Critical currents of the phase slip process in the presence of electromagnetic radiation: Rectification for time asymmetric ac signal

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We studied theoretically the effect of time symmetric and asymmetric electromagnetic (e.m.) radiation on the phase-slip process in superconducting wires in the regime where there is no stimulation of superconductivity. We found that for large amplitudes j^{ac} of the symmetric ac signal the value of the lower critical current $j_{c1}(j^{ac})$ at which the voltage vanishes in the sample oscillates as a function of j^{ac}. The amplitude of these oscillations decays with increasing power of the ac signal, and we explain it either by the existence of a maximal current j_{c3} beyond which no phase slips can be created or by a weak heat removal from the sample. Applying an asymmetric in time signal (with zero dc current) we show that it may lead to a finite voltage in the system (i.e., ratchet effect). At high enough frequencies the rectified voltage is directly proportional to the frequency of the applied e.m. radiation. These properties resemble in many aspects the behavior of a Josephson junction under e.m. radiation. The differences are mainly connected to the effect of the transport current on the magnitude of the order parameter.

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I. INTRODUCTION

At the present moment there are several well-known phenomena connected with the interaction of electromagnetic (e.m.) radiation with superconductors. These are the destruction of superconductivity by e.m. radiation with frequency $\nu > 2\Delta/\hbar$ (Δ is the superconducting gap), stimulation of superconductivity[1] (Eliashberg effect) by e.m. radiation with frequency $\nu \ll 2\Delta/\hbar$, and the appearance of Shapiro steps in the current-voltage characteristics of Josephson junctions.[2] Recently, Shapiro steps were also observed in superconducting strips in the absence of Josephson junctions.[3] These Shapiro steps were explained by the existence of phase-slip lines[4–6] in the sample and their interaction with external e.m. radiation.[3]

In this paper we investigate the effect of e.m. radiation on the critical currents of the phase-slip process in a regime when there is no stimulation of superconductivity. Surprisingly, practically no theoretical studies exist on this subject. We found that at large enough power of the ac signal in the superconductor oscillations appear in the critical currents similar to what has been observed in Josephson junctions.[7] We will also show that if the applied signal is asymmetric in time, it may lead to a ratchet effect as was shown in recent experiments[8] on annular Josephson junctions.

The paper is organized as follows. In Sec. II we introduce the model and set of equations which are numerically solved in Secs. III A and III B for the cases of time symmetrical and asymmetrical ac signals, respectively. In Sec. IV we compare our results with the well-developed theory of Josephson junctions and discuss the experimental conditions under which they may be observed experimentally.

II. THEORETICAL MODEL

For our theoretical study we use the generalized time-dependent Ginzburg-Landau (TDGL) equations

$$\frac{u}{\sqrt{1+\gamma^2|\psi|^2}}\left(\frac{\partial}{\partial t}+i\varphi+\frac{\gamma^2}{2}\frac{\partial|\psi|^2}{\partial t}\right)\psi=\frac{\partial^2\psi}{\partial x^2}+(1-T-|\psi|^2)\psi, \tag{1a}$$

$$j=\mathrm{Im}(\psi^*\nabla\psi)-\frac{\partial\varphi}{\partial x}, \tag{1b}$$

with explicit inclusion of the time relaxation τ_ϵ (through the coefficient $\gamma=2\tau_\epsilon\Delta_0/\hbar$) for the nonequilibrium quasiparticle distribution due to the electron-phonon interaction[9,10] and $\Delta_0=4k_BT_c/u^{1/2}\pi$ is the "effective" value of the order parameter at $T=0$. In Eqs. (1) all the physical quantities (order parameter $\psi=|\psi|e^{i\phi}$, electrostatical potential φ) are measured in dimensionless units: the momentum of the superconducting condensate $\mathbf{p}=\nabla\phi$ is scaled by the unit $\Phi_0/[2\pi\xi(0)]$ (where Φ_0 is the quantum of magnetic flux), the order parameter is in units of Δ_0, and the coordinates are in units of the coherence length $\xi(0)=(8k_BT_c/\pi\hbar D)^{-1/2}$ (D is the diffusion constant). In these units the current density is scaled with $j_0=\sigma_n\hbar/2e\tau_{GL}(0)\xi(0)$, time is in units of the Ginzburg-Landau relaxation time $\tau_{GL}(0)=\pi\hbar/8k_BT_cu$, and the electrostatic potential φ is in units of $\varphi_0=\hbar/2e\tau_{GL}(0)$ (σ_n is the normal-state conductivity). The vector potential is equal to zero because there is no applied magnetic field and self-induced effects are small for our system (see below).

