$  decreases with decreasing  $\gamma_1$  when we fix  $\gamma_2$ . Therefore the interface current  $j_{\parallel}$  reaches a maximum when  $\gamma_1 = \gamma_2$ .

In conclusion, we investigated the spin-dependent tunneling through a symmetric semiconductor barrier consisting of zinc-blende semiconductor material. It is interesting to notice that the spin polarization and the interface current  $j_{\parallel}$  are enhanced significantly by including the Dresselhaus effect in the contact regions, and they all reach a maximum when the strength of the Dresselhaus effect in the barrier is equal to that in the contact regions, i.e.,  $\gamma_1 = \gamma_2$ .

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