Magnetization in lateral superlattices induced by strong dc and ac electric fields

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We consider nonlinear electric field effects in lateral superlattices with Rashba spin-orbit coupling. Both strong dc and ac electric fields are applied to the superlattice. The pseudomagnetic field associated with the spin-orbit interaction gives rise to a homogeneous spin polarization under nonequilibrium conditions of a biased superlattice. The application of a strong radiation field in the terahertz domain leads to photon resonances, dynamical localization, and delocalization as well as to a reorientation of the field-induced magnetization. We stress the analogy between the field-induced spinless carrier transport and the spin polarization under intense irradiation.

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Recent interest has focused on manipulating the spin degree of freedom of conduction electrons in semiconductor heterostructures for applications in spintronics. One of the main objectives in this fast developing field is the study of all electrical nonmagnetic mechanisms for manipulating the spin degree of freedom. Currently, two effects are discussed: the spin-Hall effect and the spin accumulation. We shall focus on the latter effect, which was predicted many years ago.[1–3] Due to the application of an electric field to an electron gas with spin-orbit interaction, an effective magnetic field appears, which polarizes electron spins under nonequilibrium conditions. This mechanism is quite similar to the magnetoelectric effect in antiferromagnetic insulators, in which a magnetic moment is induced in the bulk by an external electric field. It has been shown that the electric-field-induced magnetization is not accompanied by a spin-polarized current.[4] Only recently, this effect of field-induced homogeneous spin accumulation was experimentally demonstrated by terahertz (THz) transmission experiments[5] and by optical detection in strained GaAs and (In,Ga)As epitaxial layers.[6] This kind of spin magnetization due to an electrical current can be regarded as a counterpart to the recently discovered spin-galvanic effect.[7] From a theoretical point of view, the treatment of an electric-field- or current-induced spin polarization remains up to now within the linear response regime. Therefore, it would be desirable to extend this approach to the nonlinear field regime, where Wannier-Stark localization may occur. In addition, it would be intriguing to study effects due to an oscillating electric field, because a variety of interesting time dependent phenomena have been identified in the related field of spinless carrier transport. We mention only the absolute negative conductance[8,9] and photon-assisted carrier localization and delocalization.[10] Compared to bulk semiconductors, nonlinear and high-electric-field effects are most suitably studied in semiconductor superlattices with a large lattice constant d.

Let us treat a two-dimensional electron gas with a superlattice structure described by the tight-binding dispersion relation

$$\varepsilon(\mathbf{k}) = \frac{\Delta}{2}[1-\cos(k_x d)] + \varepsilon(k_y), \tag{1}$$

with the parabolic kinetic energy $\varepsilon(k_y)=\hbar^2k_y^2/2m^*$ for carriers moving along the y axis oriented perpendicular to the time dependent electric field $\vec{\mathcal{E}}(t)=\vec{\mathcal{E}}_{dc}+\vec{\mathcal{E}}_{ac}\cos(\omega t)$ (ω denotes the frequency of the ac field). We restrict ourselves to considering the lowest miniband of width Δ. Taking into account the Rashba spin-orbit coupling, the model Hamiltonian of the biased lateral superlattice takes the form

$$H_0 = \sum_{\mathbf{k},\sigma}\varepsilon(\mathbf{k})a^+_{\mathbf{k}\sigma}a_{\mathbf{k}\sigma} + \sum_{\mathbf{k}}\sum_{\sigma,\sigma'}J_{\sigma\sigma'}(\mathbf{k})a^+_{\mathbf{k}\sigma}a_{\mathbf{k}\sigma'} - ie\vec{\mathcal{E}}(t)\sum_{\mathbf{k},\sigma}\nabla_\kappa\,(a^+_{\mathbf{k}-\kappa/2\sigma}a_{\mathbf{k}+\kappa/2\sigma})\big|_{\kappa=0}, \tag{2}$$

where $a^+_{\mathbf{k}\sigma}(a_{\mathbf{k}\sigma})$ is the creation (annihilation) operator of electrons with quasimomentum $\mathbf{k}$ and spin σ. Different spin states are coupled by the Rashba term

$$J_{\sigma\sigma'}(\mathbf{k}) = \begin{pmatrix} 0 & J_{12}(\mathbf{k}) \\ J^*_{12}(\mathbf{k}) & 0 \end{pmatrix}, \tag{3}$$