Strictly speaking, the validity of Eqs. (1) was demonstrated theoretically only near the critical temperature $T_c-T<\hbar/(k_B\tau_\epsilon)$ and for frequencies $\nu<1/\tau_\epsilon$. Using the value $\tau_\epsilon(T=T_c)\sim 3\times 3^{-10}$ s and $T_c\sim 3.8$ K which are typical for tin, we find that for this material Eqs. (1) should give reasonable results for $\nu<3$ GHz and $T_c-T<0.02$ K. From previous applications of the TDGL equations we expect that our results may be applicable, at least qualitatively, for higher frequencies and even far from T_c. We use a one-dimensional model assuming that in the considered temperature range the width of the film satisfies $W < \lambda^2/d$ with thickness $d < \lambda$ and hence the current density distribution is practically uniform over the film width. In our calculations we used $T = T_0 = 0.9T_c$ and two values for the parameter $\gamma = 40, 120$ (for tin $\gamma \sim 160$ and for lead $\gamma \sim 21$). It is more convenient to express the different quantities in units normalized at $T = T_0$. So, for example, with j_0 we mean $j_0(T = 0.9T_c) = 0.1^{3/2} j_0(T = 0)$ and with $\tau_{GL}(T = 0.9T_c) = 10\tau_{GL}(T = 0)$. The length of the sample was taken equal to $L = 20\xi(T = 0.9)$ which is larger than the penetration depth of the electric field for the used values of γ and temperature.

To simulate a real experimental situation we use "bridge" boundary conditions $|\psi(-L/2)| = |\psi(L/2)| = 1$, $\psi(\pm L/2, t + dt) = \psi(\pm L/2, t)e^{-i\varphi(\pm L/2)dt}$, and $\varphi(-L/2, t) = 0$. The Euler method is used to solve Eqs. (1) and (2). Initial conditions were $|\Psi| = 1$ and $\varphi = 0$ at moment $t = 0$. The behavior of the system is studied on a large time scale when time-averaged values do not depend on time. A phase-slip center appears (at some critical current) always in the center of the wire because there the order parameter is minimal due to the boundary conditions. Because of the relatively small length L, only one phase-slip center nucleates. With increasing L the number of phase-slip centers increases and the dynamics of the system becomes very complicated, and therefore this case is not considered.

In the experiment the induced currents are distributed uniformly over the sample because the wavelength of the applied electromagnetic radiation is usually much larger than the sample size. Therefore, in our theoretical model we may assume that the full current in the film is $j = j^{dc} + j^{ac}\sin(2\pi\nu t)$, with ν the frequency of the applied e.m. radiation. In order to include heating effects we also solved the equation for the temperature distribution in the sample:

$$C_{eff}\frac{\partial T}{\partial t} = K_{eff}\Delta T + j_n^2 - h(T - T_0), \tag{2}$$

where $C_{eff} = (D_s C_s/d_f + C_f)T_c\sigma_n/\tau_{GL}(0)j_0^2$, $K_{eff} = (D_s k_s/d_f + k_f)T_c\sigma_n/\xi^2(0)j_0^2$, $h = k_s T_c\sigma_n/D_s d_f j_0^2$, the heat transfer coefficient h governs the heat removal from the sample, and C_s, C_f, k_s, and k_f are heat capacity and heat conductivity of the substrate (subscript s) and film and sample (subscript f), respectively. Here we used a model for the temperature distribution in thin superconducting films as was previously discussed in details in Ref. 12 and we assumed that the thickness of the substrate and film, $D_s + d_f$, is much smaller than the healing length $\Lambda_h = \sqrt{K_{eff}/h} \gg D_s + d_f$. If $D_s C_s/d_f \ll C_f$ and $D_s k_s/d_f \ll k_f$, we can use the Wiedemann-Franz law as an estimate for C_f and k_f and we obtain $C_{eff} = \pi^3/48 \simeq 0.65$ and $K_{eff} = \pi^4/48u^2 \simeq 0.06$ at temperature close to T_c. Because of the uncertainty in the actual value of k_s, we use h as a parameter to distinguish the cases of complete heat removal ($h = 10^{-1}$) and weak heat removal ($h = 10^{-4}$).

