Theory of transverse spin dynamics in a polarized Fermi liquid and an itinerant ferromagnet

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Linear equations for the transverse spin dynamics in a weakly polarized degenerate Fermi liquid are derived from the Landau-Silin phenomenological kinetic equation with a general two-particle collision integral. Unlike a previous treatment where the Fermi velocity and density of states were taken to be constant independent of polarization we make no such assumption. The equations found describe the spin dynamics in a paramagnetic Fermi liquid with finite polarization as well in an itinerant ferromagnet. The results are confirmed by field theoretical calculations based on the integral equation for the vertex function. The transverse spin wave frequency in a polarized paramagnetic Fermi liquid is found to be proportional to k^2 with a complex diffusion coefficient such that the damping has a finite value proportional to the quasiparticles scattering rate at $T=0$. This behavior of a polarized Fermi liquid contrasts with the behavior of a Heisenberg ferromagnet in the hydrodynamic regime where the transverse spin-wave attenuation appears in terms proportional to k^4. The reactive part of the diffusion coefficient in a paramagnetic state at $T=0$ proves to be inversely proportional to the magnetization whereas in a ferromagnetic it is directly proportional to the magnetization. The dissipative part of the diffusion coefficient at $T=0$ in the paramagnetic state is polarization independent, whereas in the ferromagnetic state it is proportional to the square of the magnetization. Moreover, the spin wave spectrum in a ferromagnetic Fermi liquid proves to be unstable demonstrating the difficulty of applying a Fermi liquid description to itinerant ferromagnetism.

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I. INTRODUCTION

Since its appearance in the pioneering papers of Silin[1] and Leggett[2] the theory of spin dynamics in spin-polarized Fermi liquid has been intensively discussed mostly in relation to the zero-temperature transverse spin-wave attenuation. The calculations of the transverse spin-diffusion coefficient in a dilute degenerate Fermi gas with arbitrary polarization was done in the papers by Jeon and Mullin[3] in which the low-temperature saturation of the corresponding relaxation time was established. About the same time Meyerovich and Musaelyan[4,5] derived the spin kinetics in a polarized Fermi liquid from microscopic theory and came to the same conclusion. A derivation and an exact solution of the kinetic equation in the *s*-wave-scattering approximation for a dilute degenerate Fermi gas with arbitrary polarization at $T=0$ and for a small polarization at $T\neq 0$ have been obtained also in papers[6] by Golosov and Ruckenstein. For the treatment of this problem in a Fermi-liquid Matthiessen-type rule arguments and a simple relaxation-time approximation for the collision integral have been used.[7] More recently, the derivation of the transverse spin dynamics in a spin-polarized Fermi liquid from the Landau-Silin kinetic equation with a general form of a two-particle collision integral has been performed.[8] The existence of zero-temperature damping of transverse spin waves has been established. At low temperatures and polarizations γH the damping is proportional to the rate of collisions between quasiparticles

$$\frac{1}{\tau} \propto [(\gamma H)^2 + (2\pi T)^2]. \tag{1}$$

Experimentally the saturation of the transverse spin wave diffusion constant at temperatures of about several millikelvin has been registered by the spin-echo technique (see, for instance, Ref. 9). On the other hand, spin-wave experiments demonstrate the behavior characterized rather by the absence of transverse spin-wave damping in the same temperature region.[10] The latter seems to be a confirmation of the point of view of Fomin[11] who has argued that the transversal spin wave spectrum is dissipationless, derived from calculating the correction to the system energy brought about by a gauge transformation into the coordinate system in which the magnetization vector is constant. The calculation of the generalized susceptibility coefficient in the expression for the spin current found in Ref. 11 has not been performed; only the reference of such calculations[12] in superfluid ^{3}He were cited. Indeed, one can calculate susceptibility by application of a similar procedure. However, due to collisions between quasiparticles the proper Green function in a polarized Fermi liquid includes the finite imaginary self-energy part,[4,5] that inevitably leads to spin-wave attenuation. The same conclusion is obtained by the derivation of transverse spin-wave dynamics from the kinetic equation in a rotating reference frame[13] in which the matrix function of the quasiparticle distribution still contains an odd in momentum part producing the spin-current relaxation so long there is a finite rate of collisions between quasiparticles.

The derivation of linear transverse spin-wave dynamics has been undertaken in Ref. 8 at finite polarization. However, all the Fermi-liquid characteristic parameters have been taken as constants independent of polarization. The derivation partly free of this assumption (taking into consideration the polarization dependence of the Fermi velocity and the density of states) is proposed in the present article (Sec. II). It results not only in equations for the time-space variations of spin and spin current densities with more general expressions for all the coefficients but also reveals the origin of the

distinction between a spin-polarized paramagnetic Fermi liquid and a ferromagnetic Fermi liquid with spontaneous magnetization.

The transverse spin-wave frequency in a polarized paramagnetic Fermi liquid is found to be proportional to k^2 with a complex diffusion coefficient such that the damping has a finite value proportional to the scattering rate of quasiparticles at $T=0$. This behavior of a polarized Fermi liquid contrasts with the behavior of a Heisenberg ferromagnet in the hydrodynamic regime where the transverse spin-wave attenuation appears in terms proportional to k^4. The latter derivation is presented in the Appendix.

The polarization dependence of the diffusion coefficient is found to be different for a polarized Fermi liquid and for an itinerant ferromagnet. Moreover, the spin-wave spectrum in ferromagnetic Fermi liquids proves to be unstable, which demonstrates the inherent failure of the Fermi-liquid description of itinerant ferromagnetism.

It is well known that the phenomenological Landau Fermi-liquid theory has well established foundations based on microscopic theory. Namely, the transport equation for the vibrations in a Fermi liquid was derived from an integral equation for the vertex function and the general relationship between the amplitude of forward scattering and the Fermi-liquid interaction parameters was found.[14] There also exist several publications where the kinetic equation and field theoretical methods based on Landau Fermi-liquid theory have been applied to the treatment of itinerant isotropic ferromagnet.[15–17] In particular the derivation of a dissipationless (up to the terms of order $\sim k^4$) spin-wave spectrum has been announced.[16]

In the present article in the frame of microscopic theory we reconsider the problem of transverse spin waves in a spin-polarized Fermi liquid (Sec. III) and in itinerant ferromagnet taking into account the divergence of the static susceptibility (Sec. IV). It is shown that in both cases the microscopic derivation leads to the same conclusions as found by means of kinetic equation with a two-particle collision integral (Sec. II).