in which the matrix element

$$J_{12}(\mathbf{k}) = \alpha m^*[iv_x(\mathbf{k}) + v_y(\mathbf{k})], \tag{4}$$

is calculated from the components v_x and v_y of the drift velocity operator. α denotes the spin-orbit coupling constant. In the absence of external fields, the Hamiltonian in Eq. (2) can be diagonalized by a canonical transformation defined by

$$a_{\mathbf{k}\sigma} = \sum_\mu A_{\sigma\mu}(\mathbf{k})b_{\mathbf{k}\mu}, \quad \sum_\mu A^*_{\sigma\mu}(\mathbf{k})A_{\sigma'\mu}(\mathbf{k}) = \delta_{\sigma\sigma'}, \tag{5}$$

which preserves the commutation relations. The matrix, which diagonalizes the spin Hamiltonian, has the form

$$\hat{A} = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\varphi(\mathbf{k})/2} & e^{-i\varphi(\mathbf{k})/2} \\ -e^{i\varphi(\mathbf{k})/2} & e^{i\varphi(\mathbf{k})/2} \end{pmatrix}, \tag{6}$$

where the angle $\varphi(\mathbf{k})$ depends on the quasimomentum and is determined by

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$$\sin[\varphi(\mathbf{k})] = \alpha m^* \frac{\Delta d}{2\hbar} \frac{\sin(k_x d)}{J(\mathbf{k})}, \tag{7}$$

in which the amount of energy splitting due to the spin-orbit interaction is given by

$$J(\mathbf{k}) = \alpha m^* \sqrt{\left(\frac{\hbar k_y}{m^*}\right)^2 + \left(\frac{\Delta d}{2\hbar}\sin(k_x d)\right)^2}. \tag{8}$$

In the new basis, the Hamiltonian is expressed as

$$H_0 = \sum_{\mathbf{k}\mu} \varepsilon_\mu(\mathbf{k}) b^+_{\mathbf{k}\mu} b_{\mathbf{k}\mu} - e\vec{\mathcal{E}}(t)\sum_{\mathbf{k}} \mathbf{d}(\mathbf{k})(b^+_{\mathbf{k}1}b_{\mathbf{k}2} + b^+_{\mathbf{k}2}b_{\mathbf{k}1}) - ie\vec{\mathcal{E}}(t)\sum_{\mathbf{k}\mu} \nabla_\kappa b^+_{\mathbf{k}-\kappa/2,\mu} b_{\mathbf{k}+\kappa/2,\mu}\big|_{\kappa=0}, \tag{9}$$

with the spin-dependent eigenenergies $\varepsilon_\mu(\mathbf{k}) = \varepsilon(\mathbf{k}) \pm J(\mathbf{k})$ and the dipole moment $e\mathbf{d}(\mathbf{k}) = e\nabla_{\mathbf{k}}\varphi(\mathbf{k})/2$. At vanishing electric fields, the Hamiltonian in Eq. (9) becomes diagonal. We shall focus on field-induced spin polarization, which is described by the spin-dependent part of the distribution function $\mathrm{Tr}(\vec{\sigma}\cdot\mathbf{f})$ [with $\vec{\sigma}$ being the vector of Pauli matrices and $\mathbf{f}$ the density matrix with the elements $f_{\sigma\sigma'}(\mathbf{k}|t) = \langle a^+_\sigma(\mathbf{k}) a_{\sigma'}(\mathbf{k})\rangle_t$]. The calculation is carried out for the Hamiltonian in Eq. (9) by determining the related density matrix $\mathbf{F}$ from the kinetic equation

$$\left\{\frac{\partial}{\partial t} - \frac{i}{\hbar}[\varepsilon_\mu(\mathbf{k}) - \varepsilon_{\mu'}(\mathbf{k})] + \frac{e\vec{\mathcal{E}}(t)}{\hbar}\nabla_{\mathbf{k}}\right\} F^{\mu'}_{\mu}(\mathbf{k}|t) + \frac{ie\vec{\mathcal{E}}(t)}{\hbar}\sum_{\mu_1}{}' [\mathbf{d}_{\mu_1\mu}(\mathbf{k})F^{\mu'}_{\mu_1}(\mathbf{k}|t) - \mathbf{d}_{\mu'\mu_1}(\mathbf{k})F^{\mu_1}_{\mu}(\mathbf{k}|t)] = \sum_{\mathbf{k}'}\sum_{\mu_1\mu_2} F^{\mu_1}_{\mu_2}(\mathbf{k}'|t) W^{\mu_1\mu'}_{\mu_2\mu}(\mathbf{k}',\mathbf{k}). \tag{10}$$