III. NUMERICAL RESULTS

A. Symmetric ac signal

As is well known, the phase-slip process is usually hysteretic (see the review in Ref. 4 and the books in Refs. 5 and 6). There are two critical currents: j_{c1}, which characterized that for $j < j_{c1}$ the phase-slip process vanishes in the superconductor, and j_{c2}, at which the superconducting resistiveless state becomes unstable (in a defectless sample j_{c2} is equal to the depairing current density). In Ref. 11 it was argued that the first critical current is connected to the competition of two processes: the growth of the magnitude of the order parameter and the growth of the momentum p in the phase-slip center (PSC). If the order parameter grows faster than the momentum, the phase-slip process cannot occur in the sample. A finite relaxation time for the magnitude of the order parameter $\tau_{|\psi|}$ results in a voltage jump ΔV at the current $j = j_{c1}$.[11]

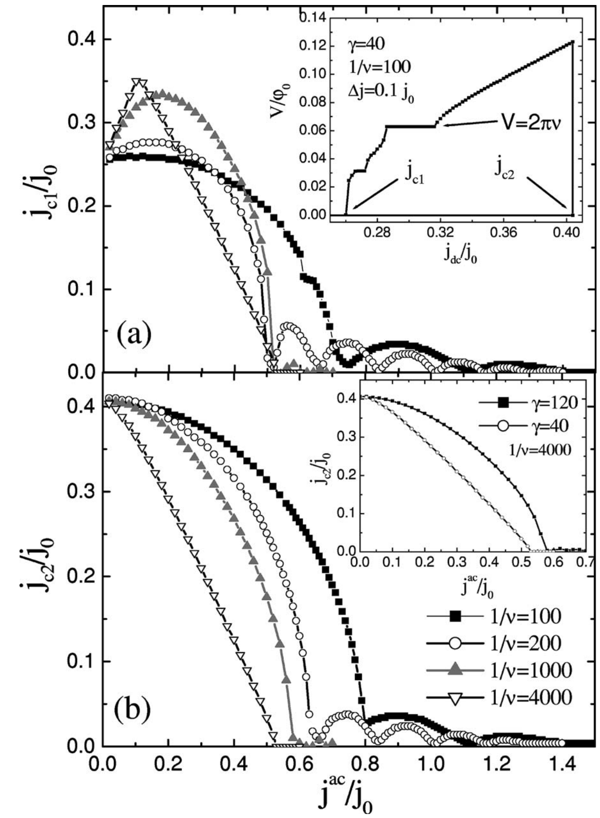

FIG. 1. Dependence of (a) j_{c1} and (b) j_{c2} on the amplitude of the ac signal for different values of the period $1/\nu$ of the e.m. radiation. $\gamma = 40$ and heat transfer coefficient $h = 0.1$. In the inset of (a) we show typical current-voltage characteristics of the superconductor for a small period of the e.m. radiation. The inset of (b) shows the dependence of $j_{c2}(j^{ac})$ on the parameter γ.

In the present paper we study how the ac component of the current affects the value of $j_{c1}(j^{ac}, \nu)$ and $j_{c2}(j^{ac}, \nu)$. In this respect the above-mentioned currents j_{c1} and j_{c2} may be considered as a particular case of zero ac current $j^{ac} = 0$ and we will use the notations $j_{c2}(0)$ and $j_{c1}(0)$ for that case. In Fig. 1 we present the dependences of j_{c1} and j_{c2} for different values of the period $1/\nu$ of the radiation, with $\gamma = 40$ and for almost perfect heat removal [$h = 0.1$ in Eq. (2)].

The main features of such a behavior may be explained by the finite relaxation time of the magnitude of the order parameter $\tau_{|\psi|}$. If built on an analogy with Josephson junctions, this time provides for the superconductor some kind of "inertia" term.

First we discuss j_{c2}. At large enough period of the e.m. radiation $1/\nu$, the order parameter has sufficient time to adopt itself to any change of the external current. When the sum $j^{dc}+j^{ac}$ becomes larger than $j_{c2}(0)$, phase slips appear in the sample and $j_{c2}=j_{c2}(0)-j^{ac}$ changes linearly with j^{ac}. With increasing frequency of the radiation it is necessary to increase the sum $j^{dc}+j^{ac}$ above $j_{c2}(0)$ because of the finite relaxation time of $|\psi|$. The smaller the period $1/\nu$, the larger this increase of the sum has to be, and it results in a nonlinear dependence of j_{c2} on j^{ac} at high frequencies.

This result gives us the possibility to determine experimentally the relaxation time ($\tau_{|\psi|}$) by applying e.m. radiation with different frequencies and measuring its $j_{c2}(j^{ac})$ dependence. For example, for the parameters used in Fig. 1 deviation of j_{c2} from a linear behavior starts at $1/\nu \lesssim 4000$. From this we may conclude that the order parameter decays from its equilibrium value to zero in about $4000\tau_{GL}(T=0.9)$ (at the considered values of γ) if we apply to the sample a dc current which slightly exceeds j_{c2}. This method allows us to obtain a rough estimate, because this time depends strongly on the value of the dc current (see Refs. 6 and 13).