II. SPIN-WAVE DISPERSION

The quasiparticle distribution function as well as quasiparticle energy are given by 2×2 matrix in spin space

$$\hat{n}_{\mathbf{k}}(\mathbf{r},t) = n_{\mathbf{k}}(\mathbf{r},t)\hat{I} + \boldsymbol{\sigma}_{\mathbf{k}}(\mathbf{r},t)\hat{\boldsymbol{\sigma}}, \tag{2}$$

$$\hat{\varepsilon}_{\mathbf{k}}(\mathbf{r},t) = \varepsilon_{\mathbf{k}}(\mathbf{r},t)\hat{I} + \mathbf{h}_{\mathbf{k}}(\mathbf{r},t)\hat{\boldsymbol{\sigma}}. \tag{3}$$

Here $\hat{\boldsymbol{\sigma}}=(\hat{\sigma}_x,\hat{\sigma}_y,\hat{\sigma}_z)$ are Pauli matrices. As long as we consider small deviations of the magnetization direction from its equilibrium direction the equation for the scalar part of the distribution function $n_{\mathbf{k}}(\mathbf{r},t)$ decouples from the equation for the vector part of the distribution function $\boldsymbol{\sigma}_{\mathbf{k}}(\mathbf{r},t)$ and we may put $n_{\mathbf{k}}$ equal to its equilibrium value, namely, the usual Fermi function. Hence, the equation for the $\boldsymbol{\sigma}_{\mathbf{k}}(\mathbf{r},t)$ has the form

$$\frac{\partial \boldsymbol{\sigma}_{\mathbf{k}}}{\partial t} + \frac{\partial \varepsilon_{\mathbf{k}}}{\partial k_i}\frac{\partial \boldsymbol{\sigma}_{\mathbf{k}}}{\partial x_i} - \frac{\partial \mathbf{h}_{\mathbf{k}}}{\partial x_i}\frac{\partial n_{\mathbf{k}}}{\partial k_i} - 2(\mathbf{h}_{\mathbf{k}}\times\boldsymbol{\sigma}_{\mathbf{k}}) = \left(\frac{\partial \boldsymbol{\sigma}_{\mathbf{k}}}{\partial t}\right)_{\mathrm{coll}}. \tag{4}$$

We divide all the matrices into equilibrium and nonequilibrium parts

$$\hat{n}_{\mathbf{k}} = \hat{n}^0_{\mathbf{k}} + \delta\hat{n}_{\mathbf{k}}, \tag{5}$$

$$\hat{\varepsilon}_{\mathbf{k}} = \hat{\varepsilon}^0_{\mathbf{k}} + \delta\hat{\varepsilon}_{\mathbf{k}}, \tag{6}$$

where

$$\hat{n}^0_{\mathbf{k}} = \bar{n}_0(\varepsilon_{\mathbf{k}})\hat{I} + \frac{1}{2}\Delta n_0(\varepsilon_{\mathbf{k}})(\hat{\mathbf{m}}\hat{\boldsymbol{\sigma}}) \tag{7}$$

is the equilibrium distribution function of the polarized Fermi liquid and

$$\hat{\varepsilon}^0_{\mathbf{k}} = \varepsilon_{\mathbf{k}}\hat{I} - \frac{1}{2}\gamma(\mathbf{B}\hat{\boldsymbol{\sigma}}) \tag{8}$$

is the equilibrium quasiparticle energy. Here, the functions

$$\bar{n}_0(\varepsilon_{\mathbf{k}}) = \frac{1}{2}(n_0^+ + n_0^-) \tag{9}$$

and

$$\Delta n_0(\varepsilon_{\mathbf{k}}) = n_0^+ - n_0^- \tag{10}$$

are determined through two Fermi distribution functions

$$n_0^{\pm}(\varepsilon_{\mathbf{k}}) = n_0\left(\varepsilon_{\mathbf{k}} \mp \frac{\gamma H}{2}\right) = \frac{1}{\exp\left(\dfrac{\varepsilon_{\mathbf{k}} \mp \dfrac{\gamma H}{2} - \mu}{T}\right) + 1}, \tag{11}$$

γ is the gyromagnetic ratio, and Planck's constant $\hbar=1$ throughout the paper. The polarization direction is determined by the unit vector $\hat{\mathbf{m}}=\mathbf{H}/H$.

We have introduced two magnetic fields $\mathbf{H}$ and $\mathbf{B}$ and shall assume that they are parallel to each other. The field $\mathbf{H}$ determining the shift of the quasiparticle distribution function corresponds to the magnetization created by the external magnetic field $\mathbf{H}_0$ and by the pumping[18] in a paramagnetic Fermi liquid. The pumped part in view of a very long time for longitudinal relaxation should be considered as equivalent to the equilibrium part of the magnetization. In a ferromagnetic Fermi liquid $\mathbf{H}$ is spontaneous magnetic field existing even in absence of an external field and pumping. The field $\mathbf{B}$ determines the shift in energy of quasiparticles consisting of the external magnetic field $\mathbf{H}_0$ and the Fermi-liquid molecular field. To define $\mathbf{B}$ we must consider the equilibrium distribution matrix (7) and equilibrium energy matrix (8) to give the deviations from the corresponding matrices for an unpolarized Fermi liquid

$$\hat{n}^0_{\mathbf{k}} = n_0(\varepsilon_{\mathbf{k}})\hat{I} + \delta\hat{n}^0_{\mathbf{k}}, \tag{12}$$

$$\hat{\varepsilon}^0_{\mathbf{k}} = \varepsilon_{\mathbf{k}}\hat{I} - \frac{1}{2}\gamma(\mathbf{B}\hat{\boldsymbol{\sigma}}) = \varepsilon_{\mathbf{k}}\hat{I} - \frac{1}{2}\gamma(\mathbf{H}_0\hat{\boldsymbol{\sigma}}) + \frac{1}{2}Sp' \int d\tau' f^{\sigma\sigma'}_{\mathbf{k}\cdot\mathbf{k}'}\delta\hat{n}'^0_{\mathbf{k}'}, \tag{13}$$

where $d\tau = 2d\mathbf{k}/(2\pi)^3$ and $f^{\sigma\sigma'}_{\mathbf{k}\cdot\mathbf{k}'}$ is the Fermi-liquid interaction matrix.