We concentrate on scattering processes accompanied by small momentum transfers, when only the scattering in and out probabilities W^{11}_{11}, W^{22}_{22}, W^{22}_{11}, and W^{11}_{22} are relevant (in the extreme case, all other matrix elements vanish). Applying the canonical transformation in Eq. (5) to the density matrix $\mathbf{f}$, we obtain the following relationship between the spin-dependent distribution functions of the original Hamiltonian in Eq. (2) and its canonically transformed version in Eq. (9)

$$f_x(\mathbf{k}|t) = -\sin[\varphi(\mathbf{k})]F_y(\mathbf{k}|t) + \cos[\varphi(\mathbf{k})]F_z(\mathbf{k}|t)$$

$$f_y(\mathbf{k}|t) = \cos[\varphi(\mathbf{k})]F_y(\mathbf{k}|t) - \sin[\varphi(\mathbf{k})]F_z(\mathbf{k}|t)$$

$$f_z(\mathbf{k}|t) = F_x(\mathbf{k}|t). \tag{11}$$

Due to symmetry properties with respect to the $\mathbf{k}$ dependence, the electric fields induce only a nonvanishing spin polarization for the y component of the density matrix $f_y = \Sigma_{\mathbf{k}}\langle f_y(\mathbf{k}|t)\rangle$ (with $\langle\ldots\rangle$ denoting the time average over the period $2\pi/\omega$). According to Eq. (11), $f_y(\mathbf{k}|t)$ contains two quite different contributions. The first one is proportional to the off-diagonal element F_y of the density matrix and results from the electric-field-induced tunneling between different spin states within the basis of the Hamiltonian in Eq. (9). This term, which disappears in the ohmic electric field region, describes quantum effects due to resonant tunneling between spin-orbit split states. It appears to be much smaller than the second contribution to f_y, which is obtained from the diagonal elements of the density matrix expressed by the eigenstates of the spin Hamiltonian. For a two-dimensional electron gas [for which $\sin\varphi(\mathbf{k}) = k_x/|\mathbf{k}|$], we obtain from this contribution the result derived previously[3,4] for a constant electric field $\mathcal{E}$ within a linear response approach. Indeed, making use of the antisymmetric part of the distribution functions for zero temperature ($T=0$) and restricting the expansion to lowest order in the spin-orbit coupling constant α, we obtain $f_y = \alpha m^* a^2 e\mathcal{E}\tau/(2\pi\hbar^2)$, with a being the lattice constant of the 2 DEG and τ an effective scattering time. We proceed by focusing on the quasiclassical limit, when the spin-splitting energy is larger than the level width [$\hbar/2\tau < J(k_F)$, k_F denotes the Fermi momentum] so that the off diagonal elements F_x, F_y in Eqs. (10) and (11) can be neglected. Switching to the Wannier-Stark ladder representation by a discrete Fourier transformation[11]

$$F_z(\mathbf{k}|t) = \sum_{l=-\infty}^{\infty} e^{ilk_x d} F_z(l,k_y|t), \tag{12}$$

we obtain from Eqs. (7) and (11) the following result for the magnetization in the y direction

$$f_y = -\alpha m^* \frac{\Delta d}{2\hbar}\sum_{k_x,k_y}\sum_l e^{ilk_x d}\frac{\sin(k_x d)}{J(k_x,k_y)}\langle F_z(l,k_y|t)\rangle. \tag{13}$$