A similar reasoning can be utilized to explain the behavior of $j_{c1}(j^{ac})$. At low frequencies, j_{c1} at first increases because the difference $j-j^{ac}$ may become less than $j_{c1}(j^{ac}=0)$ at some moment in time and hence the phase-slip process will decay and cannot occur if $j^{dc}+j^{ac}<j_{c2}(0)$. For $j^{ac}<[j_{c2}(0)-j_{c1}(0)]/2$ the j_{c1} increases linearly with j^{ac}. When $j^{ac}=[j_{c2}(0)-j_{c1}(0)]/2$ the hysteresis in the current-voltage characteristic disappears, $j_{c1}=j_{c2}$, and we recover the scenario as described above. For high frequencies the finite relaxation time of the magnitude of the order parameter starts to play an essential role. It smooths the dependence of j_{c1} and removes the maximum at low amplitudes. Concurrently, the hysteresis in the *I*-*V* characteristics persists up to much higher values of j^{ac}. The reason is that the order parameter cannot vary substantially during a short time and hence it simply does not notice small oscillations of the ac current.

Another feature becomes visible when we go to higher frequencies: an oscillating dependence of j_{c1} for large amplitudes j^{ac} (see Figs. 1 and 2). Actually these oscillations are also present at low frequencies but their amplitude and period are very small. The origin of these oscillations is the following. When the amplitude of the ac current becomes sufficiently large (the larger the period, the smaller the amplitude, but definitely larger than j_{c2}) phase slips are created in the system even for $j^{dc}=0$. However, it does not lead to a nonzero voltage because of the symmetry of the ac signal. The number of phase-slip events is the same for positive and negative currents. The smaller the period $1/\nu$, the smaller the number of these events at constant j^{ac}. For example, at $1/\nu=200$ we have one "negative" (leading to negative voltage) and one "positive" (leading to positive voltage) PS event at the first oscillation of j_{c1} [at $0.52<j^{ac}<0.68$ in Fig. 1(a)], two "positive" and two "negative" for the second one, and so on. For larger period we have already many PS events during the first oscillation.

When we add positive dc current to the original ac current, we increase the time during which the current is positive and it raises the number of positive phase slip events.

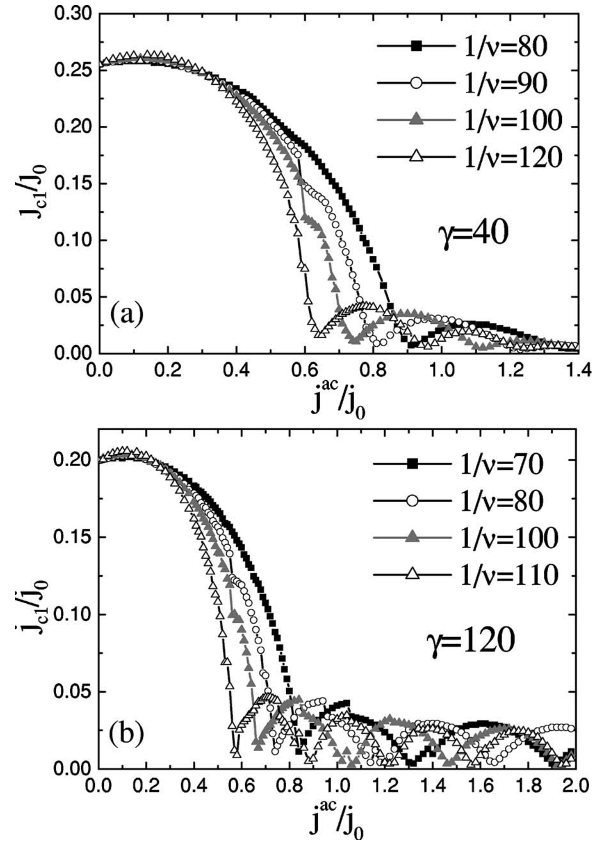

FIG. 2. Dependence of j_{c1} [for two values of the parameter γ: (a) $\gamma=40$ and (b) $\gamma=120$] in the range of e.m. radiation where oscillations in j_{c1} are most pronounced.

But there is some kind of "resistance" which wants to preserve the number of PS events. The origin of this resistance is the finite time needed for each PS event. The larger the amplitude of the ac current or the value of the dc current, the less time it takes for the system to create phase slips and hence more phase-slip events can occur within a period.[14] The "weakest" points in the system where it is easy to "insert" an extra PS event are these points which are responsible for the change from the regime of two PS events to four, six, and so on. In these points the current $j_{c1}\simeq 0$ [for $1/\nu=200$ it occurs at $j^{ac}=0.52, 0.68, 0.84, \ldots$—see Fig. 1(a)]. The most "resistive" points correspond to the middle of every such an oscillation, and the current j_{c1} is maximal there. The interplay between these factors is the reason[15] for the decreasing amplitude and period of the oscillations in j_{c1} with increasing period $1/\nu$.