As was discussed in[8] for a finite polarization and a general form of $f^{\sigma\sigma'}_{\mathbf{k}\cdot\mathbf{k}'}$ the vector $\mathbf{B}$ proves to be energy dependent. This means that a description of the spin dynamics in terms of closed system of differential equations for spin and spin current densities is not possible. To circumvent these difficulties as in the paper[8] we assume the functions $f_{\mathbf{k}\cdot\mathbf{k}'}$ to be independent of energy and take them to have the simplified form

$$f^{\sigma\sigma'}_{\mathbf{k}\cdot\mathbf{k}'} = f^s_{\mathbf{k}\cdot\mathbf{k}'}\hat{I}\hat{I}' + [f^a_0 + f^a_1(\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}')]\hat{\boldsymbol{\sigma}}\hat{\boldsymbol{\sigma}}'. \tag{14}$$

Now, from Eqs. (13) and (14) we obtain an equation that can be used to determine $\mathbf{B}$;

$$\gamma\mathbf{B} = \gamma\mathbf{H}_0 - \hat{\mathbf{m}}f^a_0\int d\tau\Delta n_0. \tag{15}$$

For small polarizations and taking for simplicity $T=0$ one can rewrite Eq. (15) as

$$\mathbf{B} = \mathbf{H}_0 - \mathbf{H}F^a_0\left[1 - \frac{1}{6}\left(\frac{\gamma H}{4\mu}\right)^2\right]. \tag{16}$$

Here and below

$$F^a_i = N_0 f^a_i, \quad i = 0,1\ldots,$$

and $N_0 = m^* k_F/\pi^2$ is the density of states at zero polarization.

In the absence of a pumped magnetization the field $\mathbf{B} = \mathbf{H}$ and Eq. (16) is just the self-consistency equation for the field $\mathbf{H}$ as a function of an external field $\mathbf{H}_0$ giving in the lowest order

$$\mathbf{H} = \frac{\mathbf{H_0}}{1 + F^a_0}. \tag{17}$$

As a particular case one can consider a ferromagnetic state when $F^a_0 = -1 - \delta$. The solution of Eq. (16),

$$\frac{1}{6}\left(\frac{\gamma H}{4\mu}\right)^2 = \frac{1 + F^a_0}{F^a_0} \tag{18}$$

then exists even in the absence of an external field. The spontaneous polarization is directly proportional to the magnetization (an order parameter) of the ferromagnet

$$\mathbf{M}^{\|} = \frac{\gamma N_0}{4}\mathbf{H}. \tag{19}$$

According to Eq. (18) it is determined by the Fermi-liquid interaction. Note also, and we shall use it below, that in a weakly polarized ferromagnetic Fermi liquid the deviation of F^a_0 from -1 is quadratic in the polarization.

When part of the magnetization is created by pumping, $\mathbf{H}$ is an independent value and the total energy shift $\gamma(\mathbf{B}\hat{\boldsymbol{\sigma}})/2$ is determined from two fields: external field $\mathbf{H}_0$ and "effective" field $\mathbf{H}$.

We discuss only perpendicular deviations from the initial equilibrium state

$$\delta\hat{n}_{\mathbf{k}} = \delta\boldsymbol{\sigma}_{\mathbf{k}}(\mathbf{r},t)\hat{\boldsymbol{\sigma}}, \quad (\hat{\mathbf{m}}\,\delta\boldsymbol{\sigma}_{\mathbf{k}}) = 0. \tag{20}$$

Then the energy deviation matrix has the form

$$\delta\hat{\varepsilon}_{\mathbf{k}} = \delta\mathbf{h}_{\mathbf{k}}\hat{\boldsymbol{\sigma}}, \quad \delta\mathbf{h}_{\mathbf{k}} = \int d\tau' f^a_{\mathbf{k}\cdot\mathbf{k}'}\delta\boldsymbol{\sigma}_{\mathbf{k}'} \tag{21}$$

and the kinetic equation (4) can be rewritten as

$$\frac{\partial\delta\boldsymbol{\sigma}_{\mathbf{k}}}{\partial t} + \frac{\partial\varepsilon^0_{\mathbf{k}}}{\partial k_i}\frac{\partial\delta\boldsymbol{\sigma}_{\mathbf{k}}}{\partial x_i} - \frac{\partial\bar{n}_0}{\partial k_i}\frac{\partial\delta\mathbf{h}_{\mathbf{k}}}{\partial x_i} - 2\left[\left(-\frac{1}{2}\gamma\mathbf{B} + \delta\mathbf{h}_{\mathbf{k}}\right) \times \left(\frac{1}{2}\hat{\mathbf{m}}\Delta n_0\hat{+}\,\delta\boldsymbol{\sigma}_{\mathbf{k}}\right)\right] = \left(\frac{\partial\boldsymbol{\sigma}_{\mathbf{k}}}{\partial t}\right)_{\text{coll}}. \tag{22}$$

To derive a closed system of equations for the spin density $\mathbf{M}$ and the spin current density $\mathbf{J}_i$ in the case of finite polarization we make an assumption which is plausible for a weakly polarized Fermi liquid that the energy dependence of $\delta\boldsymbol{\sigma}_{\mathbf{k}}(\mathbf{r},t)$ can be factorized from the space and direction $\hat{\mathbf{k}}$ dependences:

$$\delta\boldsymbol{\sigma}_{\mathbf{k}}(\mathbf{r},t) = \mathbf{A}(\mathbf{r},t)\alpha(\varepsilon) + \mathbf{B}_i(\mathbf{r},t)\hat{k}_i\beta(\varepsilon). \tag{23}$$