The time—averaged z component of the distribution function F_z is calculated from Eq. (10). The rigorous solution of this equation can only be performed numerically. However, in order to demonstrate some main physical results, we shall carry out the calculations analytically. For simplicity, let us, therefore, adopt the simple relaxation-time approximation. Although this approximation is useful for elastic scattering, it allows a treatment of high-field effects only in a rather nonrigorous fashion because only inelastic scattering can dissipate the energy provided by the electric field. Therefore, the relaxation-time approximation is an oversimplification of the different scattering processes taking place in a real structure. Nevertheless, several authors[12–14] have demonstrated that nonlinear carrier transport under THz irradiation can be very well described within this approximation, when compared to the corresponding results obtained from a full scale Monte Carlo simulation. Adopting this approximation, we obtain in the quasiclassical case

$$\left(\frac{\partial}{\partial t} + \frac{e\vec{\mathcal{E}}(t)}{\hbar}\nabla_{\mathbf{k}}\right)F_z(\mathbf{k}|t) = -\frac{1}{\tau}[F_z(\mathbf{k}|t) - F^{(dc)}_z(\mathbf{k})], \tag{14}$$

where the unknown function $F^{(dc)}_z(\mathbf{k})$ is chosen in such a way that at vanishing ac field a distribution function is recovered that governs the behavior under constant bias. This modification of the relaxation-time approximation was introduced and thoroughly discussed in Ref. 15. The main physical effect that arises beyond this approximation is field-induced tunneling, which leads to spin depolarization at tunneling resonances.[16]

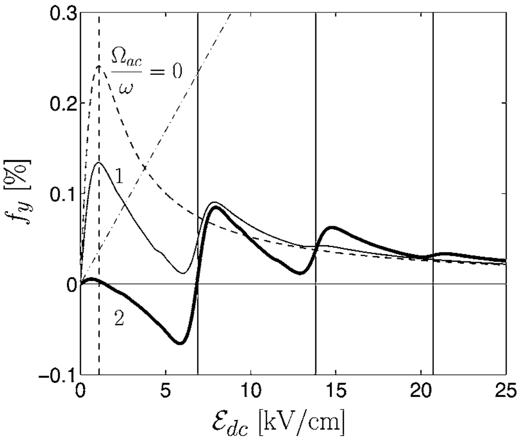

FIG. 1. Spin polarization f_y as a function of $\mathcal{E}$ for $\Omega_{ac}/\omega=0$ (dashed line), 1 (thin solid line), and 2 (thick solid line). Parameters used in the calculation are: $\tau=1$ ps, $T=4$ K, $\Delta=100$ meV, and $\hbar\alpha=5\times10^{-9}$ eV cm. The ac field has the frequency of $\nu_{ac}=1$ THz, and the Fermi energy is given by $\varepsilon_F=100$ meV. The dashed vertical line indicates the maximum at $\Omega_{dc}\tau=1$ and the position of photon resonances are marked by vertical solid lines. The low-field result of a two-dimensional electron gas is shown by the dash-dotted line.

Taking into account the periodic boundary condition $F_z(\mathbf{k}|t+2\pi/\omega)=F_z(\mathbf{k}|t)$, Eq. (14) is easily solved by

$$\langle F_z(l,k_y|t)\rangle = F_z^{(dc)}(l,k_y)\sum_{j=-\infty}^{\infty}\frac{J_j^2(l\Omega_{ac}/\omega)}{i(l\Omega_{dc}+j\omega)\tau+1}, \qquad (15)$$

with J_j denoting the Bessel function and $\Omega_{dc}[\Omega_{ac}]$ are the Bloch frequencies $e\mathcal{E}_{dc}d/\hbar[e\mathcal{E}_{ac}d/\hbar]$. Our final-result for the field-induced spin accumulation is obtained from Eqs. (13) and (15). Note that the Rashba coupling has a particular symmetry that is not observed, e.g., for Luttinger-type spin-orbit systems.[17] As a consequence, nonvanishing contributions to the l sum occur only for odd integers l. The main $l=1$ contribution to the field-induced spin polarization is expressed by

$$f_y = \sum_j J_j^2\left(\frac{\Omega_{ac}}{\omega}\right) f_y^{(dc)}(\Omega_{dc}+j\omega), \qquad (16)$$