At high frequencies a discontinuity in j_{c1} is found (most pronounced for $1/\nu=100$ in Fig. 2). Our numerical calculations show that this occurs when the superconductor transits to the zero-resistance ($V=0$) state from the Shapiro step. We connect it with the existence of the commensurability between the intrinsic frequency of the phase-slip process and the external frequency which "helps" to keep the phase-slip process even at small enough dc current in the presence of radiation. This explains why the effect exists only for $\nu \gtrsim 1/\Delta V$ (ΔV is the value of the jump of the voltage at $j=j_c1$ in absence of e.m. radiation) because $1/\Delta V$ is the mini-

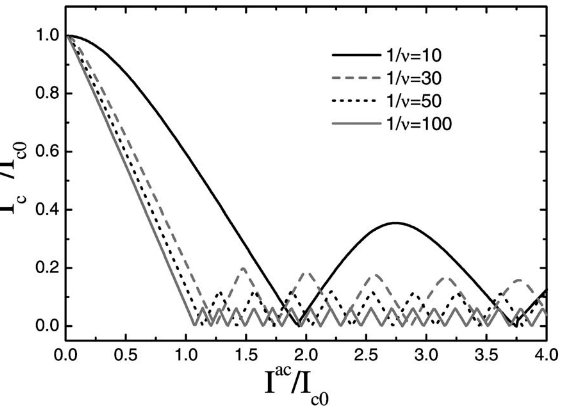

FIG. 3. Dependence of the dc critical current on the amplitude of the ac current in a Josepshon junction within the RCSJ model for a damping parameter $\beta_c=0.01$ (see Ref. 6). Current is measured in units of the Josephson critical current I_{c0}, and time is in units $\tau_J=2eI_{c0}R/\hbar$ where R is the resistance of the junction in the normal state.

mal possible intrinsic frequency of the phase-slip process.[15] We should note that these jumps also exist in the region where j_{c1} oscillates with increasing amplitude j^{ac}.

It is instructive to compare our results with the well-developed theory of Josephson junctions (see, e.g., Ref. 7). The phase-slippage process in Josephson (tunnel or weak link) junctions resembles the phase-slip centers found in uniform superconducting films. The phase changes by 2π after each phase-slip event in both systems. The main difference is that the order parameter is strongly suppressed in the Josephson junction region and hence there is no "inertia" connected with its variation (especially in a tunnel Josephson junction). The role of inertia in the case of the Josephson junction is played by the capacitance. Indeed, when the capacitance is zero (overdamped Josephson junction) there is no hysteresis in the I-V characteristic. The same is true for the Ginzburg-Landau model [Eqs. (1)] in the limit $\tau_{|\psi|}\rightarrow 0$ (when, for example, $u\rightarrow 0$).

The main properties we found for phase-slip centers may be obtained in the framework of the simple RCSJ (resistively and capacitevely shunted junction) model for the Josephson junction[6,7] (see Fig. 3). We consider the overdamped limit (damping parameter β_c is small) because of its simplicity. For low frequencies we have a linear dependence of j_c on j^{ac} (at small amplitudes of the ac current) which changes to a nonlinear dependence at high frequencies. We also have oscillations in $j_c(j^{ac})$ with the same qualitative behavior (see discussion two paragraphs above) when one changes the period or amplitude of the e.m. radiation. The main differences are (i) the absence of discontinuous jumps in j_{c1} which we found for some frequencies and (ii) there is almost no decay in the amplitude of the oscillations of j_{c1} with j^{ac} (compare Figs. 2 and 3).[16] The latter feature is connected with the existence of the third critical current at which the phase-slip process decays in the superconductor. Actually it is related to the problem of the stability of the boundary between the superconductor (S) and normal metal (N). It has been known for a long time (see the review in Ref. 4, for example) that in the presence of transport current the S-N boundary is unstable and it should move in the direction of the normal metal (and recover superconductivity in the sample) at $j<j_{c1}$ and it moves in the direction of the superconductor (and destroy superconductivity) at $j>j_{c3}$. In the range $j_{c1}<j<j_{c3}$ the phase-slip process exists.