In terms of these functions one can write the spin density

$$\mathbf{M}(\mathbf{r},t) = \frac{1}{2}\int d\tau\delta\boldsymbol{\sigma}_{\mathbf{k}} = \frac{1}{2}\mathbf{A}(\mathbf{r},t)\int d\tau\alpha(\varepsilon), \tag{24}$$

and the spin current density

$$\mathbf{J}_i(\mathbf{r},t) = \frac{1}{2}\int d\tau\left[v_i\delta\boldsymbol{\sigma}_{\mathbf{k}} - \frac{\partial n_0}{\partial k_i}\delta\mathbf{h}_{\mathbf{k}}\right] = \frac{1}{2}\psi\int d\tau v_i\delta\boldsymbol{\sigma}_{\mathbf{k}} = \frac{1}{6}\mathbf{B}_i(\mathbf{r},t)\int d\tau v_i(\varepsilon)\left[\beta(\varepsilon) - \frac{f^a_1}{3}\frac{\partial\bar{n}_0}{\partial\varepsilon}\int d\tau\beta(\varepsilon)\right], \tag{25}$$

where $v_i(\varepsilon) = \partial\varepsilon_{\mathbf{k}}/\partial k_i$ and

$$\psi = \frac{\int d\tau v(\varepsilon)\left[\beta(\varepsilon) - \frac{f^a_1}{3}\frac{\partial\bar{n}_0}{\partial\varepsilon}\int d\tau\beta(\varepsilon)\right]}{\int d\tau v(\varepsilon)\beta(\varepsilon)}. \tag{26}$$

Carrying out the integrations in kinetic equation (22), $\int d\tau/2$ and $\int d\tau v_i/2$, we obtain after some simple algebra

$$\frac{\partial\mathbf{M}}{\partial t} + \frac{\partial\mathbf{J}_i}{\partial x_i} - \mathbf{M}\times\gamma\mathbf{H}_0 = 0, \tag{27}$$

$$\frac{\partial\mathbf{J}_i}{\partial t} + \frac{w^2}{3}\frac{\partial\mathbf{M}}{\partial x_i} - \mathbf{J}_i\times\gamma\mathbf{H}_0 + \mathbf{J}_i\times\mathbf{C} = \frac{\psi}{2}\int d\tau v_i\left(\frac{\partial\boldsymbol{\sigma}_{\mathbf{k}}}{\partial t}\right)_{\text{coll}}. \tag{28}$$

Here

$$w^2 = \psi\left[\frac{\int d\tau v^2(\varepsilon)\alpha(\varepsilon)}{\int d\tau\alpha(\varepsilon)} - f_0^a\int d\tau v^2(\varepsilon)\frac{\partial\bar{n}_0}{\partial\varepsilon}\right] \quad (29)$$

and

$$\mathbf{C} = \hat{\mathbf{m}}\left[f_0^a\int d\tau\Delta n_0(\varepsilon) - \frac{f_1^a\int d\tau\beta(\varepsilon)\int d\tau v(\varepsilon)\Delta n_0(\varepsilon)}{3\int d\tau\beta(\varepsilon)v(\varepsilon)}\right]. \quad (30)$$

Equations (27) and (28) have the same form as in the case of vanishingly small polarization.[2,8] The correspondence becomes exact if we put

$$\alpha(\varepsilon) \propto \beta(\varepsilon) \propto \Delta n_0(\varepsilon). \quad (31)$$

Thus, one can work with Eq. (28) taking more specific definitions for

$$\psi = \frac{\int d\tau v(\varepsilon)\left[\Delta n_0(\varepsilon) - \frac{f_1^a}{3}\frac{\partial\bar{n}_0}{\partial\varepsilon}\int d\tau\Delta n_0(\varepsilon)\right]}{\int d\tau v(\varepsilon)\Delta n_0(\varepsilon)}, \quad (32)$$

$$w^2 = \psi\left[\frac{\int d\tau v^2(\varepsilon)\Delta n_0(\varepsilon)}{\int d\tau\Delta n_0(\varepsilon)} - f_0^a\int d\tau v^2(\varepsilon)\frac{\partial\bar{n}_0}{\partial\varepsilon}\right], \quad (33)$$

and

$$\mathbf{C} = \frac{\hat{\mathbf{m}}}{N_0}\left(F_0^a - \frac{F_1^a}{3}\right)\int d\tau\Delta n_0(\varepsilon). \quad (34)$$

At last, using the calculations of the paper[8] for the collision integral in a weakly polarized liquid we come to the equation for the spin current density

$$\frac{\partial\mathbf{J}_i}{\partial t} + \frac{w^2}{3}\frac{\partial\mathbf{M}}{\partial x_i} - \mathbf{J}_i\times\gamma\mathbf{H}_0 + \mathbf{J}_i\times\mathbf{C} = -\frac{\mathbf{J}_i}{\tau}, \quad (35)$$

where the current relaxation time is

$$\frac{1}{\tau} = \frac{m^{*3}}{6(2\pi)^5}(2\overline{W_1} + \overline{W_2})[(2\pi T)^2 + (\gamma H)^2]. \quad (36)$$

Although Eq. (35) has the form of the typical so called "relaxation time approximation" we must stress here that it is derived from kinetic equation and general form of two particle collision integral.[8]

In the frame rotating with the local Larmor frequency $\omega_L = \gamma H_0$ the quasistationary solution of Eq. (35) has the form

$$\mathbf{J}_i = D''\hat{\mathbf{m}}\times\nabla_i\mathbf{M} - D'\nabla_i\mathbf{M}. \quad (37)$$

and the dispersion law of the transverse spin waves following from Eqs. (27) and (35) is (see, for instance, Ref. 19)

$$\omega = \omega_L + (D'' - iD')k^2, \quad (38)$$

where

$$D' = \frac{w^2\tau}{3[1+(C\tau)^2]} \cong \frac{w^2}{3C^2\tau} \quad (39)$$

is the dissipative part of the diffusion coefficient and

$$D'' = C\tau D' \cong \frac{w^2}{3C} \quad (40)$$

is its reactive part. Here the approximate values of D' and D'' correspond to the limit $C\tau \gg 1$.