where $f_y^{(dc)}(\Omega_{dc})$ denotes the magnetization due to a constant dc electric field alone. This representation has an interesting analogy in the theory of photon-assisted spinless carrier transport, where it is known to be the Tien-Gordon[18] or Tucker[19] formula for the current density. This similarity results from the fact that in spinless carrier transport also the $l=1$ component of the carrier distribution function plays the most important role. Therefore, qualitatively similar results are expected to occur both in the photon-assisted spin accumulation and carrier transport. This expectation is supported by numerical results for the field-dependent spin polarization shown in Fig. 1. The model calculation was carried out for a miniband width of $\Delta=100$ meV. For weakly coupled superlattices ($\Delta/\hbar\Omega_{dc}\ll1$), in which sequential tunneling occurs, a field-induced spin polarization is not efficient. For simplicity, we replaced $F_z^{(dc)}$ by the Fermi function. In Fig. 1, the dashed line shows the spin accumulation f_y induced by a dc electric field without any irradiation ($\Omega_{ac}=0$). This curve exhibits a maximum at $\Omega_{dc}\tau=1$ (marked by the vertical dashed line) and decreases at higher field strengths according to $f_y\sim1/\mathcal{E}_{dc}$. This decrease is due to Wannier-Stark localization of carriers under strong dc electric fields. The dash-dotted line shows the field-induced spin polarization of the related two-dimensional electron gas, which was previously derived[3] for the ohmic region. In the linear response regime, the spin polarization is much larger in the superlattice than in the two-dimensional electron gas. When in addition to the dc electric field, the sample is exposed to a radiation field, photon resonances appear in the spin polarization at dc field strengths determined from $l\Omega_{dc}=j\omega$ as indicated by vertical lines in Fig. 1 for $l=1$ and $j=1,2,3$. Resonances for $l>1$ appear as weak shoulders in the solid curves of Fig. 1. The most interesting effect, however, occurs when the field strength of the THz irradiation becomes sufficiently strong (thick solid line in Fig. 1). In this case, a field-induced reorientation of the homogeneous magnetization appears at low dc field strengths. In this region, the energy provided by the radiation field is used by the carriers to align the spins oppositely to the direction that is fixed by the pseudomagnetic field of the spin-orbit interaction. This reorientation of the magnetization expressed by negative values for f_y has its complete analogy in the field of spinless carrier transport by the appearance of absolute negative currents, which are observed, whenever carriers travel against the dc field direction by exploiting the energy supplied by the radiation field.[8,9] As shown in Fig. 1 by the thick solid line, the spin flip induced by the strong THz irradiation occurs under resonance condition $\Omega_{dc}=\omega$. This analogy between effects observed in two quite different research fields such as field-induced carrier transport and spin polarization even goes one step further, since dynamical localization and delocalization studied in the transport theory[10] have also their correspondence in the field-induced spin polarization governed by Eq. (13). Due to dynamical delocalization, the THz-mediated spin polarization can exceed the values that result from a dc electric field alone. At other dc field strengths, dynamical localization leads to spin depolarization. The field dependence of the magnetization as illustrated in Fig. 1 qualitatively agrees with results obtained previously for the current density.[8,9] The main quantum effect, which is not included in our approach, results from the off-diagonal elements of the density matrix $F_{\mu\mu'}$ that gives rise to field-induced tunneling resonances and an associated spin depolarization.

The radiation-induced switching between the local maximum and minimum of the magnetization is of the same order of magnitude as the effect of a dc electric field alone namely of about 0.1%. Although the predicted field-induced homogeneous magnetization is not very pronounced, it should be sufficiently large to be measurable by optical techniques. For systematic studies of ac and dc electric-field-induced spin polarization in strongly coupled lateral superlattices, a free-electron laser can be used as a continuous tunable source in the THz domain.

In conclusion, we developed a quasiclassical approach to treat the electric-field-induced spin polarization in lateral superlattices. The application of a strong dc electric field to the superlattice structure leads to a homogeneous spin polarization in the plane, which becomes maximal at $\Omega_{dc}\tau=1$ and which turns out to be larger than the one observed in the related two-dimensional electron gas. If in addition to this constant electric field the sample is irradiated by a strong ac field in the THz domain, dynamical localization and delocalization, as well as resonant photon absorption occur giving rise to a number of interesting phenomena. The most interesting effect of the THz irradiation is the reorientation of the in-plane spin polarization. The energy for these spin flips is provided by the ac electric field. The underlying mechanism is quite similar to the generation of absolute negative currents under the condition that carriers can use the energy of the radiation field to move the Wannier-Stark ladder upwards. The predicted effects induced by strong ac electric fields should be measurable with about the same precision as the spin polarization due to a dc electric field.

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