In Ref. 11 the physical meaning of the current j_{c1} was studied. Here we try to clarify the meaning of the current j_{c3}. Using Eq. (1a) we may write the equation for the dynamics of the magnitude of the order parameter:

$$u\sqrt{1+\gamma^2|\psi|^2}\frac{\partial|\psi|}{\partial t}=\frac{\partial^2|\psi|}{\partial s^2}+|\psi|(1-T-|\psi|^2-p^2). \quad (3)$$

From this equation it follows that the order parameter decreases in time (and hence the normal phase grows) if the right-hand side of Eq. (3) is negative. We may estimate the term $-p^2|\psi|$ (the other terms lead to a positive contribution) via the equation for the electrostatic potential, Eq. (1b), $j=j_n+|\psi|^2p$, and from the knowledge that the normal current density decays in the superconductor as $j_n=je^{-x/\Lambda_Q}$ in the absence of the Andreev reflection process (see Ref. 17, for example). Λ_Q is the decay length of the charge imbalance in the superconductor and within the framework of Eqs. (1), $\Lambda_Q\sim\sqrt{\gamma/u}(1-T)^{-1/4}$ [in the limit $\gamma(1-T)^{1/2}\gg 1$]. Because the variation of the order parameter occurs on a distance of about ξ (it is the size of the S-N boundary) and in the above limit $\Lambda_Q\gg\xi$, we have $|\psi|^2p\simeq j/\Lambda_Q(j_n\sim j-j/\Lambda_Q)$ and finally

$$u\gamma|\psi|\frac{\partial|\psi|}{\partial t}=\frac{\partial^2|\psi|}{\partial s^2}+|\psi|(1-T-|\psi|^2)-\frac{j^2}{\Lambda_Q^2|\psi|^3}. \quad (4)$$

Supposing that the distribution of the order parameter slightly depends on the actual distribution of the normal current density near the S-N boundary, we find [assuming that $\psi\sim(1-T)^{1/2}$]

$$j_{c3}\simeq\Lambda_Q(1-T)^{3/2}\sim\sqrt{\gamma/u}(1-T)^{5/4}j_0(T=0). \quad (5)$$

This expression should be considered only as a rough estimate to clarify the dependence of j_{c3} on the parameters γ and u. It explains why with increasing γ the oscillations decay slower (see Fig. 2) and the partially superconducting (resistive) state could exist at higher currents with decreasing u as recently found numerically in Ref. 18.

Because of the existence of a finite j_{c3} (which is absent in the simple RCSJ model of the Josephson junction), oscillations in j_{c1} decay for large enough amplitudes of the ac current. Another reason for the fast (in comparison to the Josephson junction case) decay may be related to heating effects. Due to heat dissipation, the local temperature of the superconductor may strongly deviate from the bath temperature. In Fig. 4 we present the dependences of j_{c1} and j_{c2} for different regimes of heat removal from the sample. It is clear that for the case of weak heat removal oscillations decay much faster. This is due to the local increase of temperature (which is not necessary above T_c) and hence to a decrease of j_{c3} according to Eq. (5).

B. Asymmetric ac signal

Interesting effects occur when we apply e.m. radiation which is asymmetric in time. Namely, we considered the

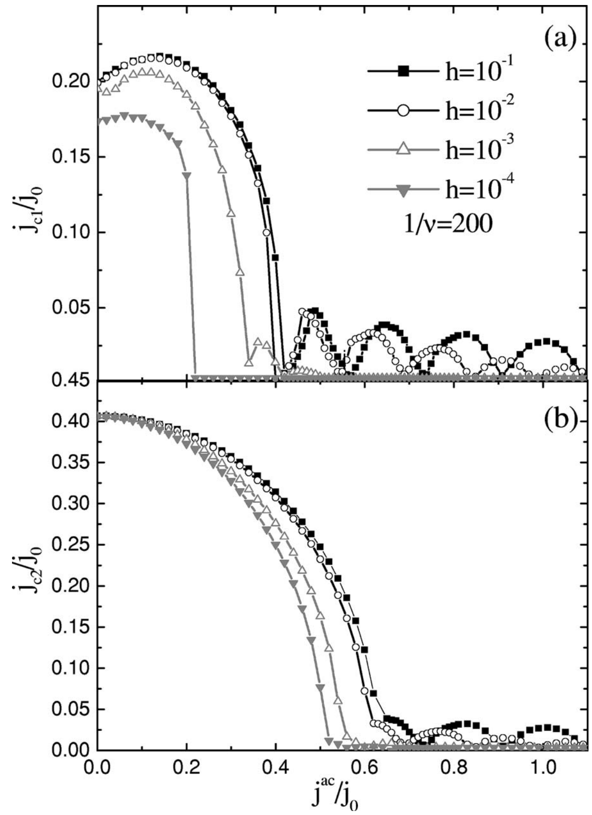

FIG. 4. Dependences of j_{c1} (a) and j_{c2} (b) on the amplitude of the ac current at different values of the heat removal coefficient. Current j_{c2} depends on the heat dissipation (for $j^{ac}<0.5$) due to the presence of the ac electric field in the superconducting state (even in the absence of the phase slips) and hence normal component of the current.