For a weakly polarized fluid $C = (F_0^a - F_1^a/3)\gamma H$ and $\psi = 1 + F_1^a/3$. The expression for w^2 depends on the state. One can find it analytically in the case of weak polarization. In a paramagnetic Fermi liquid it is

$$w^2 = v_F^2(1+F_0^a)\left(1+\frac{F_1^a}{3}\right), \quad (41)$$

where v_F is the Fermi velocity in the unpolarized liquid. In a ferromagnetic Fermi liquid (if an external field is smaller than the spontaneous field) we find from Eq. (33) with help of Eq. (15)

$$w^2 = -v_F^2\left(1+\frac{F_1^a}{3}\right)\left(\frac{\gamma H}{4\mu}\right)^2. \quad (42)$$

Thus, the reactive part of the diffusion coefficient in a paramagnetic state at $T=0$ proves to be inversely proportional to the magnetization

$$D'' = \frac{v_F^2(1+F_0^a)(1+F_1^a/3)}{3(F_0^a - F_1^a/3)\gamma H}, \quad (43)$$

whereas in ferromagnetic state it is directly proportional to the magnetization

$$D'' = \frac{v_F^2\gamma H}{3(4\varepsilon_F)^2}. \quad (44)$$

The latter is in exact correspondence with the known result obtained by Moriya in framework of the Stoner-Hubbard model.[20]

The dissipative part of the diffusion coefficient given by Eq. (39) at $T=0$ in a paramagnetic state is polarization independent, whereas in a ferromagnetic state it is proportional to the square of the magnetization. More importantly, the imaginary part of the dispersion law in a Fermi liquid with spontaneous magnetization proves to be positive. This means that there is an intrinsic instability in a Fermi liquid with spontaneous magnetization. The latter rules out a description of itinerant ferromagnetism based on a polarized Fermi liquid or, in other words, on the Hubbard-Stoner model with short range repulsion between Fermi particles with the opposite spins.

Physically the instability found originates from the Fermi liquid interaction. Indeed, the negative value of $1+F_0^a$ causes the appearance of spontaneous magnetization and also (in isotropic case) it stimulates a transverse deviation of magnetization from equilibrium direction.

The transverse spin waves frequency in a polarized paramagnetic Fermi liquid is found to be proportional to k^2 with a complex diffusion coefficient. This behavior contrasts with the behavior in a Heisenberg ferromagnet in the hydrodynamic regime[21–23] (see the Appendix), where the transverse spin wave attenuation appears in terms proportional to k^4. The damping in a polarized Fermi liquid for

$$\gamma H\tau \gg 1 \tag{45}$$

has a finite value proportional to the scattering rate of quasiparticles at $T=0$. As it was pointed out in Ref. 8 the latter is formally analogous with ultrasound attenuation in the collisionless regime. It is worth noting, however, that the parameter $\gamma H\tau$ has no relation to the establishment of local equilibrium.

The results (38)–(44) are valid in assuming (i) the quasistationarity condition

$$Dk^2\tau \ll 1, \tag{46}$$

which is the case both in spin-echo[9] and spin-wave[10] experiments, and (ii)

$$Dk^2 \ll \gamma H \tag{47}$$

that is the condition of validity of two moment approximation for the solution of the kinetic equation.[2]

III. MICROSCOPIC DERIVATION OF SPIN-WAVE SPECTRUM IN A POLARIZED FERMI LIQUID

The Landau-type derivation of transverse spin dynamics in a weakly spin-polarized Fermi liquid from microscopic theory has been performed in the paper.[5] Here we make a similar derivation to stress the conditions required for its validity, to compare the answer with that obtained from the kinetic equation at nonzero temperatures, and to juxtapose this with the derivation for a ferromagnetic Fermi liquid[17] which we also reproduce afterwards.

As in the original paper by Landau[14] we consider a system of fermions at $T=0$, with arbitrary short range interaction forces. The presence of polarization means that subsystems of spin-up and spin-down particles have different chemical potentials $\mu_\pm=\mu\pm\gamma B/2$ and the distribution functions are based on different Fermi momenta $p_\pm=p_0\pm\gamma H/2v_F$. The polarization in general is nonequilibrium and, as in previous section, we shall distinguish the fields **H** and **B**. Here, we shall not consider the polarization dependence of the Fermi velocity and density of states and limit the discussion to a weakly polarized paramagnetic Fermi liquid. The ferromagnetic case shall be discussed in the next section. So, the Fermi velocity is $v_F=\partial\varepsilon(p)/\partial p|_{p=p_0}$ and $p_0=(p_++p_-)/2$.

The Green functions near $|\mathbf{p}|=p_\pm$ and $\varepsilon(\mathbf{p})=\mu_\pm$ have the form

$$G_\pm(\mathbf{p},\varepsilon)=\frac{a}{\varepsilon-\varepsilon(\mathbf{p})+\mu_\pm+ibv_F^2(p-p_\pm)|p-p_\pm|}. \tag{48}$$

We assume a weak polarization $v_F(p_+-p_-)\ll\varepsilon_F$ and also that both the Fermi distributions are characterized by the same Landau Fermi-liquid parameters. We introduce here the general form of the imaginary part of self-energy[24] which is a quadratic function of the difference $(p-p_\pm)$ and changes its sign at $p=p_\pm$ correspondingly. The assumption of small polarization means in particular that G_+ is given by the expression (48) not only near $|\mathbf{p}|=p_+$ and $\varepsilon(\mathbf{p})=\mu_+$ but in the entire intervals $p_-<p<p_+$ and $\mu_-<\varepsilon(\mathbf{p})<\mu_+$ and also near $|\mathbf{p}|=p_-$ and $\varepsilon(\mathbf{p})=\mu_-$. The same is true for G_-.

Following Landau, let us write the equation for the vertex function describing the scattering of two particles with opposite spin directions and a small transfer of four-momentum $K=(\mathbf{k},\omega)$

$$\Gamma(P_1,P_2,K)=\Gamma_1(P_1,P_2)-\frac{i}{(2\pi)^4}\int\Gamma_1(P_1,Q)\times G_+(Q)G_-(Q+K)\Gamma(Q,P_2,K)d^4Q. \tag{49}$$

If K is small and the polarization is also small, the poles of the two Green functions are close to each other. Let us assume that all other quantities in the integrand are slowly varying with respect to Q: their energy and momentum scales of variation are larger than $\max\{\gamma H,\omega\}$ and $\max\{\gamma H/v_F,k\}$ correspondingly. Then one can perform the integration in Eq. (49) at fixed values of $q=p_0$, $\varepsilon=0$ in the arguments of the Γ and Γ_1 functions. In other words, one can substitute in Eq. (49)

$$\begin{aligned}G_+(Q)G_-(Q+K)&=G_+(\mathbf{q},\varepsilon)G_-(\mathbf{q}+\mathbf{k},\varepsilon+\omega)\\&=\frac{2\pi ia^2}{v_F}\delta(\varepsilon)\delta(|\mathbf{q}|-p_0)\\&\quad\times\frac{\gamma H+\mathbf{k}\cdot\mathbf{v}_F}{\omega-\omega_L+\gamma HF_0^a+ib(\gamma H)^2/2-\mathbf{k}\cdot\mathbf{v}_F+ib\gamma H\mathbf{k}\cdot\mathbf{v}_F}\\&\quad+\Phi_{\mathrm{reg}}.\end{aligned} \tag{50}$$

For elimination Γ_1 from Eq. (49) we shall rewrite this equation in the operator form

$$\Gamma=\Gamma_1-i\Gamma_1(i\Phi+\Phi_{\mathrm{reg}})\Gamma, \tag{51}$$

where product is interpreted as an integral, and $i\Phi$ denotes the first term from the right-hand side Eq. (50). In Eq. (51), we transpose the term involving Φ_{reg} to the left-hand side, and then apply the operator $(1+i\Gamma_1\Phi_{\mathrm{reg}})^{-1}$, obtaining

$$\Gamma=\Gamma^\omega+\Gamma^\omega\Phi\Gamma, \tag{52}$$

where

$$\Gamma^\omega=(1+i\Gamma_1\Phi_{\mathrm{reg}})^{-1}\Gamma_1. \tag{53}$$

As it is known,[14] $\Gamma^\omega(H=0)$ is directly related to the function determining the Fermi-liquid interaction

$$\Gamma^\omega(H=0)=\Gamma[(|\mathbf{k}|/\omega)\to 0,H=0]=\frac{F_{\mathbf{n}\cdot\mathbf{n}'}}{a^2N_0}. \tag{54}$$

At finite H the Γ^{ω} function can be expanded over the polarization as

$$a^2 N_0 \Gamma^{\omega} = F_{\mathbf{n}\cdot\mathbf{n}'} + ib\gamma H C_{\mathbf{n}\cdot\mathbf{n}'} + O(H^2). \tag{55}$$

From Eqs. (52) and (55), we come, according to a well known procedure,[14] to the kinetic equation

$$\left(\omega - \omega_L + \gamma H F_0^a + \frac{ib(\gamma H)^2}{2} - \mathbf{k}\cdot\mathbf{n}v_F + ib\mathbf{k}\cdot\mathbf{n}v_F\gamma H\right)\nu(\mathbf{n})$$
$$= (\gamma H + \mathbf{k}\cdot\mathbf{n}v_F)\int \frac{d\mathbf{n}'}{4\pi}(F_{\mathbf{n}\cdot\mathbf{n}'} + ib\gamma H C_{\mathbf{n}\cdot\mathbf{n}'})\nu(\mathbf{n}'). \tag{56}$$

We limit ourself to the first two harmonics in the Landau interaction function $F_{\mathbf{n}\cdot\mathbf{n}'} = F_0^a + (\mathbf{n}\cdot\mathbf{n}')F_1^a$ and $C_{\mathbf{n}\cdot\mathbf{n}'} = C_0 + (\mathbf{n}\cdot\mathbf{n}')C_1$. To obtain the spectrum of the spin waves (see below) obeying the Larmor theorem: the system of spins in a homogeneous magnetic field executes precessional motion at the Larmor frequency $\omega_L = \gamma H_0$, the coefficient C_0 has to be chosen[25] equal to $1/2$.

Introducing the expansion of the distribution function $\nu(\mathbf{n})$ over spherical harmonics of direction $\mathbf{n} = \mathbf{v}_F/v_F$, one can find from Eq. (56) that the ratio of amplitudes of the successive harmonics with $l \geqslant 1$ is of the order of $kv_F/\gamma H$. Hence if this ratio is assumed to be a small parameter one can work with distribution function taken to have the form[2] $\nu(\mathbf{n}) = \nu_0 + (\mathbf{n}\cdot\hat{\mathbf{k}})\nu_1$. The functions ν_0 and ν_1 obey the following system of linear equations:

$$(\omega - \omega_L)\nu_0 - \frac{kv_F}{3}\left[1 + \frac{F_1^a}{3} - ib\left(1 - \frac{C_1}{3}\right)\gamma H\right]\nu_1 = 0, \tag{57}$$

$$-kv_F\left(1 + F_0^a - \frac{ib\gamma H}{2}\right)\nu_0$$
$$+\left[\omega - \omega_L + \left(F_0^a - \frac{F_1^a}{3}\right)\gamma H + ib\left(\frac{1}{2} - \frac{C_1}{3}\right)(\gamma H)^2\right]\nu_1 = 0. \tag{58}$$

The vanishing of the determinant of this system gives the spin waves dispersion law. At long enough wavelengths when the dispersive part of the $\omega(k)$ dependence is much less than ω_L and neglecting the terms $\sim(b\gamma H)^2$, we have

$$\omega = \omega_L + (D'' - iD')k^2, \tag{59}$$

where

$$D'' = \frac{v_F^2(1 + F_0^a)(1 + F_1^a/3)}{3(F_0^a - F_1^a/3)\gamma H} \tag{60}$$

is a reactive part of the diffusion coefficient. It is exactly the same as given by Eq. (43). The dissipative part of the diffusion coefficient is

$$D' = \frac{bv_F^2[(1 - C_1/3)(1 + F_0^a)^2 - (1 + F_1^a/3)^2/2]}{3(F_0^a - F_1^a/3)^2}. \tag{61}$$

It is polarization independent and proportional to the quasiparticle scattering rate in correspondence with Eq. (39). We derived Eqs. (60) and (61) assuming of $(F_0^a - F_1^a/3) \neq 0$.