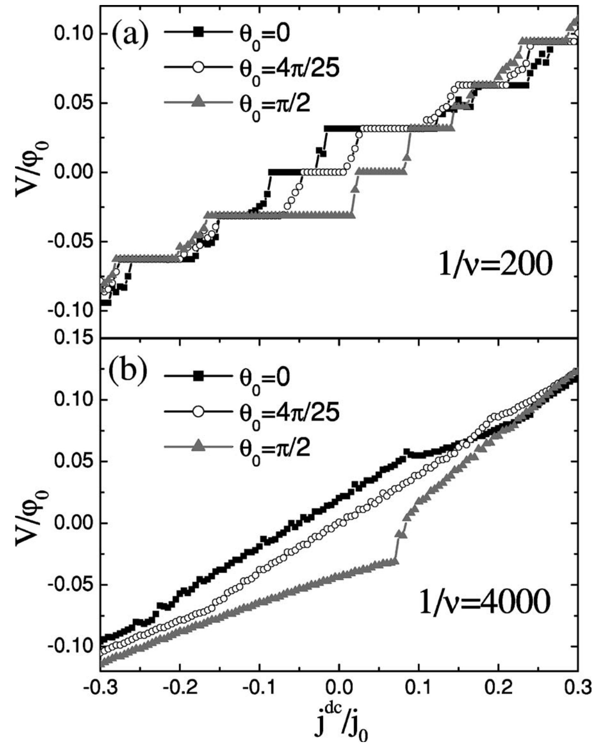

FIG. 5. Current-voltage characteristics of the superconducting sample in the presence of a time asymmetric ac signal, Eq. (6) ($j_1=0.5$, $j_2=0.3$, $\gamma=40$), with period $1/\nu=4000$ (a) and $1/\nu=200$ (b). At chosen values for j_1, j_2 and period $1/\nu=200$ phase slips do not nucleate at zero dc current if we start from the superconducting state. Therefore, at some moment in time we locally suppress superconductivity which leads to the nucleation of phase slips.

case in which the ac current contains a second harmonic which is phase shifted with respect to the first harmonic:

$$j(t)=j^{dc}+j_1\sin(2\pi\nu t)+j_2\sin(4\pi\nu t+\theta_0). \tag{6}$$

We found that such a current may lead to a nonzero-voltage response even if the dc current is equal to zero. This effect is similar to the one which was theoretically predicted in Refs. 19 and 20 and experimentally found in Ref. 8 for the ratchetlike behavior of a single Josephson vortex in an annular Josephson junction. In Fig. 5 we present the dependence of the voltage induced in the superconductor by the current (6) as a function of the dc current j^{dc} for $j_1=0.5$, $j_2=0.3$, and $1/\nu=4000$ [Fig. 5(a)] and $1/\nu=200$ [Fig. 5(b)].

If we apply a current (6) with no dc component and with a high enough frequency, then for relatively small amplitudes j_1 and j_2 no phase slips are created in the sample (if we take the initial condition $|\psi|=1$ or equivalently we start from a pure superconducting state in the experiment) and we have $V=0$. If now due to some external influence (e.g., a pulse of a laser beam or in numerical calculations we put $\psi=0$ in the center of the sample at $t=0$), superconductivity is locally weakened or destroyed, a phase slip process is induced, and as a consequence we will observe a nonzero-voltage response. Alternatively, we can apply a high dc current (to nucleate phase slips) and decrease it to zero.

The value of the induced (or rectified) voltage at $j^{dc}=0$ strongly depends on the phase θ_0 (see Fig. 6). In comparison to the work of Ref. 8 the rectified voltage is nonzero even for $\theta_0=0$ (for $1/\nu<10^4$). The reason is that for phase-slip processes the "damping" parameter (using the terminology for Josephson junctions) and the characteristic times are large and the system behaves in an adiabatic way only for relatively low frequencies (for the parameters of Fig. 6 at $1/\nu>10^4$).

IV. DISCUSSION

Let us estimate the experimental conditions under which the predicted effects can be observed. In the case for low-temperature superconductors the critical temperature is about several kelvins and $\tau_{GL}(0)\sim 5.2\times 10^{-13}$ s$/T_c$. The predicted effects therefore should be observable in the GHz range of e.m. radiation at temperatures not too close to T_c (to exclude heating of the sample by large currents). Actually part of our theoretical results was recently observed in an experiment[23] on tin bridges (with length 5 μm and width 2 μm) at a temperature close to $T_c\sim 3.8$ K and for not very large incident power of radiation (in our units for $j^{ac}<0.5j_0$). The nonmonotonous dependence of the $j_{c1}(j^{ac})$ [similar to that shown in

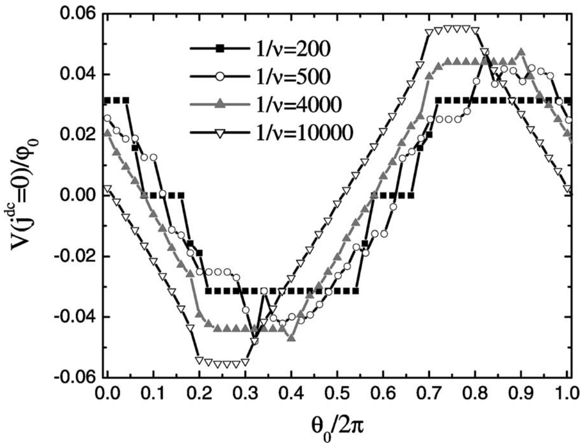

FIG. 6. Dependence of the induced voltage for $j^{dc}=0$ on the phase shift between the two ac signals θ_0 for different frequencies ν of the ac signal (parameters as in Fig. 5). In comparison with Ref. 8 a nonzero voltage appears already for $\theta_0=0$ in the case of high frequencies. This is connected with the strong nonlinearity of the studied system and the dependence of the different relaxation times on the applied current. It is interesting to note that for frequencies close to, or larger than, $\nu_c=2e\Delta V/\hbar$ the induced voltage is proportional to the frequency of the applied signal.

Fig. 1(a)] was found at $\nu=200$ MHz and $T=3.67$ K, and the critical current j_{c2} decayed always with increasing j^{ac} unless stimulation of superconductivity occurred at $\nu>3$ GHz. Besides, discontinuous jumps in the dependence $j_{c1}(j^{ac})$ (see Fig. 2) were observed at high frequencies $\nu_c \simeq 1/\Delta V$ (in experiment $\nu_c \sim 9.3$ GHz) when the transition to the zero-resistance state occurs in the Shapiro step. The rest of our theoretical results (for $j^{ac} \gtrsim 0.5 j_0$) still need experimental verification.

We showed that applying a time asymmetric ac signal to the superconducting film may induce a finite voltage even in an unbiased system $j^{dc}=0$ (Fig. 6). As a consequence one may use it as a rectifier or voltage source of high precision at high frequency $\nu_c > 2e\Delta V/\hbar$ of radiation. The actual value of the induced voltage is in the range of several μV per phase-slip center (for low-temperature superconductors at T close to T_c). In our calculations we restricted ourselves to the case when only one phase-slip center arises in the system. If several phase-slip centers coexist, they will start to interact with each other and such a system may be synchronized.[5] Consequently the rectified quantized voltage may increase several times in magnitude. Such a quantized voltage was already observed experimentally in unbiased tunnel Josephson junctions[21,22] and used for measuring the ratio $\hbar/e$ and as a standard for the voltage.[7]

As we already pointed out above, the behavior of the phase-slip center under the influence of the e.m. radiation is very similar to the behavior of a Josephson junction. The critical currents decay and oscillate with increasing the amplitude of the ac current and have a similar dependence on the frequency of the applied radiation (compare Figs. 2 and 3). The main differences are the faster decay and jumps in the dependence $j_{c1}(j^{ac})$ (Fig. 2) in comparison to the case of a Josephson junction. We may understand this by the influence of the transport current on the value of the order parameter (instability of the N-S boundary at finite current j_{c3}) and the existence of the finite-voltage jump ΔV in the current-voltage characteristics and hence the low threshold for the intrinsic frequency of the phase-slip process. Both these effects are absent in the RSJ model with fixed Josephson critical current I_{c0} and zero capacitance.

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[14] The time between PS events, τ_{PS}, is inversely proportional to the voltage via the Josephson relation $\tau_{PS} \sim 1/V$. Both for Josephson junctions and for phase-slip centers the voltage increases with the applied current and hence the time τ_{PS} decreases.

[15] Oscillations in the current j_{c1} starts to be pronounced when two PS events (one "positive" and one "negative") can be "fit" into a period of the e.m. radiation (at $j^{dc}=0$). Normally it occurs when the frequency of the e.m. radiation is proportional to the jump of the voltage ΔV at current j_{c1}: $\nu_c \simeq 2e\Delta V/\hbar$. The reason is that ΔV provides a lower threshold for the time between two PS

events $\tau_{PS}=\hbar/2e\Delta V$. When $\nu \ll \nu_c$ within a period $1/\nu$ many PS events occur and oscillations become invisible.

[16] If one consider the underdamped case, then hysteresis and a finite-voltage jump will appear in the current-voltage characteristics of the Josephson junction and hence property (i) may appear. This limit should be investigated separately and is left for future investigation.

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