The expressions for D'' and D' have been obtained first by the same method by Meyerovich and Musaelyan.[5] The former result literally coincides with that found in this paper, the latter has the same parametric dependence but depends in a different way on the Fermi-liquid parameters. This is perhaps due to "off-shell" vertex corrections[26] taken into account in Ref. 5 where one can also find a comparison with the solution obtained for the weakly interacting gases.

Thus, the general microscopic derivation confirms the statement about the existence of zero-temperature spin waves attenuation in a polarized Fermi liquid. The value of the dissipative part of spin diffusion D' is determined by the amplitude "b" of the imaginary part of self-energy. It originates from collisions between quasiparticles.

IV. MICROSCOPIC DERIVATION TAKING INTO ACCOUNT THE TRANSVERSE STATIC SUSCEPTIBILITY DIVERGENCY

There are several known investigations of an isotropic itinerant ferromagnetic state as some peculiar type of Fermi liquid. This subject was discussed phenomenologically by Abrikosov and Dzyaloshinskii[15] and then microscopically by Kondratenko.[16] These authors did not include in the theory a finite scattering rate between quasiparticles and as result they obtained dissipationless transverse spin wave dispersion laws as expected in an isotropic ferromagnet. The derivation[15] was criticized by Herring[27] who pointed out the existence of a finite scattering rate. Later Dzyaloshinskii and Kondratenko[17] rederived the spin-wave dispersion law in ferromagnets. Making use as the starting point the Landau equation for the vertex function for the scattering of two particles with opposite spins and a small transfer of four-momentum they have redefined the product of two Green functions G_+G_- in such a manner that its resonant part was taken equal to zero at $\omega = 0$. This trick allows one to use the $1/k^2$ divergency of transverse static susceptibility, which is an inherent property of degenerate systems and occurs both in an isotropic ferromagnet, and in a spin-polarized paramagnetic Fermi liquid in the absence of interactions violating total magnetization conservation. As in the previous papers,[15,16] the authors of Ref. 17 did not introduce a finite quasiparticle attenuation in momentum space between the Fermi surfaces for the particles with opposite spins.

Similar derivation including calculation of a reactive part spin diffusion constant has been performed by Moriya[20] in the framework of Stoner-Hubbard microscopic model. The dissipative part of dispersion law has not been found.

Let us see now what kind of modifications appear if we reproduce the derivation proposed in Ref. 17 with the Green functions (48) taking into account the finite quasiparticle attenuation in the whole interval $p_- < p < p_+$. We discuss an isotropic ferromagnet at equilibrium $\mathbf{B} = \mathbf{H}$ first in the absence of external field. Following[17] we write

$$G_+(Q)G_-(Q+K) = G_+(\mathbf{q},\varepsilon)G_-(\mathbf{q}+\mathbf{k},\varepsilon+\omega) = \frac{2\pi i a^2}{v_F}\delta(\varepsilon)\delta(|\mathbf{q}|-p_0) \times \frac{\omega}{\omega-\gamma H+ib(\gamma H)^2/2-\mathbf{k}\cdot\mathbf{v}_F+ib\mathbf{k}\cdot\mathbf{v}_F\gamma H}+\tilde{\Phi}_{\mathrm{reg}}. \tag{62}$$

Now Eq. (49) is written as

$$\Gamma=\Gamma_1-i\Gamma_1(i\tilde{\Phi}+\tilde{\Phi}_{\mathrm{reg}})\Gamma, \tag{63}$$

where $i\tilde{\Phi}$ denotes the first term on the right-hand side of Eq. (62). The equivalent form of this equation is

$$\Gamma=\Gamma^{\mathbf{k}}+\Gamma^{\mathbf{k}}\tilde{\Phi}\Gamma, \tag{64}$$

where

$$\Gamma^{\mathbf{k}}=\Gamma\left(\frac{\omega}{|\mathbf{k}|}\to 0\right)=(1+i\Gamma_1\tilde{\Phi}_{\mathrm{reg}})^{-1}\Gamma_1. \tag{65}$$

The isotropic part of $\Gamma^{\mathbf{k}}$ is proportional to the static transverse susceptibility. Hence it has a singular form[17]

$$\Gamma^{\mathbf{k}}\propto-\frac{1}{N_0(ck)^2}. \tag{66}$$

Here, c is a parameter with the dimensions of length. One can show by direct calculation of static transverse susceptibility in the Stoner ferromagnet[20] that c is the polarization independent

$$c\sim\frac{1}{p_0}. \tag{67}$$

At the same time similar calculations for a polarized paramagnetic Fermi liquid gives the value of c inversely proportional to polarization

$$c\sim\frac{v_F}{\gamma H} \tag{68}$$

such that the divergency (66) disappears in an unpolarized liquid when $\gamma H\to 0$.

Substitution of Eq. (66) into Eq. (64) gives the transverse spin-wave dispersion law

$$\omega=\gamma H(ck)^2\left(1-\frac{ib\gamma H}{2}\right). \tag{69}$$

One can take into consideration a static external field, by working in the rotating with Larmor frequency frame that is equivalent to the substitution $\omega\to\omega-\omega_L$ (see also Ref. 17). As a result, we obtain the dispersion law

$$\omega=\omega_L+\gamma H(ck)^2\left(1-\frac{ib\gamma H}{2}\right) \tag{70}$$

that has the same form as Eq. (38). Taking into account the relations (67) and (68) one can verify that the polarization dependences of reactive and dissipative parts of the diffusion constants in ferromagnetic Fermi liquids and in polarized paramagnetic Fermi liquids coincide with those found at the end of the Sec. II. The present derivation does not contain a self-consistent calculation of the "b" coefficient, that is why, unlike to the Sec. II, here we cannot comment on the instability in a Fermi liquid with spontaneous magnetization.

V. CONCLUSION

In conclusion we stress once again that the transverse spin wave dispersion in a polarized paramagnetic Fermi liquid is found to be attenuating. The spin wave frequency is proportional to k^2 with complex diffusion coefficient such that the damping at $T=0$ has a finite value proportional to the scattering rate of quasiparticles. This behavior of a polarized paramagnetic Fermi liquid contrasts with the behavior of a Heisenberg ferromagnet in the hydrodynamic regime where the transverse spin wave attenuation appears in terms proportional to k^4 (see the Appendix). At the phenomenological level this difference originates from the diffusive current which exists in the mixture of spin-up and spin-down Fermi liquids even at zero temperature. Unlike for a polarized paramagnetic Fermi liquid the spectrum of transverse spin waves in a ferromagnetic Fermi liquid has an inherent instability that a pure Fermi-liquid description of itinerant ferromagnetism is not necessarily possible.

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APPENDIX: TRANSVERSE SPIN WAVES IN HEISENBERG FERROMAGNET

The treatment of transverse spin waves hydrodynamics has been undertaken by several authors (see, e.g., Refs. 21–23). We rederive it here in a different manner. With this purpose we shall use a phenomenological expression for the free energy

$$F=\int d\mathbf{r}\left\{f(M)-\mathbf{M}\cdot\mathbf{H_0}+\frac{a}{2}(\boldsymbol{\nabla}_i\mathbf{M})^2\right\}, \tag{A1}$$

where the minimum of homogeneous energy density $f(M)$ gives the equilibrium value of magnetization M in ferromagnetic state and $\mathbf{H_0}$ is an external magnetic field. Being interested in the dispersion law of small transverse vibrations of magnetization $\delta M_\alpha=e_{\alpha\beta\gamma}\Theta_\beta M_\gamma$, where $\boldsymbol{\Theta}$ is a vector of infinitesimal rotation lying in the plane perpendicular to $\mathbf{M}$, we rewrite the free energy in terms of these angles

$$F=\int d\mathbf{r}\left\{f(M)-\mathbf{M}\cdot\mathbf{H_0}+\frac{a}{2}\mathbf{M}^2(\boldsymbol{\nabla}_i\boldsymbol{\Theta})^2\right\}. \tag{A2}$$

Then, by introducing the magnetization current as

$$\mathbf{J}_i = -\frac{\delta F}{\delta \boldsymbol{\nabla}_i \boldsymbol{\Theta}} = a(\mathbf{M} \times \boldsymbol{\nabla}_i \mathbf{M}), \tag{A3}$$

we obtain the equation of motion of magnetization or the equation of spin-density conservation

$$\frac{\partial \mathbf{M}}{\partial t} + \frac{\partial \mathbf{J}_i}{\partial x_i} - \mathbf{M} \times \gamma \mathbf{H_0} = 0 \tag{A4}$$

known as Landau-Lifshits equation.[28] The simple derivation from Eqs. (A3) and (A4) results in dispersion law of linear transverse spin waves

$$\omega = \omega_L + aMk^2. \tag{A5}$$

Here $\omega_L = \gamma H_0$ is the Larmor frequency.

So, the reactive part of spin-wave dispersion proves to be directly proportional to the magnetization value. This general property of Landau-Lifshits equation is sometimes formulated as a result of finite domain wall rigidity.

The dissipation can be also taken into consideration. It is only necessary to generalize the spin-current expression

$$\mathbf{J}_i = -\frac{\delta F}{\delta \boldsymbol{\nabla}_i \boldsymbol{\Theta}} + \mathbf{J}_i^{\mathrm{diss}}, \quad \mathbf{J}_i^{\mathrm{diss}} = -\frac{\delta R}{\delta \boldsymbol{\nabla}_i \boldsymbol{\Theta}}, \tag{A6}$$

where

$$R = \frac{1}{2} \iint d\mathbf{r} d\mathbf{r}' b(\mathbf{r} - \mathbf{r}') e_{\alpha\beta\gamma} M_\alpha \boldsymbol{\nabla}_i \boldsymbol{\nabla}_j \Theta_\beta(\mathbf{r}) \boldsymbol{\nabla}_i' \boldsymbol{\nabla}_j' \Theta_\gamma(\mathbf{r}') \tag{A7}$$

is the dissipation function which according to general rules is chosen being quadratic on gradients of spin velocity $\boldsymbol{\nabla}_i \boldsymbol{\Theta}$ (it is variable conjugated to the spin current) and such that the Onsager principle

$$b(\mathbf{r} - \mathbf{r}') e_{\alpha\beta\gamma} M_\alpha = b(\mathbf{r}' - \mathbf{r}) e_{\alpha\gamma\beta}(-M_\alpha) \tag{A8}$$

has to be fulfilled. Here $b(\mathbf{r})$ is a short-range even function.

Now the total spin current density is

$$\mathbf{J}_i = a(\mathbf{M} \times \boldsymbol{\nabla}_i \mathbf{M}) + b \boldsymbol{\nabla}_i \Delta \mathbf{M}, \tag{A9}$$

and taking it into account we obtain from Landau-Lifshits equation the transverse spin-wave dispersion law with dissipation

$$\omega = \omega_L + aMk^2 - ibk^4. \tag{A10}$$

The microscopic calculation[29,30] gives the value of coefficient $b \propto \ln(T/k^2)/|M|^3$ meaning the nonanalytic wave vector dependence of dispersion law. We stress also that all the results found here are valid in hydrodynamic or local equilibrium regime that is under the following condition:

$$aMk^2 \tau \ll 1. \tag{A11}$$

Unlike Heisenberg ferromagnet (A9) the spin-current density in a ferromagnetic Fermi liquid has the following form [see Eq. (37)]

$$\mathbf{J}_i = D'' \hat{\mathbf{m}} \times \boldsymbol{\nabla}_i \mathbf{M} - D' \boldsymbol{\nabla}_i \mathbf{M}. \tag{A12}$$

The first reactive (time reversal invariant) terms in two cases just coincide. While the second terms describing the dissipative flow and odd in respect of time reversal are different. The dissipative current of the first order in gradients is absent in a Heisenberg ferromagnet but it is always present in spin polarized Fermi liquid as diffusion current in the solution of Fermi liquids with up and down spins.

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